

Current-induced magnetization changes in a spin
valve
due to incoherent emission of non-equilibrium
magnons

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Abstract

We describe spin transfer in a ferromagnet/normal metal/ferromagnet spin-valve point contact. Spin is transferred from the spin-polarized device current to the magnetization of the free layer by the mechanism of incoherent magnon emission by electrons. Our approach is based on the rate equation for the magnon occupation, using Fermi's golden rule for magnon emission and absorption and the non-equilibrium electron distribution for a biased spin valve. The magnon emission reduces the magnetization of the free layer. For anti-parallel alignment of the magnetizations of the layers and at a critical bias a magnon avalanche occurs, characterized by a diverging effective magnon temperature. This critical behavior can result in magnetization reversal and consequently to suppression of magnon emission. However the magnon-magnon scattering can lead to saturation of magnon concentration at large but finite value. The further behavior depends on the parameters of the system. In particular, gradual evolution of magnon concentration followed by the magnetization reversal is possible. Another scenario corresponds to step-like increase of magnon concentration followed by slow decrease. In the latter case the spike in differential resistance is expected due to contribution of electron-magnon scattering. A comparison of the obtained results to existing experimental data and theoretical approaches is given.

1 Introduction

The giant magnetoresistance[1] (GMR) of magnetic multilayers is a strong drop of the electric resistance of such a multilayer on application of an external magnetic field. This effect arises from the different scattering strength experienced by the two spin channels making up the spin-polarized sense current through the multilayer and from a change from antiparallel alignment of the magnetization of the ferromagnetic layers to parallel alignment. For antiparallel alignment appreciable scattering of electrons occurs in both spin channels, for each spin type in those layers where that type is a minority-spin[2]. For parallel alignment the majority-spin channel experiences negligible scattering, so that it is a low resistance channel that short-circuits the high resistance minority-spin channel.

The dual effect of the GMR is a change of the magnetization state of a magnetic layered structure induced by a spin-polarized bias current traversing the layers. The mechanism of such a change is transfer of electron spin from the current to the magnetization of the layers, as proposed by Slonczewski for a ferromagnet/normal metal/ferromagnet (FM/NM/FM) spin valve[3]. This is a trilayer structure, with a NM layer sandwiched between two FM layers. The spin transfer is predicted[3] to lead to steady, coherent precession of the magnetization of the layers and, in the presence of a uniaxial anisotropy, to switching of the magnetization. These effects should occur for high current densities ($j = 10^6 - 10^7$ A/cm²) and small lateral dimensions (diameter=100-1000 nm). Recently, Katine *et al.*[4] have reported magnetization switching in a spin valve operating in this regime, which they interpret in the framework of Slonczewski's theory. However while at small external magnetic fields the behavior was hysteretic just in accordance to Slonczewskii theory at higher magnetic fields the deviations from Slonczewskii predictions were observed. Some later, the extended studies of current-driven magnetic evolution were made by Urazdin *et al.*[5]; the authors reported the hysteretic switching at low external magnetic fields and the reversible behavior in high fields. It is also important to note that while Slonczewskis theory predict switching-like evolution of magnetization, our studies of a biased nanometer-scale Co/Cu/Co spin-valve point contact[6] demonstrated a possibility of gradual evolution; similar features were observed by Katine *et al.*[4]

The approach of Ref. [3] is semi-classical: the electron spin is treated quantum mechanically, but spin transfer is derived from the classical law of angular momentum conservation. Further, in Ref. [3] it is ignored that in the spin-transfer regime the electron system is strongly out of equilibrium. Actually, for the electron system the spin-transfer regime in a spin valve is very similar to the regime of point-contact spectroscopy (PCS) of the electron-magnon interaction in ferromagnetic metals. PCS of this interaction was theoretically developed by Kulik and Shekhter [10] for homogeneous ferromagnetic point contacts, which are short and narrow constrictions between a pair of three-dimensional ferromagnetic electrodes. The idea is that in a biased point contact a bias-dependent non-equilibrium electron distribution is created. This distribution enables energy relaxation of the electrons by incoherent emission of magnons,

the elementary excitations of magnetization. Magnon excitation can be detected in the second derivative of the current-voltage characteristic of the contact[11], which is proportional to the magnon density of states. In the same spirit, a non-equilibrium electron distribution generated in a current-biased spin valve should lead to incoherent emission of magnons in the magnetic layers. To some extent Berger [12] discusses the effect of a non-equilibrium distribution, but his intuitive approach lacks a general quantum mechanical basis.

Further theoretical studies of the non-equilibrium dynamics of magnetic multilayers were presented in Refs.[13] where in particular the steps to generalize approaches by Slonczewskii and Berger were made. However these studies were also based on semiclassical considerations similar to the ones exploited in [3].

In this article we present a consistent quantum mechanical description of spin transfer in a spin-valve point contact, based on Fermi's golden rule and taking into account the non-equilibrium electron distribution. Inspired by the PCS results[10],[11] we consider emission and absorption of magnons. However, contrary to PCS, which is concerned with the direct effect of electron-magnon processes on the electrical transport, we here focus on the effect of these processes on the magnetization, which can be strongly reduced by a non-equilibrium population of emitted magnons. A change of the magnetization due to electron-magnon processes can be probed with the GMR of the contact, as we recently demonstrated for a biased nanometer-scale Co/Cu/Co spin-valve point contact[6]. We will show that different scenarios of the magnetization evolution are possible including switching, gradual evolution of magnetization, hysteretic and reversible behavior.

2 Incoherent emission of magnons

We consider the spin-valve point contact. This device comprises two metallic electrodes, which are electrically connected through a small circular orifice (diameter 10-50 nm) in a thin insulating layer. Adjacent to the insulator, embedded in the left electrode, there is a spin valve with structure FM($t_a \approx 2$ nm)/NM($t_{sp,1} \approx t_a$)/FM($t_p \approx 100$ nm), the layer thicknesses being indicated in brackets. The thick FM layer is the spin-polarizer. Its fixed saturation magnetization $\mathbf{M}_{s,p}$ points in the positive z -direction. The thin FM layer is the spin-analyzer with saturation magnetization $\mathbf{M}_{s,a}$. The direction of $\mathbf{M}_{s,a}$ is in the plane of the layer. It can be parallel or anti-parallel to $\mathbf{M}_{s,p}$. Polarizer and analyzer are separated by a NM spacer of thickness $t_{sp,1} \approx 2$ nm. This is much smaller than the spin-flip diffusion length (and any other scatter length) of a NM spacer of a spin valve, so that spin is preserved between polarizer and analyzer. The analyzer, in turn, is separated from the insulating layer by a NM spacer of thickness $t_{sp,2}$ ($t_{sp,2} \approx t_{sp,1}$). The different scattering experienced by the minority- and majority-spin channels in the FM layers is reflected in different resistivities ρ_{FM}^{\min} and ρ_{FM}^{maj} , which obey $\rho_{NM} \ll \rho_{FM}^{maj} < \rho_{FM}^{\min}$ (ρ_{NM} is the NM resistivity). Outside the spin valve the left electrode is continued with NM, which is also the material inside the orifice and of the right electrode. The

elastic mean free path of the electrons, both in the NM and the FM, is supposed small compared to the size of the orifice, so that transport is diffusive on this scale.

