

# Proximity fingerprint of $s_{\pm}$ -superconductivity

A. E. KOSHELEV and V. STANEV

*Materials Science Division, Argonne National Laboratory - Argonne, IL 60439, USA*

received 17 June 2011; accepted in final form 6 September 2011

published online 7 October 2011

PACS 74.45.+c – Proximity effects; Andreev reflection; SN and SNS junctions

PACS 74.70.Xa – Pnictides and chalcogenides

PACS 74.20.Rp – Pairing symmetries (other than  $s$ -wave)

**Abstract** – We suggest a straightforward and unambiguous test to identify possible opposite signs of the superconducting order parameter in different bands proposed for iron-based superconductors ( $s_{\pm}$ -state). We consider the proximity effect in a weakly coupled sandwich composed of a  $s_{\pm}$ -superconductor and a thin layer of the  $s$ -wave superconductor. In such system the  $s$ -wave order parameter is coupled differently with different  $s_{\pm}$ -gaps and it typically aligns with one of these gaps. This forces the other  $s_{\pm}$ -gap to be anti-aligned with the  $s$ -wave gap. In such situation the aligned band induces a peak in the  $s$ -wave density of states (DoS), while the anti-aligned band induces a dip. Observation of such contact-induced negative feature in the  $s$ -wave DoS would provide a definite proof for  $s_{\pm}$ -superconductivity.

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The report of superconductivity at  $T_c = 26$  K in fluorine-doped LaFeAsO [1] followed by the discovery of several new classes of iron-based superconductors [2–5] with transition temperatures up to 56 K generated enormous interest in the condensed-matter community, see reviews [6–8]. In spite of extensive research, the symmetry of the order parameter in these materials remains a prominent unresolved issue. The Fermi surface of the iron-based superconductors is composed of several electron and hole sheets. Theory strongly suggests that Cooper pairing in these materials has electronic origin and the superconducting order parameter has opposite signs in the electron and hole bands ( $s_{\pm}$ -state) [9–12]. The experimental evidence for this state, however, remains rather limited.

The ARPES measurements [13–15] indicate full gaps on both hole and electron bands, at least in some compounds, but they cannot probe the relative signs of the gaps. At present, the strongest support in favor of the sign-changing order parameters is coming from the observation of a resonance in the spin excitation spectrum developing below the superconducting transition temperature. This resonance has been observed by the inelastic neutron scattering in most iron-based superconducting compounds [16–21]. However, the straightforward interpretation of these experiments was questioned in ref. [22].

Several recent experiments provide substantial indirect support in favor of the  $s_{\pm}$ -state. In particular, the behavior of the quasiparticle interference with the magnetic field in the Fe(Se,Te) compound probed by STM is consistent with this state [23]. Also, the observation of the microscopic

coexistence of superconductivity and spin density wave in some iron pnictides is most naturally explained assuming opposite signs of the order parameters in the electron and hole bands [24].

The most convincing demonstration of the  $s_{\pm}$ -state could come from phase-sensitive experiments. These kinds of experiments [25,26] played a decisive role in convincing the superconductivity community that the order parameter in the cuprate superconductors has  $d$ -wave symmetry. Even though the theoretical proposals for similar experiments have been made for iron-based superconductors [27,28], they have not been realized yet. The only phase-sensitive experiment reported so far is the observation of half-integer flux-quantum jumps of the magnetic flux through the loop formed by niobium and polycrystalline iron-pnictide sample [29]. It is desirable, however, to design an experiment with better control and more predictable outcome.

In this letter we propose an alternative straightforward test for the relative sign of the order parameter in the electron and hole bands in the case when the absolute values of the gaps are different. We consider a sandwich composed of  $s_{\pm}$  and  $s$ -wave superconductors, see inset in fig. 1. Peculiar properties of  $s$ - $s_{\pm}$  Josephson junctions and point contacts were recognized and studied in several theoretical papers [30–36]. Nevertheless, to our knowledge, the proximity effect we describe here was never mentioned.

