Charge transfer between a superconductor and a hopping insulator

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A theory of the low-temperature charge transfer between a superconductor and a hopping insulator is analyzed, and the corresponding interface resistance is calculated. This resistance is dominated by proposed electron-hole processes similar to Andreev reflection, but involving localized states in the insulator. The possibility of a new type of qubit where one of the quantum states is split between two spatially separated centers is discussed.

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The transmission of the charge through the normal metal-superconductor interface occurs via the electron-hole conversion known as the Andreev reflection process: an electron incident from the metal side with an energy smaller than the energy gap in the superconductor is converted into a hole which moves backward with respect to the electron. The missing charge $2e$ (an electron has charge $-e$ and a hole $+e$) propagates as an electron pair into the superconductor and joins the Cooper pair condensate [1]. Correspondingly, a Cooper pair transfer from the superconductor is described as the Andreev reflection of a hole. This Andreev transport channel is characterized by the so-called Andreev interface contact resistance. Since transport current is introduced into a superconductor via normal leads, the Andreev reflection phenomenon is a foundation for most applications of superconductors (see Ref. [2] for a review).

There exists however an important experimental situation of the hopping insulator coupled to a measuring circuit via superconducting leads (see, for example, [3]). The transport in hopping semiconductors occurs via localized *non-propagating* single particle states [4]. A single particle transport through the interface though is exponentially suppressed, $\propto e^{-\Delta/T}$, where Δ is the superconductor gap, the temperature, T, being measured in energy units. At the same time the hopping transport occurs via single-electron hops. Thus one needs Andreev-type processes capable to facilitate two particle transport through the hopping insulator/superconductor interface. However, the conventional Andreev reflection picture does not literally apply. The transport through such an interface has been discussed in Ref. [5] but no quantitative theory of hopping transport - supercurrent conversion was presented.

In this Letter we propose a theory for the transport through the hopping insulator-superconductor interface and calculate the corresponding contact resistance. We show that the low-temperature charge transfer occurs via the correlated processes mediated by the *pairs* of hopping centers located near the interface. We demonstrate that while this process resembles the conventional Andreev electron-to-hole reflection into a normal metal, the exponential suppression of transport specific to a single-particle processes is lifted. Thus, despite the limitation in the number of coherent hopping centers that are suitable for Andreev transport, the resulting contact resistance can become low as compared to the resistance of the hopping insulator.

Let a superconductor (S) and a hopping insulator (HI) to occupy the adjacent 3D semi-spaces separated by a tunneling barrier (B). The presence of the barrier simplifies calculations which will be made in the lowest non-vanishing approximation in the tunneling amplitude T_0. This models the Schottky barrier usually presenting at a semiconductor-metal interface. In the linear response theory the conductance is determined by the Kubo formula [7] for the susceptibility,

$$\chi(\omega) = i \int_{-\infty}^{t} \left\langle \left[\hat{I}^{+}(t'), \hat{I}(t) \right] \right\rangle e^{i\omega t'}\, dt' \qquad (1)$$

as $\mathcal{G} = \lim_{\omega\to 0} \omega^{-1} \operatorname{Im} \chi(\omega)$. Here the current operator $\hat{I}(t)$ is defined as [6]:

$$\hat{I}(t) = ied\,T_0 \int d^2r\, [a^{+}(\mathbf{r},t) b(\mathbf{r},t) - \text{h.c.}]\,,$$

where $\mathbf{r}$ is the coordinate in the interface plane, $a^{+}(\mathbf{r},t)$ and $b(\mathbf{r},t)$ are creation and annihilation operators in the semiconductor and superconductor, respectively, d is the electron localization length under barrier. The susceptibility, $\chi(\omega)$, is calculated by analytical continuation of the Matsubara susceptibility [8],

$$\chi_M(\Omega) = \int_0^{\beta} \langle T_\tau I(\tau) I(0)\rangle\, e^{i\Omega\tau}\, d\tau\,. \qquad (2)$$