The device is biased in the positive direction at a voltage V ($V > 0$), applying $-V/2$ to the left electrode or reservoir and $+V/2$ to the right reservoir. The resulting electron current flowing from the polarizer to the orifice is spin-polarized. In zeroth order, *i.e.* without inelastic processes (inelastic diffusion length exceeds orifice diameter), the resulting spin-dependent electron-distribution function in the plane of the orifice is

$$f_{\mathbf{k},\sigma} = \frac{1}{2} \left\{ \left[1 - \alpha \left(\frac{1}{2} + \sigma \right) \right] f_0 \left(\epsilon_{\mathbf{k},\sigma} + \frac{eV}{2} \right) + \left[1 + \alpha \left(\frac{1}{2} + \sigma \right) \right] f_0 \left(\epsilon_{\mathbf{k},\sigma} - \frac{eV}{2} \right) \right\} \quad (1)$$

In Eq. (1) the degree of the spin-polarization induced by the polarizer is given by $\alpha = \Delta R_p / R_M = (\rho_{FM}^{\min} - \rho_{FM}^{maj}) t_p / (\pi a^2 R_M)$. This is the difference in polarizer resistance seen by minority-spin and majority-spin electrons, normalized to the Maxwell resistance R_M , which is the resistance of the corresponding diffusive, homogeneous NM point contact. σ denotes the electron spin in the polarizer ($\sigma = +1/2$ for majority-spin electrons and $\sigma = -1/2$ for minority-spin electrons, defined with respect to $\mathbf{M}_{s,p}$). Further, f_0 is the Fermi-Dirac distribution, $\epsilon_{\mathbf{k},\sigma}$ is the total energy of an electron in state \mathbf{k} and with spin σ , *i.e.* inclusive the electrostatic energy, and e is the elementary charge ($e > 0$). The distribution in the analyzer, since it is so close to the orifice, to a good approximation is also given by Eq. (1). The non-equilibrium distribution is similar to that of a homogeneous diffusive NM point contact[14] or mesoscopic diffusive NM wires[15]. It is the average of two Fermi step functions displaced with respect to each other by an energy eV , which is the difference of the chemical potentials of the reservoirs. In this case, however, the weight of the functions is spin-dependent, so that for the energy range of values intermediate between $f_{\mathbf{k},\sigma} = 1$ and $f_{\mathbf{k},\sigma} = 0$ two values exist: $f_{\mathbf{k},+1/2} = (1 + \alpha)/2$ and $f_{\mathbf{k},-1/2} = 1/2$, for majority- and minority-spin electrons, respectively. To arrive at Eq. (1), we assume that only the potential drop across the polarizer, because of its large thickness and high resistivity, is responsible for the modification of the electron distribution with respect to the one corresponding to the homogeneous NM point contact (which would correspond to $\alpha = 0$ in Eq.1). Thus the effect of the analyzer is assumed negligible. Further, we concentrate on the spin-dependent contribution of the polarizer to the electron distribution of Eq.1, neglecting the average over the electron spins.

Analogous to point-contact spectroscopy of magnons[10], [11], the electron distribution prepared in a biased spin-valve point contact enables magnon emission by electrons in the analyzer, up to a maximum magnon energy eV . Relaxation of created magnons is dominated by absorption by electrons. As usual for ferromagnets, we assume that electronic transport is primarily due to *sp*-electrons, so that these electrons control the magnon distribution, irrespective the strength of electron-magnon coupling. We further assume that the analyzer region exposed to the current is single domain. Finally, at the present level,

we neglect escape of created magnons from the region exposed to the current and corrections to the distribution given by Eq. (1) due to strong magnon absorption by electrons. Thus, applying Fermi's golden rule to magnon emission and absorption, and carrying out integrations over initial and final states in spherical coordinates, we find the rate equation for the occupation number of magnons N_ω with energy $\hbar\omega_{\mathbf{q}}$ in the analyzer:

$$\frac{dN_\omega}{dt} = \frac{1}{2\pi\hbar} \int d\epsilon D(\epsilon) \int d\epsilon' D(\epsilon') |\tilde{g}|^2 [f_{\epsilon,\sigma} (1 - f_{\epsilon',-\sigma}) (1 + N_\omega) \delta(\epsilon - \epsilon' - \hbar\omega_{\mathbf{q}}) - f_{\epsilon',-\sigma} (1 - f_{\epsilon,\sigma}) N_\omega \delta(\epsilon' - \epsilon + \hbar\omega_{\mathbf{q}})]. \quad (2)$$

Here $D(\epsilon)$ is the electron density of states normalized with respect to the unit cell, \tilde{g} is an effective matrix element for electron-magnon coupling, *i.e.* renormalized with respect to wave-vector-nonconserving scattering processes (see Appendix), and the distribution f is given by Eq. (2). Note that the orientation of σ in Eq.2 corresponds to minority spins in the analyzer. The first term of the integrand applies to emission, the factor $(1 + N_\omega)$ denoting the sum of spontaneous and stimulated processes. By nature, this magnon emission is incoherent, so that incoherent magnetization precessions result. This is in contrast to the current-induced coherent magnetization precession described in Ref. [3]. Magnons are spin unity quanta. According to spin conservation, magnon emission thus is a spin-flip process, which must increase the net spin of the electron system by unity by converting analyzer minority-spin electrons into majority-spin electrons. Accordingly, the first energy integration in Eq. (2) involves minority-spin electrons, while the second integration involves majority-spin electrons (ϵ and ϵ' no longer depend on the wave-vector, as angular integrations were done already). The second term of the integrand applies to absorption, so that the role of minorities and majorities is reversed. When $\mathbf{M}_{s,p}$ and $\mathbf{M}_{s,a}$ are antiparallel, a minority-spin current in the analyzer was a majority-spin current in the polarizer, so that for this configuration $\sigma = +1/2$ in Eq. (1) gives the minority distribution in the analyzer and $\sigma = -1/2$ the majority distribution. For the parallel configuration, minorities and majorities conserve their character when they enter the analyzer, so that in Eq. (1) the usual sign convention applies.

