Journal of Physics: Conference Series **150** (2009) 052124 doi:10.1088/1742-6596/150/5/052124

Resonant Terahertz Radiation from Layered Superconductors: Mechanisms of Damping and Structure of Dynamic States

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Abstract. Intrinsic Josephson-junction stacks realized in high-temperature superconductors may generate powerful electromagnetic radiation in terahertz frequency range. A major challenge is to synchronize phase oscillations in many junctions. A promising way of efficient synchronization is to excite an internal cavity mode, with the frequency set by the stack lateral size. We discuss several issues relevant for this mechanism: (i) damping of the resonance mode due to radiation into free space and into the base crystal, (ii) mechanisms of coupling to the internal mode, (iii) structures and stability of coherent states.

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1. Introduction

Large density of intrinsic Josephson junctions and large value of the gap in $Bi_2Sr_2CaCu_2O_8$ (BSCCO) and other related high-T_c superconducting compounds make them very attractive for developing coherent sources of electromagnetic radiation in the terahertz frequency range [1]. Powerful radiation can be achieved only if the oscillations of the superconducting phases are synchronized in a large number of junctions. A very promising way to efficient synchronization is to excite an internal cavity resonance in finite-size samples (mesas)[2, 3]. The frequency of such in-phase Fiske mode is set by the lateral size of the mesa. The experimental demonstration of this mechanism [2] significantly advanced the quest for superconducting terahertz sources.

In this Proceeding we summarize our current understanding of several important issues relevant for resonant generation of electromagnetic radiation using intrinsic Josephson junctions.

2. Phase dynamics in Josephson junction stacks

We consider a rectangular mesa fabricated on the top of a bulk single crystal, see Fig. 1a. The dynamics of stack of intrinsic Josephson junctions can be described by reduced coupled equations for the phase differences, φ_n, and reduced magnetic fields, h_n,

$$\frac{\partial^2\varphi_n}{\partial\tau^2} + \nu_c\frac{\partial\varphi_n}{\partial\tau} + g(x)\sin\varphi_n - \ell^2\frac{\partial h_n}{\partial x} = 0, \tag{1}$$

$$\left[\ell^2\nabla_n^2 - \left(1+\nu_{ab}\frac{\partial}{\partial\tau}\right)\right]h_n + \left(1+\nu_{ab}\frac{\partial}{\partial\tau}\right)\frac{\partial\varphi_n}{\partial x} = 0. \tag{2}$$

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Here units of length, time and magnetic field are the Josephson length λ_J, inverse plasma frequency $1/\omega_p$, and $\Phi_0/(2\pi s\lambda_J)$, where s is the interlayer period. The equations depend on three parameters, $\nu_c = 4\pi\sigma_c/(\varepsilon_c\omega_p) \approx 0.002 - 0.01$, $\nu_{ab} = 4\pi\sigma_{ab}/(\varepsilon_c\omega_p\gamma^2) \approx 0.1 - 0.2$, and $\ell = \lambda/s$, where σ_c and σ_{ab} are components of the quasiparticle conductivity, ε_c is the c-axis dielectric constant, λ is the in-plane London penetration depth, and γ is the anisotropy parameter. Function $g(x)$ describes possible modulation of the Josephson critical current to promote coupling to the resonance mode. We assume that the mesa contains N junctions and located at $|x| < L/2$. Regions $n \geq 1$ and $n \leq 0$ correspond to the mesa and crystal respectively. We consider mesa in the resistive state, $\varphi_n \approx \omega\tau + \alpha_n(x) + \mathrm{Re}[\theta_n(x)\exp(-i\omega\tau)]$ for $n \geq 1$ and $\varphi_n \approx \mathrm{Re}[\theta_n(x)\exp(-i\omega\tau)]$ for $n \leq 0$. Here $\omega = \omega_J/\omega_p$ is the reduced Josephson frequency proportional to dc voltage drop at one junction. We will focus on the situation when the lowest in-phase resonance mode is excited inside the mesa in the vicinity of the resonance frequency, $\omega_1 = \pi\ell/L$, corresponding to $\theta_n(x) \approx \psi\sin(\pi x/L)$ and $h_n = \bar{h}\cos(\pi x/L)$ with $\bar{h} = \pi\psi/L$.

3. Radiation properties

The resonance mode excited in the mesa looses its energy due to internal quasiparticle dissipation and due to electromagnetic radiation. Two radiation channels are important for the mesa on the top of bulk crystal: radiation into free space and radiation into the base crystal. The latter channel was not considered before. We will demonstrate, however, that it gives large contribution to the resonance damping. Distribution of the radiated electric fields for both channels is illustrated in Fig. 1b.

