Radiation due to Josephson Oscillations in Layered Superconductors

L. N. Bulaevskii[1] and A. E. Koshelev[2]

[1]Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

[2]Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA

(Received 20 February 2007; published 2 August 2007)

We derive the power of direct radiation into free space induced by Josephson oscillations in intrinsic Josephson junctions of layered superconductors. We consider the superradiation regime for a crystal cut in the form of a thin slice parallel to the c axis. We find that the radiation correction to the current-voltage characteristic in this regime depends only on crystal shape. We show that at a large number of junctions oscillations are synchronized providing high radiation power and efficiency in the terahertz frequency range. We discuss the crystal parameters and bias current optimal for radiation power and crystal cooling.

DOI: 10.1103/PhysRevLett.99.057002 PACS numbers: 85.25.Cp, 42.25.Gy, 74.50.+r

Josephson junctions (JJs), as sources of tunable continuous electromagnetic (EM) radiation, were discussed for a long time after the prediction of the ac Josephson effect [1]. Early measurements [2] demonstrated that emittence from a single JJ has very low power, typically $\sim 10^{-6}$ μW. Since then significant effort has been devoted to develop JJ arrays as coherent sources of radiation; see, e.g., Refs. [3,4]. A major challenge is to force all JJs in array to emit coherently, so that power increases proportionally to the square of the total number of junctions [3]. In particular, for an array of 500 junctions a maximum power of the order of 10 μW at discrete frequencies ≤ 0.4 THz has been achieved so far in the superradiation regime [5]. The difficulties in synchronizing many artificial JJs are related mainly to the fact that artificial junctions always have slightly different parameters, especially the Josephson critical current, and that one cannot put many of them at distances smaller than a wavelength but needs to distribute them over a wavelength or more [3,4]. Also, as the maximum frequency is limited by the superconducting gap, it cannot exceed several hundred gigahertz for structures fabricated out of conventional superconductors.

Layered high-temperature superconductors like $Bi_2Sr_2CaCu_2O_8$ (BSCCO) offer a very attractive alternative for developing radiation sources [6]. A large value of the gap (up to 60 meV) allows for very high Josephson frequencies, which can be brought into the practically important terahertz range. Moreover, intrinsic JJs (IJJs) have much closer parameters than artificial ones as these parameters are controlled by the atomic crystal structure rather than by amorphous dielectric layer in artificial JJs. Also, layered superconductors provide a very high density of IJJs (1 per 15.6 Å along the c axis) and thus it is easy to reach the superradiation regime with many junctions on the scale of radiation wavelength. In this regime the radiated EM field effectively couples JJs and helps to synchronize them. In this Letter we demonstrate that the superradiation regime indeed results in the synchronization of Josephson oscillations in IJJs in zero external magnetic field. Thus the c-axis current biased crystal may work as a source of Josephson coherent emission of radiation. We calculate the radiation power and IV characteristics and discuss an optimal crystal geometry accounting for heating due to quasiparticle dissipation.

So far mostly radiation from the flux flow of the Josephson vortices has been discussed in the literature [6]. The inductive interlayer coupling typically promotes formation of the *triangular vortex lattice*. However, to generate noticeable outside radiation, oscillations induced by the moving lattice have to be in phase in different layers, which is realized only if the moving vortices form a *rectangular lattice*. No regular way to prepare such a lattice is known at present. In addition, it seems to be unstable in most of parameter space [7,8]. Here we consider the synchronization of the Josephson oscillations by radiation field in the simplest case: when dc magnetic field is not applied and only radiation itself introduces the in-plane phase gradients.

