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Plasma resonance at low magnetic fields as a probe of vortex line meandering in layered superconductors

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We consider the magnetic-field dependence of the plasma resonance frequency in pristine and in irradiated $Bi_2Sr_2CaCu_2O_8$ crystals near T_c. At low magnetic fields we relate linear in field corrections to the plasma frequency to the average distance between the pancake vortices in the neighboring layers (wandering length). We calculate the wandering length in the case of thermal wiggling of vortex lines, taking into account both Josephson and magnetic interlayer coupling of pancakes. Analyzing experimental data, we found that (i) the wandering length becomes comparable with the London penetration depth near T_c and (ii) at small melting fields (<20 G) the wandering length does not change much at the melting transition. This shows existence of the line liquid phase in this field range. We also found that pinning by columnar defects affects weakly the field dependence of the plasma resonance frequency near T_c.

Josephson plasma resonance (JPR) measurements in highly anisotropic layered superconductors provide unique information on the interlayer Josephson coupling and on the effect of pancake vortices on this coupling. The squared c-axis plasma resonance frequency, ω_p^2, is proportional to the average interlayer Josephson energy, $\omega_p^2 \propto J_0\langle\cos\varphi_{n,n+1}(\mathbf{r})\rangle$, where J_0 is the Josephson critical current, $\varphi_{n,n+1}(\mathbf{r})$ is the gauge-invariant phase difference between layers n and $n+1$, and $\mathbf{r}$ is the in-plane coordinate. Here $\langle\cdots\rangle$ means average over thermal disorder and pinning. Thermal fluctuations and uncorrelated pinning lead to misalignment of pancake vortices induced by the magnetic field applied along the c axis. Misalignment results in nonzero phase difference and in the suppression of Josephson coupling and plasma frequency.[1] Thus, the ω_p dependence on the c-axis magnetic field measures the c-axis correlations of pancakes in the vortex state.

The JPR measurements performed in the liquid vortex phase at relatively high magnetic fields, $B>B_J=\Phi_0/\lambda_J^2$, revealed that the plasma frequency drops approximately as $1/\sqrt{B}$.[2] Here $\lambda_J=\gamma s$ is the Josephson length, γ is the anisotropy ratio, and s is the interlayer distance. The above dependence is characteristic for the pancake liquid weakly correlated along the c axis.[3,4] Here a pancake in a given layer is shifted by a distance of the order of vortex spacing $a=(\Phi_0/B)^{1/2}\ll\lambda_J$ from the nearest pancake in the neighboring layer. Thus at high fields many pancake vortices contribute to the suppression of the phase difference at a given point $\mathbf{r}$, because λ_J determines the decay length for the phase difference induced by misaligned pancakes of a given vortex line.[5] In contrast, in the vortex solid a lattice of vortex lines forms as shown by neutron scattering and μ^+SR data.[6] JPR measurements in $Bi_2Sr_2CaCu_2O_{8-\delta}$ (Bi-2212) crystals[7–9] have shown that in the fields above 20 Oe the interlayer phase coherence changes drastically at the transition line [$\langle\cos\varphi_{n,n+1}(\mathbf{r})\rangle$ jumps], implying the decoupling nature of the first-order melting transition, see discussion in Ref. 10.

In the following we focus on the low magnetic fields, $B<B_J$, near the T_c regime. Here the intervortex distance is much larger than λ_J, and the Josephson coupling in the region occupied by a given vortex is not suppressed by other vortices. In such a single vortex regime the Josephson energy increases linearly with displacements of nearest pancakes in neighboring layers, when $|\mathbf{u}_{n,n+1}|\equiv|\mathbf{r}_n-\mathbf{r}_{n+1}|>\lambda_J$, see Ref. 5 (here $\mathbf{r}_n$ is the coordinate of a pancake in the layer n). This leads to the confinement of pancakes in neighboring layers, and c-axis correlated pancakes (i.e., vortex lines) may be preserved above the melting transition. We show that it is the linear decrease of ω_p^2 with B that characterizes such a vortex state. The linear dependence was observed experimentally in Refs. 7–9 in both solid and liquid vortex states in Bi-2212 crystals in fields below 20 Oe near T_c providing evidence for a line structure of the vortex liquid state at low fields.

