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Single-electron transport through the vortex core levels in clean superconductors

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We develop a microscopic theory of single-electron low-energy transport in normal-metal–superconductor–normal-metal hybrid structures in the presence of applied magnetic field introducing vortex lines in a superconductor layer. We show that vortex cores in a thick and clean superconducting layer are similar to mesoscopic conducting channels where the bound core states play the role of transverse modes. The transport through not very thick layers is governed by another mechanism, namely by tunneling via vortex core levels. We apply our method to calculation of the thermal conductance along the magnetic field.

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I. INTRODUCTION

Electron transport through various hybrid structures is the focus of current nanoscale physics research. Of special interest are normal-metal (N)–superconductor (S)– normal-metal (N) trilayers where a superconducting gap Δ_0 suppresses single-particle transport, making charge transfer transparency very sensitive to the external controlling parameters. If the thickness d of the superconducting slab is much larger than the coherence length ξ, the electrons with low energies $\epsilon<\Delta_0$ incident on the slab are reflected as holes, and the normal current converts into the supercurrent. Single-electron tunneling through an NSN structure decays exponentially with the slab thickness, giving rise, in particular, to the exponential drop off of the electronic contribution to the thermal conductance.[1,2] A single-particle transport recovers by applying a magnetic field $\mathbf{H}$ that creates vortex lines where the gap in the spectrum is suppressed. Since the single-particle contribution to electric conductivity is short-circuited by supercurrent, we focus on the thermal conductivity, which is the experimentally accessible characteristic of the one-electron transport.

Taking the simplest view of a vortex core as a normal conductor, we arrive at a single-electron Sharvin conductance per vortex $G_{\rm Sh}=(e^2/\pi\hbar)N_{\rm Sh}$, where $k_F=p_F/\hbar$ is the Fermi wave vector and $N_{\rm Sh}\sim(k_F\xi)^2$ is the number of conducting channels in a normal wire with a radius ξ. As we already mentioned, the single-electron transport determines the thermal conductivity. The Wiedemann-Franz law would result in $\kappa\sim TG/e^2\sim(T/\hbar)N$ for the thermal conductance. A more attentive consideration shows that only those trajectories contribute to a single-particle conductivity that do not hit vortex core "walls": An electron with $\epsilon<\Delta_0$ flying into the core boundary is Andreev reflected, which blocks the single-electron transport along such a trajectory. The trajectories that traverse freely the normal region are confined to a solid angle ξ^2/d^2, resulting in an "Andreev-wire" single-electron conductance[3] $G_{\rm A}\sim(e^2/\pi\hbar)N_{\rm A}$ with the heat conductance $\kappa_{\rm A}\propto(T/\hbar)N_{\rm A}$ where the number of channels $N_{\rm A}\sim(k_F\xi)^2(\xi/d)^2$ is decreased against $N_{\rm Sh}$.

The low-energy transport obviously saturates for very thick superconducting slabs where it is associated with the Caroli–de Gennes–Matricon (CdGM) states[4] propagating along the core. The CdGM spectrum $\epsilon_\mu(k_z)$ as a function of the quantized angular momentum $\mu=n+1/2$ varies from $-\Delta_0$ to $+\Delta_0$, crossing zero as the impact parameter $b=-\mu/k_r$ varies from $-\infty$ to $+\infty$. Here $k_r=\sqrt{k_F^2-k_z^2}$ is the wave vector in the plane perpendicular to the vortex, $(r,\ \phi,\ z)$ is a cylindrical coordinate system with the z axis chosen along the vortex line. For small ϵ the spectrum is $\epsilon_\mu(k_z)\simeq-\mu\Delta_0/(k_r\xi)$. Transport carried by the quantized transverse modes is described by the Landauer formula. In the limit $\Delta_0/(k_F\xi)\ll T\ll\Delta_0$, the number of modes is $\sim(T/\Delta_0)k_F\xi$; thus one gets for the single-particle conductance of one vortex[3]

$$G_{\rm L}=(e^2/\pi\hbar)\sum_\mu \mathcal{T}_\mu\sim(e^2/\pi\hbar)(T/\Delta_0)(k_F\xi).$$