In the resistive state phases φ_n oscillate at the Josephson frequency, $\omega_J = 2eV/\hbar$, where the voltage V between the neighboring layers is induced by interlayer dc current. For uniform oscillations and identical junctions the voltage V is the same in all junctions (except, possibly, top and bottom junctions in the stack) because the same current flows between all layers. To reach a resistive state the pulse of dc current exceeding the Josephson critical current should be applied, but then current may be diminished to reach the necessary voltage. Transport measurements in BSCCO have shown that the resistive state on a current-descending branch is preserved down to the voltage $V_p \sim \hbar\omega_p/2e$, where ω_p is the Josephson plasma frequency; see, e.g., Ref. [9] [$\omega_p/(2\pi) \approx 0.15$ THz in optimally doped BSCCO]. We consider crystal with sizes L_x, w, and $L_z = Ns$ in the directions x, y, and z, respectively; see Fig. 1. The conditions $L_x, L_z \lesssim k_\omega^{-1} = c/\omega_J$ are necessary for the superradiation regime. The boundary conditions for the oscillating phase are sensitive to distribution of outside EM fields which, in turn, depend on geometry of the stack and electric contacts. We consider the simplest geometry assuming that (i) $w \gg k_\omega^{-1}$ so that all quantities are y

 © 2007 The American Physical Society

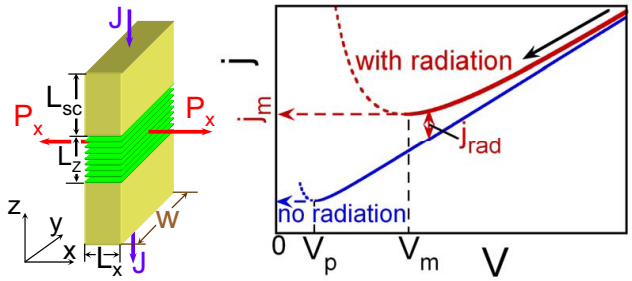

FIG. 1 (color online). Left: Schematic picture of layered superconductor placed in between platelike leads serving as screens. The directions of the dc transport current J, and of the radiation Poynting vectors P_x, are shown. Right: Influence of radiation on current-descending branch of the IV dependence.

independent, and (ii) JJ stack is bounded by metallic or superconducting contacts with the same lateral sizes as the stack which extend in the z direction over distance $L_{sc} \gg k_\omega^{-1}$ (see Fig. 1). We also assume that the contact material has a very small surface impedance so that the ac electric field at the contact surface is negligible. Such contacts serve as "screens," restricting radiation to half-infinite spaces $|x| > L_x/2$. This greatly simplifies the analytical consideration. As dissipation increases with L_x and radiation does not, we argue that optimal crystal should be platelike with $L_x < L_z$.

To find the phase differences $\varphi_n(x, t)$ inside the crystal we solve finite-difference differential equations (see, e.g., Ref. [8]),

$$\frac{\partial^2 \varphi_n}{\partial \tau^2} = (\alpha \nabla_n^2 - 1)\left(\nu_c \frac{\partial \varphi_n}{\partial \tau} + \sin\varphi_n - \nabla_u h_{y,n}\right), \quad (1)$$

$$(\nabla_n^2 - \ell^{-2}\hat{T}_{ab})h_{y,n} + \hat{T}_{ab}\nabla_u \varphi_n = 0. \quad (2)$$

