

Rabi oscillations of a qubit coupled to a two-level system

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Abstract. – The problem of Rabi oscillations in a qubit coupled to a fluctuator and in contact with a heat bath is considered. A scheme is developed for taking into account both phase and energy relaxation in a phenomenological way, while taking full account of the quantum dynamics of the four-level system subject to a driving AC field. Significant suppression of the Rabi oscillations is found when the qubit and fluctuator are close to resonance. The effect of the fluctuator state on the read-out signal is discussed. This effect is shown to modify the observed signal significantly. This may be relevant to recent experiments by Simmonds *et al.* (*Phys. Rev. Lett.*, **93** (2004) 077003).

Recent experiments have demonstrated Rabi oscillations in macroscopic quantum systems (qubits) [1–7]. These oscillations decay rather quickly due to interaction with the environment even in pure systems. An interesting behavior was observed in spectroscopic experiments with Josephson qubits [1]. Both the decay rate and the oscillation pattern were strongly dependent on the qubit eigenfrequency ramped by external parameters. Namely, the oscillations were significantly suppressed in the vicinity of certain eigenfrequencies. This was interpreted [1] as an influence of some two-level systems (fluctuators) located in the qubit environment. The suppression is strong when the fluctuator's and qubit's energy splittings are very close.

Below we present a theory of a qubit interacting with an external AC field and a single fluctuator coupled to the qubit. In addition, we assume that the system interacts with some thermal bath providing both phase and energy relaxation. The account of the energy relaxation distinguishes our model from a similar approach used in ref. [8], whereas in the recent numerical work of ref. [9] no account is taken of the phase relaxation. By solving the kinetic equation for the density matrix we compute level populations *vs.* time. The results

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explain the strong influence of the resonant fluctuator on the Rabi oscillations of the qubit in agreement with experimental findings. An alternative interpretation of the same experiments was offered in ref. [10] and we do not believe that this can be ruled out on the basis of current experimental knowledge.

We will characterize the qubit by a spin \mathbf{S} interacting with a fluctuator represented by a spin \mathbf{s} . The Hamiltonian in the form

$$\tilde{\mathcal{H}}(t) = \mathcal{H}_q + \mathcal{H}_f + \mathcal{H}_{q-f} + \mathcal{H}_{\text{man}}(t) \quad (1)$$

takes into account the qubit and the fluctuator (\mathcal{H}_q and \mathcal{H}_f) as well as the interaction between them (\mathcal{H}_{q-f}). We also included into $\tilde{\mathcal{H}}(t)$ the “manipulation” part, $\mathcal{H}_{\text{man}}(t) = S_x F \cos \omega t$, which in spin terms is an oscillating magnetic field in the x -direction applied to the qubit (F is the Rabi frequency). The other parts of the Hamiltonian can be written as

$$\mathcal{H}_q = \frac{E}{2} S_z, \quad \mathcal{H}_f = \frac{e}{2} s_z, \quad \mathcal{H}_{q-f} = \frac{u}{2} (S_x s_x + S_y s_y). \quad (2)$$

Here E is the distance between the qubit levels, e is the distance between the fluctuator’s levels, S_i and s_i are the Pauli matrices acting, respectively, in the spaces of the qubit and fluctuator, u is the off-diagonal coupling constant⁽¹⁾. Below we will use the so-called rotating wave approximation replacing $\cos \omega t \rightarrow (1/2) \exp[i\omega t]$, see, e.g., Slichter [12]. This approximation, which neglects higher harmonics of the response, is valid close to the resonance, i.e., when $|E - \omega|, |e - \omega|, |u| \ll \omega$. We also omit the diagonal qubit-fluctuator interaction $\propto S_z s_z$ which is important only when the qubit and the fluctuator are far from the resonance. On the contrary, the off-diagonal part is important only when the qubit and the fluctuator have close energy splitting, $|E - e| \ll E, e$. The importance of this interaction was stressed in ref. [1]. We will also assume that

$$|u| \ll \omega \lesssim T, \quad (3)$$

where ω and T are measured in energy units. Only in this case does the qubit act as a resonant system.

We are not going to take the thermal bath explicitly into account. Instead we introduce damping phenomenologically into the equation for the density matrix evolution.