μ numerates the transverse modes with transparencies $\mathcal{T}_\mu$ open in the core. From the Wiedemann-Franz law,

$$\kappa_{\rm L}\sim(T/\hbar)(T/\Delta_0)(k_F\xi). \tag{1}$$

This estimate can also be obtained from the Sharvin conductance provided the group velocity is taken as $v_g=\partial\epsilon_\mu/\hbar\partial k_z\sim\epsilon_\mu/\hbar k_F$ instead of the velocity $v_z\sim v_F$ as in a normal tube. The number of channels $N_{\rm L}$ in Eq. (1) is by a factor $(T/T_c)(k_F\xi)^{-1}\ll1$ smaller than $N_{\rm Sh}$. One thus expects that the Andreev-wire thermal conductance $\kappa_{\rm A}$ of the vortex core transforms into Eq. (1) with increasing d.

In the vicinity of H_{c2}, the thermal conductivity has been studied theoretically in a number of papers (see, for instance, Refs. 5–7). In dirty superconductors, the electron contribution to the thermal conductance along the vortices[8] is simply proportional to the area occupied by the cores $\kappa(B)\simeq(B/H_{c2})\kappa_N$, where κ_N is the electron thermal conductivity in the normal state. Unfortunately, in clean superconductors this simple estimate fails to describe the experimental data:[9,10] the thermal conductance along the vortices appears to be two orders of magnitude smaller. It was noted first in Ref. 10 that this obvious conflict can be caused by a very small group velocity of the CdGM modes as discussed above. Analysis of quantum transport through individual vortices is of particular importance for understanding mesoscopic superconductors whose exotic vortex states are nowadays the focus of a considerable attention.[11–13]

Hereafter we concentrate on the low-energy $\epsilon \ll \Delta_0$ single-particle transport through an isolated vortex core in clean (both elastic and inelastic mean free path $\ell \gg d$) type-II superconductors in the low-field limit of separated vortices ($H \ll H_{c2}$) and develop a systematic approach for calculation of the thermal conductance along vortex cores. We study the transmission of an electron wave incident on the superconducting slab placed between two bulk normal-metal electrodes assuming ideally transparent boundaries and neglecting the normal scattering. Considering two extremes of infinite and finite slab thicknesses we confirm the intuitive picture discussed above. For a not very thick slab, the transmission is determined by the semiclassical resonant tunneling through the energy gapped region. The transmission is proportional to the large Sharvin conductance and is dominated by the trajectories that go almost parallel to the vortex axis. However, the drop-off of the conductance is more rapid than that found in Ref. 3 for a model of a normal core: it is proportional to d^{-6} for not very large d and to d^{-2} but with a much smaller temperature prefactor $(T/T_c)^4$ for larger d. Finally, it goes over into a thickness-independent expression, Eq. (1), with further increase in the slab thickness.

II. SINGLE-ELECTRON TRANSPORT: QUASICLASSICAL APPROACH

A. Transmission and reflection probabilities

We start with the Bogoliubov–de Gennes equations

$$\left[\frac{1}{2m}\left(-i\hbar\nabla-\frac{e}{c}\mathbf{A}\right)^2-E_F\right]u+\Delta v=\epsilon u, \tag{2}$$

$$\left[\frac{1}{2m}\left(-i\hbar\nabla+\frac{e}{c}\mathbf{A}\right)^2-E_F\right]v-\Delta^* u=-\epsilon v, \tag{3}$$

