

Radiation from a Single Josephson Junction into Free Space due to Josephson Oscillations

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We consider electromagnetic emission from a Josephson junction (JJ) in a resistive state in an external magnetic field and derive the radiation power from the dielectric layer inside a JJ directly into outside dielectric media. Matching the electric and magnetic fields at the JJ edges, we find dynamic boundary conditions for the phase difference in JJ. We find that the fraction of the power transformed into radiation is determined by the dissipation inside the JJ. It tends to unity as dissipation vanishes independently of the mismatch of the junction and dielectric media impedances.

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Renewed interest in electromagnetic radiation from Josephson junctions (JJs) is motivated by two independent recent developments. First, intrinsic JJs in layered superconductors have been proposed as a source of electromagnetic radiation in the terahertz frequency range [1]. Experimentally, radiation from the high-temperature superconductor $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ was detected by Hechtfischer *et al.* [2] only at relatively low frequencies, 7–16 GHz. Therefore, it would be interesting to estimate the possible radiation power in such a system at higher frequencies and find the optimal conditions for generation. Second, there is the prospect of using a single JJ in the quantum regime as a qubit for quantum computation [3], which requires long decoherence times. Radiation into free space limits decoherence time in this system. For both the above cases direct radiation from the dielectric layer inside JJ into outside dielectric media is important. In addition to such direct radiation, there may also be radiation from the external part of the circuit containing JJ (load). For this part JJ plays the role of oscillator providing ac current. Such an indirect radiation from a load induced by JJ was in the focus of many studies; for a review see, e.g., Ref. [4]. A mathematical description of JJ with the capacitively coupled load was discussed in Ref. [5]. In the following we discuss only the direct radiation important for problems outlined above and we ignore radiation and dissipation in the external part of the circuit. Though the study of radiation from Josephson oscillations has a long history, little attention was paid to such direct radiation.

In the case of a single JJ, weak radiation from flux flow directly into the waveguide with the power $\mathcal{P}_{\text{rad}} \sim 10^{-12}$ W was detected a long time ago [6,7] soon after the prediction by Josephson. It was understood [8] that flux flow induces electromagnetic (Swihart) waves inside a JJ, and they are partially transmitted outside when they hit the JJ edges. From measurements of the current-voltage (I - V) characteristics, Langenberg *et al.* [7] found the power $\mathcal{P} = IV$ fed into the junction and the power conversion efficiency, $Q = \mathcal{P}_{\text{rad}}/\mathcal{P} \approx 10^{-5}$. It was argued that Q is mainly determined by a ratio of the impedances of the JJ modeled as a strip line and the waveguide [7,9]:

$$Q_Z \sim (32\lambda d/\varepsilon_i w^2)^{1/2}, \quad (1)$$

where λ is the London penetration length of the superconducting leads, d is the thickness of the insulating layer, ε_i is its dielectric constant, and w is the junction width. Equation (1) gives $Q_Z \approx 10^{-5}$ for the studied junction in agreement with the experimental value. Thus the low radiation power from JJ was attributed to the mismatch between the impedances of the junction and the waveguide. To the best of our knowledge, neither deeper theoretical treatment of direct radiation nor quantitative analysis of experimental radiation from JJ directly into the dielectric media was made after that. On the other hand, significant progress has been made in the practical extraction of JJ-generated microwave power into waveguides and striplines so that it became possible to use Josephson oscillators in high-frequency devices; see, e.g., the recent review [10] and references therein.

Meanwhile, the impedance-matching approach in its simplest and most common form [9] implicitly assumes that the electromagnetic wave propagating inside JJ has only one attempt to escape decaying before the reflected wave reaches another edge. This approach is satisfactory only in the case of JJ with a high level of dissipation. At low dissipation rate reflections lead to the formation of almost standing Swihart waves inside a JJ. In this case Q strongly depends on the dissipation and approaches unity as dissipation vanishes. Then the question turns out to be what are the limitations on \mathcal{P} rather than on Q . For that, the electromagnetic field should be expressed via the phase difference φ and equations for φ should be solved.

On the other hand, standard analysis of transport properties of finite-size JJs uses the sine-Gordon equation for φ and zero-derivative boundary conditions for the oscillating phase at the edges [9,11,12]. For such boundary conditions direct radiation is absent.