The Hamiltonian (1) in the resonant approximation can be expressed as a 4×4 matrix:

$$\mathcal{H} = \frac{1}{2} \begin{pmatrix} -E - e & 0 & Fe^{i\omega t} & 0 \\ 0 & -E + e & u & Fe^{i\omega t} \\ Fe^{-i\omega t} & u^* & E - e & 0 \\ 0 & Fe^{-i\omega t} & 0 & E + e \end{pmatrix}. \quad (4)$$

Shown in fig. 1 are the energy terms of the Hamiltonian *vs.* the qubit energy splitting E for a fixed fluctuator splitting e and $u = 0.1e$. Near the resonance, $E = e$, all 4 levels are involved in the AC-induced transitions. In this region one can expect strong influence of the fluctuator on the qubit response.

The 4-level system qubit+fluctuator is characterized by a 4×4 density matrix $\tilde{\rho}_{\mu\nu}(t)$ whose diagonal matrix elements $n_{\downarrow\downarrow}, n_{\downarrow\uparrow}, n_{\uparrow\downarrow}, n_{\uparrow\uparrow}$ describe the occupations of each of the levels. In

⁽¹⁾We have taken into account only the off-diagonal coupling, $(u/2)(\mathbf{S}_\perp \cdot \mathbf{s}_\perp)$, between the qubit and fluctuator. One can also include $\mathcal{H}_z = (v/2)(\mathbf{S}_z \cdot \mathbf{s}_z)$. This interaction contributes to the decoherence of the qubit via random modulation of the Rabi frequency [11]. However, as we checked, it does not specifically affect the visibility of Rabi oscillations as long as the fluctuator is decoupled from the AC field, $\mathcal{H}_{12} = \mathcal{H}_{34} = 0$.

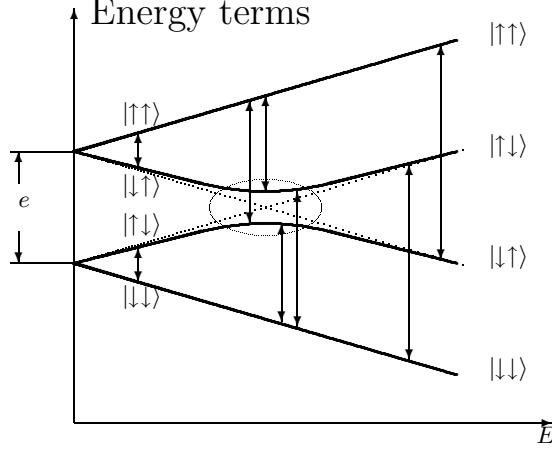


Fig. 1 – Diagram of the terms of the 4-level system consisting of the qubit and the fluctuator. Transitions induced by the AC field are indicated with arrows.

the presence of the high-frequency driving force $\cos \omega t$ the off-diagonal matrix elements also rapidly oscillate:

$$\begin{pmatrix} n_{\downarrow\downarrow} & -if e^{i\omega t} & -ig e^{i\omega t} & -ij e^{2i\omega t} \\ if^* e^{-i\omega t} & n_{\downarrow\uparrow} & -ik & -il e^{i\omega t} \\ ig^* e^{-i\omega t} & ik^* & n_{\uparrow\downarrow} & -im e^{i\omega t} \\ ij^* e^{-2i\omega t} & ig^* e^{-i\omega t} & im^* e^{-i\omega t} & n_{\uparrow\uparrow} \end{pmatrix}. \quad (5)$$

It is convenient to introduce a matrix with elements that all vary slowly with time:

$$\hat{\rho}_{\mu\nu} \equiv \begin{pmatrix} n_{\downarrow\downarrow} & -if & -ig & -ij \\ if^* & n_{\downarrow\uparrow} & -ik & -il \\ ig^* & ik^* & n_{\uparrow\downarrow} & -im \\ ij^* & il^* & im^* & n_{\uparrow\uparrow} \end{pmatrix}. \quad (6)$$

The matrix (6) is just the density matrix in the frame rotating with frequency ω around the z -axis in the qubit spin space. In this frame the von Neumann equation, $\partial_t \hat{\rho} = -i[\mathcal{H}, \hat{\rho}]$, involves a time-independent Hamiltonian,

$$\mathcal{H} = \frac{F}{2} \begin{pmatrix} -\mathcal{E} - \epsilon & 0 & 1 & 0 \\ 0 & -\mathcal{E} + \epsilon & \eta & 1 \\ 1 & \eta^* & \mathcal{E} - \epsilon & 0 \\ 0 & 1 & 0 & \mathcal{E} + \epsilon \end{pmatrix}. \quad (7)$$

Here $\mathcal{E} = (E - \omega)/F$, $\epsilon = (e - \omega)/F$, $\eta = u/F$. The matrix elements $\mathcal{H}_{13} = \mathcal{H}_{31}^*$ and $\mathcal{H}_{24} = \mathcal{H}_{42}^*$ correspond to transitions between the qubit levels, while $\mathcal{H}_{23} = \mathcal{H}_{32}^*$ describe simultaneous “flips” of the qubit and the fluctuator.