We use a reduced x coordinate, $u = x/\lambda_J$, normalized to $\lambda_J = \gamma s$, reduced time, $\tau = \omega_p t$ and $\omega = \omega_J/\omega_p$, with $\omega_p = c/(\lambda_c\sqrt{\epsilon_c})$, and reduced magnetic field $h_{y,n} = B_{y,n}/B_c$ with $B_c = \Phi_0/(2\pi\lambda_{ab}\lambda_c)$, where $B_{y,n}$ is the magnetic field between the layers n and $n+1$. Here ϵ_c is the c-axis dielectric constant inside the superconductor, λ_{ab} and λ_c are the London penetration lengths, $\gamma = \lambda_c/\lambda_{ab}$ is the anisotropy ratio. In terms of these parameters the Josephson critical current is $J_c = \Phi_0 c/(8\pi^2 s\lambda_c^2)$. Further, $\hat{T}_{ab} \equiv 1 + \nu_{ab}\partial/\partial\tau$, $\ell \equiv \lambda_{ab}/s$, ∇_n^2 notates the discrete second derivative operator, $\nabla_n^2 A_n = A_{n+1} + A_{n-1} - 2A_n$, and $\alpha \sim 0.1$–1 is the parameter of the capacitive coupling [10]. The dissipation parameters, $\nu_{ab} = 4\pi\sigma_{ab}/(\gamma^2\epsilon_c\omega_p)$ and $\nu_c = 4\pi\sigma_c/(\epsilon_c\omega_p)$, are determined by the quasiparticle conductivities, σ_{ab} and σ_c, along and perpendicular to the layers, respectively. The electric field inside the superconductor between the layers n and $n+1$ is given by

$$(1 - \alpha\nabla_n^2)E_{zn} = (B_c\ell/\sqrt{\epsilon_c})(\partial\varphi_n/\partial\tau). \quad (3)$$

The typical parameters of optimally doped BSCCO at low temperatures are $\epsilon_c = 12$, $s = 15.6$ Å, $\gamma = 500$, $\lambda_{ab} = 200$ nm, $J_c = 1700\,\mathrm{A/cm^2}$, $\sigma_c(0) = 2\times10^{-3}$ $(\Omega\,\mathrm{cm})^{-1}$, $\sigma_{ab}(0) = 4\times10^4$ $(\Omega\,\mathrm{cm})^{-1}$ [11]. This gives $\ell \approx 130$, $\nu_{ab} \approx 0.2$, and $\nu_c \approx 2\times10^{-3}$. An important feature of BSCCO is that $\nu_{ab} \gg \nu_c$.

The boundary conditions, i.e., relations between time and space derivatives of φ_n at the edges parallel to (y, z), are determined by the relations between the electric and magnetic fields in the outside media. As the y and z sizes of the system (crystal and screens) are assumed to be larger than the wavelength, the dielectric media can be treated as infinite in these directions. Such a half-infinite space geometry allows us to find the boundary conditions analytically. From the Maxwell equations in the free space we find the relationship between the magnetic, $\mathbf{B} = (0, B_y, 0)$, and the electric, $\mathbf{E} = (E_x, 0, E_z)$, fields at the boundaries. We assume that there are only outgoing waves from the crystal (y, z) edges, meaning that the fields have the coordinate and time dependence $\exp(ik_x|x| + ik_z z - i\omega\tau)$, where $k_x = \mathrm{sgn}(\omega)(k_\omega^2 - k_z^2)^{1/2}$ for $k_z^2 < k_\omega^2$ and $k_x = i(k_z^2 - k_\omega^2)^{1/2}$ for $k_z^2 > k_\omega^2$. The relations between fields at $u = \pm\tilde{L}_x/2$ $(\tilde{L}_x = L_x/\lambda_J)$ are [12]

$$B_y(\omega, k_z) = \mp\zeta_\omega(k_z)E_z(\omega, k_z),$$
$$\zeta_\omega(k_z) = \begin{cases} |k_\omega|(k_\omega^2 - k_z^2)^{-1/2}, & \text{for } k_z^2 < k_\omega^2, \\ -ik_\omega(k_z^2 - k_\omega^2)^{-1/2}, & \text{for } k_z^2 > k_\omega^2. \end{cases} \quad (4)$$

Inverse Fourier transform with respect to k_z gives nonlocal relation between the magnetic and electric fields at the edges. As we assume that the screen material has small surface impedance, we can neglect the electric field at $|z| > L_z/2$ and, using Eq. (3), we obtain the reduced boundary condition connecting $h_{y,n}$ with the phases at the edges [$h_{y,n}$ is expressed via $\nabla\varphi_n$ by Eq. (2)],

$$\pm h_{y,n,\omega} = \frac{is\ell\omega}{2\sqrt{\epsilon_c}}\sum_m (1 - \alpha\nabla_m^2)^{-1}\varphi_{m,\omega}[|k_\omega|J_0(k_\omega s|n-m|) + ik_\omega N_0(k_\omega s|n-m|)], \quad (5)$$

where $J_0(x)$ and $N_0(x)$ are the Bessel functions.