where (u,v) are the particlelike and holelike parts of the quasiparticle wave function, $\mathbf{A}$ is the vector potential of the magnetic field $\mathbf{B}=B(r)\hat{\mathbf{z}}$, and Δ is the gap function for a vortex line. The gap function can be written in the form $\Delta=|\Delta|e^{i\phi}$, where the absolute value of the gap $|\Delta|$ depends only on the distance r from the vortex axis. Inside the core, $r \ll \xi$, it is $|\Delta|\sim\Delta_0 r/\xi$. For distances r larger than the coherence length ξ the gap saturates at its value in the bulk: $|\Delta|\to\Delta_0$. Choosing the gauge $A_\phi=A_\phi(r)$, $A_z=A_r=0$ we search for the solution with a given angular momentum μ:

$$u=e^{i\phi/2+i\mu\phi}U,\quad v=e^{-i\phi/2+i\mu\phi}V.$$

The equation for $\hat{\mathcal{U}}=(U,V)$ reads

$$\frac{\hbar^2}{2m}\left[-\hat{\mathcal{U}}''_{zz}-\hat{\mathcal{U}}''_{rr}-\frac{1}{r}\hat{\mathcal{U}}'_r+\left(\frac{\mu}{r}+\hat{\sigma}_z\frac{m}{\hbar}\mathcal{V}_s\right)^2\hat{\mathcal{U}}-k_F^2\hat{\mathcal{U}}\right]+i\hat{\sigma}_y|\Delta|\hat{\mathcal{U}}$$
$$=\hat{\sigma}_z\epsilon\hat{\mathcal{U}},$$

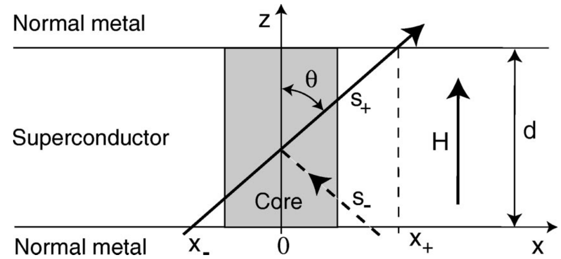

FIG. 1. Quasiclassical trajectories in a superconducting slab with a single vortex line for incident, s_-, (dashed line) and scattered, s_+, waves (solid line, positive-x region) together with the extended trajectory (solid line, entire x axis).

where $\hat{\sigma}_y,\hat{\sigma}_z$ are the Pauli matrices, and $\mathcal{V}_s=(\hbar/2m)(1/r+2eA_\phi/\hbar c)$ is the superfluid velocity field. The characteristic length scale for the magnetic field around the vortex is of the order of the London penetration depth λ_L, which is $\lambda_L\gg\xi$ in extreme type-II superconductors. For $\lambda_L\gg\xi$ the magnetic field is almost uniform near the core and the vector potential is $A_\phi=Br/2\sim(H/H_{c2})r/\xi^2$. For $H\ll H_{c2}$, it can be neglected in the expression for $\mathcal{V}_s$ as compared to the large gradient of the order parameter phase $\nabla\phi\propto 1/r$.

Let us consider a cylindrical electronic wave incident on a superconducting slab placed between two normal metal half-spaces (see Fig. 1). An external magnetic field is applied along the z axis perpendicular to the normal-metal/superconductor boundaries. Restricting ourselves to the low magnetic field limit we study the scattering problem in the presence of a single isolated vortex line. To calculate the transmission and reflection probabilities we use a quasiclassical approach and look for $\hat{\mathcal{U}}$ in the form

$$\hat{\mathcal{U}}=e^{ik_z z}H_l^{(1)}(k_r r)\hat{w}^{(+)}+e^{ik_z z}H_l^{(2)}(k_r r)\hat{w}^{(-)},$$

where $H_l^{(1,2)}$ are the Hankel functions, $l=\sqrt{\mu^2+1/4}$, $k_r^2+k_z^2=k_F^2$, and $\hat{w}=(w_1,\ w_2)$ are slow functions of r and z satisfying the equation