Our next step is to rewrite the von Neumann equation

$$\dot{\rho} = \hat{L} \rho \quad (8)$$

as an equation for the vector ρ with 16 components:

$$\{n_{\downarrow\downarrow}, n_{\downarrow\uparrow}, n_{\uparrow\downarrow}, n_{\uparrow\uparrow}, g, g^*, l, l^*, f, f^*, m, m^*, k, k^*, j, j^*\}. \quad (9)$$

The 16×16 evolution matrix \hat{L} is read off from the commutator $[\mathcal{H}, \rho]$.

When no heat bath is coupled to the 4-level system, the matrix \hat{L} has rank 12. This reflects the fact that in the absence of dissipation there are four conserved quantities, namely the occupation numbers of the eigenstates of the Hamiltonian equation (7).

The system reaches a stationary state only in the presence of dissipation. We take the dissipation into account in the following way. First, we assume that at large time the system will reach a stationary, however non-equilibrium, state ρ^∞ , where the matrix elements depend on the relationship between the pumping amplitude, F , and the phase and energy relaxation rates. Given the “dissipation operator” $\hat{\Gamma}$, which takes into account both phase and energy relaxation, one can determine ρ^∞ as a solution of the equation

$$(\hat{L} + \hat{\Gamma})\rho^\infty = 0. \quad (10)$$

The dissipation operator $\hat{\Gamma}$ is in general non-linear and is determined by the particular model chosen for the thermal bath. We are mostly interested diagonal matrix elements of the stationary density matrix

$$\rho_{\text{diag}}^\infty = \{n_{\downarrow\downarrow}^\infty, n_{\downarrow\uparrow}^\infty, n_{\uparrow\downarrow}^\infty, n_{\uparrow\uparrow}^\infty\}, \quad (11)$$

which can be specified phenomenologically in a model-independent way. The dynamics of the remaining 12 matrix elements $\rho_{\text{off-diag}}^\infty$ is rather insensitive to the details of the model.

We therefore suggest the following approximative strategy: $\hat{\Gamma}$ is replaced by some linear operator (16×16 matrix) $\hat{\Gamma}_0$ and the operator $\hat{L} + \hat{\Gamma}_0$ is presented as

$$\hat{L} + \hat{\Gamma}_0 = \begin{pmatrix} \hat{M}_2 & \hat{M}_1^\dagger \\ \hat{M}_1 & \hat{M}_0 \end{pmatrix} \quad \begin{matrix} 4 \\ 12 \end{matrix}; \quad \begin{matrix} 4 \\ 12 \end{matrix} \quad ; \quad (12)$$

$\rho_{\text{diag}}^\infty$ is then chosen phenomenologically and approximate values for the off-diagonal and diagonal elements of the density matrix ρ^∞ are chosen so that they are stationary with respect to the time evolution given by $\hat{L} + \hat{\Gamma}_0$:

$$\rho_{\text{off-diag}}^\infty = -\hat{M}_0^{-1}\hat{M}_1\rho_{\text{diag}}^\infty. \quad (13)$$

The situation here is somewhat similar to nonlinear AC conductivity in disordered metals: when the scattering of itinerant electrons is predominantly elastic, the details of the inelastic relaxation are not important in spite of the Joule heat. This inelastic relaxation rate enters usually only through the effective field-dependent temperature, while the temperature-dependent conductivity can be determined without taking energy relaxation into account [13].

We write $\rho = \rho^\infty + \rho_1$ and determine the dynamics of the nonstationary part by the equation $\dot{\rho}_1(t) = (\hat{L} + \hat{\Gamma}_0)\rho_1(t)$. The solution can be presented as an expansion,

$$\rho(t) = \rho^\infty + \sum_{i=1}^{16} c_i e_i e^{\lambda_i t}, \quad (14)$$

over the eigenvectors, e_i , of the matrix $\hat{L} + \hat{\Gamma}_0$, λ_i being the corresponding eigenvalues. Note that it is important to choose $\hat{\Gamma}_0$ in such a way that all components of ρ_1 decay to 0. That is, such that we have $\text{Re } \lambda_i < 0$ for all i .