We consider high-frequency Josephson oscillations, $\omega = \omega_J/\omega_p \gg 1$, in the case of layered crystals with a large number of junctions $N \gtrsim \ell$. This allows us to neglect finite-size effects along the z axis. The equation for uniform solution $\varphi_n(u, \tau) = \varphi(u, \tau)$ is

$$\partial^2\varphi/\partial\tau^2 + \nu_c\partial\varphi/\partial\tau + \sin\varphi - \ell^2\nabla_u^2\varphi = 0. \quad (6)$$

In the limit $\omega \gg 1$ we look for the solution in the form $\varphi(u, \tau) = \omega\tau + \phi(u, \tau)$ with $\phi \ll 1$. Equation (5) gives the boundary conditions for ϕ at $u = \pm\tilde{L}_x/2$ [12],

$$\nabla_u\phi = \pm i\omega\zeta\phi,$$
$$\zeta = \frac{L_z}{2\ell\sqrt{\epsilon_c}}[|k_\omega| - ik_\omega\mathcal{L}_\omega], \qquad \mathcal{L}_\omega \approx \frac{2}{\pi}\ln\left[\frac{5.03}{|k_\omega|L_z}\right]. \quad (7)$$

The solution is $\phi(u, \tau) = \mathrm{Im}[\phi_\omega(u)\exp(-i\omega\tau)]$, where

$$\phi_\omega = -\frac{1}{\omega^2 + i\omega\nu_c} + \frac{i\zeta\cos(\bar{k}_\omega u)/(\omega + i\nu_c)}{\bar{k}_\omega \sin(\bar{k}_\omega \tilde{L}_x/2) + i\zeta\omega\cos(\bar{k}_\omega \tilde{L}_x/2)}, \tag{8}$$

with $\bar{k}_\omega = \omega/\ell$ and $|\zeta| \ll 1$. The first term in ϕ_ω is the amplitude of the Josephson oscillations, while the second term describes the EM waves propagating inside the junctions. They are generated at the boundaries due to the radiation field.

Next we show that coherent radiation field, similar to all junctions, in combination with intralayer dissipation stabilizes the uniform Josephson oscillations. For that we have to consider a small perturbation to the uniform solution, $\varphi_n(u,\tau) = \omega\tau + \phi(u,\tau) + \vartheta_n(u,\tau)$, and verify that there are no perturbations, $\vartheta_n(u,\tau)$, increasing with time. The general analysis is rather cumbersome. Results in a closed form may be obtained only by use of approximations valid in the limiting case considered here, $L_x, L_z \ll k_\omega^{-1}$, $\omega \gg 1$, and $\ell \gg 1$.

Equations for $\vartheta_n(u,\tau)$ are obtained by linearization of Eqs. (1) and (2). The term $\cos[\phi(\tau)]\vartheta_n(u,\tau) \approx \cos(\omega\tau)\vartheta_n(u,\tau)$ in the linearized equation couples harmonics with small frequency Ω with the high-frequency terms $\Omega \pm \omega$. At $\omega \gg 1$ we can neglect coupling to the higher frequency harmonics $\Omega \pm m\omega$ with $m > 1$ and represent the phase perturbation (and field) as

$$\vartheta_n \approx \sum_q \left[\bar{\vartheta}_q + \sum_{\beta=\pm 1} \tilde{\vartheta}_{q,\beta}\exp(i\beta\omega\tau)\right]\sin(qn)\exp(-i\Omega\tau),$$