We calculate the plasma frequency at low magnetic fields B and near T_c assuming that only vortices induced by the applied magnetic field $\mathbf{B}\|c$ contribute to the field dependence of ω_p. We, thus, ignore the contribution of thermally excited vortices and antivortices to the field dependence of the plasma frequency.

The JPR absorption is described by a simplified equation[11] for small oscillations of the phase difference $\varphi'_{n,n+1}(\mathbf{r},\omega)$ induced by an external microwave electric field with the amplitude $\mathcal{D}$ and the frequency ω applied along the c axis:

$$\left[\frac{\omega(\omega+i\Gamma_c)}{\omega_0^2}-1+\lambda_J^2\hat{L}\nabla_{\mathbf{r}}^2-\mathcal{V}_n(\mathbf{r})\right]\varphi'_{n,n+1}=-\frac{i\omega\mathcal{D}}{4\pi J_0}. \tag{1}$$

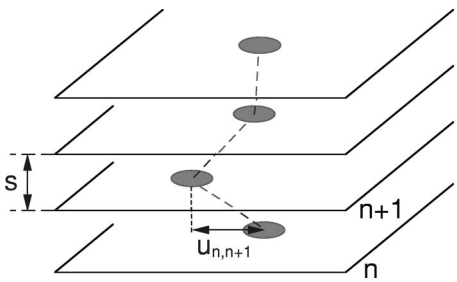

FIG. 1. Meandering of pancakes along the vortex line in the single vortex regime at low magnetic fields in highly anisotropic layered superconductors.

Here $\mathcal{V}_n(\mathbf{r})=\cos\varphi_{n,n+1}(\mathbf{r})-1$ is the effective potential, and $\varphi_{n,n+1}(\mathbf{r})$ is the phase difference induced by vortices misaligned due to the thermal fluctuations and pinning in the absence of a microwave field. In Eq. (1) we neglect the time variations of $\varphi_{n,n+1}(\mathbf{r},t)$ because the plasma frequency is much higher than the characteristic frequencies of vortex fluctuations, see below. Further, $\omega_0(T)=c/\sqrt{\epsilon_0}\lambda_c(T)$ is the zero-field plasma frequency,[12] ϵ_0 is the high-frequency dielectric constant, λ_{ab} and $\lambda_c=\gamma\lambda_{ab}$ are the components of the London penetration depth, $E_J=E_0/\lambda_J^2$ is the Josephson energy per unit area, and $E_0=s\Phi_0^2/16\pi^3\lambda_{ab}^2$ is the characteristic pancake energy. The inductive matrix $\hat{L}$ is defined as $\hat{L}A_n=\Sigma_m L_{nm}A_m$ with $L_{nm}\approx(\lambda_{ab}/2s)\exp(-|n-m|s/\lambda_{ab})$. The parameter $\Gamma_c=4\pi\sigma_c/\epsilon_0$ describes dissipation due to quasiparticles. Here σ_c is the c-axis quasiparticle conductivity in the superconducting state. Practically it coincides with conductivity right above T_c.

The absorption in the uniform ac electric field is defined by the imaginary part of the inverse dielectric function

$$\mathrm{Im}\frac{1}{\epsilon(\omega)}=\frac{1}{\epsilon_0}\sum_{\alpha n}\int d\mathbf{r}\frac{\langle\Psi^*_{\alpha n}(0)\Psi_{\alpha n}(\mathbf{r})\rangle\omega^3\Gamma_c}{(\omega^2-\omega_\alpha^2)^2+\omega^2\Gamma_c^2}, \tag{2}$$

where $E_\alpha=1-\omega_\alpha^2/\omega_0^2$ and $\Psi_{\alpha n}(\mathbf{r})$ are the eigenvalues and eigenfunctions of the operator $-\lambda_J^2\hat{L}\nabla_{\mathbf{r}}^2+\mathcal{V}_n(\mathbf{r})$.