Here T_τ means ordering in the imaginary time, $\beta \equiv 1/T$. In the expression for $\langle T_\tau I(\tau) I(0)\rangle$ one should keep terms up to the second order with respect to the tunneling Hamiltonian,

$$H_T(\tau) = dT_0 \int d^2r\, \left[a^{+}(\mathbf{r},\tau) b(\mathbf{r},\tau) + \text{h.c.}\right]\,. \qquad (3)$$

Keeping only those second order terms that contain $\langle T_\tau b(\mathbf{r},\tau)b(\mathbf{r}_0,0)\rangle\,\langle T_\tau b^+(\mathbf{r}_1,\tau_1)b^+(\mathbf{r}_2,\tau_2)\rangle$ products and thus represent the Andreev-type processes, one arrives at the expression

$$\left\langle T_\tau \hat{I}(\tau)\hat{I}(0)\right\rangle = e^2|T_0|^4\int d\tau_1\,d\tau_2\prod_i d^2r_i(A+B)\,;$$
$$A(\{x_i\}) = F(x-x_0)F^+(x_1-x_2)G(x_1,x)G(x_2,x_0)\,,$$
$$B(\{x_i\}) = F(x-x_1)F^+(x_0-x_2)[G(x_0,x)G(x_2,x_1) -G(x_0,x_1)G(x_2,x)]\,. \quad (4)$$

Here $x\equiv\{\mathbf{r},\tau\}$, $x_0\equiv\{\mathbf{r}_0,0\}$, $x_i\equiv\{\mathbf{r}_i,\tau_i\}$; $F(x-x')=\langle T_\tau b(\mathbf{r},\tau)b(\mathbf{r}',\tau')\rangle$ is the anomalous Green function in the superconductor while $G(x,x')=-\langle T_\tau a(\mathbf{r},\tau)a^+(\mathbf{r}',\tau')\rangle$ is the Green function in the hopping insulator. One can show that the Andreev-type process we are interested in is given by the first term of $B(\{x_i\})$ in Eq. (4). The relevant diagram is shown in Fig. 1. Keeping only this

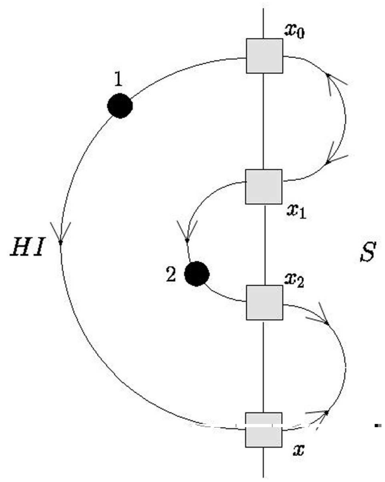

FIG. 1: The diagram describing processes of Andreev type. Lines with one arrow correspond to the Green's functions in the hopping insulator. They are associated either with center 1, or with center 2. Lines with two arrows correspond to anomalous Green's functions, see [8]. Squares correspond to matrix elements of the tunneling Hamiltonian (3).

term and using the Matsubara frequency representation one obtains

$$\chi_M(\Omega) = 2Te^2|T_0|^4d^4\int\prod_i d^2r_i\sum_{\omega_n}F(\mathbf{r}-\mathbf{r}_1,\omega_n)$$
$$\times F^+(\mathbf{r}_0-\mathbf{r}_2,\omega_n)G(\mathbf{r}_0,\mathbf{r},\omega_n-\Omega_m)G(\mathbf{r}_2,\mathbf{r}_1,\omega_n).\,(5)$$

Here $\Omega_m=2\pi mT$, $\omega_n=(2n+1)\pi T$. The normal Green's functions can be expressed through the wave functions of the localized states, $\varphi_s(\mathbf{r})=(\pi a^3)^{-1/2}\exp(-|\mathbf{r}-\mathbf{r}_s|/a)$, as

$$G(\mathbf{r},\mathbf{r}',\omega_n)=\sum_s\frac{\varphi_s^*(\mathbf{r})\varphi_s(\mathbf{r}')}{i\omega_n-\varepsilon_s}\,. \quad (6)$$

Here we have assumed that for all of the sites under consideration the voltage drops between the site and the superconductor are the same. This is true when the partial interface resistance due to an electron-hole pair is much larger than the typical resistance of the bond forming the percolation cluster. This situation resembles that considered by Larkin and Shklovskii for the tunnel resistance between the hopping conductors [9].