with $q = \pi k/(N+1)$, $k = 1, 2, \ldots, N$. Here the complex eigenfrequency $\Omega = \Omega(q)$ is assumed to be small, $|\Omega| \ll \omega$, and has to be found. Stability means that $\mathrm{Im}[\Omega] < 0$ for all q. Substituting this presentation into the linearized equations (1) and (2), excluding oscillating magnetic fields, and separating the fast and slow parts, we obtain coupled equations

$$\left[\frac{\Omega^2}{1+\alpha_q} + i\nu_c\Omega - \bar{C}\right]\bar{\vartheta}_q + G_q^{-2}\nabla_u^2\bar{\vartheta}_q = \frac{\tilde{\vartheta}_{q,+} + \tilde{\vartheta}_{q,-}}{2}, \tag{9}$$

$$\left[\frac{(\Omega \mp \omega)^2}{1+\alpha_q} + i\nu_c(\Omega \mp \omega)\right]\tilde{\vartheta}_{q,\pm} + G_{q,\pm}^{-2}\nabla_u^2\tilde{\vartheta}_{q,\pm} = \frac{\bar{\vartheta}_q}{2}. \tag{10}$$

Here $\bar{C} \equiv \langle\cos\phi\rangle_\tau \approx \mathrm{Re}[\phi_\omega]/2$, $\alpha_q \equiv \alpha\tilde{q}^2$ with $\tilde{q}^2 = 2(1-\cos q)$, $G_{q,\beta}^2 = \tilde{q}^2/[1 - i(\Omega - \beta\omega)\nu_{ab}] + \ell^{-2}$, and $G_q = G_{q,0}$. Using Eqs. (2) and (4) (in the limit $k_z^2 > k_\omega^2$ due to $k_\omega L_z \ll \pi$) and (5) we get the boundary conditions for slow and fast components at $u = \pm\tilde{L}_x/2$ for $q \gg \pi/N$,

$$\nabla_u\bar{\vartheta}_q = \pm\kappa_0\bar{\vartheta}_q, \quad \kappa_0 \approx G_q^2\Omega^2/[(1+\alpha_q)\epsilon_c q\gamma], \tag{11}$$

$$\nabla_u\tilde{\vartheta}_{q,\beta} = \pm\kappa_\beta\tilde{\vartheta}_{q,\beta}, \qquad \kappa_\beta \approx \frac{(\Omega - \beta\omega)^2 G_{q,\beta}^2}{(1+\alpha_q)\epsilon_c q\gamma}. \tag{12}$$

Because of the condition $|\Omega| \ll \omega$, in most cases one can neglect Ω in equation and boundary conditions for $\tilde{\vartheta}_{q,\beta}$. We also assume $\nu_c \ll 1 \ll \omega$ and neglect dissipation when it is not essential. As $\bar{\vartheta}_q$ varies at the typical length scale $\sim 1/G_q\Omega$, which is much larger than L_x, the coordinate-dependent part of $\bar{\vartheta}_q$ can be treated as a small perturbation. Neglecting the coordinate dependence of $\bar{\vartheta}_q$ in the equation for $\tilde{\vartheta}$, we obtain the approximate solution of Eqs. (10) and (12). Substituting it into Eq. (9), we obtain the Mathieu equation for the slow-varying component

$$\left(\frac{\Omega^2}{1+\alpha_q} + i\nu_c\Omega - \frac{\alpha_q}{2\omega^2} - V(u) + G_q^{-2}\nabla_u^2\right)\bar{\vartheta}_q = 0, \tag{13}$$

where $V(u) = V_1(u) + V_2(q,u)$,

$$V_1(u) \approx \frac{1}{2\omega^2}\mathrm{Re}\left[\frac{i\zeta\omega\cos(\bar{k}_\omega u)}{\bar{k}_\omega\sin(\bar{k}_\omega\tilde{L}_x/2) + i\zeta\omega\cos(\bar{k}_\omega\tilde{L}_x/2)}\right],$$

