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Mixed state in dirty two-band superconductors: application to MgB_2

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Abstract

We investigate a two-band superconductor with strong intraband and weak interband electronic scattering rates in the framework of coupled Usadel equations. We calculate the upper critical field for different field orientation. Due to a very large difference between the *c*-axis coherence lengths in two-bands, the GL theory is applicable only in the extremely narrow temperature range. This leads to the strong temperature dependence of the H_{c2} anisotropy and large deviations of the angular dependence of H_{c2} from a simple effective-mass law. In the case of field along *c*-direction we solved coupled nonlinear Usadel equations numerically to find field evolution of the pair potentials and local densities of states for two-bands. The existence of two distinct length and field scales corresponding to different bands is demonstrated.

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1. Introduction

The multigap superconductivity in the recently discovered compound MgB_2 has been theoretically predicted [1] and confirmed by many experiments. Superconductivity in MgB_2 resides in two distinct groups of bands: strongly superconducting quasi-2D σ-bands and weakly superconducting 3D π-bands. Intraband impurity scattering in both bands may vary in large limits, while interband scattering remains weak due to the disparity of σ- and π-band wave functions [2]. Therefore, the two-gap superconductivity in MgB_2 persists even in the intraband dirty limit. One of the most spectacular consequences of the two-band superconductivity is the unusual behavior of anisotropy factors. In clean MgB_2 samples the anisotropy of the London penetration depth [3], γ_λ, has to be different from the anisotropy of the upper critical field [4] γ_{c2}. Both anisotropy factors strongly depend on temperature and have opposite dependencies: γ_λ increases and γ_{c2} decreases with temperature, in agreement with experiment (see, e.g., Ref. [6]).

In this proceeding we summarize results of our recent theoretical study of the mixed state in MgB_2 [5] based on analysis of the multiband Usadel equations assuming weak interband scattering (see also [7]). We consider the upper critical field for different field orientations and structure of vortex state for field along *c*-direction. We demonstrate that the strong temperature dependence of the H_{c2}-anisotropy exists also in the dirty case and represents a general property of a two-band superconductor. The main reason for this dependence is the strong reduction of the in-plane upper critical field by the weak π-band in the very narrow temperature region near T_c. The angular dependence of H_{c2} deviates from the "effective-mass" dependence predicted by the anisotropic Ginzburg–Landau theory (AGLT) and these deviations are strongest near T_c. This illustrates the breakdown of the AGLT for this superconductor. Due

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to the large difference between the microscopic coherence lengths in the c-direction for the two-bands, the AGLT is applicable only within the extremely narrow temperature range near T_c.

For field along c-direction we solved coupled nonlinear Usadel equations numerically to find the field evolution of the pair potentials and local densities of states (DoS) for two-bands. Superconductivity in the two-bands is characterized by different energy and length scales. The c-axis vortex structure in MgB_2 was studied by STM [8], which probes mainly the weakly superconducting π-band. A large vortex core size compared to estimates based on H_{c2} and the rapid suppression of the apparent tunneling gap by small magnetic fields has been reported. We show that these observations can be naturally explained within the two-band model [5,9].

2. Model

We consider a two-band superconductor with weak interband scattering and strong intraband scattering (dirty limit), which is described by the coupled Usadel equations [5]

$$\omega F_\alpha - \sum_j \frac{\mathscr{D}_{\alpha,j}}{2}[G_\alpha \Pi_j^2 F_\alpha - F_\alpha \nabla_j^2 G_\alpha] = \Delta_\alpha G_\alpha, \quad (1)$$

$$W_\alpha \Delta_\alpha - W_{\alpha\beta}\Delta_\beta = 2\pi T \sum_{\omega>0}\left(F_\alpha - \frac{\Delta_\alpha}{\omega}\right) + \Delta_\alpha \ln\frac{T_c}{T}, \quad (2)$$

where $\alpha = 1,2$ is the band index (1 [2] corresponds to σ- [π-] band), $j = x, y, z$ is the coordinate index, $\Pi_j \equiv \nabla_j - \frac{2\pi i}{\Phi_0}A_j$, $\mathscr{D}_{\alpha,j}$ are diffusion constants, which determine the coherence lengths $\xi_{\alpha,j} = \sqrt{\mathscr{D}_{\alpha,j}/2\pi T_c}$, G_α and F_α are the Green's functions, Δ_α are the pair potentials, and $\omega = 2\pi T(s + 1/2)$ are the Matsubara frequencies. The degenerate matrix $W_{\alpha\beta}$ is related to the coupling constants [5] and in the case of MgB_2 can be estimated as $W_1 \approx 0.088$, $W_2 \approx 2.56$, $W_{12} \approx 0.535$, $W_{21} \approx 0.424$. The relative role of the π-band is characterized by the small ratio $S_{12} \equiv W_1/W_2 \approx 0.034$. All bands are isotropic in the xy plane, $\mathscr{D}_{\alpha x} = \mathscr{D}_{\alpha y}$ and anisotropic in the xz plane with the anisotropy ratios $\gamma_\alpha = \sqrt{\mathscr{D}_{\alpha x}/\mathscr{D}_{\alpha z}}$, with $\gamma_1 \approx 6.3$ and $\gamma_2 \approx 0.82$.