The anomalous Green function, $F(R,\omega_n)$, is

$$F(R,\omega_n) = \frac{\Delta}{(2\pi\hbar)^3}\int\frac{d^3p}{(2\pi\hbar)^3}\frac{\Delta}{\Delta^2+\xi_{\mathbf{p}}^2+\omega_n^2}\,e^{i\mathbf{p}\cdot\mathbf{R}/\hbar}$$
$$= \frac{\pi g_m\Delta}{2\sqrt{\Delta^2+\omega_n^2}}\frac{\sin(Rk_F)}{Rk_F}e^{-\frac{R}{\pi\xi}\frac{\sqrt{\Delta^2+\omega_n^2}}{\Delta}}\,. \quad (7)$$

Here $\xi_{\mathbf{p}}=(p^2-p_F^2)/2m$, $g_m=mp_F/\pi^2\hbar^3$ is the density of states in a metal, $k_F=p_F/\hbar$, while ξ is the coherence length in a superconductor. Since $F(R)$ oscillates with the period $2\pi/k_F$ integration over spatial coordinates along the interface yields the factor $a^4/k_F^6|\boldsymbol{\rho}_{ls}|^2$. Here $\boldsymbol{\rho}_{ls}$ is projection of the vector $\mathbf{R}_{ls}$ connecting the centers on the interface plane.

The summation over the Matsubara frequencies, ω_n, is standard,

$$T\sum_{\omega_n}f(\omega_n)=\oint\frac{d\varepsilon}{4\pi i}\,f(\varepsilon)\tanh\frac{\varepsilon}{2T}\,.$$

The contour of integration rounds the cuts $|\varepsilon|>\Delta$ along the real axis. After the proper analytical continuation one ends up with the following expression for the conductance:

$$\mathcal{G}=\frac{\pi}{2}\frac{e^2}{\hbar}\frac{g_m^2|T_0|^4d^4}{Tk_F^6a^2}\sum_{s\neq l}\frac{n(\varepsilon_s)n(\varepsilon_l)}{|\boldsymbol{\rho}_{ls}|^2}\left(\frac{\Delta^2}{\Delta^2-\varepsilon_s^2}\right)$$
$$\times e^{-2(r_{s\perp}+r_{l\perp})/a}\,e^{-2|\boldsymbol{\rho}_{ls}|\sqrt{\Delta^2-\varepsilon_s^2}/\pi\xi\Delta}\delta\left(\varepsilon_s+\varepsilon_l+U_c\right)\,.$$

Here $n(\varepsilon)\equiv\left(e^{\varepsilon/T}+1\right)^{-1}$ is the Fermi function, while U_c is the energy of the inter-site Coulomb repulsion.

In the following we replace $\sum_{l,s}$ by $g^2\int d^3r_l\,d^3r_s\,d\varepsilon_l\,d\varepsilon_s$ where g is the the effective density of states in the hopping insulator. This is the density of states in the layer adjacent to the interface. Due to screening by the superconductor it is not affected by the Coulomb gap and can be considered as constant. Since we are dealing with the pairs close to the interface the Coulomb repulsion is suppressed by screening. This screening can be conveniently regarded as an interaction of the charged particle with its image having the opposite charge. Thus the Coulomb correlation manifest themselves as the dipole-dipole interaction and for $\rho_{sl}\gg a$ one arrives at $U_c=e^2a^2/\kappa\rho_{sl}^3$. Requiring it to me smaller than T one obtains a cut-off $\rho_{sl}\equiv\rho_{sc}>a\zeta^{2/3}=r_h/\zeta^{1/3}$. Here $\zeta\gg1$ is the critical hopping exponent [10], while $r_h=a\zeta$ is the typical hopping distance in the bulk.