To evaluate Eq. (2) for $T = 0$, where the energy of created magnons is limited to $\hbar\omega_{\mathbf{q}} < eV$, one has to recognize the possible emission and absorption processes in the analyzer and to take into account the energy range and distribution function (together termed "phase volume") of the electron states involved. Magnon emission takes an initial state with $f_{\epsilon,+1/2} = (1 + \alpha)/2$ to a final state with $f_{\epsilon',-1/2} = 1/2$, the phase volume being $(1 + \alpha)(eV - \hbar\omega_{\mathbf{q}})/4$. For absorption there is a complementary process from a state with $f_{\epsilon',-1/2} = 1/2$ to a state with $f_{\epsilon,+1/2} = (1 + \alpha)/2$, with a phase volume $(1 - \alpha)(eV - \hbar\omega_{\mathbf{q}})/4$. The sum of these contributions is $(\alpha/2)(eV - \hbar\omega)$; the corresponding term give rise to a net emission magnons (which corresponds to our choice of the sign of α). In addition there are absorption processes from states with $f_{\epsilon',-1/2} = 1$ to

states with $f_{\epsilon,+1/2} = (1 + \alpha)/2$ and from states with $f_{\epsilon',-1/2} = 1/2$ to states with $f_{\epsilon,+1/2} = 0$, with phase volumes $\hbar\omega_{\mathbf{q}}/2$ and $(1 - \alpha)\hbar\omega_{\mathbf{q}}/2$, respectively. The total phase volume is $\hbar\omega(1 - \alpha/2)$. Note that this contribution to absorption exists for purely equilibrium state of the electron system ($V = 0, \alpha = 0$). In the parallel configuration, *mutatis mutandis*, similar processes with similar phase volumes occur. For either configuration Eq. (2) thus goes over into

$$\frac{dN_{\omega}}{dt} = -\frac{1}{\tau_{m-e}} \left[N_{\omega} \left(1 + \frac{eV}{\hbar\omega_{\mathbf{q}}} S_z \right) - \frac{eV - \hbar\omega_{\mathbf{q}}}{4\hbar\omega_{\mathbf{q}}} (1 - 2S_z) \right]. \quad (3)$$

$S_z = \alpha(\mathbf{M}_{s,a} \cdot \mathbf{M}_{s,p}/2M_{s,a}M_{s,p})$ is a projection of $\mathbf{M}_{s,a}$ on $\mathbf{M}_{s,p}$, weighed by the current polarization. Whereas the spin polarization is defined with respect to $\mathbf{M}_{s,p}$, S_z takes into account the sensitivity of magnon-electron processes to the polarization with respect to the direction of $\mathbf{M}_{s,a}$. In Eq. (3) $\tau_{m-e} \approx \hbar^{-1} |\tilde{g}|^2 [D(\epsilon_F)]^2 \hbar\omega_{\mathbf{q}}$ is the characteristic time for magnon-electron processes, $D(\epsilon_F)$ being the electron density of states at the Fermi level.

Eqs. (2) and (3) describe transfer of spin from the spin-polarized current to the magnon system. Due to spin transfer the population of non-equilibrium magnons increases until a steady state is reached where magnon emission and absorption balance each other, *i.e.* where $dN_{\omega}/dt = 0$. For magnons of energy $\hbar\omega_{\mathbf{q}}$ the steady state is characterized by an effective magnon temperature $T_{m,\omega}^{eff}$, obtained by equating the number of such magnons to the average population as given by the Planck distribution (k_B is Boltzmann's constant):

$$T_{m,\omega}^{eff} = \frac{1}{k_B} \frac{eV - \hbar\omega_{\mathbf{q}}}{4} \frac{1 - 2S_z}{1 + (eV/\hbar\omega_{\mathbf{q}})S_z}. \quad (4)$$

In the limit of weak polarization and for magnon energies $\hbar\omega_{\mathbf{q}} \ll eV$ one obtains $T_{m,\omega}^{eff} \approx eV/4k_B$. The effective temperature is larger for $S_z < 0$ (antiparallel configuration) than for $S_z > 0$ (parallel configuration). This is natural, because the phase volume for magnon creation processes is larger when $S_z < 0$. Moreover, in the antiparallel configuration $T_{m,\omega}^{eff}$ diverges at a critical voltage given by

$$eV_c S_z = -\hbar\omega_{\mathbf{q}} \quad (5)$$

This is interpreted as an unlimited increase of the number of magnons due to stimulated emission, resembling a magnon avalanche. This highly excited state of the analyzer goes along with a strongly suppressed magnetization and may lead to a kind of critical behavior with similarities to the phase transition to the normal state at the Curie temperature, which results from strong thermal excitation of magnons. As seen in Eq. (4), at the voltages larger than the critical one dN_{ω}/dt is positive so that N_{ω} has a positive increment.

Let us introduce the magnon concentration n_m normalized with respect to elementary cell

$$n_m = \int d\omega \nu_m N_{\omega}$$

where $\nu_m(\omega)$ is magnon density of states normalized by elementary cell volume. It is clear that $n_m = 1$ would correspond to complete suppression of magnetization. Note that a decrease of total magnetization of the ferromagnet $|\mathbf{M}|$ with an increase of n_m for small n_m is a well-established fact and describes in particular a decrease of $|\mathbf{M}|$ with temperature increase. This behavior can be interpreted as a result of uncertainty of the orientation of $\mathbf{M}_{s,a}$ within the angle $\theta \sim n_m$ with respect to its orientation at $n_m = 0$. As for the situation of very high magnon occupation numbers when $n_m \sim 1$, to the best of our knowledge it still has not met a consequent theoretical treatment. One could speculate that the magnon avalanche leading to $n_m \sim 1$ corresponds to switching since for $n_m \simeq 1$ the average, time-independent magnetization of the sample would be completely suppressed. At the same time any local fluctuation with the with the opposite direction of magnetization would be magnified due to the fact that the sign of the projection of the incoming spins on this direction entering the driving terms in Eq.3 would correspond to magnon absorption rather than to magnon emission.

However one can expect that the values of n_m can be stabilized at some $n_m \ll 1$ due to mechanisms not included in Eq. (4). To look for such mechanisms we should consider magnon kinetics at large occupation numbers (but still within the framework of conventional magnon theory implying $n_m \ll 1$).