Consider magnetic fields $B\ll\Phi_0/4\pi\lambda_{ab}^2, B_J$ (single vortex regime). The phase difference near a given vortex line is induced by displacements of pancakes in neighboring layers along this vortex line (see Fig. 1). The potential $\mathcal{V}_n(\mathbf{r})$ at distances $u_{n,n+1}\ll r\ll\lambda_J$ is determined by the phase difference produced by nearest pancakes in neighboring layers n and $n+1$ relatively displaced at distance $\mathbf{u}_{n,n+1}$:

$$\varphi_{n,n+1}(\mathbf{r})\approx[\mathbf{r}\times\mathbf{u}_{n,n+1}]/r^2. \tag{3}$$

At large distances $r\gg\lambda_J$ the potential drops exponentially and at small distances $r\lesssim u_{n,n+1}$ it tends to a constant. This potential is attractive and there are localized and delocalized states. At $a\gg\lambda_J, u_{n,n+1}$ the main contribution to absorption is coming from the most homogeneous delocalized state. Such a state determines the center of JPR line $\omega_p(B,T)$. Other states lead to inhomogeneous line broadening in addition to broadening caused by quasiparticle dissipation described by Γ_c. The latter mechanism of broadening dominates at low magnetic fields near T_c.

The strength of the potential with respect to the kinetic term, $\lambda_J^2\hat{L}\nabla_{\mathbf{r}}^2$, is characterized by the dimensionless parameter r_w^2/λ_J^2 which we assume to be small in the following calculations. Here r_w is the elemental wandering length of vortex lines, $r_w^2\equiv\langle\mathbf{u}_{n,n+1}^2\rangle$. Then we use perturbation theory with respect to the potential $\mathcal{V}_n(\mathbf{r})$ to find the energy of the most uniform delocalized state. The unperturbed wave function of this state is given by a constant. The first-order correction to the energy of this most homogeneous delocalized state is given by the space average of the potential, averaging is over $\mathbf{r}$ and n. This space average is equivalent to the thermal average of $\cos\varphi_{n,n+1}(\mathbf{r})-1\approx-\varphi_{n,n+1}^2(\mathbf{r})/2$. Using Eq. (3) we obtain a simple relation connecting field-induced suppression of the plasma frequency $\omega_p(B,T)$ with r_w for the case $r_w<\lambda_J$:

$$\frac{\omega_0^2(T)-\omega_p^2(B,T)}{\omega_0^2(T)}\approx\frac{\langle\varphi_{n,n+1}^2(\mathbf{r})\rangle}{2}\approx\frac{\pi B r_w^2}{2\Phi_0}\ln\frac{\lambda_J}{r_w}. \tag{4}$$

The relation (4) is very general and does not depend on the mechanism of the vortex wandering. It allows one to extract r_w^2 from the plasma resonance measurements. The field dependence of the resonance temperature, $T_r(B)$, is determined by the equation $\omega_p^2(B,T_r)=\omega^2$. According to Eq. (4) this gives a linear dependence at small fields, $T_r(B)\approx T_r(0)+(dT_r/dB)B$, and r_w^2 is directly related to the slope of this dependence

$$r_w^2\ln\frac{\lambda_J}{r_w}=\frac{2\Phi_0}{\pi}\frac{d\ln\omega_0^2(T)}{dT}\left(\frac{dT_r}{dB}\right)_{B\to 0}. \tag{5}$$

We now calculate r_w^2 when wandering of the vortex lines is caused by thermal fluctuations. In the single vortex regime r_w^2 is determined by the wandering energy consisting of the Josephson and magnetic contributions,

$$\mathcal{F}_w\approx\frac{s}{2}\sum_n\left[\varepsilon_{1J}\left(\frac{\mathbf{r}_{n+1}-\mathbf{r}_n}{s}\right)^2+w_M\mathbf{r}_n^2\right], \tag{6}$$

where $\varepsilon_{1J}\approx[\Phi_0^2/(4\pi\lambda_c)^2]\ln(\lambda_J/r_w)$ is the line tension due to the Josephson coupling and $w_M\approx[\Phi_0^2/(4\pi\lambda_{ab}^2)^2]\ln(\lambda_{ab}/r_w)$ is the effective cage potential, which appears due to strongly nonlocal magnetic interactions between pancake vortices in different layers.[13,10] Assuming Gaussian fluctuations we have

