Attenuation of two-dimensional plasmons

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We calculate the attenuation of gapless plasmons in single- and double-layer electron systems by including higher-order corrections to the electron polarization operator from the Coulomb interaction. Both particle-hole excitations and plasmons in the intermediate states contribute to the relaxation. The obtained results are in good agreement with experimental findings for temperatures above 30 K. We also make predictions for plasmon attenuation at low temperatures where no experimental data are currently available.

Among the low-dimensional electron systems semiconductor double-quantum-well structures occupy an exclusive position and attract intense current attention. The reason is that being well studied theoretically such structures offer a unique test ground for a study of cooperative phenomena in the low-dimensional structures and especially for the investigation of plasmon modes which play a primary role in formation of electronic properties in a wealth of strongly correlated systems ranging from semiconducting bilayer structures to high-T_c superconductors.

In recent experiments[1] the inelastic light scattering was used to probe plasmon properties of a double two-dimensional (2D) electron gas (2DEG) structure. The chosen technique allowed for not only measuring the spectrum of the in-phase and the out-of-phase plasmon modes, but also for finding the temperature dependence of the attenuation for different values of the wave vector, thus providing a direct access to subtle many-body effects.

The results on the temperature dependence of the plasmon attenuation were described by Landau damping.[2] However, for Landau damping to be effective the plasmon branch of the spectrum should cross the particle-hole branch, whereas the plasmon spectrum measured in Ref. 1 apparently lies *above* the particle-hole continuum. This means that at low temperatures Landau damping in the degenerate system is exponentially small and vanishes for $T\to 0$. Yet no theoretical mechanism for the temperature-dependent attenuation of gapless plasmon modes in the single- or double-layer electron systems alternative to Landau damping was proposed. Thus while at sufficiently high temperatures (above 30 K for the system and parameters used in Ref. 1, as we shall show below) the plasmon damping can indeed be attributed to Landau damping, the origin of the plasmon attenuation at lower temperatures remains an open question.

Motivated by the findings of Ref. 1, we propose a mechanism for the attenuation of gapless plasmons due to the Coulomb interaction in high orders of perturbation theory and show that both particle-hole excitations and plasmons in the intermediate states contribute to the relaxation. The obtained results are general and can be easily extended to 2D superconducting-wires networks, which became recently a subject of extensive experimental attention[3], and to plasmon excitation in 2D superconductors.[4,5]

Definition of plasmon attenuation. In a single layer the plasmon spectrum is determined from the equation $V_0(Q)P_0^R(Q,\Omega)=1$, where the bare polarization operator

$$P_0^R(Q,\Omega)=\frac{\nu}{2}\left(\frac{Qv_F}{\Omega+i0}\right)^2,\quad Qv_F<\Omega, \tag{1}$$

and $V_0(Q)=2\pi e^2/Q$ is the two-dimensional Coulomb potential. The plasmon spectrum thus is $\Omega_0(Q)=v_F(\kappa Q/2)^{1/2}$, where $\kappa=2\pi e^2\nu$ is the screening momentum and $\nu=m/\pi$ is the two-dimensional two-spin density of states. In the plasmon region, $Qv_F<\Omega$, the polarization operator P_0 does not have an imaginary part for $T=0$. This implies the absence of Landau damping and the absence of plasmon attenuation in zero approximation. The imaginary part appears upon taking into account corrections to the polarization operator, $P_i(Q,\Omega)$, due to the Coulomb interaction. In other words one have to include into consideration the higher-order relaxation processes. A similar problem for a three-dimensional nondegenerate electron plasma was considered in Ref. 6.