In this Letter we reconsider this problem and derive the power of direct radiation from a single JJ in the classical regime. Our rigorous approach is based on the solution of the Maxwell equations inside the superconducting leads and in outside space, which allowed us to find ac electric

and magnetic fields in terms of $\bar{\varphi}$. Then we formulate an accurate dynamic boundary condition for the oscillating phase inside the JJ. Next, we solve analytically the equation for $\bar{\varphi}$ with dynamic boundary conditions in the linear regime and find the radiation power.

To derive direct radiation due to Josephson oscillations, one has to match oscillating fields inside the junction and in the superconducting leads with the wave solution outside the JJ. We consider a JJ with the length $l \gg \lambda$ located at $-l < x < 0$ and bounded by a dielectric with dielectric constant ε_d ; see Fig. 1. The strength of the coupling in the JJ is characterized by the Josephson current density J_c and related parameters: the Josephson length $\lambda_J = \sqrt{c\Phi_0/(8\pi^2\Lambda J_c)}$ and plasma frequency $\omega_p = \sqrt{8\pi^2 dc J_c/(\varepsilon_i \Phi_0)}$, where $\Lambda = 2\lambda + d$. We consider the simplest situation when the JJ width along the z direction, w , is much larger than both λ_J and the wavelength of the outgoing electromagnetic wave. We consider the JJ in resistive state and assume that its phase $\bar{\varphi}$ oscillates with the Josephson frequency ω generating oscillating electric (E_x and E_y) and magnetic (B_z) fields inside the JJ and in the superconducting leads. Our task is to find spatial distribution of these fields and match them with outside fields to find the equation and boundary conditions for $\bar{\varphi}$.

We first derive the equation for the oscillating magnetic field inside the superconducting leads. We use a complex representation for the oscillating fields and phase, e.g., $\bar{\varphi}(x, t) = \langle \bar{\varphi}(x, t) \rangle_t + \sum_{\omega > 0} \text{Re}[\bar{\varphi}_\omega(x) e^{-i\omega t}]$, where $\langle \dots \rangle_t$ means time average. The phase gradient is connected with the magnetic field inside the junction and supercurrents flowing along the junction at the opposite sides as

$$\nabla_x \bar{\varphi} = (8\pi^2 \lambda^2 / c \Phi_0) [J_{x+} - J_{x-}] - 2\pi d B_z / \Phi_0. \quad (2)$$

From Maxwell equations, the material equation for supercurrent, $\mathbf{J} = (c/4\pi\lambda^2)[(\Phi_0/2\pi)\nabla\phi - \mathbf{A}] + \sigma_q \mathbf{E}$, the London relation for the electric field, $\mathbf{E} = -(4\pi\lambda^2 i\omega/c)\mathbf{J}$, and Eq. (2), we derive the following equation for the oscillating magnetic field inside the leads ($-l < x < 0$) at $d \ll \lambda$:

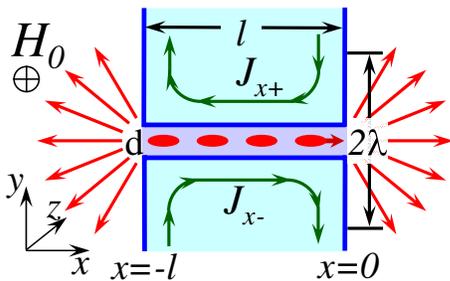


FIG. 1 (color online). A finite-size Josephson junction opened into free space at both edges. The dc magnetic field H_0 is applied along the z axis. Arrows show radiation from the dielectric layer inside the junction. Ellipses illustrate the moving vortex lattice and “arrowed” lines show the screening currents inside the superconducting leads.

$$(\nabla_x^2 + \nabla_y^2)B_z(\omega) - \frac{B_z(\omega)}{\lambda_\omega^2} = \frac{\Phi_0}{2\pi\lambda^2} \frac{\partial \bar{\varphi}_\omega}{\partial x} \delta(y), \quad (3)$$

where $\lambda_\omega^{-2} = \lambda^{-2} + 4\pi i k_\omega \sigma_q / c + \varepsilon_s k_\omega^2$ with $k_\omega \equiv \omega/c$. We ignore small contribution to the magnetic field from the dc current flowing via the JJ. The total ac electric field inside the JJ is composed of the field inside the dielectric layer and the field inside the leads,

$$E_y(\omega, x, y) = -\frac{i\omega\Phi_0}{2\pi c} \bar{\varphi}_\omega(x) \delta(y) + i\lambda_\omega^2 k_\omega \nabla_x B_z(\omega, x, y). \quad (4)$$