3 Role of magnon-electron processes

First let us discuss a possible role of magnon-electron interactions. So far we assumed that the electron distribution given by Eq.1 exists even at high magnon concentration. However one can expect that a high magnon emission rate leads to decay in the analyzer of the spin-dependent part of the electron distribution. The latter consequently can no longer support further strong emission. To understand the evolution of this distribution with an increase of the magnon concentration we make use of the electron-magnon collision operator:

$$I_{e-m} = \int d\omega \nu_\omega |\tilde{g}|^2 (f_{\varepsilon,\sigma}(1 - f_{\varepsilon',-\sigma})[(1 + N_\omega)\delta(\varepsilon - \varepsilon' - \hbar\omega) + N_\omega\delta(\varepsilon' - \varepsilon + \hbar\omega)] - f_{\varepsilon',-\sigma}(1 - f_{\varepsilon,\sigma})[(1 + N_\omega)\delta(\varepsilon' - \varepsilon - \hbar\omega) + N_\omega\delta(\varepsilon - \varepsilon' + \hbar\omega)]) \quad (6)$$

Here $\nu(\omega)$ is the magnon density of states. As it is seen, for $N_\omega \gg 1$ with a neglect of the spontaneous processes this operator tends to establish an electron distribution of the sort $f_{min}(\varepsilon) = f_{maj}(\varepsilon - \hbar\omega)$. In addition, the electron-magnon processes even at very large N_ω can only take place between of the electronic majority and minority states separated by the energy $\hbar\omega$ since the minority electron arising as a result of magnon absorption by the majority electron can not further absorb magnons and vice versa, the minority electron turning to the majority electron by an emission of magnon can not further emit magnons. Having these facts in mind one sees that, in particular, the integral of Eq.6 is

cancelled by the electron distribution

$$\begin{aligned} f_{maj} &= \frac{1}{2} \left(f_0\left(\varepsilon + \frac{eV}{2} + \hbar\omega\right) + f_0\left(\varepsilon - \frac{eV}{2}\right) \right) \\ f_{maj} &= \frac{1}{2} \left(f_0\left(\varepsilon + \frac{eV}{2}\right) + f_0\left(\varepsilon - \frac{eV}{2} - \hbar\omega\right) \right) \end{aligned} \quad (7)$$

So the electron-magnon coupling does not lead to a total decay of spin polarization, however the total density of polarized spins supported by the distribution resulting from the coupling in question is $(\alpha/2)\hbar\omega D$ while the initial electron distribution given by Eq.[1] corresponds to the density of spins $\alpha eV/2D$. So one concludes that for $V > V_c$ the electron-magnon coupling within the polarizer whatever strong it is can not completely suppress the spin polarization.

4 Role of magnon-magnon processes

Now let us discuss a possible role of magnon-magnon processes which can become important at high phonon occupation numbers. Note that these processes can be considered as a precursor of the general nonlinearity of the magnon physics at high magnon excitation levels. One discriminates between 3-magnon processes (originating due dipolar-dipolar interactions and non-conserving total spin) and 4-magnon processes related to purely exchange interactions and conserving total spin and total number of magnons (see e.g. [17]). For the simplicity we will consider purely 3D magnon spectrum ($\nu_m(q) \propto q^2$) while generalization for the case of more complex spectrum is straightforward). Let us start from 3-magnon processes. For the convolution of 2 magnons of the modes 1 and 2 into a magnon of the mode 3 the efficiency rate can be estimated as [17]

$$\frac{1}{\tau_3} \sim \frac{2\pi}{\hbar} \frac{(\mu_B M)^2}{\Theta_C} \gamma_3(1, 2) N_2 (N_3 + 1) \quad (8)$$

where Θ_C is the Curie temperature, M is magnetic moment density while $\gamma_3(1, 2)$ is a dimensionless parameter depending on the details of magnon spectrum. Note that the parameter γ crucially depends on the details of magnon spectrum and on the momentum conservation law. For standard 3D magnon spectrum $\gamma \sim q_2 a$ where q_2 is the wave vector of the mode 2. However if $\omega(q \rightarrow 0)$ does not depend on the direction of \mathbf{q} momentum conservation law allows coalescence processes only if $\omega_3 > 3\omega(q \rightarrow 0)$ (see e.g. [17]). However in general case

$$\omega_q^2 = (\mu_B H_i + 2JS(qa)^2)(\mu_B H_i + 2JS(qa)^2 + 4\pi\mu_B M \sin^2 \theta_{\mathbf{q}}) \quad (9)$$

where $H_i = H_0 - 4\pi\mu_B M + H_a$, $\theta_{\mathbf{q}} = \angle(\mathbf{q}, \mathbf{M})$ while H_0 and H_a are external magnetic field and anisotropy field, respectively. As it is seen, in this case the restrictions related to momentum conservation are not as severe due to a presence of the term depending on $\theta_{\mathbf{q}}$. One also notes that while the magnon-magnon processes in the bulk of a perfect crystal definitely obey momentum

conservation law, it is not necessarily the case for imperfect systems as it is shown in particular in the Appendix for electron-magnon interactions. One can expect that the efficiency of processes violating the momentum conservation can be as high as the for non-violating ones if the spatial scale of inhomogeneity is comparable with magnon wavelength.

In its turn, the rate of 4-magnon process ($1, 2 \rightarrow 3, 4$) can be estimated as

$$\frac{1}{\tau_4} \sim \gamma_4(1, 2, 3)N_2(1 + N_3)(1 + N_4) \quad (10)$$

where $\gamma_4(1, 2, 3)$ is the corresponding dimensionless parameter which for standard 3D spectrum and wave vectors of the same order of magnitude is equal to $(qa)^8$. One notes that since 4-magnon processes simultaneously conserve the total number of magnons and the total energy, they can not efficiently modify the magnon distribution if it is initially concentrated at lowest possible energies (ensuring the largest gain for the magnon avalanche). So we will mainly concentrate on 3-magnon processes. The magnon occupation numbers N can be written in terms of our normalized parameters n_m as

$$N_\omega \sim n_{m,\omega} \mathcal{V}_\omega^{-1} \quad (11)$$

where \mathcal{V} is the phase volume available for the corresponding modes (for standard 3D spectrum $\omega(\mathbf{q})$ $\mathcal{V} \sim (qa)^3$). So one concludes that despite of relatively small efficiency of dipolar-dipolar interactions (related to small matrix element $\mu_B M$), it can be of importance for small magnon wave vectors where the corresponding smallness can be compensated even at $n_m < 1$ - by large factors \mathcal{V}_ω^{-1} resulting from the phase volume considerations.

First let us consider a situation when the occupation numbers of the resulting magnons N_3 are small. In this case one can rewrite Eq.3 with an account of 3-magnon processes as

$$\frac{dN_\omega}{dt} = -\frac{1}{\tau_{m-e}\hbar\omega}(S_z eV + \hbar\omega)N_\omega - \frac{2\pi(\mu_B M)^2}{\hbar\Theta_C}n_m\frac{\gamma_3}{\mathcal{V}_\omega}N_\omega \quad (12)$$

So one sees that the magnon avalanche is stabilized at

$$\tilde{n}_m = (|S_z eV| - \hbar\omega)\frac{\hbar\Theta_C}{2\pi(\mu_B M)^2\tau_{m,e}\hbar\omega}\frac{\mathcal{V}_\omega}{\gamma_3} \quad (13)$$

One notes that in this case the gradual evolution of n_m is possible with an increase of V (starting from V_c) following by the reversal of magnetization at some $V = V_{c1}$ corresponding to $n_m(V) \sim 1$. As for the numerical estimates, one notes that for typical ferromagnets $\Theta_C/2\pi\mu_B M \sim 10^3$. Then, according to the estimates given in Appendix $\tau_{m,e}\omega \sim 10 - 100$. Thus, assuming $\gamma_3 \sim 10^{-2}$ one obtains

$$n_m \sim (10^{-3} - 10^{-2})\frac{(|S_z eV| - \hbar\omega)}{\hbar\omega}\frac{\hbar\omega}{\mu_B M}$$

where $\mu_B M \sim 0.01 mV$. Thus the values $n_m < 1$ are still expected for $|V - V_c| \sim 1 mV$. However it may occur that for small values of γ_3 V_{c1} is too close to V_c to allow a wide region for the gradual evolution.