$$r_{wT}^2=\int\frac{dq_z}{2\pi}\frac{4T(1-\cos q_z s)}{2(\varepsilon_{1J}/s^2)(1-\cos q_z s)+w_M}=\frac{2sT}{\varepsilon_{1J}}f(\zeta), \tag{7}$$

$$f(\zeta)=\frac{\zeta}{1+\zeta+\sqrt{1+\zeta}},$$

where the parameter $\zeta(T)=4\lambda_{ab}^2(T)/\lambda_J^2$ describes the relative roles of the Josephson and magnetic interactions. Substituting this result into Eq. (4) we obtain

$$\frac{\omega_0^2(T)-\omega_p^2(B,T)}{\omega_0^2(T)}\approx\frac{B}{B_0}f(\zeta) \tag{8}$$

with $B_0=\Phi_0^3/16\pi^3\lambda_c^2 sT=B_J(E_0/T)$. We stress that this result of the single vortex regime is valid in both solid and liquid vortex states for $B\ll B_J$, because in this field range wandering of lines at short scales does not change much at

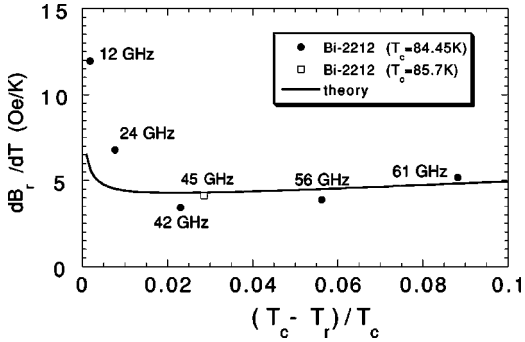

FIG. 2. Comparison of the experimental temperature dependence of dB_r/dT obtained in Ref. 7 using different microwave frequencies with theoretical dependence from Eq. (9) (see text). We also show a data point from Ref. 9 obtained for Bi-2212 with slightly different T_c.

the melting point. The difference between these states appears only in the second order in the magnetic field. At $B=0$ the resonance occurs at the temperature $T_r(\omega)$. The slope of the curve $B_r(T)$ at small B_r is

$$\frac{dB_r}{dT}=\frac{1}{f(\zeta_r)}\left(\frac{dB_0}{dT}\right)_{T=T_r}, \quad \zeta_r=\frac{4\lambda_{ab}^2(T_r)}{\lambda_J^2}. \tag{9}$$

The crossover region from the line liquid, $r_w^2 \ll a^2$, to the pancake liquid, where $r_w^2 \approx a^2$, takes place at the magnetic field $B \approx \min[\pi B_0/2f(\zeta), B_J]$.

To compare our calculations with experiment we plot in Fig. 2 dependence of dB_r/dT vs reduced resonance temperature at $B=0$, $t_r=(T_c-T_r)/T_c$, obtained in Refs. 7 and 8 using different microwave frequencies (shown in the plot) for Bi-2212 with $T_c=84.45$ K. We also show a data point obtained by Matsuda[9] for Bi-2212 with close T_c, $T_c=85.7$ K. To calculate the dependence dB_r/dT from Eq. (9) we need dependencies $\lambda_c(T)$ and $\lambda_{ab}(T)$, which determine dependencies $B_0(T)$ and $\zeta(T)$. $\lambda_c(T)$ was found from temperature dependence of ω_0 at zero field, which we fit as $\omega_0(t)/2\pi \approx (133.5\ \text{GHz})t^{0.32}$, and taking $\epsilon_c=11$. Matsuda[9] obtained similar temperature dependence of ω_0 using direct frequency scan. $\lambda_{ab}(T)$ was obtained assuming temperature-independent γ, which we adjusted to obtain the best agreement between the theoretical and experimental curves giving $\gamma=480$. Nonmonotonic temperature dependence of dB/dT_r arises from competition between two factors in Eq. (9): increase at low temperatures is due to the factor $1/f(\zeta) \propto \lambda_{ab}^{-2}$ at $\zeta \ll 1$, and increase at temperatures close to T_c is due to nonlinearity of the dependence $\lambda_c^{-2}(T)$, which leads to the divergency of dB_0/dT at $T \to T_c$. As one can see from the plot, our theory describes satisfactorily the field dependence of ω_p not very close to T_c, i.e., in the region where the critical fluctuations are not very strong. The region of critical fluctuations is beyond applicability of our theory.