The boundary condition for $\bar{\varphi}_\omega(x)$ follows from the boundary condition for the electric field, $-i\omega[\varepsilon_d E_x(+0, y) - \varepsilon E_x(-0, y)] = 4\pi J_x(-0, y)$. As $E_x(-0, y) = -i\omega(4\pi\lambda^2/c^2)J_x(-0, y)$, we obtain

$$4\pi(1 - \varepsilon k_0^2 \lambda^2) J_x(-0, y) = -i\omega \varepsilon_d E_x(+0, y). \quad (5)$$

As the electric field $E_x(+0, y)$ is continuous at $y = 0$, this means that $J_x(0, y)$ also must be continuous and from Eq. (2) in the limit $d \ll \lambda$ we obtain

$$\nabla_x \bar{\varphi}_\omega(0) = \nabla_x \bar{\varphi}_\omega(-l) = 0. \quad (6)$$

We can represent the solution of Eq. (3) near the edge $x = 0$ as $[c_\pm = \sqrt{(x \pm x')^2 + y^2}/\lambda_\omega]$

$$B_z(\omega, \mathbf{r}) = B_b(\omega, \mathbf{r}) - \frac{\Phi_0}{(2\pi\lambda)^2} \int_{-\infty}^0 dx' \nabla_{x'} \bar{\varphi}_\omega(x') \times [K_0(c_-) - K_0(c_+)], \quad (7)$$

where $K_0(z)$ is the modified Bessel function and $B_b(x, y)$ is the solution of the homogeneous equation $(\nabla_x^2 + \nabla_y^2)B_b - \lambda_\omega^{-2}B_b = 0$, with the boundary condition $B_b(-0, y) = B_z(+0, y)$. As a function of x , $B_b(x, y)$ decays at a distance $\sim \lambda$ from the boundary. For its Fourier transform along the y direction, we obtain $[\kappa_x(k_y) = (\lambda_\omega^{-2} + k_y^2)^{1/2}]$

$$B_b(\omega, x, k_y) = B_z(+0, k_y) \exp(-\kappa_x(k_y)|x|). \quad (8)$$

Using the Maxwell equation $\nabla_x B_z = (i\omega \varepsilon_i / c) E_y - (4\pi/c) J_y$, the Josephson relation $E_y(x, 0) = -i\omega \Phi_0 \bar{\varphi}_\omega(x) / (2\pi c d)$, and Eq. (7), we find the nonlocal equation for the oscillating phase

$$\left(\frac{\omega^2}{\omega_p^2} + \alpha_t \frac{i\omega}{\omega_p} \right) \bar{\varphi}_\omega(x) + \frac{\lambda_J^2}{\pi\lambda} \int_{-\infty}^0 dx' \nabla_{x'} \left[K_0\left(\frac{x-x'}{\lambda_\omega} \right) + K_0\left(\frac{x+x'}{\lambda_\omega} \right) \right] \nabla_{x'} \bar{\varphi}_\omega(x') - \frac{c \nabla_x B_b(x, 0)}{4\pi J_c} = s_\omega(x), \quad (9)$$

where $s_\omega(x)$ is the complex amplitude of the oscillating Josephson current, $\sin[\bar{\varphi}(x, t)] = \sum_{\omega > 0} \text{Re}[s_\omega(x) \times \exp(-i\omega t)]$, and $\alpha_t = \omega_p \sigma_t \Phi_0 / (2\pi J_c c d)$ is the damping due to the tunneling quasiparticle conductivity. A similar nonlocal equation has been derived by Gurevich [13].

At this stage we have coupled Eqs. (7)–(9) for $B_z(\omega, \mathbf{r})$ and $\bar{\varphi}_\omega$ with boundary conditions (6). To solve them, we need additional boundary conditions for $B_z(\omega, \mathbf{r})$. The

problem can be strongly simplified in the case $\lambda \ll \lambda_J$. In this case at $|x| \gg \lambda$ nonlocality is not essential and Eq. (9) can be reduced to the usual local approximate equation,

$$\left(\frac{\omega^2}{\omega_p^2} + \alpha_t \frac{i\omega}{\omega_p}\right)\varphi_\omega + \lambda_J^2 \left(1 - \alpha_q \frac{i\omega}{\omega_p}\right) \nabla_x^2 \varphi_\omega = s_\omega(x), \quad (10)$$