3. Upper critical field

Linearization of Eq. (1) leads to the following equation for the upper critical field in c-direction H_{c2}^c:

$$\ln\frac{1}{t} - g\left(\frac{H_{c2}^c}{tH_1}\right) = -\frac{W_1[\ln(1/t) - g(H_{c2}^c/tH_2)]}{W_2 - [\ln(1/t) - g(H_{c2}^c/tH_2)]},$$

with $g(x) \equiv \psi(1/2 + x) - \psi(1/2)$, and $\psi(x)$ is a digamma function. Due to small ratio W_1/W_2 the π-band has only weak influence on H_{c2}^c.

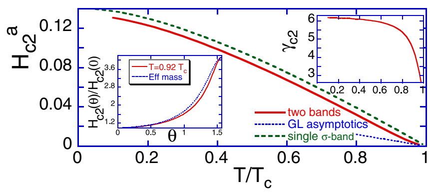

Fig. 1. The temperature dependence of H_{c2}^a normalized to $2T_c\Phi_0/\sqrt{\mathscr{D}_{1z}\mathscr{D}_{1x}}$ for $\mathscr{D}_{2z}/\mathscr{D}_{1z} = 300$. Upper inset shows the T-dependence of the H_{c2} anisotropy. Lower inset shows the angular dependence of H_{c2} at $T = 0.92\ T_c$ and comparison with the effective mass dependence.

The problem of the upper critical field in the a-direction, H_{c2}^a, does not have an exact solution, because the bands have different sets of Landau levels. Expanding with respect to the σ-band Landau levels, one can derive a coupled set of equations, which can be solved numerically [5]. The exact solution can be obtained only at $T \to T_c$ (GL region), where H_{c2}^a is determined by the averaged coherence lengths, $\xi_{x,z}^2 = (\xi_{1x,z}^2 + S_{12}\xi_{2x,z}^2)/(1 + S_{12})$, and the π-band strongly reduces H_{c2}^a. However, the validity criterion for the GL theory, $\xi_z^{GL}(T) > \xi_{2,z}$, is satisfied only within very narrow temperature range $(T_c - T)/T_c < \max(\xi_{1z}^2/\xi_{2z}^2, S_{12})$. Fig. 1 shows the example of the computed temperature dependencies of H_{c2}^a and γ_{c2}. We calculated the angular dependence of H_{c2} and found that it deviates from the "effective mass" dependence

$$H_{c2}(\theta) = H_{c2}^c/\sqrt{\cos^2\theta + \gamma_{c2}^{-2}\sin^2\theta}.$$

The deviations are strongest in the vicinity of T_c (see lower inset in Fig. 1).

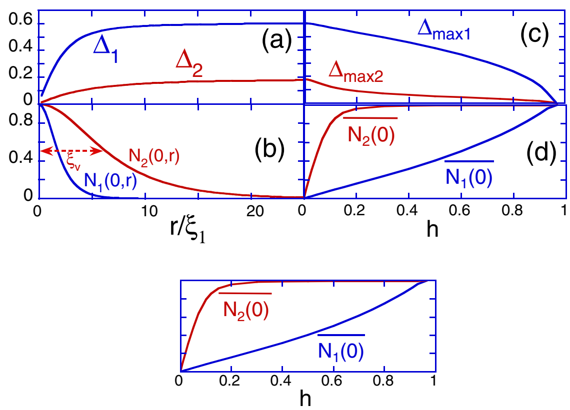

Fig. 2. Left column: spatial dependencies of (a) pair potentials and (b) partial DoS at $E = 0$ for isolated vortex for $\mathscr{D}_{1x} = 0.2\mathscr{D}_{2x}$. Right column: field dependencies of (c) maximum pair potentials and (d) averaged DoS at $E = 0$.

4. Vortex structure for $H\|c$

For $H\|c$ we solved Eqs. (1) and (2) numerically for different fields. The obtained results for $\Delta_\alpha(r)$ and the partial DoS, N_α, are summarized in Fig. 2. One can see that the π-band DoS, $N_2(0,r)$, has longer spatial range than the σ-band DoS, $N_1(0,r)$. The large core size in the weakly superconducting band is the general property of a two-band superconductors. This yields two different field scales. Fig. 2 shows that the average DoS in the π-band reaches its normal value at fields considerably smaller than H_{c2}^c. Since *c*-axis measurements probe the π-band, this result explains the STM data from Ref. [8].

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