For the illustration of the effect we consider the simplest situation, *i.e.* when the thickness of the  $s$ -wave superconductor is small compared with the coherence lengths,

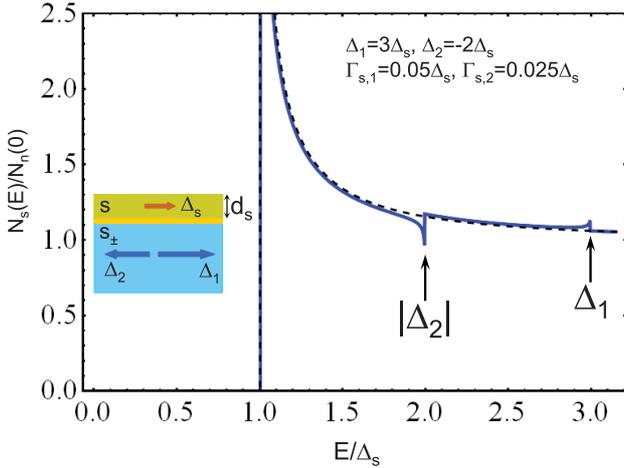


Fig. 1: (Colour on-line) Representative density of states for a thin layer of the  $s$ -wave superconductor on the top of the  $s_{\pm}$ -superconductor as illustrated in the inset. The dashed line shows the bulk  $s$ -wave DoS. The dip caused by the anti-aligned band provides a definite fingerprint of the  $s_{\pm}$ -superconductivity.

and the coupling between the superconductors is weak. We also assume a simple two-band model for the  $s_{\pm}$ -superconductor and a dirty limit for both materials. While these assumptions allow for a simple analytical treatment of the problem, none of them is actually essential for the proposed effect.

The first microscopic description of the proximity sandwich composed of two thin superconductors was elaborated by McMillan [37]. The modern treatment of this problem in the dirty limit is based on the Usadel equations [38,39] which were generalized for the multiband case in ref. [40]. The advantage of this approach with respect to more microscopic models is that it describes the properties of junctions via a minimum number of the most relevant and physically transparent parameters.

For the  $s$ -superconductor located within  $0 < x < d_s$  the Green's function  $\Phi_s$  obeys

$$\frac{D_s}{2\omega G_s} [G_s^2 \Phi_s']' - \Phi_s = -\Delta_s, \quad G_s = \frac{\omega}{\sqrt{\omega^2 + \Phi_s^2}}, \quad (1)$$

where  $D_s$  is the diffusivity and  $\omega = 2\pi(n + 1/2)T$  stands for the Matsubara frequencies. We will be interested only in the density of states of the  $s$ -wave material and will not need equations for the  $s_{\pm}$  Green's functions.

The boundary condition for the top boundary is simple,  $\Phi_s' = 0$  at  $x = d_s$ . The boundary conditions for the contact of two dirty superconductors have been derived in ref. [41] and generalized to multiband case in ref. [40],

$$\xi_s G_s^2 \Phi_s' = \sum_{\alpha} \frac{\xi_{\alpha}}{\gamma_{\alpha}} G_{\alpha}^2 \Phi_{\alpha}', \quad \text{with } \gamma_{\alpha} = \frac{\rho_{\alpha} \xi_{\alpha}}{\rho_s \xi_s}, \quad (2)$$

$$\gamma_{B\alpha} \xi_{\alpha} G_{\alpha} \Phi_{\alpha}' = G_s (\Phi_s - \Phi_{\alpha}), \quad \text{with } \gamma_{B\alpha} = \frac{R_B^{\alpha}}{\rho_s \xi_{\alpha}}. \quad (3)$$

Here  $\rho_s$  is the resistivity of the  $s$ -wave superconductor,  $\alpha = 1, 2$  is the band index,  $\Phi_{\alpha}$  and  $G_{\alpha} = \omega / \sqrt{\omega^2 + \Phi_{\alpha}^2}$  are the  $s_{\pm}$  Green's functions,  $\rho_{\alpha}$  stands for the partial resistivities for the bands of the  $s_{\pm}$ -superconductor<sup>1</sup>,  $R_B^{\alpha}$  stands for the partial resistances of the boundary which determine electrical coupling between the  $s$ -wave superconductor and  $s_{\pm}$  bands. In the case of weak coupling,  $\gamma_{B\alpha} \gg 1$ , we can use approximations  $\Phi_s \approx \Delta_s$  and  $\Phi_{\alpha} \approx \Delta_{\alpha}$  and obtain the approximate boundary conditions for the  $s$ -wave Green's function,

$$\xi_s G_s \Phi_s' = \sum_{\alpha} \frac{G_{\alpha}}{\tilde{\gamma}_{B\alpha}} (\Delta_s - \Delta_{\alpha}) \quad (4)$$

with  $\tilde{\gamma}_{B\alpha} = \gamma_{\alpha} \gamma_{B\alpha} = R_B^{\alpha} / \rho_s \xi_s$ . This condition together with eq. (1) allows us to obtain the correction to the  $s$ -wave Green's function imposed by the contact with  $s_{\pm}$ -superconductor. In general, the gap values  $\Delta_s$  and  $\Delta_{\alpha}$  have to be found self-consistently but in the case of weak coupling they can be well approximated by the bulk gaps, which we assume to be known.