$$\mp\frac{i\hbar^2 q_r}{m}\frac{\partial\hat{w}^{(\pm)}}{\partial r}-\frac{i\hbar^2 k_z}{m}\frac{\partial\hat{w}^{(\pm)}}{\partial z}-\hat{\sigma}_z\left(\epsilon-\frac{\mu\hbar^2}{2mr^2}\right)\hat{w}^{(\pm)}$$
$$+i\hat{\sigma}_y|\Delta|\hat{w}^{(\pm)}=0, \tag{4}$$

where $q_r=\sqrt{k_r^2-\mu^2/r^2}$. We now introduce new coordinates[14] $x=\sqrt{r^2-b^2}$, $b=-\mu/k_r$ so that $q_r=xk_r/r$, $\partial/\partial r=(r/x)\partial/\partial x$. Finally, we define the trajectories $z=z_0\pm x\cot\theta$ for $\hat{w}^{(\pm)}$, respectively. One has $dx=\pm ds_\pm\sin\theta$, $dz=ds_\pm\cos\theta$, where $k_z=k_F\cos\theta$ and $ds_\pm$ is the distance along the corresponding trajectory (see Fig. 1). The differential operators in Eq. (4) transform into the operator $i(\hbar^2 k_F/m)(d/ds_\pm)$ along the corresponding trajectory. The order parameter is now a function of the distance s on the trajectory $|\Delta|=|\Delta[r(s)]|$, which is simply $|\Delta(x)|$ since $|\Delta|$ is independent of z. Projecting the trajectory on the plane perpendicular to the vortex axis we replace $k_F(d/ds_\pm)$ with $\pm k_r(d/dx)$ and arrive at the equation

$$\mp\frac{i\hbar^2 k_r}{m}\frac{d\hat{w}^{(\pm)}}{dx}-\hat{\sigma}_z\left(\epsilon+\frac{\hbar^2 k_r b}{2m(x^2+b^2)}\right)\hat{w}^{(\pm)}+i\hat{\sigma}_y|\Delta|\hat{w}^{(\pm)}$$
$$=0. \quad (5)$$

$\hat{\mathcal{U}}$ is a superposition of a wave $w_{1,2}^{(+)}$ radiating from the vortex into the bulk and a wave $w_{1,2}^{(-)}$ incident on the vortex. The regularity at $r=0$ requires $\hat{w}^{(+)}(0,z)=\hat{w}^{(-)}(0,z)$ at the classical turning point, $x=0$. The coordinate x has been defined positive so far. Let us now put $\hat{w}^{(+)}(x,z)=\hat{w}(x,z)$, $\hat{w}^{(-)}(x,z)=\hat{w}(-x,z)$. The functions $w_{1,2}(x,z)$ are continuous and satisfy Eq. (5) with the upper sign along the entire x axis (the extended solid-line trajectory in Fig. 1).

We introduce new functions η and ζ through $w_1=e^{\zeta+i\eta/2}$, $w_2=e^{\zeta-i\eta/2}$ and arrive at the equations

$$\frac{d\eta}{dx}=\frac{2m\epsilon}{\hbar^2 k_r}+\frac{b}{(x^2+b^2)}-\frac{2m|\Delta|}{\hbar^2 k_r}\cos\eta, \quad (6)$$

$$\frac{d\zeta}{dx}=-m|\Delta|\hbar^{-2}k_r^{-1}\sin\eta. \quad (7)$$

The requirement that w vanishes at $x\to\pm\infty$ is $\eta=\pm\pi/2-(\epsilon/|\Delta_0)+2\pi k$. These values, however, are not stable for a general choice of the integration constants. A general solution for not very large *positive* x is

$$\eta=\arctan\frac{x}{b}+\eta_0 e^{2K(x)}$$
$$+\frac{2m}{\hbar^2 k_r}\int_0^x\left[\epsilon-|\Delta(x')|\frac{b}{|x'|}\right]e^{2K(x)-2K(x')}\,dx', \quad (8)$$

where η_0 is a constant and

$$K(x)=m\hbar^{-2}k_r^{-1}\int_0^x|\Delta(x')|\,dx'. \quad (9)$$

For $\eta_0\ll 1$ and $\epsilon\ll\Delta_0$, the function η is close to $\pi/2$ for $b\ll x\lesssim x_0$ where $x_0\sim\xi\ln(1/|\gamma|)$ and

$$\gamma=2m\hbar^{-2}k_r^{-1}\int_0^\infty[\epsilon-(b/x)|\Delta|]e^{-2K(x)}\,dx. \quad (10)$$