Defining $\gamma(Q,\Omega)=-\mathrm{Im}P_i^R(Q,\Omega)/\nu$ we can immediately write down the equation for the plasmon spectrum with attenuation

$$\Omega^2(Q)=\Omega_0^2(Q)\left(1+2i\gamma[Q,\Omega_0(Q)]\frac{\Omega_0^2(Q)}{(Qv_F)^2}\right). \tag{2}$$

Introducing plasmon attenuation as $\Gamma(Q,\Omega)=\mathrm{Im}\Omega$ and assuming that $\Gamma[Q,\Omega_0(Q)]\ll\Omega_0(Q)$ we find

$$\Gamma(Q,\Omega)=\gamma[Q,\Omega_0(Q)]\frac{\Omega_0^3(Q)}{(Qv_F)^2}. \tag{3}$$

Plasmon attenuation in a single layer electron system. In this section we calculate the correction to the polarization operator $P_i^R(Q,\Omega)$ arising from the Coulomb potential $V^A(q,\omega)$ in a single layer. By definition $Qv_F<\Omega$, but as functions of (q,ω) corrections may lie both in the particle-hole region, $|\omega|<qv_F$, and in the plasmon region of electron excitation spectrum, $qv_F<|\omega|$. The screened Coulomb potential is

$$V^A(q,\omega)=\frac{V_0(q)}{1-V_0(q)P_0^A(q,\omega)},\quad V_0(q)=\frac{2\pi e^2}{q}, \tag{4}$$

and its imaginary part in the particle-hole region reads

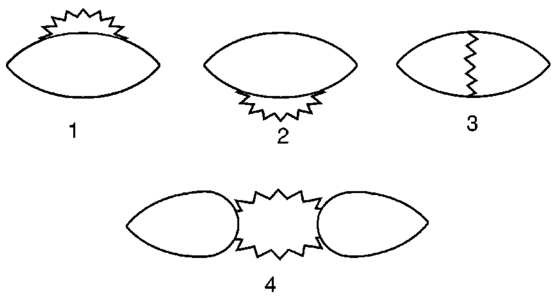

FIG. 1. Diagrams for the correction to the electron polarization operator contributing to attenuation of two-dimensional plasmons.

$$\mathrm{Im}V^A(q,\omega)=\mathrm{Im}\frac{1}{\nu}\frac{\kappa}{q+\kappa(1-i\omega/qv_F)}\approx\frac{1}{\nu}\left(\frac{\kappa}{q+\kappa}\right)^2\frac{\omega}{qv_F}. \tag{5}$$

In the plasmon region one encounters the following integral when calculating corrections:

$$\nu\int_0^{|\omega|/qv_F}dq q^2\mathrm{Im}V^A(q,\omega)$$
$$=\kappa\omega^2\mathrm{Im}\int_0^{|\omega|/qv_F}\frac{dq}{(\omega-i0)^2-\kappa v_F^2 q/2}$$
$$=\frac{2\pi}{v_F^2}\frac{\omega^3}{|\omega|}\left(\frac{2\omega^2}{\kappa v_F^2}\right)^2. \tag{6}$$

Shown in Fig. 1 are the relevant diagrams giving interaction corrections to the polarization operator P_i. In order to calculate the imaginary part of the diagrams most straightforwardly, it is convenient to make use of the unitary theorem, see, e.g., Ref. 7. The first three diagrams give

$$\mathrm{Im}P^R_{1-3}(Q,\Omega)=8\int\frac{d\epsilon}{2\pi}\frac{d^2p}{(2\pi)^2}\int\frac{d\omega}{2\pi}\frac{d^2q}{(2\pi)^2}$$
$$\times\frac{n(\epsilon+\Omega+\omega)N(-\omega)n(-\epsilon)}{N(\Omega)}$$
$$\times\mathrm{Im}G^A(p,\epsilon)\mathrm{Im}G^A(p+Q+q,\epsilon+\Omega+\omega)$$
$$\times\mathrm{Im}V^A(q,\omega)|G(p+Q,\epsilon+\Omega)$$
$$+G(p+q,\epsilon+\omega)|^2, \tag{7}$$

where the electron Green's function is defined as $G^A(p,\epsilon)=(\epsilon-\xi_p-i0)^{-1}$, $\xi_p=p^2/2m-E_F$, E_F is the Fermi energy, and $n(\epsilon)$ and $N(\Omega)$ are Fermi and Bose distributions correspondingly. Performing first the integration over the electron energy one obtains

$$\int d\xi\mathrm{Im}G^A(p,\epsilon)\mathrm{Im}G^A(p+Q+q,\epsilon+\Omega+\omega)$$
$$=\pi^2\delta\left(\Omega+\omega-\mathbf{v}\cdot(\mathbf{Q}+\mathbf{q})+\frac{(\mathbf{Q}+\mathbf{q})^2}{2m}\right). \tag{8}$$