In the case of the thin layer,  $d_s \ll \xi_s$ , we can expand the Green's functions,  $\Phi_s(x) \approx \bar{\Phi}_s + (a_s/2)(x - d_s)^2$ , where the parameters  $a_s$  and  $\bar{\Phi}_s$  can be related by eq. (1),

$$\frac{D_s}{2\omega} G_s a_s \approx \bar{\Phi}_s - \bar{\Delta}_s. \quad (5)$$

Matching at  $x = 0$  using eq. (4) gives

$$\xi_s G_s a_s d_s = - \sum_{\alpha} \frac{G_{\alpha}}{\tilde{\gamma}_{B\alpha}} (\Delta_s - \Delta_{\alpha}). \quad (6)$$

Solving the last two equations, we obtain

$$\bar{\Phi}_s - \bar{\Delta}_s \approx - \sum_{\alpha} \frac{\Gamma_{s,\alpha} (\Delta_s - \Delta_{\alpha})}{\sqrt{\omega^2 + \Delta_{\alpha}^2}}, \quad (7)$$

where, following ref. [37], we introduced the coupling parameters,

$$\Gamma_{s,\alpha} \equiv \frac{D_s}{2\xi_s d_s \tilde{\gamma}_{B\alpha}} = \frac{\rho_s D_s}{2d_s R_B^{\alpha}} = \frac{1}{2e^2 \nu R_B^{\alpha} d_s}, \quad (8)$$

which have dimensionality of energy. This correction is similar to the McMillan result [37] for a single-band  $s$ -wave superconductors in the linear with respect to  $\Gamma_{s,\alpha}$  order.

The density of states of the  $s$ -wave superconductor is given by

$$N_s(E) = \text{Re} \left[ \frac{E}{\sqrt{E^2 - \Phi_s^2}} \right]. \quad (9)$$

Performing the analytic continuation of eq. (7),  $i\omega \rightarrow E - i\delta$ ,

$$\bar{\Phi}_s \approx \bar{\Delta}_s + \sum_{\alpha} \frac{\Gamma_{s,\alpha} (\Delta_{\alpha} - \Delta_s)}{\sqrt{\Delta_{\alpha}^2 - E^2}}, \quad (10)$$

<sup>1</sup>Note that the resistivity  $\rho$  is related to the corresponding diffusion coefficient  $D$  by  $1/\rho = e^2 \nu D$ , where  $\nu$  is the normal DoS.

and expanding, we finally obtain

$$N_s(E) = \text{Re} \left[ \frac{E}{\sqrt{E^2 - \Delta_s^2}} + \frac{E\Delta_s}{(E^2 - \Delta_s^2)^{3/2}} \sum_{\alpha} \frac{\Gamma_{s,\alpha}(\Delta_{\alpha} - \Delta_s)}{\sqrt{\Delta_{\alpha}^2 - E^2}} \right]. \quad (11)$$

This result is valid for any gap parameters.

To proceed further we have to make assumptions about the gap magnitudes and their signs. We assume that  $|\Delta_1| > |\Delta_2| > |\Delta_s|$ ,  $\Delta_1 > 0$ ,  $\Delta_2 = -|\Delta_2| < 0$ . The sign of  $\Delta_s$  marks the alignment with one of the  $s_{\pm}$  bands and it is determined by the relation between the corresponding Josephson couplings. The partial Josephson coupling energy between the  $s$ -wave superconductor and the  $\alpha$  band,  $E_{J,\alpha}$ , is proportional to the corresponding Josephson current density,  $j_{J,\alpha}$ , which is given by the same formula as for a tunneling contact between two different superconductors, see, *e.g.*, ref. [42],

$$E_{J,\alpha} \propto j_{J,\alpha} = \frac{2\hbar}{eR_B^{\alpha}} \frac{|\Delta_{\alpha}||\Delta_s|}{|\Delta_{\alpha}| + |\Delta_s|} K \left( \frac{|\Delta_{\alpha}| - |\Delta_s|}{|\Delta_{\alpha}| + |\Delta_s|} \right) \quad (12)$$

for  $T \ll T_c^s$ . Here  $K(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 x)^{-1/2} dx$  is the complete elliptic integral of the first kind. In the case of strong inequalities  $|\Delta_s| \ll |\Delta_{\alpha}|$  we have  $E_{J,\alpha} \propto (\Delta_s/R_B^{\alpha}) \times \ln(4|\Delta_{\alpha}|/|\Delta_s|)$  meaning that the ratio of the coupling energies is mostly determined by the ratio of the partial resistivities and only weakly depends on the gap magnitudes  $|\Delta_{\alpha}|$ .