We can now check the validity of our approximations: the static approximation for the potential in Eq. (1) and the perturbation theory with respect to the potential. The maximum frequency of vortex fluctuations in the single vortex regime is $\omega_{fl} \approx \varepsilon_{1J}/s^2\eta$, where the vortex viscosity η estimated from flux flow resistivity is in the interval 10^{-6}–10^{-7} g/cm s (see Ref. 14). This gives $\omega_{fl}/\omega_p \approx \Phi_0^2\sqrt{\epsilon_0}/16\pi^2 s^2 \lambda_c c\eta \lesssim 0.1$ near T_c. Condition $r_w^2/\lambda_J^2 \lesssim 1$ for applicability of the perturbation theory is satisfied at $t>0.02$.

Near T_c the experimental curve $T_r(B)$ shows no change of slope when it crosses the melting line, $H_m \approx 1.3[\text{Oe/K}](T_c-T)$.[7] This gives evidence that the parameter r_w^2 does not change much at melting. Using Eq. (5) we estimate that, near the melting line $r_w^2/a^2 \lesssim 0.1$ at temperatures studied. This confirms the line structure of the vortex liquid above the melting line at low fields, though vortex lines wander over extended distances already in the solid phase due to high temperatures, see Fig. 1. The estimated value, $r_w \approx 1$ μm at 77 K, is comparable with both λ_J and λ_{ab}. In Bi-2212 crystals near optimal doping crossover to a pancake liquid occurs in the field interval $\approx 10-15$ Oe. In less anisotropic high-T_c materials one anticipates a larger region of the line liquid on the vortex phase diagram.

Next we calculate the effect of columnar defects (CD's) on the parameter r_w^2 at high temperatures. Columnar defects always straighten vortex lines and reduce r_w. At low temperatures this effect is very strong. Each vortex line is localized near one defect and its wandering is much smaller than in an unirradiated superconductor. At high temperatures lines start to distribute over a large number of defects and the effect of CD's progressively decreases. We consider the extreme case of very high temperatures when the effect of defects can be considered within perturbation theory. This approach is applicable at temperatures higher than the pinning energy of pancakes by CD, $T > \pi E_0 \ln(b/\xi_{ab})$, where b is the CD radius and ξ_{ab} is the superconducting correlation length. This situation corresponds to experiments.[7–9]

The free-energy functional is

$$\mathcal{F}=\mathcal{F}_w+\mathcal{F}_p, \quad \mathcal{F}_p=\sum_n U(\mathbf{r}_n), \tag{10}$$

where $U(\mathbf{r})=\Sigma_i V(\mathbf{r}-\mathbf{R}_i)$ is the pinning potential of CD's, $\mathbf{R}_i$ are the positions of CD's, and $V(\mathbf{r})=\pi E_0 \ln(1-b^2/r^2)\exp(-r/\lambda_{ab})$ is the pinning potential of the individual CD.[15] Expanding with respect to disorder up to second-order terms we obtain the correction, $(-r_{wD}^2)$, to the zero-order term, r_{wT}^2, due to pinning by CD's:

$$\begin{aligned} -r_{wD}^2 &= r_w^2 - r_{wT}^2 \\ &= -\frac{s}{T}\frac{\partial}{\partial \varepsilon_{1J}} \sum_m [\langle K(\mathbf{r}_m-\mathbf{r}_0)\rangle_0 - \langle\langle K(\mathbf{r}_0'-\mathbf{r}_0)\rangle_0\rangle_0'], \end{aligned} \tag{11}$$

where $\langle \cdots \rangle_0$ stands for statistical average for a system without disorder and $K(\mathbf{r})$ is the correlation function of disorder,

$$K(\mathbf{r}'-\mathbf{r})=\langle U(\mathbf{r}')U(\mathbf{r})\rangle_D - \langle U(\mathbf{r})\rangle_D^2. \tag{12}$$