$$V_2(q,u) \approx \frac{1}{2\omega^2}\mathrm{Re}\left[\frac{\kappa_+\cos p_+ u}{p_+\sin(p_+\tilde{L}_x/2) + \kappa_+\cos(p_+\tilde{L}_x/2)}\right],$$

and $p_+ = \omega G_{q,+}$. In the regime $\bar{k}_\omega\tilde{L}_x \ll 1$, the part $V_1(u)$ reduces to a constant, $V_1(u) \approx \mathcal{K}_\omega/(2\omega^2)$ with $\mathcal{K}_\omega = [\mathcal{L}_\omega(\mathcal{L}_\omega + \epsilon_c a) + 1]/[(\mathcal{L}_\omega + \epsilon_c a)^2 + 1]$ and $a = L_x/L_z$.

Equation (13) and the boundary conditions (11) determine the spectrum of small perturbations to the uniform solution. Treating the coordinate-dependent part of $\bar{\vartheta}_q$ as a small perturbation, we derive the expression for $\Omega(q)$,

$$\Omega^2 + i\nu_c\Omega \approx [\alpha_q + \mathcal{K}_\omega - W_2(q)]/(2\omega^2),$$
$$W_2(q) = \mathrm{Re}(2/\{p_+\tilde{L}_x[p_+/\kappa_+ + \cot(p_+\tilde{L}_x/2)]\}). \tag{14}$$

From this result we can conclude that the main contribution to stabilization of uniform oscillations comes from the term $\mathcal{K}_\omega$, describing effective coupling of junctions due to the radiation. Its stabilization effect increases with L_z as $\mathcal{K}_\omega \approx \mathcal{L}_\omega L_z/\epsilon_c L_x$ for $L_z < \epsilon_c L_x$ and $\mathcal{K}_\omega \to 1$ for $L_z > \epsilon_c L_x$. The charging-effect term α_q also contributes to stabilization. The term W_2 describes the effect of modes $\tilde{\vartheta}_{q,\pm}$ induced inside the crystal due to radiation. Formally, the W_2 term leads to instabilities *in the limit of zero dissipation* because its denominator vanishes near the resonance values of q given by $2(1-\cos q) = [2\pi m/(\omega\tilde{L}_x)]^2 \gg 1/\ell^2$, where m is an integer. These instabilities correspond to parametric excitation of the Fiske resonances described by Eq. (10). However, they are suppressed already by very small dissipation. Indeed, at small dissipation we estimate the maximum value of W_2 as $[q\gamma\epsilon_c\mathrm{Im}[p_+]\tilde{L}_x^2]^{-1}$, where $\mathrm{Im}[p_+] \approx \sin(q/2) \times (\nu_c + \nu_{ab}\omega^2)$. As $\nu_{ab}\omega^2 \gg \nu_c$, we see that $|W_{2,\max}| \ll 1$ for $\nu_{ab} \gg 1/(2\pi^2\gamma\epsilon_c) \sim 10^{-5}$. For a realistic level of dissipation in BSCCO, $\nu_{ab} \sim 0.2$, the resonance features in W_2 are completely washed out and $|W_2| \ll 1$ for all q's. Thus the intralayer dissipation stabilizes uniform oscillations, but it does not affect them in any other way.

We derive now the radiation power and IV characteristics. The Poynting vector P_x at $x = \pm L_x/2$ in terms of the oscillating phase (8) is given by [12]

$$P_x(\omega) = \pm \frac{\Phi_0^2 \omega_J^3}{64\pi^3 c^2 s N} \sum_{n,m} J_0(k_\omega s|n-m|)|\varphi_\omega(\pm L_x/2)|^2.$$