For definiteness, we assume that  $E_{J,1} > E_{J,2}$  and  $\Delta_s > 0$  is aligned with the larger gap  $\Delta_1$ , as is illustrated in the inset of fig. 1. In this case we can rewrite the proximity correction to the DoS as

$$\delta N_s(E) = \frac{E\Delta_s}{(E^2 - \Delta_s^2)^{3/2}} \left[ \frac{\Gamma_{s,1}(\Delta_1 - \Delta_s)}{\sqrt{\Delta_1^2 - E^2}} \Theta(\Delta_1 - E) - \frac{\Gamma_{s,2}(|\Delta_2| + \Delta_s)}{\sqrt{\Delta_2^2 - E^2}} \Theta(|\Delta_2| - E) \right], \quad (13)$$

where  $\Theta(x)$  is the step function. We immediately see that the *aligned* band induces a *positive* correction and the *anti-aligned* band induces a *negative* correction. While the positive correction is a standard feature of the proximity between two superconductors [37], the negative anomaly is unique to  $s/s_{\pm}$  proximity. The amplitude of the peak is proportional to the gap difference  $\Delta_1 - \Delta_s$ , while the amplitude of the dip is proportional to the gap sum  $|\Delta_2| + \Delta_s$ . An example of the  $s$ -wave DoS for representative parameters is shown in fig. 1. We can also see that the  $s_{\pm}$ -superconductor can both enhance and suppress the  $s$ -wave DoS at energies  $E \sim \Delta_s$ ; the sign of the total correction in this energy range is determined by the sign of the combination  $\Gamma_{s,1} \sqrt{\frac{\Delta_1 - \Delta_s}{\Delta_1 + \Delta_s}} - \Gamma_{s,2} \sqrt{\frac{|\Delta_2| + \Delta_s}{|\Delta_2| - \Delta_s}}$ . The

simple analytical result (13) is obtained in the linear order with respect to the coupling parameters  $\Gamma_{s,\alpha}$  and does not describe the energy region close to the gap values  $|E - \Delta_{\alpha}| \sim \Gamma_{s,\alpha}^2/\Delta_{\alpha}$ . In particular, the vanishing of the correction at energies larger than the corresponding gap leading to a very asymmetric shape of the correction is not an exact result but just a consequence of this linear approximation.

The  $s$ -wave DoS can be experimentally accessed in a standard way by measuring the tunneling conductance from the top surface of the sandwich using scanning tunneling microscopy, point contacts, or making a planar tunnel junction. The proposed test only works if there is a noticeable difference between the absolute values of the  $s_{\pm}$ -gaps so that their features are sufficiently separated in energy (voltage). A good possible choice for the  $s$ -wave material may be amorphous thin films, such as  $\text{Mo}_x\text{Ge}_{1-x}$ , because, due to completely incoherent tunneling, these materials are expected to have comparable coupling with all bands of the  $s_{\pm}$ -superconductor. The film thickness has to be smaller or at least comparable with the coherence length. In a real experiment the anomalies induced by the  $s_{\pm}$ -gaps are expected to be less sharp than in the illustrated ideal situation. They will be smeared, *e.g.*, by temperature, finite transparency of the interface, and pair-breaking scattering. The optimum coupling strength between the superconductors for the observation of the effect has to be in the intermediate range: it should not be too weak so that the DoS corrections do not vanish in the noise but it should be also not too strong so that the gaps at the surface are close to the bulk values. This corresponds to the coupling parameters in the range  $\Gamma_{s,\alpha}/\Delta_s = 0.01-0.1$ .

In summary, we proposed a straightforward test for the  $s_{\pm}$  superconducting state using the proximity-induced correction to the density of state of a conventional superconductor. The coupling between  $s$ - and  $s_{\pm}$ -superconductors typically aligns the  $s$ -wave gap with one of the  $s_{\pm}$ -gaps. In this case the anti-aligned gap induces a *negative* correction to the  $s$ -wave DoS which which can serve as a definite fingerprint for the  $s_{\pm}$ -state. We analytically evaluated the DoS corrections in the linear order with respect to the coupling strength.

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The authors would like to thank TH. PROSLIER for useful discussions. This work was supported by UChicago Argonne, LLC, operator of Argonne National Laboratory, a U.S. Department of Energy Office of Science laboratory, operated under contract No. DE-AC02-06CH11357, and by the ‘‘Center for Emergent Superconductivity’’, an Energy Frontier Research Center funded by the U.S. Department of Energy, Office of Science, Office of Basic Energy Sciences under Award No. DE-AC0298CH1088.

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