The ϵ integration gives in its turn

$$\int d\epsilon\frac{n(\epsilon+\Omega+\omega)N(-\omega)n(-\epsilon)}{N(\Omega)}$$
$$=\int d\epsilon[n(\epsilon)-n(\epsilon+\Omega)][n(\epsilon+\Omega+\omega)+N(\omega)]$$
$$=\Omega[n(\Omega+\omega)+N(\omega)]. \tag{9}$$

In the particle-hole region the angular integration is straightforward and yields

$$\int\frac{d\phi}{2\pi}\delta\left(\Omega+\omega-\mathbf{v}\cdot(\mathbf{Q}+\mathbf{q})+\frac{(\mathbf{Q}+\mathbf{q})^2}{2m}\right)|G(p+Q,\epsilon+\Omega)$$
$$+G(p+q,\epsilon+\omega)|^2$$
$$=\frac{1}{qv_F}\left(\frac{\mathbf{q}\cdot\mathbf{Q}}{m\Omega^2}\right)^2,\quad \cos\phi=\frac{\mathbf{v}\cdot\mathbf{q}}{v_F q}. \tag{10}$$

Finally, we arrive at the contribution to the attenuation in a form

$$\gamma^{p-h}_{1-3}(Q,\Omega_0)=\frac{1}{8\pi}\frac{Q^2}{m\Omega^3p_F^2}\int_0^{2p_F}dqq\left(\frac{\kappa}{q+\kappa}\right)^2$$
$$\times\int_0^\infty d\omega\omega[n(\omega+\Omega)+n(\omega-\Omega)+2N(\omega)]. \tag{11}$$

In the high-temperature limit, $\Omega_0\ll T$, Eq. (11) becomes

$$\gamma^{p-h}_{1-3}(Q,\Omega_0)=\frac{\pi}{8}\left(\frac{Qv_F}{\Omega_0}\right)^2\frac{T^2}{E_F\Omega}\left(\frac{\kappa}{p_F}\right)^2$$
$$\times\left[\ln\left(1+\frac{2p_F}{\kappa}\right)-\frac{2p_F}{2p_F+\kappa}\right] \tag{12}$$

and at low temperatures, $T\ll\Omega_0$,

$$\gamma^{p-h}_{1-3}(Q,\Omega_0)=\frac{1}{8\pi}\frac{(Qv_F)^2}{E_F\Omega}\left(\frac{\kappa}{p_F}\right)^2\left[\ln\left(1+\frac{2p_F}{\kappa}\right)-\frac{2p_F}{2p_F+\kappa}\right]. \tag{13}$$

In the plasmon region the δ function in Eq. (8) takes care of the ω integration, and as a result

$$\gamma^{pl}_{1-3}(Q,\Omega_0)=\frac{\pi}{2}\left(\frac{Q}{\kappa}\right)^2\left(\frac{\Omega}{E_F}\right)^3[1/2+N(\Omega)]. \tag{14}$$