3.1. Radiation into free space

Distribution of the oscillating electric field for the fundamental mode is given by $E_z(\mathbf{r}) \approx E_{z,1}\sin(\pi x/L_x)$, where $E_{z,1}$ is related with the amplitude of oscillating phase by the Josephson relation, $E_{z,1} = [i\omega_J\Phi_0/(2\pi cs)]\psi$. For short mesa, $k_\omega L_z \ll 1$, radiation into free space is mostly determined by the oscillating electric field at the edges and can be found similar to the capacitor plates and patch antennas [4, 3]. Simple analytical result can be derived for a long mesa, $k_\omega L_y \gg 1$,

$$\mathcal{P}_{\mathrm{edge}} = A\frac{\omega_J L_y L_z^2|E_{z,1}|^2}{8\pi} = A\frac{\Phi_0^2\omega_J^3 L_y N^2}{32\pi^3 c^2}|\psi|^2. \tag{3}$$

Here A is the geometrical factor, for the mesa on the top of bulk crystal $A \approx 1 + J_0(k_\omega L)$, where $J_0(x)$ is the Bessel function. The Bessel-function term appears due to a constructive interference of waves coming from opposite sides of the crystal along the x-axis.

3.2. Radiation into base crystal

3.2.1. Boundary condition at the mesa/crystal boundary We assume that in the crystal plasma waves propagate only away from the mesa, $n \to -\infty$, i.e., the oscillating magnetic field at $n \leq 0$ can be represented as $h_n(x) = \int_{-\infty}^{\infty}\frac{dk}{2\pi}\exp(-iq_k n + ikx)h_k$ where $q_k(\omega)$ is the propagation wave vector along the c direction, $\cos q_k = 1 + \left(1 - k^2/k_0^2\right)/(2\ell_\omega^2)$, with $\mathrm{Im}[q_k] > 0$, $k_0 \equiv \sqrt{\omega^2 - 1 + i\nu_c\omega}/\ell$, and $\ell_\omega^2 \equiv \ell^2/(1 - i\nu_{ab}\omega)$. This means that the in-plane Fourier components of the fields in 0-th and 1-st junctions are connected by relation $h_{0,k} = \exp(iq_k)h_{1,k}$. We are interested in behavior near the lowest resonance when the coordinate dependence of the magnetic field inside the mesa is given by $h_n(x) \approx \bar{h}_n\cos(\pi x/L)$ for $|x| < L/2$ and outside the mesa (in free space) the magnetic field is much smaller. In this case we can derive the approximate condition for the mode amplitudes of the magnetic field at the border in between the mesa and crystal

$$\bar{h}_1 - \bar{h}_0 = \zeta\bar{h}_1 \text{ with } \zeta \approx (0.57 + 0.31i)/\ell_\omega. \tag{4}$$

with As $|\zeta| \ll 1$, the variation of magnetic field across the boundary is small, $|h_0 - h_1| \ll |h_0|$.

3.2.2. Power radiated into crystal For monochromatic oscillations, we evaluate the energy loss due to the leaking radiation into the crystal as

$$\mathcal{P}_{\rm bot} = P^{(0)}_{\rm bot}\frac{\ell^4\omega}{2}\int_{-L/2}^{L/2} dx {\rm Im}\left[\frac{h_0^*\left(h_1-h_0\right)}{1-i\nu_{ab}\omega}\right]$$

with $P^{(0)}_{\rm bot} = \omega_p\Phi_0^2 L_y/(16\pi^3\lambda_c\lambda)$ and $\lambda_c = \gamma\lambda$. When the coordinate dependence of $h_n(x)$ is mostly determined by the lowest mode, $h_n(x) \approx \bar{h}\cos(\pi x/L)$, and the mode amplitudes are related by the boundary condition (4), we obtain

$$\mathcal{P}_{\rm bot} \approx P^{(0)}_{\rm bot}\frac{\ell^4\omega L}{4}{\rm Im}\left[\frac{\zeta}{1-i\nu_{ab}\omega}\right]|\bar{h}|^2 = \frac{C_{ab}\Phi_0^2\lambda\omega_J}{64\pi s^2 L}|\psi|^2 \tag{5}$$

with $C_{ab} = {\rm Im}\left[\frac{0.57+0.31i}{\sqrt{1-i\nu_{ab}\omega}}\right]$. It is interesting to compare this result with radiation losses from the edges (3),

$$\mathcal{P}_{\rm bot}/\mathcal{P}_{\rm edge} = 2C_{ab}\varepsilon_c\lambda L/(AL_z^2). \tag{6}$$

For parameters of experiment [2], $\varepsilon_c = 12$, $\lambda = 0.3$ μm, $L = 60-100$ μm, $L_z = 1$ μm, $\nu_{ab}\omega \sim 1$, we estimate $\mathcal{P}_{\rm bot}/\mathcal{P}_{\rm edge} = 30-50$, i. e., leaking of radiation to crystal significantly exceeds the radiation into free space.