Now let us consider a situation when the occupation numbers N_3 can be large which lead to stimulated emission of the corresponding magnons.

For the mode ω_3 one has

$$\frac{dN_{\omega_3}}{dt} = -\frac{1}{\tau_{m-e}\hbar\omega_3} \left((S_z eV + \hbar\omega_3) N_{\omega_3} - \frac{eV - \hbar\omega_3}{4\hbar\omega_3} (1 - 2S_z) \right) + \frac{2\pi (\mu_B M)^2 \gamma_3}{\hbar \Theta_C \mathcal{V}_\omega^2} n_m^2 (N_{\omega_3} + 1) \quad (14)$$

We will assume that, since ω_3 is at least twice larger than ω_1 , $S_z eV + \hbar\omega_3 > 0$. So in stationary situation one has

$$N_{\omega_3} \sim \frac{n_m^2 + \beta}{n_c^2 - n_m^2} \quad (15)$$

where

$$n_c^2 = \frac{1}{\tau_{m-e}\hbar\omega_3} (S_z eV + \hbar\omega_3) \left(\frac{2\pi (\mu_B M)^2 \gamma_3}{\hbar \Theta_C \mathcal{V}_\omega^2} \right)^{-1} \quad (16)$$

while

$$\beta = \frac{eV - \hbar\omega_3}{4\tau_{m,e}\hbar\omega_3} (1 - 2S_z) \left(\frac{2\pi (\mu_B M)^2 \gamma_3}{\hbar \Theta_C \mathcal{V}_\omega^2} (N_{\omega_3} + 1) \right)^{-1} \quad (17)$$

So if $\beta < n_m^2$ the stationary solution for N_ω corresponds to

$$(|S_z eV| - \hbar\omega) \frac{\hbar\Theta_C \mathcal{V}_\omega^2}{2\pi(\mu_B M)^2 \tau_{m,e}\omega} \left(1 - \frac{n_m^2}{n_c^2} \right) = n_m \quad (18)$$

or

$$n_m = -\frac{n_c^2}{2\tilde{n}_m} + \frac{n_c^2}{2\tilde{n}_m} \left(1 + \frac{4\tilde{n}_m^2}{n_c^2} \right)^{1/2} \quad (19)$$

As it is seen, the behavior depends on the ratio

$$\frac{n_c}{\tilde{n}_m} = N_{\omega_3}^{-1} \sim \frac{(S_z eV + \hbar\omega_3)^{1/2}}{(|S_z eV - \hbar\omega|)} \frac{\mu_B M}{\Theta_C^{1/2} (\tau_{m-e}\omega)^{1/2}} \gamma_3^{-1/2} \quad (20)$$

Namely, in the first scenario $\tilde{n}_m < n_c$

$$n_m \sim \tilde{n}_m \quad (21)$$

In this case the gradual evolution of n_m is possible provided $V_{c1} - V_c$ is large enough. In the second scenario $\tilde{n}_m > n_c$ one has

$$n_m \sim n_c \quad (22)$$

While at $V - V_c \rightarrow 0$ one always has $n_m \sim \tilde{n}_m$, at higher V the condition $\tilde{n}_m > n_c$ can be reached at some $V = V_{c2}$ provided $V_{c2} < V_{c1}$. It is important to note that when $n_m \sim n_c$ the value of n_m *decreases* with an increase of $|V|$ since the magnon generation corresponds to $S_z eV < 0$.

Thus, the non-monotonous evolution of magnetization is expected: at critical bias $S_z eV = -\hbar\omega$ the value of $n_m = n_c(V = V_c)$ is reached while the further increase of V leads to a decrease of the magnons number. One notes that in this regime

$$N_{\omega_3} \sim \frac{\tilde{n}_m}{n_c}$$

and thus N_{ω_3} increases with the increase of V . The situation again becomes critical when the values of N_3 becomes comparable with N and a situation of energy diffusion is established. However we will not discuss this situation in more detail.

Note that the actual physical picture with an account of magnon-magnon processes is rather complex and sensitive to details of magnon spectrum etc. Thus our simplified analysis aims only to reveal the main features. We can conclude that the magnon-magnon processes can lead to saturation of magnon emissions at $n_m < 1$, that when no switching occurs. In this case the further gradual evolution of magnetization with an increase of V is possible. The scenario depends on the relation between n_c and \tilde{n}_m .

5 Comparison to previous theoretical approaches

A short comparison with the approach of Slonczewski[3] is in place. In the latter case the spin transfer from the incident electrons to the layer is proportional to the amplitude of magnetization precession and is zero for \mathbf{M} being parallel or antiparallel to \mathbf{M}_0 while in our case the spontaneous processes exists as well allowing spin relaxation even for the spins parallel or antiparallel to \mathbf{M}_0 . In the case of mechanism by Slonczewskii the spin transfer from the incident electrons is equal to $j_{s,inj} \sin \theta$ provided $d > l_p \sim (\hbar/p_F)\varepsilon_F/E_{ex}$ where E_{ex} is the exchange energy, l_p is the spin precession length and $j_{s,inj}$ is the spin current. Since E_{ex} is not too small with respect to the Fermi energy this condition holds for practical values of t_a . Thus this mechanism is restricted by the near-surface layer with a thickness l_p . The evolution of the precession angle is described by Eq.17, the key equation of the paper, which can be in our terms be written as

$$\frac{d\theta}{dt} = -(\alpha\omega - \frac{j_{s,inj}}{t_a}) \sin \theta \quad (23)$$

where α is the Gilbert parameter while $j_{s,inj}$ is the spin current in the analyzer. (Note, that, strictly speaking, the applicability of Gilbert damping, suggesting the equilibrium state of the ferromagnet, is questionable for the non-equilibrium electron distribution). To compare with our considerations we first note that for small θ the number of "coherently excited magnons" can be related to θ as $n_m \sim \theta^2/2$. Then, one can consider the product $\alpha\omega$ as the magnon relaxation

rate, $1/\tau_m$. We believe that the dominant mechanism of the magnon relaxation is related to magnon-electron coupling discussed above. Thus one can rewrite this equation as

$$\frac{dn_m}{dt} = \left(\frac{j_{s,inj}}{t_a} - \frac{1}{\tau_m} \right) n_m \quad (24)$$

As it is seen, the criterion for the current-driven evolution of magnetization can be written for the Slonczewskii mechanism as

$$j_{s,inj} > \frac{t_a}{\tau_m} \quad (25)$$

which can be rewritten in terms of the bias with an account of the estimate of τ_m given above as

$$V > \eta^{-1} V_c \quad (26)$$

where $\eta = j_{s,inj}/j_{s,inj}^{ball}$ while $j_{s,inj}^{ball}$ is the spin current which would exist under the same bias provided the structure would be ballistic. One notes that this criterion differs from that given by Eq.5 by a factor $\eta^{-1} > 1$ in the r.h.s. of the inequality which for the diffusive electron transport is much larger than unity. Thus there is a broad region of biases

$$\hbar\omega < |S_z|eV < \eta^{-1}\hbar\omega$$

where the excitation of magnons according to our scenario is possible while the mechanism by [3] is not efficient.