At $\bar{k}_\omega \tilde{L}_x \ll 1$, we obtain for the total radiation power $\mathcal{P}_{\rm rad}(\omega) = P_x(\omega) L_z w$ going from one side,

$$\mathcal{P}_{\rm rad}(\omega_J)/w \approx [\Phi_0^2 \omega_p^4 N^2/(64\pi^3 c^2 \omega_J)]\mathcal{L}(a), \quad \mathcal{L}(a) = a^2\epsilon_c^2/[(a\epsilon_c + \mathcal{L}_\omega)^2 + 1]. \tag{15}$$

For small L_x, $L_x \ll L_z/\epsilon_c$, $\mathcal{P}_{\rm rad} \propto L_x^2$ and it is N independent, while for larger L_x the geometrical factor $\mathcal{L}(a) \to 1$, meaning that $\mathcal{P}_{\rm rad} \propto N^2$ and it is L_x independent.

The dc interlayer current density J consists of quasiparticle contribution, $\sigma_c V/s$, and the Josephson part $J_c \sin\varphi$ averaged over τ and u. We derive for $j = J/J_c$,

$$j = \nu_c \omega + \frac{1}{2}\langle {\rm Im}[\phi_\omega]\rangle_u = \nu_c \omega + \frac{\nu_c}{2\omega^3} + \frac{\mathcal{L}(a)}{2\omega^2 \epsilon_c a}. \tag{16}$$

The last term, $j_{\rm rad}$, describes contribution to the dc current due to radiation losses. This part of the current multiplied by the total voltage gives the total radiation power $2\mathcal{P}_{\rm rad}$. As a function of ω the current j has a minimum at $\omega = \omega_m \approx [\mathcal{L}(a)/\epsilon_c \nu_c a)]^{1/3} \gg 1$. The influence of radiation on the IV dependence is illustrated in Fig. 1. Only part of the IV characteristics at $\omega > \omega_m$ is stable. At the voltage $V_m = \hbar\omega_m/2e$, corresponding to the current $j_m = (3/2)\nu_c \omega_m$, the stack jumps back to the static state. At this "retrapping" current the dissipation power is twice that of the radiation power; i.e., for the conversion efficiency we obtain $2\mathcal{P}_{\rm rad}/(\mathcal{P}_{\rm dis} + 2\mathcal{P}_{\rm rad}) \le 1/3$.

An important issue is the stability of the coherent state with respect to parameter variations from layer to layer, which may include the crystal width δL_{xn} and the Josephson current density δJ_{cn}. The coherent solution with the same voltage drop in all junctions V must satisfy the current conservation condition. Such a solution can be built as $\varphi_n(u,\tau) = \omega\tau + \phi_{\omega,n}(u,\tau) + \beta_n$, where the additional phase shifts β_n compensate for parameter variations. In the case of smooth parameter variations and $\alpha \ll 1$, we can derive correction to the local current and obtain

$$\sin(\beta_n) - \langle \sin(\beta_n)\rangle \approx \frac{\delta L_{xn}}{\langle L_{xn}\rangle}\frac{\nu_c \omega}{j_{\rm rad}} - \frac{\delta J_{cn}}{\langle J_{cn}\rangle}\langle \sin(\beta_n)\rangle,$$

where $\langle \cdots \rangle$ is average over n. We see that the coherent state survives until $\delta L_{xn}/\langle L_{xn}\rangle < j_{\rm rad}/(\nu_c \omega)$ and $\delta J_{cn} < \langle J_{cn}\rangle$. These conditions do not impose too demanding restrictions on the acceptable range of parameter variations.