It is convenient to introduce the reduced parameters of radiation damping, $\nu_{\rm edge}$ and $\nu_{\rm bot}$, which determine the damping of the mode in the same way as the quasiparticle-dissipation damping ν_c, $\mathcal{P}_{\rm c} = \nu_{\rm c}L_xL_yL_z\left(\varepsilon_c\omega_p/16\pi\right)|E_{z,1}|^2$,

$$\nu_{\rm edge} = 2A\omega_J L_z/(\varepsilon_c\omega_p L), \quad \nu_{\rm bot} = \pi C_{ab}\lambda\lambda_c/(2L_zL). \tag{7}$$

In contrast to the c-axis dissipation, the radiation damping parameters are very sensitive to mesa geometry. For above parameters and $\lambda_c = 200$ μm, we estimate $\nu_{\rm bot} \approx 0.3-0.5$, which is much higher than both ν_c and $\nu_{\rm edge}$. This means that *the dissipation due to the radiation into crystal significantly exceeds both the dissipation due to the free-space radiation and the quasiparticle dissipation and dominates in the resonance damping.*

4. Dynamic states near resonance

4.1. Homogeneous state

The simplest state is the homogeneous phase oscillation identical in all junctions [3]. For such state the cavity mode can only be excited in presence of external asymmetric modulation of the Josephson coupling given by the modulation function $g(x)$. In this case the mode amplitude, ψ, is directly proportional to the strength of modulation which is determined by the coupling parameter, g_1,

$$\psi = \frac{ig_1}{\omega^2-\omega_1^2+i\nu\omega}, \; g_1 = \frac{2}{L}\int_0^L dx\cos(\pi x/L)g(x), \tag{8}$$

where ν is the total damping parameter, $\nu = \nu_c+\nu_{\rm edge}+\nu_{\rm bot}$. Unfortunately, our recent analysis shows that this simple state is typically unstable with respect to the alternating along the c axis deformations.

4.2. Alternating state

More interesting possibility has been found recently by numerical simulations [5]. It occurs that the resonance promotes the formation of an alternating coherent state, in which the system spontaneously splits into two subsystems with different phase-oscillation patterns. The analytical solution describing this state was also found.

In this state the phases have static coordinate-dependent contribution, which alternates from layer to layers, see Fig. 1c, $\varphi_n(x,\tau) \approx \omega\tau + (-1)^n\alpha(x) + \mathrm{Re}[\theta(x)\exp(-i\omega\tau)]$, where $\alpha(x)$ has a kink near the center where it rapidly varies from 0 to π, similar to the static soliton of the sine-Gordon equation. The width of this soliton shrinks with approaching the resonance. Due to this contribution, the average Josephson current density acquires a self-generated modulation given by $g(x) = \cos[\alpha(x)]$, which varies from 1 to -1 in the narrow region near the center. This leads to quite efficient excitation of the resonance cavity oscillations described by the average phase $\theta(x)$. The amplitude of the mode is again given by Eq. (8) with $g_1 \approx 4/\pi$. The oscillating electric and magnetic fields are almost homogeneous in all the junctions. The formation of this state promotes efficient pumping of the energy into the cavity resonance leading to strong resonance features in the current-voltage dependence.

5. Summary

In summary, we analyzed properties of the dynamic resonance state in mesocrystals of layered superconductors. For a mesa fabricated at the top of a bulk single crystal, the resonance is mostly damped due to the leaking of radiation into the crystal. The energy may be efficiently pumped into the resonance mode via formation of the inhomogeneous alternating state.

Acknowledgments

The authors would like to acknowledge useful discussions with U. Welp, K. Gray, M. Tachiki, X. Hu, and K. Kadowaki. This work was supported by the U.S. DOE, Office of Science, under contract # DE-AC02-06CH11357.

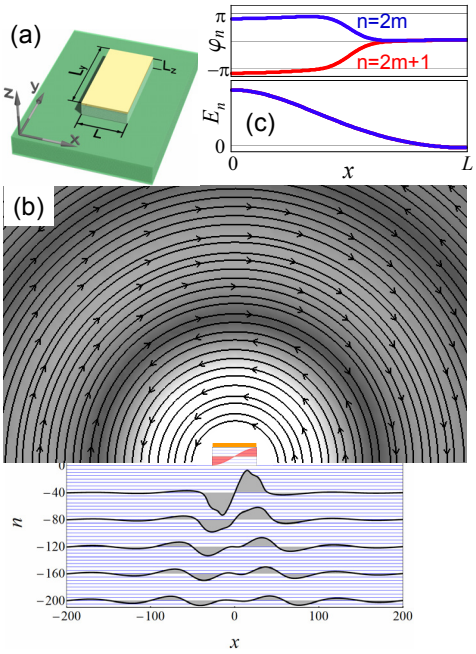

Figure 1. (a) Geometry of mesa fabricated on the top of bulk crystal. (b) Illustration of distribution of the radiated electric field in free space and in the base crystal. Both radiations are induced by the fundamental cavity mode sketched inside the mesa. The mesa has metallic contact on the top. (c) Snapshots of the phases in odd and even junctions and the electric field identical in all junctions for the alternating state. The key feature is the static coordinate-dependent phase shift between the phases in two subsystems, which generates coupling of the Josephson oscillations to the resonance mode.

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