Note however that the approach suggested in our paper is completely self-consistent one and is limited by quantum-mechanical treatment while the approach of [3] exploits a combination of quantum-mechanical treatment of electron spin evolution and semi-classical transfer of x - component of electron spin to the layer as a whole described with a help of classical Landau-Lifshits equation. As a result, some questions do not meet unequivocal treatment - in particular a mechanism leading to a change of z -component of spin by portions equal $(1/2)$ which governs the spin evolution in a consequent quantum-mechanical picture and is expected to have "non-coherent" character. It is also not clear in what way the single-electron spin is transferred to the excited region as a whole since any given electron is coupled only to its closest surrounding.

To some extent these questions were noted by Berger [12] who exploited the similar mechanism of boundary-induced spin transfer to describe emission of spin waves by incoming electron spin flux. He attempted to write the equation for z -component of electron spin, but he simply postulated it in the most simple relaxation time approximation form to fit a quantum-mechanical principle that z -component of spin can be changed by portions of $1/2$ (Eqs. (9-12) in [12]). In this way he also came to a conclusion concerning instability of magnon system for the current exceeding some threshold value similar to one we have formulated on the base of our consistent quantum-mechanical treatment. Note however that, as the paper [3], this approach does not take into account a possibility of non-equilibrium electron distribution, relating the chemical potential difference

majority and minority subbands to a difference in pumping of electron momenta at the boundary due to a difference in drift currents. To our opinion such an approach is equivalent to an assumption that the energy diffusion length is equal to elastic mean free path and strongly underestimates the real potential difference.

Since one notes similarities between our results and results of [3], [12] including a possibility of switching discussed above, magnon escape factor etc. one can consider our scheme as a generalization of approaches by Slonzevskii and Berger. But we also predict a possibility of stabilization in magnon system at the levels still much less than corresponding to switching - in contrast to results by [3], [12].

As for high excitation levels $V - V_c \geq 1$ we would like to note that there is still a problem whether the real magnetization evolution is in accordance to our "incoherent" scenario or it has features of "coherent" classical magnetization evolution of the sort considered in [3] (with some correction of the Gilbert parameter α for the nonequilibrium case). In any case one expects that if initially \mathbf{M} and \mathbf{M}_0 are parallel or antiparallel an initial stage is described by our scenario since it is related to magnon excitation which can be considered as quantum fluctuations and exist even for $\theta = 0$. The further behavior is not as clear since, as we have noted above, the situation of very high magnon occupation numbers is still not perfectly clear. In particular one can expect a sort of "producing of coherency" originated due to effective magnon-magnon processes and establishing the "coherent" evolution. Note that a similar problem was discussed for acoustoelectric generators of sound waves where the coherent signal arose (or not arose) from initial electron-drift driven emission of incoherent phonons ([18]). Certainly this choice can be crucially dependent on the factor of boundaries which can support - or destroy - such coherency, and on the excitation level (that is on the product $|S_z|eV$).

We would like also to emphasize that for non-ballistic limit where the electron-magnon coupling in our case has a bulk character (see Appendix) in contrast to mechanisms of spin transfer considered in [3], [12] our "bulk" mechanism is expected to dominate in any case.

To conclude this Section, we would like in addition to consider a situation when the exciting current passes only through small area of the analyzing layer which takes place when the planar multilayered structure is excited with a help of a point contact attached to such a structure. This geometry corresponds in particular to experimental situation of the papers [7], [9] which will be to some extent discussed in what follows. In contrast to the picture suggested above, the emitted magnons have a possibility to escape the excited region propagating along the plane of the analyzing layer (assumed to have an infinite area). If one denotes a size of the excited region (being equal to the size of the point contact) as a , the escape time τ_m^{esc} is expected to be equal to a/v_m where $v_m = \hbar k_m/m_m$; here v_m is a magnon velocity while k_m is the magnon wave vector with a smallest possible value of $\sim 1/a$. Correspondingly, the largest possible value for τ_m^{esc} is obviously $a^2 m_m/\hbar$.

This escape can be taken into account by adding to Eq. (3) the relax-

ation term $-N_\omega/\tau_m^{esc}$, where τ_m^{esc} is the magnon escape time for the point-contact geometry. This gives τ_{m-e}/τ_m^{esc} as extra term in the denominator of Eq. (4). Divergence of $T_{m,\omega}^{eff}$ is not prevented by magnon escape, but fast escape ($\tau_{m-e}/\tau_m^{esc} \gg 1$) leads to a strong increase of the critical voltage according to

$$eV_c S_z \approx -(\tau_{m-e}/\tau_m^{esc})\hbar\omega_{\mathbf{q}} \quad (27)$$

Taking into account that $\tau_{m-e}^{-1} \simeq \omega/(k_F d)$ one obtains

$$\tau_{m-e}/\tau_m^{esc} \geq \frac{k_F d}{\hbar\omega} \frac{\hbar^2}{a^2 m_m} \quad (28)$$

where the equality corresponds to the smallest magnon in-plane wave vector $\sim 1/a$. As it is seen, in this case the criticality starts at the lowest possible in-plane wave vector. So the criticality criterion is in principle independent of the magnon frequency provided the ratio given by Eq.28 is larger than unity.

Note that according to Eq.27 in the corresponding regime $V_c \propto 1/(a^2)$. For diffusive electron transport the critical current is $I_c = V_c/R_M$ is thus independent of the orifice size which agrees with prediction by Slonczewski for the similar geometry although in terms of coherent excitation of spin waves [19]

6 Comparison to experimental situation

Thus we have shown that the incoherent stimulated emission of magnons trigger an avalanche evolution of magnetization at biases smaller than ones necessary for the switching behavior according to the mechanism suggested by Slonczewski. However the final stage at these moderate biases corresponds to the state with very high magnon temperature but still with the initial direction of magnetization. Such a behavior is different from "switching" predicted in [3]. In our scenario the first step-like evolution of magnetization with the applied bias (at $V = V_c = |\hbar\omega_m/eS_z|$) can be followed by the gradual increase of the number of magnons and, correspondingly, by a gradual decrease of the z -component of total magnetization. Note that neither the approach by Slonczewski nor the approach by Berger have predicted such a gradual evolution. Unfortunately the actual physical picture at this stage which is controlled by magnon-magnon processes is rather complex and sensitive to details of magnon spectrum etc. Thus our simplified analysis allows only to reveal the main features and does not allow direct quantitative comparison to experimental data.