In what follows we restrict ourselves to the low-temperature case, $T \ll \Delta$, because otherwise the effect would be dominated by a quasiparticle transfer. In this case the combination $(2T)^{-1}\cosh^{-2}(\varepsilon_s/2T)$ can be replaced by $\delta(\varepsilon_s)$.

As a crude estimate, we will assume that $d^4 \sim k_F^{-4}$ while $T_0 \approx T_p e^{-\Lambda}$ with $T_p \approx \varepsilon_F$. Bearing this in mind one has $g_m^2 T_p^2/k_F^6 \sim g_m^2 \varepsilon_F^2/k_F^6 \sim 1$. As a result,

$$\mathcal{G} \sim \frac{e^2}{\hbar}\left[gaS\frac{T_p}{(ak_F)^2}\right]\left[ga\rho_{sc}^2\frac{T_p}{(ak_F)^2}\right]e^{-4\Lambda}. \quad (8)$$

Here S is the contact area.

One can compare the estimate (8) with the conductance of a boundary between a normal metal and a hopping insulator,

$$G_n \sim \frac{e^2}{\hbar}(gaS)\frac{T_p}{(ak_F)^2}\,e^{-2\Lambda}.$$

Taking into account that $T_p/(ak_F)^2 \sim \varepsilon_s$ where ε_s is the typical energy of a localized state we interpret the product $gaST_p/(ak_F)^2$ as the number of localized centers within the layer of a thickness a near the boundary interface. The ratio G/G_n is

$$\frac{\mathcal{G}}{\mathcal{G}_n} = \left[ga\rho_{sc}^2\frac{T_p}{(ak_F)^2}\right]e^{-2\Lambda} \approx ga\rho_{sc}^2\varepsilon_s e^{-2\Lambda}. \quad (9)$$

The first factor is just the probability to find a close pair of localized centers which dominate the Andreev processes discussed above.

The above approach holds, as we have already mentioned, only when the resistance of the typical Andreev-type resistor is much larger then the critical hopping resistor, $R_h = (h/e^2\gamma)\,e^{\zeta}$. Here γ is a dimensionless factor depending on the mechanism of electron-phonon interaction. This inequality holds with the exponential accuracy as long as $4\Lambda > \zeta$.

There are many realistic situations where the barrier strength, Λ, is not too large. In particular, that may be the case of the Schottky barrier at the natural interface [5]. Consequently, if $\zeta \gg 1$, i. e. for a situation remote from a metal-to-insulator transition, the procedure of summation over the localized states should be modified. Namely, the choice of the pairs facilitating the charge transport depends on structure of the bonds connecting these pairs to the percolation cluster.

Let us note that according to the above considerations the voltage applied to the hole is concentrated on the bonds connecting the superconductor to the percolation cluster in the HI. Correspondingly, these bonds end at the sites forming the Andreev-type resistor.

To find the typical Andreev-type resistor let us specify the thickness z of the layer adjacent to the interface containing the electron-hole pairs with the typical spatial separation $r_{12} > z$ between the two sites forming the pair. Correspondingly, r_{12} describes also the typical distance between the "ending" sites of the bonds belonging to the percolation cluster. According to the percolation theory [10], if one specifies the exponents for the critical resistance within the bonds as $\zeta + \delta\zeta$ then the typical distance between the "ending" sites of the bonds in question is $\mathcal{L} \sim a\zeta(\zeta/\delta\zeta)$. Equating this length to r_{12} we relate r_{12} to $\delta\zeta$. The relation between z and r_{12} can be found by optimizing the total resistance of the circuit including the Andreev resistor and the bond connected in series. According to the estimates given above the conductance of the Andreev-type resistor is

$$\delta G_{12} \geq \frac{e^2}{\hbar}\left(\frac{a}{r_{12}}\right)^2 \exp\left(-4\Lambda - \frac{4z}{a}\right). \quad (10)$$

Hence, the condition for the involved resistance to be smaller then or at least equal to the resistance of the bonds reads as

$$\left(\frac{a}{r_{12}}\right)^2 \exp\left(-4\Lambda - 4\frac{z}{a}\right) \leq \gamma \exp\left(-\zeta - \zeta^2\frac{a}{r_{12}}\right). \quad (11)$$

The sum of the resistances represented by the left hand and right hand sides of Eq. (11) should be minimized with respect to parameters z and r_{12}. Thus one estimates $z \approx a\zeta/4$. With the logarithmic accuracy one can neglect both Λ and $\ln\gamma$ and obtain the ratio $\varkappa = r_{12}/a$ from the equation

$$\varkappa \ln \varkappa = \zeta^2. \quad (12)$$

As it follows from the above considerations, $\delta\zeta/\zeta \approx \ln\varkappa/\zeta \ll 1$.