where $\alpha_q = 2\pi\lambda^2\omega_p\sigma_q/c^2$ is the dissipation due to quasiparticles inside the superconductor. Near the boundary the situation is more complicated because, in addition to a smoothly changing part, the phase has a component decaying at distances of order λ from the boundary. This extra phase is smaller than the smooth phase by the factor λ/λ_J but it has a comparable derivative. To derive the boundary condition for the smooth phase $\varphi_\omega(x)$, we integrate Eq. (9) from intermediate distance $-x_i$ with $\lambda \ll x_i \ll \lambda_J$ up to the boundary $x = 0$. Neglecting small terms proportional to x_i and $B_b(-x_i, 0)$ [for $B_z(0, y) \ll \Phi_0/(4\pi\lambda^2)$], using local approximation at $x = -x_i$, we obtain the boundary condition for φ_ω ,

$$\nabla_x \varphi_\omega(0) \approx \nabla_x \bar{\varphi}_\omega(-x_i) = -\frac{\lambda}{\lambda_\omega} \frac{4\pi\lambda}{\Phi_0} B_z(\omega, \mathbf{r} = 0), \quad (11)$$

instead of Eq. (6) for the total phase. Equation (11) allows us to reduce the problem to the solution of local Eq. (10) for the smooth phase with modified boundary conditions and avoid solving the exact integral equation for the total phase, Eqs. (6) and (9).

Now we express $B_z(\omega, \mathbf{r} = 0)$ via φ_ω . From Eq. (4) follows the relation between boundary fields and φ_ω ,

$$E_y(\omega, 0, k_y) = i\lambda_\omega^2 k_\omega \kappa_x B_z(\omega, 0, k_y) - (ik_\omega \Phi_0/2\pi) \times \left[\varphi_\omega(0) - \frac{\lambda_\omega^2}{\lambda^2} \int_{-\infty}^0 \exp(\kappa_x x) \nabla_x \varphi_\omega(x) dx \right]. \quad (12)$$

As $\nabla_x \varphi_\omega(x) \sim \varphi_\omega(x)/\lambda_J$ the integral term in Eq. (12) is smaller than $\varphi_\omega(0)$ by the parameter λ/λ_J and we neglect it in the following. The relation between the electric and magnetic field at the boundary is determined by properties of outside media. For dielectric media in the situation $wk_\omega \gg 1$ the outside fields are z independent. Assuming no incoming electromagnetic wave and using Maxwell equations, we derive the relation between the field Fourier components at the boundary

$$B_z(\omega, 0, k_y) = \zeta(\omega, k_y) E_y(\omega, 0, k_y), \quad (13)$$

$$\zeta(\omega, k_y) = \begin{cases} |k_\omega| \varepsilon_d / \sqrt{\varepsilon_d k_\omega^2 - k_y^2}, & \text{for } |k_y| < \sqrt{\varepsilon_d} k_\omega, \\ -ik_\omega \varepsilon_d / \sqrt{k_y^2 - \varepsilon_d k_\omega^2}, & \text{for } |k_y| > \sqrt{\varepsilon_d} k_\omega. \end{cases}$$

Here the upper (lower) part of $\zeta(\omega, k_y)$ corresponds to the case of propagating (decaying) electromagnetic wave inside the dielectric. Equations (12) and (13) allow us to obtain the boundary fields in the typical case $\zeta \lambda_\omega^2 k_\omega \kappa_x \sim \lambda\omega/c \ll 1$ accounting only for the direct radiation coming

out of a very narrow dielectric layer with thickness d , i.e., for $E_y(0, y) \propto \delta(y)$,

$$E_y(\omega, 0, k_y) = \frac{B_z(\omega, 0, k_y)}{\zeta(\omega, k_y)} \approx -\frac{\Phi_0}{2\pi} ik_\omega \varphi_\omega(0). \quad (14)$$

Within this approximation, the magnetic field at the junction edge which determines the boundary condition (11) is

$$B_z(\omega, \mathbf{r} = 0) \approx -\frac{\Phi_0}{2\pi} ik_\omega \varphi_\omega(0) Z(\omega), \quad (15)$$

$$Z(\omega) \approx \frac{\varepsilon_d |k_\omega|}{2} - i \frac{\varepsilon_d k_\omega}{\pi} \ln \frac{\pi/d}{\sqrt{\varepsilon_d} |k_\omega|}.$$

Finally, we obtain the boundary conditions for the smooth oscillating phase at both edges in a finite-length JJ:

$$\nabla_x \varphi_\omega(x) \approx \pm 2i\lambda k_\omega Z(\omega) \varphi_\omega(x), \quad \text{for } x = 0, -l. \quad (16)$$