Evaluating the Joule heating, we find that the cooling rate Q per unit area of each crystal side $\parallel yz$ should be

$$Q \approx \frac{\Phi_0^2 \omega_p \nu_c}{32\pi^3 \lambda_c^2 s^2}\omega^2 L_x \approx \left(\frac{\omega}{\omega_m}\right)^3 \frac{2\mathcal{P}_{\rm rad}}{L_z w}. \tag{17}$$

The maximum efficiency is reached at $\omega \approx \omega_m$. To achieve this at frequencies close to 1 THz, one needs to maximize $\omega_m \propto [\mathcal{L}(a)/a]^{1/3}$ by optimizing the crystal shape. At $\mathcal{L}_\omega \sim 1$ we get $\max[\mathcal{L}(a)/a] \approx \epsilon_c/4$ at $a \approx 1/\epsilon_c$. This gives $\omega_m \approx 5$ corresponding to $\omega_J/(2\pi) = 0.75$ THz. At this frequency, assuming $L_z = 40\ \mu$m, the optimum lateral sizes are $L_x \approx 4\ \mu$m and $w > 300\ \mu$m. Biased with the current density $\approx 0.01 J_c$, such a crystal radiates with the power $\mathcal{P}_{\rm rad}/w \approx 30$ mW/cm from each side, while it should be cooled with the rate $Q \approx 15$ W/cm^2 at each side [13]. As L_x increases, $\mathcal{P}_{\rm rad}/w$ saturates to ≈ 0.2 W/cm at $\omega \sim \omega_m \propto L_x^{-1/3}$, while Q increases linearly with L_x. We note that increasing the number of layers also promotes synchronization of oscillations by the radiation field.

In conclusion, we have shown that uniform Josephson oscillations in IJJs of layered superconductors are stable in the superradiation regime at high frequencies $\omega_J \gg \omega_p$. They lead to coherent radiation into free space with significant power and efficiency as high as 1/3 at frequencies $\sim$1 THz. An important point is that to have a reasonable cooling rate and strong radiation the crystal should be in the form of a thin plate along the c axis with a large number of layers $N \sim 10^4$.

The authors thank M. Maley, I. Martin, K. Kadowaki, M. Tachiki, R. Kleiner, A. Ustinov, and V. Kurin for numerous useful discussions. This research was supported by the U.S. DOE under Contract No. W-7405-Eng-36 (LANL) and No. DE-AC02-06CH11357 (ANL).

[1] B. D. Josephson, Phys. Lett. **1**, 251 (1962).
[2] D. N. Langenberg *et al.*, Phys. Rev. Lett. **15**, 294 (1965); I. K. Yanson *et al.*, Zh. Eksp. Teor. Fiz. **48**, 976 (1965) [Sov. Phys. JETP **21**, 650 (1965)].
[3] A. K. Jain *et al.*, Phys. Rep. **109**, 309 (1984).
[4] J. Lukens, *Superconducting Devices* (Plenum, New York, 1990), p. 135; P. Barbara *et al.* Phys. Rev. Lett. **82**, 1963 (1999).
[5] S. Y. Han *et al.*, Appl. Phys. Lett. **64**, 1424 (1994).
[6] T. Koyama and M. Tachiki, Solid State Commun. **96**, 367 (1995); G. Hechtfischer *et al.*, IEEE Trans. Appl. Supercond. **7**, 2723 (1997); Yu. I. Latyshev *et al.*, Phys. Rev. Lett. **87**, 247007 (2001); M. Tachiki *et al.*, Phys. Rev. B **71**, 134515 (2005).
[7] S. N. Artemenko and S. V. Remizov, Physica (Amsterdam) **362C**, 200 (2001); Phys. Rev. B **67**, 144516 (2003).
[8] A. E. Koshelev and I. Aranson, Phys. Rev. B **64**, 174508 (2001).
[9] Yu. I. Latyshev *et al.*, Phys. Rev. Lett. **82**, 5345 (1999).
[10] T. Koyama and M. Tachiki, Phys. Rev. B **54**, 16 183 (1996).
[11] Yu. I. Latyshev *et al.*, Phys. Rev. B **68**, 134504 (2003).
[12] L. N. Bulaevskii and A. E. Koshelev, J. Supercond. Novel Magn. **19**, 349 (2006).
[13] For effective cooling and mechanical stability the crystal may be placed on the dielectric at one of the yz sides. The modification of the boundary condition for this case is straightforward.