Denoting the resistance for the i-th pair, which can be estimated from Eq. (10), as $R_{A,i}$ and the bond resistance as $R_{c,i}$ one can define the Andreev boundary resistance as, see [5],

$$\begin{aligned} R_A \;&\equiv\; \left(\sum_i \frac{1}{(R_{T,i}+R_{c,i})}\right)^{-1} - \left(\sum_i \frac{1}{R_{c,i}}\right)^{-1} \\ &= \frac{\displaystyle\sum_i \frac{R_{A,i}}{R_{c,i}(R_{A,i}+R_{c,i})}}{\displaystyle\sum_i \frac{1}{R_{c,i}} \sum_i \frac{1}{R_{c,i}+R_{A,i}}}. \end{aligned} \quad (13)$$

The physical meaning of R_A is the difference between the values of the boundary resistance in the superconducting and normal states of the lead. Since $R_{A,i}$ are exponentially scattered only those with $R_{A,i} \approx R_{c,i}$ effectively contribute. The relative part of such Andreev-type resistors is just ζ^{-1}. Taking into account that the total number of the bonds is $\sim S/\mathcal{L}^2$, while the typical bond conductance, R_c, is of the order of the r.h.s. of Eq. (11) the resulting Andreev resistance can be estimated as

$$R_A \sim R_{\rm cr}\frac{a^2}{S}\frac{\varkappa^4}{\zeta}. \quad (14)$$

Here $R_{\rm cr} = (h/e^2\gamma)e^{\zeta}$ is the critical hopping resistance while $\varkappa$ is given by Eq. (12). The resistance estimated above can be experimentally measured as a magnetoresistance in magnetic fields higher than the critical field for superconductivity (similar effect for quasiparticle channel was studied in [5]).

Note that, in principle, the transport involving double occupied localized states is possible. However, such a transport would require an additional activation exponential factor, $\propto e^{-U/T}$, where U is the on-site correlation energy. One can also consider processes where a double occupied center (so-called D^--center) serves as an intermediate state for the phonon-assisted two-electron tunneling. This channel is unfavorable (at least in the case of a large interface barrier) because of the abovementioned exponential factor and a small preexponential factor due to phonon-assisted tunneling. For the weak tunnel barriers the conductance is controlled by "typical" hopping sites. In this case D^- channel is suppressed either by the additional tunneling exponential, $\propto e^{-4r_h/a}$, or by a small probability to form a close triple of hopping sites. Thus we believe that in this case the D^- channel can be also neglected.

It is worth mentioning that here we operate with a *coherent* superposition of two quantum states: (i) two empty centers and N Cooper pairs in a superconductor, and (ii) two center occupied by electrons with opposite spins and $N-1$ Cooper pairs in a superconductor. These two states, in principle, can act as a qubit in which both the longitudinal and transverse splitting can be controlled by properly designed gates. An interesting feature of such structure is that one of the quantum states involves spatially separated centers. Thus knowing the state of one center we automatically obtain information about the state of the second center. The qubit can be manipulated by two gate electrodes each of which being attached to one dot.

To conclude, we have analyzed low-temperature charge transfer between a superconductor and a hopping insulator and calculated the interface resistance. This resistance is dominated by Andreev-type processes involving localized states in the insulator. We emphasize that only these processes allow low-temperature measurements of hopping transport using superconducting electrodes. Even in the case when the interface contribution is less than the typical resistance of the hopping system the former can be separated by a relatively weak magnetic field which drives the superconductor to the normal state, but doe not affect the hopping transport.

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