One can specify the initial conditions assuming that the external AC field is switched on at $t = 0$:

$$\rho_{11} \equiv n_{\downarrow\downarrow}(0) = 1, \quad \rho_{i \neq 1, k \neq 1}(0) = 0. \quad (15)$$

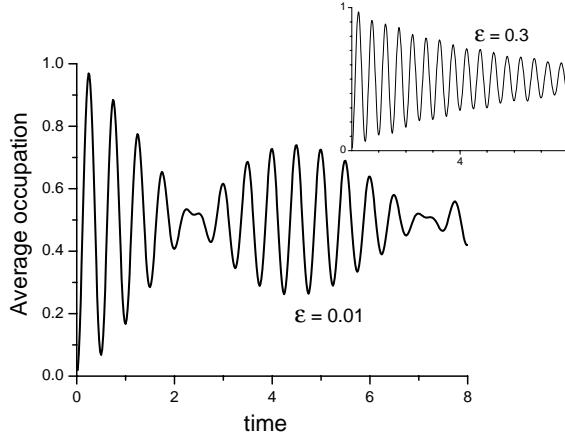


Fig. 2 – The qubit upper level occupation, $n_{\uparrow\downarrow} + n_{\uparrow\uparrow}$, as a function of time measured in Rabi periods, F^{-1} . The parameters are: $\gamma = 0.03F$, $\eta = 0.1$, $\mathcal{E} = 0$ and $n_{\uparrow\uparrow}/n_{\uparrow\downarrow} = 0.43$. Values of the fluctuator's deviation from the resonance, ϵ , are shown near the curves.

We have checked numerically that the results are only weakly sensitive to initial conditions. It is more important to determine ρ^∞ , which strongly depends on the pumping strength, F , as well as on the relaxation times of the phase, τ_2 , and energy, τ_1 . For strong pumping $F^2\tau_1\tau_2 \gg 1$ the levels with different orientations of the qubit have almost the same occupations. The stationary populations of the fluctuator level depend on the interplay between its coupling with the qubit and the energy relaxation rate. If the coupling is weak, the stationary occupancies of the levels are given by a thermal distribution:

$$n_{\uparrow\uparrow} = n_{\downarrow\downarrow}, \quad n_{\uparrow\downarrow} = n_{\downarrow\uparrow}, \quad n_{\uparrow\uparrow}/n_{\uparrow\downarrow} = e^{-e/kT}.$$

Since we do not focus on the temperature dependence, we will set the ratio $n_{\uparrow\uparrow}/n_{\uparrow\downarrow}$ equal to some constant which in principle depends on the Rabi frequency F , interaction strength u and the fluctuator relaxation rates.

The advantage of this method is that as soon as stationary level populations are specified the results only weakly depend on the specific form of the matrix $\hat{\Gamma}_0$. For simplicity, we adopt a model similar to the one conventionally used to describe relaxation in various two-level systems, see, *e.g.*, ref. [12]. If the qubit does not directly interact with the environment, while the fluctuator has a finite dephasing rate γ and energy relaxation rate 2γ , then $\hat{\Gamma}_0$ is diagonal, $(\hat{\Gamma}_0)_{ik} = \gamma_i \delta_{ik}$, where

$$\gamma_i = \begin{cases} -2\gamma, & 1 \leq i \leq 4, \\ -\gamma, & 9 \leq i \leq 12. \end{cases}$$

This damping matrix differs from the one used in ref. [8] by taking into account relaxation of the diagonal elements of the density matrix.

We have calculated the probability to find the qubit in its upper state, $n_{\uparrow\uparrow}(t) + n_{\uparrow\downarrow}(t)$, from eq. (14) and plotted it as a function of time in fig. 2.

The qubit is assumed to be in resonance with the pumping field, $\mathcal{E} = 0$. We observe clear Rabi oscillations with frequency F and a decay in time due to finite relaxation, $\gamma = 0.03F$. The oscillations are perturbed by the fluctuator coupled to the qubit with the strength $\eta = 0.1$. Two curves correspond to different shifts of the fluctuator from the resonance, $\epsilon = \pm 0.3$ (these

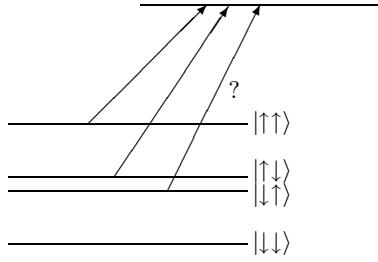


Fig. 3 – Energy levels and transitions.

two curves are practically the same) and $\epsilon = 0.01$. One can see that close to the resonance there are beats in the Rabi oscillations. This should be expected from fig. 1, since at the resonance both transitions participate. Also, for times shorter than the inverse beating frequency or larger damping the resonance effect amounts to a suppression of the Rabi oscillations. It is clear that the closer the fluctuator is to resonance the stronger it suppresses Rabi oscillations. This fact confirms that the suppression of Rabi oscillations observed in ref. [1] can indeed be induced by coupling to a fluctuator close to resonance. Large damping, certainly, makes the beats less visible.