Direct radiation to the right is given by the Poynting vector,

$$\mathcal{P}_{\text{rad}} = \frac{wc}{4\pi} \int_{-d/2}^{d/2} dy \langle E_y(0, y, t) B_z(0, y, t) \rangle_t \approx (\varepsilon_d w \omega^3 \Phi_0^2 / 64\pi^3 c^2) |\varphi_\omega(0)|^2. \quad (17)$$

In addition, the oscillating current flowing along the lead's edges away from the JJ may induce oscillating charges in the external circuit which will lead to additional (indirect) radiation. The effect of the external circuit is also described by the term of the type $Z_e(\omega) \varphi_\omega$ in the right hand side of the boundary condition (16) [5]. An important point is that effects of both external circuit and of direct radiation add in the boundary condition because the Maxwell equations are linear and this allows us to describe them separately.

Now we solve analytically Eq. (9) neglecting the effects of the external circuit and using the perturbation theory with respect to the Josephson current [11,12] in the limit $b \gg \tilde{\omega}$, 1 [14]. Taking the solution as $\varphi(\tau, u) = \tilde{\omega}\tau - bu + \theta(\tau, u)$ with $\theta(\tau, u) \ll 1$ and expanding $\sin[\varphi(\tau, u)]$, we see that $\theta_\omega(u) \equiv \theta(\tilde{\omega}, u)$ obeys Eq. (10) in reduced form with $s_\omega = -e^{ibu}/i$,

$$[\nabla_u^2 + \tilde{\omega}^2 + i\tilde{\omega}(\alpha_t - \alpha_s \nabla_u^2)]\theta_\omega(u) = -e^{ibu}/i. \quad (18)$$

Solution for $\theta(\tilde{\omega}, u)$, given by $\theta_\omega(u) \approx e^{-ibu}/(ib^2) + a_1 e^{ip_\omega u} + a_2 e^{-ip_\omega u}$, describes the moving vortex lattice (the first term) and reflected Swihart waves propagating to the right and left. Here $p_\omega \approx \tilde{\omega} + i\alpha/2$, where $\alpha = \alpha_t + \alpha_s \tilde{\omega}^2$. Finding a_1 and a_2 from the boundary conditions, Eqs. (16), we obtain

$$\theta_\omega(0) \approx [\cos(p_\omega \tilde{l}) - \exp(-ib\tilde{l})]/(\tilde{\omega}bD), \quad \tilde{l} = l/\lambda_J, \\ \theta_\omega(-l) \approx [1 - \exp(-ib\tilde{l}) \cos(p_\omega \tilde{l})]/(\tilde{\omega}bD), \quad (19)$$

where $D \approx \sin(\tilde{\omega} \tilde{l}) + i(\beta + \alpha\tilde{l}/2) \cos(\tilde{\omega} \tilde{l})$ and $\beta = d\varepsilon_d \tilde{\omega}/(\varepsilon_i \lambda_J)$. We kept only the Fiske-resonance terms and neglected $\text{Im}[Z(\omega)]$, which only slightly shifts resonance positions. Radiation to the right and left, $\mathcal{P}_{\text{rad}}^{r,l}(\tilde{\omega}, b)$, is determined by the values $|\theta_\omega(0)|^2$ and $|\theta_\omega(-l)|^2$. At low dissipation, $\alpha\tilde{l} \ll 1$, we derive

$$\mathcal{P}_{\text{rad}}^{r,l} \approx \frac{\Phi_0^2 \omega_p^2 w}{64\pi^3 c^2 b^2} \frac{1 - 2 \cos(b\tilde{l}) \cos(\tilde{\omega}\tilde{l}) + \cos^2(\tilde{\omega}\tilde{l}) \pm \rho}{|D|^2},$$

$$\rho = 2(\alpha\tilde{l} + \beta)[1 - \cos(b\tilde{l}) \cos(\omega\tilde{l})]/b. \quad (20)$$

The radiation reaches a maxima at frequencies $\tilde{\omega} = \tilde{\omega}_n = \pi n/\tilde{l}$ with $n = 1, 2, \dots$. The resonance width is determined by both the dissipation $\alpha\tilde{l}$ and by direct radiation β . The perturbation theory is valid in resonance for $|\theta_\omega(0)| \sim (b\alpha\tilde{l}\tilde{\omega})^{-1} < 1$ and the radiation power in this linear regime is small, e.g., $\mathcal{P}_{\text{rad}}/w \lesssim 10^{-6} \mu\text{W}/\text{cm}$ for $\nu = 10$ GHz.