First we note that qualitatively many experiments on current-driven excitations in magnetic multilayers [7],[8], [4],[9], [6] unambiguously exhibited signatures of a gradual evolution of magnetic state with an increase of bias.

In the papers [8], [4] current-induced switching of magnetic moments in Co/Cu/Co pillar structures was reported. The switching exhibited hysteretic behavior as function of injected current and external magnetic field and was interpreted in terms of theory by Slonczewski [3]. Note however that the initial

phase of the process clearly indicated a gradual evolution with an increase of the current following step-like increase of differential resistance. At high external magnetic fields instead of switching the data exhibited spikes for one of the directions of the current which were attributed to emission of spin waves. Since the theory [3] exploiting a constant value of Gilbert parameter does not allow an existence of stable spin-precessing mode, the authors of [4] explained the observed behavior as a result of a dependence of α on the angle of precession. Although the latter mechanism can in principle explain spike-like features, it can hardly explain the gradual evolution. Note that the gradual evolution of contact resistance with an increase of the current depending on the current direction was also observed in [6].

The transition from hysteretic behavior to the spike-like one with an increase of magnetic field was also reported in [5] for permalloy-based nanopillars. We would like to note that authors of [5] have suggested a possible role of nonequilibrium magnons with high temperatures to explain the behavior observed. The also observed a presence of telegraph noise in the vicinity of the spike. It is important to note that the noise was pronounced only at very narrow region of biases and has not revealed any trace of hysteretic behavior. This fact could not be easily understood suggesting magnetization reversal since in the latter case the anisotropy field would give rise to a hysteretic behavior.

To our opinion, the spike can be related to stabilization of the magnon occupation numbers due to magnon-magnon scattering according to the scenario (2) considered above. Indeed, for the resulting "incoherent precession state" the direction of magnetization is only slightly "smeared" with respect to its original direction and thus no hysteretic behavior is expected. In contrast, the variation of the bias around the corresponding threshold value leads to a reversible behavior of the magnon occupation number. The step-like increase of resistance leading to the spike in differential resistance can be related to additional resistance related to effective electron-magnon scattering rather than to magnetization reversal.

Correspondingly, the transition from hysteretic behavior to spike-like behavior can be related to the transition from scenario (1) to scenario (2). Indeed, with an increase of magnetic field a role of the angle-dependent term in 9 decreases with respect to the term $\mu_B H_i$. As a result, the the phase volume for 3-magnon processes decreases thus leading to a decrease of γ_3 . As it is seen, it is favorable for regime (2) where an increase of bias does not lead to an increase of n_m at least in some interval of biases.

In the papers [7], [9] magnetic multilayers were excited with a help of point contact. Correspondingly, the excitation took place only at small area of the multilayer concentrated around the point contact which is in agreement with model discussed at the end of our Sec.2. The differential resistance spectra obtained in [7] for Co/Cu multilayers excited by the metallic point contact clearly demonstrated a gradual increase of resistance following an initial spike which the authors ascribed to stimulated emission of magnons according to scenario [12].

To our opinion, the experimental data mentioned above evidence the inco-

herent character of magnons emission (at least at the initial stage) since it is the latter mechanism that can - according to our model - explain co-existence of "critical" spike-like and "switching" behavior with a gradual evolution.

7 Conclusions

To conclude, we have given a consequent quantum-mechanical treatment of spin-polarized transport in spin-valve point contact with an account of non-equilibrium electron distribution is given. It is shown that at large biases an avalanche-like creation of low-frequency magnons within the ferromagnetic layer up to is possible which agrees with earlier predictions based on semi-phenomenological models. However in contrast to earlier studies [3] it is found that the stabilization within the magnon system can be achieved at finite (although large) magnon temperatures and the gradual evolution of such a state with the bias is possible. These results are in agreement with existing experimental data.

8 Appendix

Let us consider an effect of boundaries and disorder on the magnon-electron matrix elements. Following Mills et al. [16] we will write the element corresponding to a transition from the electron state \mathbf{k} to the electron state \mathbf{k}' in a presence of created magnon in the state \mathbf{q} as

$$\mathcal{M}|_{\mathbf{k}\rightarrow\mathbf{k}'} = J \frac{\mathbf{s}\mathbf{M}}{|\mathbf{s}||\mathbf{M}|} \frac{a^{3/2}}{V^{3/2}} \int d\mathbf{r}^3 \exp i(\mathbf{k} - \mathbf{k}' - \mathbf{q})\mathbf{r} \quad (29)$$

where \mathbf{s} and \mathbf{M} are the electron spin and sample magnetization, J is an exchange constant, a is the lattice constant, \mathbf{k} , \mathbf{k}' and \mathbf{q} are the wave vectors of initial and final states of electrons and the magnon wave vector, respectively, while V is the normalizing volume. One notes that typically the integration gives a standard momentum conservation law. However now we are interested in a case when the momentum conservation is violated which in particular is related to a finite size of the layer with a thickness t_a in x -direction. Thus we will assume that in plane of the layer the momentum conservation holds in a standard way and will concentrate on the integration over x . The corresponding factor arising in the expression for $|\mathcal{M}|^2$ can be readily written as

$$\frac{a}{t_a^3 (k_x - k'_x - q_x)^2} \quad (30)$$

For a given initial and final electron energies $\varepsilon = \varepsilon_{\mathbf{k},-\sigma}$ and $\varepsilon' = \varepsilon_{\mathbf{k}',\sigma}$ One notes that the Fermi surfaces are typically separated by relatively large gap $\Delta k_F \simeq k_F E_{ex}/\varepsilon_F$. For long-wave magnons we expect $q \ll \Delta k_F$ which allows us to neglect q in our estimates. We will first integrate over angular variables

$\vartheta, \vartheta'; \varphi, \varphi'$. Having in mind that the difference $\varepsilon - \varepsilon'$ (controlled by the distribution functions) is much less than E_{ex} we also will neglect this difference in course of the angular integration. Thus one has $k_x = k_{F,-} \cos \vartheta$ and $k'_x = k_{F,+} \cos \vartheta'$. The momentum conservation in perpendicular direction eliminates integrations over φ' and ϑ' with an obvious result $\sin \vartheta' = (k_{F,-}/k_{F,+}) \sin \vartheta$ (which is related to a conservation of the in-plane component of the momentum). Correspondingly, one obtains

$$\cos \vartheta' = \left(1 - (k_{F,-}/k_{F,+})^2 \sin^2 \vartheta\right)^{1/2}$$