The imaginary part of the last diagram, P_4, consists of two terms

$$\mathrm{Im}P^R_{4a}(Q,\Omega)=-32\int\frac{d\epsilon}{2\pi}\frac{d^2p}{(2\pi)^2}\int\frac{d\epsilon'}{2\pi}\frac{d^2p'}{(2\pi)^2}\int\frac{d\omega}{2\pi}\frac{d^2q}{(2\pi)^2}\frac{n(\epsilon-\omega)n(-\epsilon)n(-\epsilon'+\omega)n(\epsilon'+\Omega)}{N(\Omega)}$$
$$\times\mathrm{Im}G^A(p,\epsilon)\mathrm{Im}G^A(p-q,\epsilon-\omega)\mathrm{Im}G^A(p'-q,\epsilon'-\omega)$$
$$\times\mathrm{Im}G^A(p'+Q,\epsilon+\Omega)V(q,\omega)V(q+Q,\omega+\Omega)|G(p+Q,\epsilon+\Omega)G(p'\epsilon')|, \quad (15)$$

and

$$\mathrm{Im}P^R_{4b}(Q,\Omega)=16\int\frac{d\omega}{2\pi}\frac{d^2q}{(2\pi)^2}\frac{N(\omega+\Omega)N(-\omega)}{N(\Omega)}\times\mathrm{Im}V^A(q,\omega)\mathrm{Im}V^A(q+Q,\omega+\Omega)\times\left|\int\frac{d\epsilon}{2\pi}\frac{d^2p}{(2\pi)^2}G(p,\epsilon)G(p+Q,\epsilon+\Omega)\times G(p-q,\epsilon-\omega)\right|^2. \quad (16)$$

The correction $\mathrm{Im}P^R_{4b}(Q,\Omega)$ describes plasmon-plasmon scattering. The straightforward analysis shows that the energy and momentum conservation laws [hidden in $\mathrm{Im}V^A(q,\omega)\mathrm{Im}V^A(q+Q,\omega+\Omega)$] cannot be satisfied for plasmons with the dispersion relation $\Omega=\Omega_0=v_F(\kappa Q/2)^{1/2}$, therefore the correction (16) is zero. Turning to $\mathrm{Im}P^R_{4a}(Q,\Omega)$ one finds upon integrating over the electron energy:

$$\int d\xi\mathrm{Im}G^A(p,\epsilon)\mathrm{Im}G^A(p+Q+q,\epsilon+\Omega+\omega)=\pi^2\delta(\Omega+\omega-\mathbf{v}(\mathbf{Q}+\mathbf{q})),$$
$$\int d\xi'\mathrm{Im}G^A(p'-q,\epsilon'-\omega)\mathrm{Im}G^A(p'+Q,\epsilon'+\Omega)=\pi^2\delta(\Omega+\omega-\mathbf{v}'(\mathbf{Q}+\mathbf{q})). \quad (17)$$

By their very structure, the Coulomb potentials in Eq. (15) cannot have singularities for real variables, thus Eq. (15) makes sense only in the particle-hole region where in accordance with Eq. (17)

$$V(q,\omega)V(q+Q,\omega+\Omega)|G(p+Q,\epsilon+\Omega)G(p'\epsilon')|=\frac{1}{\nu^2}\left(\frac{\kappa}{q+\kappa}\right)^2\frac{1}{\Omega^2}. \quad (18)$$

The product of distribution functions in Eq. (15) can be transformed into the following form:

$$\frac{n(\epsilon-\omega)n(-\epsilon)n(-\epsilon'+\omega)n(\epsilon'+\Omega)}{N(\Omega)}=\omega\Omega\left(-\frac{\partial n(\epsilon)}{\partial\epsilon}\right)\left(-\frac{\partial n(\epsilon')}{\partial\epsilon'}\right)[1+N(\omega)]\frac{n(-\epsilon'+\omega)}{n(-\epsilon')}, \quad (19)$$

and finally one arrives at (compare with Ref. 8)

$$\gamma^{p-h}_{4a}(Q,\Omega_0)=\frac{1}{8\pi^2E_F\Omega_0}\int d\omega\omega[1+N(\omega)]n(\omega)\int_{|\omega|/v_F}^{2p_F}\frac{dq}{q}\left(\frac{\kappa}{q+\kappa}\right)^2=\frac{1}{32}\frac{T^2}{\Omega_0E_F}\left[\ln\left(\frac{4E_F}{T}\right)-\ln\left(\frac{2p_F+\kappa}{\kappa}\right)-\frac{2p_F\kappa}{(2p_F+\kappa)\kappa}\right]. \quad (20)$$

Note that, unlike γ^{p-h}_{1-3}, the contribution γ^{p-h}_{4a} is *not* proportional to Q^2. The temperature dependences of attenuation are shown in Fig. 2.