When r_w is much larger than the distance between columns we can transform Eq. (11) to a simpler form

$$r_{wD}^2=\frac{2sK_0}{T}\frac{\partial}{\partial \varepsilon_{1J}}\sum_m \left[\frac{2}{\langle(\mathbf{r}_m-\mathbf{r}_0)^2\rangle_0}-\frac{1}{\langle \mathbf{r}_0^2\rangle_0}\right], \tag{13}$$

where $K_0 \equiv \int d\mathbf{r} K(\mathbf{r}) = n_\phi [\pi^2 b^2 E_0 \ln(\lambda_{ab}/b)]^2$. Here n_ϕ is the concentration of the CD's. The correction is determined by the lateral line displacement $\langle (\mathbf{r}_m - \mathbf{r}_0)^2 \rangle_0$ which we calculate as

$$\langle (\mathbf{r}_m - \mathbf{r}_0)^2 \rangle_0 = \int \frac{dq_z}{2\pi} \frac{4T(1-\cos m q_z s)}{2(\varepsilon_{1J}/s^2)(1-\cos q_z s) + w_M} = \frac{4sT}{\varepsilon_{1J}} \frac{v(1-v^m)}{(1-v^2)}, \quad v = \frac{\zeta + 2 - 2\sqrt{\zeta+1}}{\zeta}. \tag{14}$$

Combining Eqs. (13) and (14) we finally obtain for r_{wD}^2

$$r_{wD}^2 = \frac{2K_0}{T^2} f_D(v). \tag{15}$$

Here the dimensionless function

$$f_D(v) \equiv \frac{(1-v)^3}{1+v} \frac{d}{dv} \left(\frac{1+v}{1-v} \sum_{n=1}^{\infty} \frac{1}{v^{-n} - 1} \right)$$

has the limits $f_D(0) = 1$ and $f_D(v) \approx 2.16 - 2\ln(1-v)$ at $v \to 1$. Comparing this equation with Eq. (7) we obtain

$$\frac{r_{wD}^2}{r_{wT}^2} \approx \frac{\pi^2 n_\phi b^4}{\lambda_J^2} \left(\ln^2 \frac{\lambda_{ab}}{b} \right) \left(\frac{E_0}{T} \right)^3 \ln \frac{\lambda_J}{b}. \tag{16}$$

In the crystals studied $\lambda_J \approx 1$ μm, $\lambda_{ab}(0) \approx 2000$ Å, $b \approx 70$ Å. For the temperature range $T > 77$ K explored in Refs. 8 and 9 by the plasma resonance this gives a very small correction ($r_{wD}^2 / r_{wT}^2 \approx 10^{-3}$). Therefore, the effect of pinning by CD's on the field dependence of the plasma frequency near T_c is negligible.

It was found in Refs. 8 and 9 that after irradiation with the matching field $B_\phi = \Phi_0 n_\phi \approx 1$ T the value $(dB/dT_r)_{B \to 0}$ near T_c increases about two times in comparison with that in pristine crystals. As was estimated above, pinning due to CD's cannot give such a strong effect. One may think that irradiation reduces the value of the anisotropy parameter γ, probably due to damage of the crystal structure around the heavy-ion tracks. This assumption is consistent with recent measurements of the Josephson current in irradiated Bi-2212 mesas by Yurgens *et al.*[16] It was found that irradiation approximately doubles the Josephson current at zero field.

In conclusion, we have calculated the field dependence of the JPR frequency in the single vortex regime at low magnetic fields near T_c and demonstrated that the JPR provides a direct probe for meandering of individual lines. We have shown that the JPR data in highly anisotropic Bi-2212 crystals give evidence that at high magnetic fields $B \gg B_J$ pancakes are uncorrelated along the c axis in the vortex liquid (pancake liquid), while at lower fields, $B \leqslant B_J$, pancakes form vortex lines (line liquid). These lines, however, strongly meander in both solid and liquid vortex states due to thermal fluctuations. We have shown also that JPR data provide evidence that irradiation by heavy ions causes a significant decrease of the effective anisotropy.

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