One clearly see a gap preventing small values of $\cos \vartheta'$ (since the modulus of x -component of electron wave vector for majority electron is larger than for minority) and finally obtains for the denominator in Eq.30:

$$t_a^3 k_F^2 \left(\xi - \sqrt{\xi^2 \left(1 - 2 \frac{\Delta k_F}{k_F}\right) + 2 \frac{\Delta k_F}{k_F}} \right)^2 \simeq t_a^3 k_F^2 \left(\frac{\Delta k_F}{k_F} \left(\xi - \frac{1}{\xi^2} \right) \right)^2 \quad (31)$$

where $\xi = \cos \vartheta$. Thus performing the integration over ξ one has a factor

$$\sim \frac{a}{t_a^3 \Delta k_F^2}$$

Integrating over angular variables corresponding to the initial and final states one notes that the effective matrix element can be estimated as

$$|\tilde{\mathcal{M}}|^2 = J^2 (k_F / (\Delta k_F)^2 t_a) \quad (32)$$

The effective rate equation can be rewritten as

$$\begin{aligned} \frac{d\mathcal{N}_\omega}{dt} = \frac{1}{2\pi\hbar} \sum_\sigma \int d\varepsilon \nu(\varepsilon) \int d\varepsilon' \nu(\varepsilon') |\tilde{\mathcal{M}}|^2 & (f_{\varepsilon,\sigma} (1 - f_{\varepsilon',-\sigma}) (1 + \mathcal{N}_\omega) \delta(\varepsilon - \varepsilon' + \hbar\omega) - \\ & - (1 - f_{\varepsilon,\sigma}) f_{\varepsilon',-\sigma} \mathcal{N}_\omega \delta(\varepsilon - \varepsilon' - \hbar\omega)) = 0 \quad (33) \end{aligned}$$

Here ν is electron density of states normalized with respect to elementary cell. One notes that in a view of the normalizing factors of electron wave functions we have used in course of matrix element calculation one should also put $\sum_{\mathbf{k}} \propto t_a$, $\sum_{\mathbf{k}'} \propto t_a$ and the corresponding factors eliminate the normalizing factor t_a^2 arising in course of our estimates of the matrix element which is taken into account in Eq.33.

Note than if one would write the similar equation (integrated over the angular variables) in the case when momentum conservation over x is allowed (when $\Delta k_F = 0$ the resulting value of denominator in Eq.30 would be limited only by the smallest possible wave vector along x that is $k_x \simeq \pi/t_a$ and as a result of integration over ξ one would have

$$\frac{a}{t_a^2 k_F}$$

Correspondingly, an integration over angular variables would give a factor

$$\frac{k_F}{q}$$

because in the limit $q \rightarrow 0$ in the elastic limit one has a divergence of the corresponding cross-section. Thus the matrix element within the equation 33 would have a form

$$|\tilde{\mathcal{M}}|^2 = J^2 \frac{k_F}{q} \quad (34)$$

that is a decrease of the magnon-electron rate with respect to a case when momentum conservation is obeyed is described by a factor

$$\frac{q}{(\Delta k_F)} t_a$$

One notes that the finite electron-magnon coupling strength for low magnon frequencies (and relatively small biases) is in our case achieved due to boundary-induced momentum non-conservation. One expects qualitatively similar effect from any factors breaking momentum conservation. The effect of alloy disorder significantly increasing an efficiency of magnon-electron processes in low-frequency region was considered in [16], although in this case the disorder was related to on-site exchange constant. We believe that the elastic scattering of electrons can also enhance the coupling.

Let us first consider a role of this scattering when the analyzer is still thin with respect to the elastic mean free path l_e . One notes that a presence of elastic scatterers leads to a modification of electron wave functions with respect to plane waves, due to the scattering. Each of the scatterers produces the scattered wave, so the plane wave in the lowest approximation with respect to the scattering processes (which holds when $t_a < l_e$) as

$$e^{i\mathbf{k}\mathbf{r}} \rightarrow e^{i\mathbf{k}\mathbf{r}} + \sum_i f_i \frac{\exp i\mathbf{k}\mathbf{R}_i k |\mathbf{R}_i \mathbf{r}|}{|\mathbf{R}_i \mathbf{r}|} \quad (35)$$

where i numerates the scatterers, \mathbf{R}_i is the coordinate of a scatterer while f is the scattering amplitude. One notes that the replacement of the plane waves in Eq.29 by the modified ones gives a non-zero result even with no account of the boundaries. Since the magnon wave vector is assumed to be much less than the electron ones the integrand in the Eq.29 for each of the scatterer has a form

$$e^{i\mathbf{k}\mathbf{r}} \cdot e^{-i(\mathbf{k}\mathbf{R}_i + k|\mathbf{r}\mathbf{R}_i|)} = e^{i(\mathbf{k}\mathbf{r} + k\mathbf{r})} \cdot e^{i\Delta k \mathbf{r}} \cdot e^{i(\mathbf{k}\mathbf{k})\mathbf{R}_i} \quad (36)$$

where $\mathbf{r} = \mathbf{r}\mathbf{R}_i$. With an assumption $\Delta k \ll k$ the integration over \mathbf{r} gives

$$\frac{A}{k\Delta k} \exp i(\mathbf{k}\mathbf{k})\mathbf{R}_i \quad (37)$$

where $|A| \sim 1$. The total matrix element is a sum over all the scatterers in the sample. However in the square of this element the products of contributions

of different scatterers vanish due to strongly oscillating factor in Eq.37 depending on the scatterer coordinate. As a result, one finally obtains the following estimate:

$$|\mathcal{M}|^2 \sim \frac{\varepsilon_F^2}{k_F l_e} \quad (38)$$

For this estimate we have taken into account that $f^2 N_i \sim l_e^{-1}$ and that $J^2 \sim \varepsilon_F^2 (k_F / \Delta k)^2$. As it is seen, for $t_a < l_e$ the contribution of scattering is smaller than the effect of boundaries. However one expects that the result of Eq.38 holds even if for the diffusive transport in the analyzer. Indeed, one notes that actually the contribution of each of the scatterer is formed at distances $\sim 1/(\Delta k)$ from the scatterer. So the derivation implies a ballistic transport only at this spatial scale which is assumed to be less than t_a .

Note that in this case the electron-induced magnon relaxation rate with an account of the fact that $\delta k \sim k_F (J/\varepsilon_F)$ can be estimated as

$$\tau_m^{-1} \simeq \frac{1}{k_F l_e} \omega \quad (39)$$

and thus the factor $(1/k_F l_e)$ plays a role of Gilbert damping parameter.

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