Next we derive the dc current at voltage $V = \Phi_0 \omega / (2\pi c)$ and estimate Q accounting for direct radiation only. The current I via a JJ is given by the tunneling quasiparticle contribution, $I_t = \sigma_t V l w / d$, and the Josephson current contribution,

$$I_s(\omega) \approx J_c \lambda_J w \int_{-\tilde{l}}^0 du \langle \cos(\tilde{\omega}\tau - bu) \theta(\tau, u) \rangle_\tau. \quad (21)$$

The latter contribution consists of the dissipation and radiation parts $I_s = I_{s,\text{dis}} + I_{s,\text{rad}}$. The radiation part plays the same role as dissipation because in both cases energy is transferred from the moving vortex lattice to other degrees of freedom (to photons in the case of radiation). In the lowest order in $\lambda/\lambda_J \ll 1$ at the resonance frequencies we get

$$I_{s,\text{dis}} \approx \frac{\alpha \varepsilon_i \omega_p l}{2\varepsilon_d \omega d} I_{s,\text{rad}} \approx \frac{\Phi_0 c w \alpha \tilde{l} \sin^2(b\tilde{l}/2)}{32\pi^2 \lambda \lambda_J b^2 \tilde{\omega} |D|^2}. \quad (22)$$

Losses due to direct radiation are equivalent to those caused by a resistor with $Rw = 2\pi/(\varepsilon_d \omega)$ attached parallel to a JJ ($Rw \approx 90 \Omega \text{ cm}$ for $\varepsilon_d = 1$ and $\nu = 10$ GHz). The power fed into the JJ is $\mathcal{P} = IV$. Part of it, $(I_t + I_{s,\text{dis}})V$, is dissipated inside the JJ, while another part, $I_{s,\text{rad}}V$, is radiated. Neglecting the nonresonant part, I_r , we obtain for the radiated fraction $Q = \mathcal{P}_{\text{rad}}/\mathcal{P}$,

$$Q = \frac{r}{1+r}, \quad r = \frac{I_{s,\text{rad}}}{I_{s,\text{dis}}} = Q_f \mathcal{N}, \quad Q_f = \frac{2\varepsilon_d d \omega}{\varepsilon_i \lambda_J \omega_p}. \quad (23)$$

Here $\mathcal{N} = 1/\alpha\tilde{l}$ is the number of reflections before the Swihart wave decays inside the JJ. For radiation into waveguide at $wk_\omega \sim 1$ we get $Q_f \approx Q_Z$ and our result is larger than that given by Eq. (1) by the factor \mathcal{N} . As dissipation inside the JJ decreases (σ_t and σ_q drop), \mathcal{N} increases. In the linear regime $b\alpha\tilde{l}\tilde{\omega} > 1$, we get $\mathcal{N} < b\tilde{\omega}$. Because of the limitation $b < \lambda_J/\lambda$ we obtain $Q \lesssim (d/\lambda)\tilde{\omega}^2 \sim 10^{-2}(\omega/\omega_p)^2$. Hence, Q may become of order unity in the linear regime. Nevertheless, the radiation power per unit width, $\mathcal{P}_{\text{rad}}/w$, given by Eq. (17), is always small in this regime because the condition $|\theta_\omega| \lesssim 1$ also restricts the power fed into the JJ. The open question is whether it is possible to get $|\theta_\omega| \gg 1$ and larger \mathcal{P}_{rad} in strongly nonlinear regime when dissipation is very low. To answer this question more numerical studies are needed.

In conclusion, we derived the dynamic boundary conditions for the oscillating phase in a single JJ which account for direct radiation from the dielectric layer inside JJ into dielectric outside media. We computed the power of direct radiation in the linear regime of Josephson oscillation when effect of external circuit may be ignored. In this regime the power conversion efficiency Q is determined by the number of multiple reflections (i.e., dissipation rate inside the JJ) and the transmission coefficient Q_f from a JJ into free space. Even if $Q \rightarrow 1$, radiation power per unit width of the JJ remains small in the linear regime. Theoretical limits of the radiation power for large phase oscillations, in the nonlinear regime, can be obtained only by numerical simulations. Controlled measurements of direct radiation from the junctions with the lowest possible dissipation are needed to establish real limits on the radiation power from a single JJ.

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 - [14] See EPAPS Document No. E-PRLTAO-98-041701 for exact formulas in the linear regime for the phase distribution, current, and radiated fraction and their implementation in MATHEMATICA. For more information on EPAPS, see <http://www.aip.org/pubservs/epaps.html>.