Experimentally, the qubit state is probed spectroscopically by inducing transitions from the upper state to some high-energy state having small lifetime due to tunneling [1]. One can start from the assumption that the transition probability is independent of the fluctuator state. This is why we present the results for the sum $n_{\uparrow\downarrow} + n_{\uparrow\uparrow}$ in fig. 2, see also ref. [8].

In reality, the transition probabilities from the states $|\uparrow\uparrow\rangle$ and $|\uparrow\downarrow\rangle$ can significantly differ. Moreover, transitions from the qubit down states $|\downarrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ can be induced. For example, if the qubit is resonant with the fluctuator, the levels $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ will be close in energy, while the level $|\uparrow\uparrow\rangle$ will be removed from this close pair (see fig. 3).

It is then hard to expect that the driving force is resonant with transitions from the levels

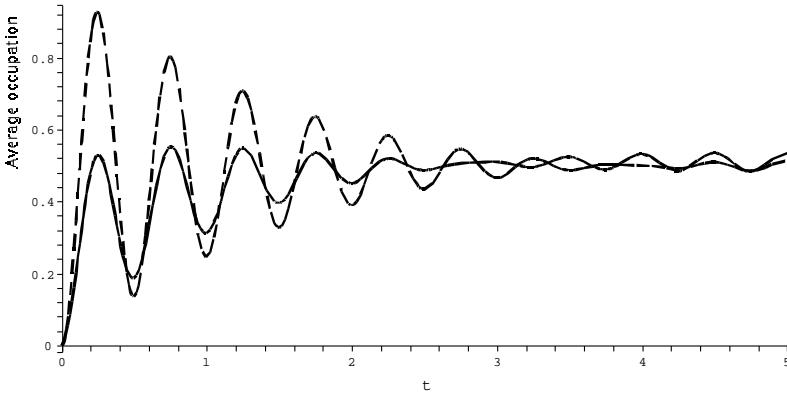


Fig. 4 – Solid curve: weighted average of the qubit upper state population, $0.5n_{\uparrow\downarrow} + n_{\uparrow\uparrow}$, for $\epsilon = 0.01$. Other parameters are the same as in fig. 2. For comparison, the sum $n_{\uparrow\downarrow} + n_{\uparrow\uparrow}$ is shown by the dashed curve.

$|\uparrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$ without also being coupled to the $|\downarrow\uparrow\rangle$ state. Different response of the different levels to the measurement pulse can significantly change the shape of the curve. To illustrate this point we plot in fig. 4 a weighted average, $0.5n_{\uparrow\downarrow} + n_{\uparrow\uparrow}$, assuming that the escape from the state $|\uparrow\uparrow\rangle$ is twice as probable as the escape from the state $|\uparrow\downarrow\rangle$ (solid curve). Interestingly, this curve is much more similar to the experimentally observed one [1] than the simple sum shown for comparison (dashed line). Namely, the plotted quantity oscillates never exceeding 1/2 and tends to $\approx 1/2$ at large time. As far as we understand it, it is unclear what combination of the occupation numbers of the states is actually probed in the experiment. In the second set of experiments [2] a different readout scheme was applied, but no Rabi oscillation traces were presented.

In conclusion, we develop a general scheme of analytical analysis of time evolution of the density matrix of a multi-level quantum system coupled to a heat bath. In this scheme the dissipation is taken into account phenomenologically, while the quantum dynamics in the presence of arbitrarily strong driving force is evaluated explicitly. We applied this scheme to the system of a qubit coupled to a fluctuator which probably was experimentally realized in ref. [1]. We demonstrated that if the fluctuator is close to resonance with the qubit, the Rabi oscillations of the qubit are suppressed at short times and demonstrate beatings when the damping is weak enough. We also found that if the read-out signal depends on the state of the fluctuator, the visibility of the Rabi oscillations can be substantially reduced. This effect can naturally explain the experimental results of ref. [1].

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