γ measures the distance from a CdGM level: $\gamma=0$ when $\epsilon=\epsilon_\mu(k_z)$. Equation (8) is valid for $\gamma\ll 1$, which generally holds if $\epsilon\ll\Delta_0$. The function η grows with $|x|$ at distances $x\gtrsim x_0$. Its behavior in the region $\eta\sim 1$ is found from Eq. (6) neglecting small ϵ and b/x:

$$\tan\left(\frac{\eta}{2}-\frac{\pi}{4}\right)=Ce^{2K(x)}. \quad (11)$$

Matching with Eq. (8) at $\xi\ll x\ll x_0$ gives $C=(\gamma+\eta_0)/2$. For $\gamma+\eta_0>0$, the function $\eta\to 3\pi/2$ while w diverges exponentially as $x\to\infty$. If $\gamma+\eta_0<0$, the function η approaches $-\pi/2$, and w diverges again. However, if $\gamma+\eta_0=0$, the value $\eta=\pi/2$ is stable (see Fig. 2) and the wave function decays for $x\to\infty$.

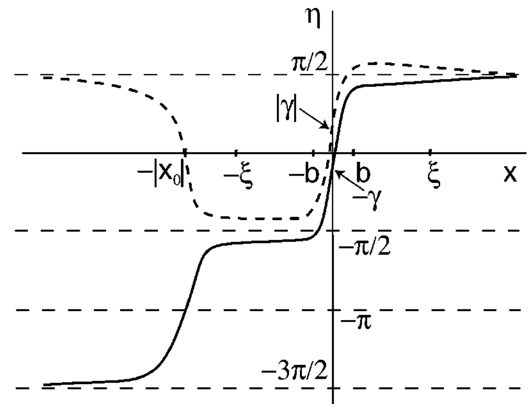

FIG. 2. The coordinate dependence of η for $\gamma+\eta_0=0$. The full line is for $\gamma>0$ while the dashed line is for $\gamma<0$.

The solution at $|x|\ll x_0$ for *negative* x is obtained from Eq. (8) by replacing $K(x)$ with $-K(x)$. The function η is close to $-\pi/2$ for $b\ll|x|\ll x_0$. Its behavior for $|x|\gtrsim x_0$ is determined by Eq. (11) where $C=2/(\gamma-\eta_0)$. This yields $-3\pi/2<\eta(-\infty)<-\pi/2$ if $\gamma-\eta_0>0$. The function η exhibits a solitonlike behavior shown by the solid line in Fig. 2: It slips down from $-\pi/2$, crosses $-\pi$ and finally approaches $\eta(-\infty)\approx-3\pi/2$. The wave function w diverges. Similarly, $-\pi/2<\eta(-\infty)<\pi/2$ if $\gamma-\eta_0<0$ so that w also diverges (dashed line in Fig. 2). The value $\eta=-\pi/2$ is stable if only $\gamma-\eta_0=0$. The wave function thus decays at both ends if $\gamma=\eta_0=0$, which corresponds to a standard CdGM state.