Plasmon attenuation in a double-layer electron system. Now we consider a system of two identical electron layers separated by the distance d. The nonscreened interlayer Coulomb potential is $U_0=V_0\exp(-qd)$. The screened potentials are given by

$$V=\frac{1}{2}\frac{V_0+U_0}{1-P(V_0+U_0)}+\frac{1}{2}\frac{V_0-U_0}{1-P(V_0-U_0)}, \quad (21)$$

$$U=\frac{1}{2}\frac{V_0+U_0}{1-P(V_0+U_0)}-\frac{1}{2}\frac{V_0-U_0}{1-P(V_0-U_0)}, \quad (22)$$

The plasmon spectrum is determined by the poles of the potentials. There are in-, (Ω_+), and out-of-phase, (Ω_-), plasmon modes, which in the long-wave limit, $Qd\ll1$, assume the form

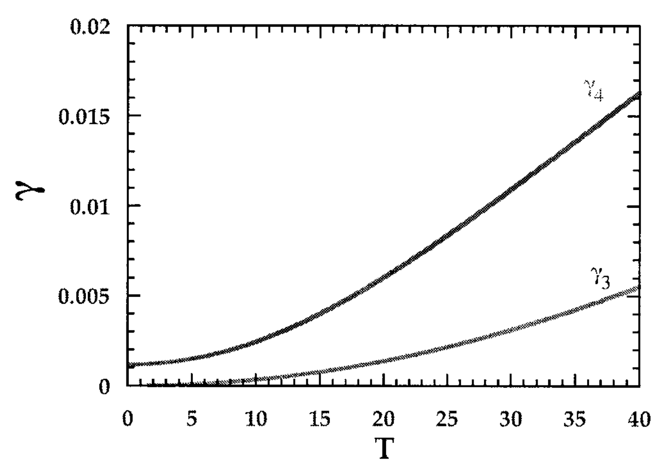

FIG. 2. Temperature dependence of attenuation of two-dimensional plasmons in a single layer.

$$\Omega_+(Q)=v_F(\kappa Q)^{1/2},\quad \Omega_-(Q)=v_F Q\frac{1+\kappa d}{(1+2\kappa d)^{1/2}}. \tag{23}$$

For $\kappa d \gg 1$ the out-of-phase mode becomes $\Omega_-(Q) = v_F Q(\kappa d/2)^{1/2}$.

One can easily see that for the diagrams of Fig. 1 only the intralayer potential V is relevant. In the particle-hole region the static intralayer potential is

$$V(q,0)=\frac{1}{\nu}\frac{\kappa(1+2\kappa d)}{q+2\kappa(1+\kappa d)}, \tag{24}$$

and as a result

$$\gamma_{1-3}^{p-h}(Q,\Omega_\pm)=\frac{\pi}{16}\left(\frac{Qv_F}{\Omega_\pm}\right)^2\frac{T^2}{E_F\Omega_\pm}A,\quad \Omega_\pm \ll T, \tag{25}$$

and

$$\gamma_{1-3}^{p-h}(Q,\Omega_\pm)=\frac{1}{16\pi}\frac{(Qv_F)^2}{E_F\Omega_\pm}A,\quad T\ll\Omega_\pm, \tag{26}$$

where

$$A=\left(\frac{\kappa}{p_F}\right)^2(1+2\kappa d)^2\left[\ln\left(1+\frac{p_F}{\kappa(1+\kappa d)}\right) - \frac{p_F}{p_F+\kappa(1+\kappa d)}\right]. \tag{27}$$