The solution for a slab is a superposition $\hat{w}=A_>\hat{w}^>+A_<\hat{w}^<$ where $A_{>,<}$ are constants. The functions $w_{1,2}^>$ have $\eta_0=-\gamma$; they decay for $x\gg\xi$ with $\eta_>(+|x|)=\pi/2$, while $\eta_>(-|x|)$ for negative x obeys

$$\tan\left[\frac{1}{2}\eta_>(-|x|)-\frac{\pi}{4}\right]=\gamma^{-1}e^{-2K(|x|)} \quad (12)$$

according to Eq. (11). For $\gamma\neq 0$ the phase $\eta_>=-3\pi/2+2\pi n$ at $x=-|x|\to-\infty$. Equations (7) and (11) yield

$$\zeta_>(-|x|)=K(|x|)+\frac{1}{2}\ln\left(\frac{\gamma^2+e^{-4K(|x|)}}{1+\gamma^2}\right).$$

For $x=|x|$ it is simply $\zeta_>(+|x|)=-K(|x|)$. The other function $w_{1,2}^<(x)=w_{1,2}^{>*}(-x)$ grows at $x\to+\infty$.

The particle transmission D_e and hole (Andreev) reflection R_h probabilities, $1=R_h+D_e$, are determined in such a way that $D_e=|w_1(z=d)/w_1(z=0)|^2$ provided there are no transmitted holes, $w_2=0$ at $z=d$. We denote x_- and x_+ the x coordinates of the end points of the trajectory at $z=0$ and $z=d$, respectively, such that $d\tan\theta=x_+-x_-$ (see Fig. 1). For trajectories crossing the vortex axis, $0<x_+<d\tan\theta$, $x_-=-|x_-|$, we find

$$D_e=(\gamma^2+a^2)^{-1}\cosh^{-2}[K(|x_+|)+K(|x_-|)]. \quad (13)$$

The half-width of the energy level $\gamma=0$

$$a=\cosh[K(|x_+|)-K(|x_-|)]/\cosh[K(|x_+|)+K(|x_-|)]$$

is proportional to the escape rate of excitations through the gapped region. For $\gamma\to 0$, the transmission becomes

$$D_e=\cosh^{-2}[K(|x_+|)-K(|x_-|)].$$

It is $D_e=1$ for resonant trajectories that go through the middle of the vortex, $|x_-|=|x_+|=(d/2)\tan\theta$. For trajectories that do not cross the vortex axis, $x_-,x_+<0$ or $x_->0,x_+>d\tan\theta$, we find

$$D_e=\cosh^{-2}[K(x_+)-K(x_-)].$$

The largest transmission takes place for a resonance tunneling when the energy coincides with one of the CdGM levels in Eq. (13). D_e is then unity if the trajectory crosses the vortex at the half of its length. However, the number of transmitted particles is small since the width of the resonance a is exponentially narrow. The exponent though corresponds to only a half of the slab thickness $d/2$, not to the entire d as it would be without vortices.

The transmission is not exponential for trajectories that go close to the vortex axis almost parallel to it, i.e., at small θ. It is this contribution that determines the transport replacing the exponential dependence on d with a power-law behavior. Consider these trajectories in more detail. Their wave functions are localized in x within $\lambda=\hbar[k_F/\pi m\Delta'(0)]^{1/2}\theta^{1/2}\sim\xi\theta^{1/2}$ [see Eq. (9)]. If a trajectory crosses the vortex axis,

$$K(|x_+|)-K(|x_-|)=x_0\ (md/\hbar^2k_F)\Delta'(0)\sim x_0d/\xi^2,$$

$$K(|x_+|)+K(|x_-|)\sim\theta\ (md^2/\hbar^2k_F)\Delta'(0)\sim d^2\theta/\xi^2,$$

where $x_0=(|x_-|-|x_+|)/2$ is the middle point of the trajectory. Therefore, the transmission coefficient $D_e\sim 1$ only for trajectories that deflect from the vortex axis by not more than $x_0\sim\xi^2/d$ and have very small incident angles $\theta\sim(\xi/d)^2$. This behavior holds as long as $\gamma\ll 1$. The estimate shows that $\gamma\sim(\epsilon/\Delta)\theta^{-1/2}$, which becomes $\gamma\sim 1$ for $d/\xi\gtrsim(\Delta/\epsilon)$. Therefore, for larger d, the behavior of the transmission coefficient changes.