In the plasmon region both plasmon modes contribute to the relaxation

$$\gamma_{1-3}^{pl}(Q,\Omega_\pm)=2\pi\left(\frac{Q}{\kappa}\right)^2[1/2+N(\Omega_\pm)]\left(\frac{\Omega_\pm}{E_F}\right)\left[\frac{1}{4}\left(\frac{\Omega_\pm}{E_F}\right)^2 + \frac{\kappa}{p_F^2 d}\right]. \tag{28}$$

The contribution from the last diagram for $\Omega_\pm < T$ is

$$\gamma_{4a}^{p-h}(Q,\Omega_\pm)=\frac{T^2}{128(1+\kappa d)^2\Omega_\pm E_F}[(1+2\kappa d)^2+2] \times\left[\ln\left(\frac{4E_F}{T}\frac{pd}{p_F d}\right)-\ln\left(\frac{pd+2\kappa d\big(1+\kappa d\big)}{2\kappa d(1+\kappa d)}\right)\right] - \frac{4[(1+2\kappa d)^2+1]\kappa d(pd)(1+\kappa d)}{[pd+2\kappa d(1+\kappa d)][2\kappa d(1+\kappa d)]}, \tag{29}$$

where $pd=\min\{2p_F d,\ 1\}$. The numerical evaluation of Eqs. (27)–(29) shows that the temperature dependence of attenuation in a double-layer system is quantitatively very close to that shown in Fig. 2.

Note that the presence of two types of plasmons with linear and square-root dispersions makes possible the plasmon-plasmon scattering processes. However, the resulting relaxation still appears to be much less effective than the processes considered above.

We have calculated the attenuation of gapless plasmons in a two-dimensional electron system due to the Coulomb electron-electron interaction. The attenuation may be described as relaxation process due to plasmon-electron-hole scattering. The main results of the present paper are given by Eqs. (12)–(14), and (20) for one layer and by Eqs. (28) and (29) for a two-layer system. In experiment[1] the temperature interval used for measurements was 30 K$<T<$100 K, while the Fermi energy was $E_F=78$ K ($p_F=1.1\times10^5\mathrm{cm}^{-1}$). Under these conditions the electron gas is almost nondegenerate and the Landau damping mechanism used in Ref. 1 holds. Note that the frequencies of out-of-phase plasmons studied in[1], $\Omega_-=20$ K for $q=1.1\times10^5$ cm^{-1} and $\Omega_-=30$ K for $q=1.6\times10^5$ cm^{-1}, satisfy condition $\Omega_-<T$ where our Eqs. (28) and (29) also qualitatively describe experimental data. Note that condition $\Gamma\ll\Omega$ is always satisfied.

We have demonstrated that the kinetic contribution due to plasmon-electron-hole scattering dominates plasmon damping at low temperatures, $T<30$ K and for small wave vectors, where Landau damping proportional to $\exp(-1/Q^2T)$ becomes exponentially small.

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[1] D. S. Kainth, D. Richards, H. P. Hughes, M. Y. Simmons, and D. A. Ritchie, Phys. Rev. B **57**, 2065 (1998).
[2] E. M. Lifshits and L. P. Pitaevskii, *Physical Kinetics* (Pergamon, New York, 1981).
[3] F. Parage, M. M. Doria, and O. Buisson, Phys. Rev. B **58**, R8921 (1998).
[4] F. J. Dunmore, D. Z. Liu, H. D. Drew, S. Das Sarma, Qi Li, and D. B. Fenner, Phys. Rev. B **52**, R731 (1995).
[5] H. A. Fertig and S. Das Sarma, Phys. Rev. Lett. **65**, 1482 (1990).
[6] V. S. Kapitonov and V. N. Popov, Zh. Éksp. Teor. Fiz. **63**, 143 (1972) [Sov. Phys. JETP **36**, 75 (1973)].
[7] R. Mattuck, *A Guide to Feynman Diagrams in Many-Body Problems* (Dover, New York, 1992).
[8] M. Yu. Reizer and J. W. Wilkins, Phys. Rev. B **55**, 7363 (1997).