The new dependence can be easily found for the limit $d/\xi\gg(\Delta/\epsilon)$. In this case Eq. (5) can be solved in a WKB approximation: $\hat{w}=A_+\hat{w}_++A_-\hat{w}_-$ where

$$\hat{w}_\pm=\begin{pmatrix}\beta_\pm\\ \beta_\mp\end{pmatrix}\exp\left(\pm i\int_0^x\Lambda\ dx\right),$$

$$\beta_\pm=[1\pm\sqrt{\epsilon^2-|\Delta|^2}/\epsilon]^{1/2},\qquad \Lambda=m\sqrt{\epsilon^2-|\Delta|^2}/\hbar^2k_r.$$

Here $|\Delta|\equiv|\Delta(x)|$. This solution applies for $|x|<x_\epsilon$ where $|\Delta(x_\epsilon)|=\epsilon$. Therefore, $x_\epsilon\sim(\Delta')^{-1}\epsilon$. The angles for the traversing trajectories are $\theta\sim x_\epsilon/d\sim(\xi/d)(\epsilon/\Delta)$. One can easily check that the centrifugal energy and the term with the order-parameter derivative in Eq. (5) are of the order of $v_F\theta/x_\epsilon\ll\epsilon$ and can be neglected. Requiring $w_2=0$ at the exit point $x=x_+$ we find the transmission coefficient

$$D_e=\frac{[\beta_+^2(x_+)-\beta_-^2(x_+)]^2}{|\beta_+(x_-)\beta_+(x_+)-\beta_-(x_-)\beta_-(x_+)e^{i\Phi}|^2},$$

$$\Phi=2\int_{x_-}^{x_+}\Lambda\ dx.$$

D_e oscillates as a function of Φ whose magnitude is $\Phi\sim(\epsilon/|\Delta|)(d/\xi)\gg 1$. Oscillations thus average out after summing up various trajectories yielding an average $D_e\sim 1$. For $x>x_\epsilon$, particles experience Andreev reflection from vortex core boundaries, and $D_e=0$.

B. The heat current

The energy current along z is

$$I_{\mathcal{E}}=\int d^2r\sum_\mu\int\frac{dk_z}{2\pi m}\left[\epsilon_\mu\, u^*_{\mu k_z}\left(\hbar k_z-\frac{e}{c}A_z\right)u_{\mu k_z}n(\epsilon_\mu)\right.$$
$$\left.-\epsilon_\mu\, v^*_{\mu k_z}\left(\hbar k_z+\frac{e}{c}A_z\right)v_{\mu k_z}[1-n(-\epsilon_\mu)]\right].\qquad(14)$$

Particles u^*u with the distribution $n(\epsilon)$ carry the energy $+\epsilon$ while the holes v^*v with the distribution $1-n(-\epsilon)$ carry the energy $-\epsilon$. If the electrodes are in equilibrium, $1-n(-\epsilon)=n(\epsilon)$ in each electrode. For a finite slab thickness, $I_{\mathcal{E}}$ can be expressed through the transmission and reflection coefficients,

$$I_{\mathcal{E}}=\sum\int d\epsilon|v_z|[\epsilon n_1(\epsilon)-\epsilon R_h[1-n_1(-\epsilon)]-\epsilon D_e n_2(\epsilon)]$$
$$=\nu_F\int_{v_z>0}\frac{d\Omega}{2\pi}|v_z|\int d^2r\int D_e\ \epsilon(n_1-n_2)d\epsilon,\qquad(15)$$

where the sum is over all the trajectories; ν_F is the single-spin density of states at the Fermi level, and $n_1=[e^{\epsilon/T_1}+1]^{-1}$ and $n_2=[e^{\epsilon/T_2}+1]^{-1}$ are the distribution functions in the electrodes 1 and 2. The first two terms in the upper line are due to incoming particles and Andreev reflected holes on one side of the slab. The third term is due to transmitted particles from the other side.

Only those trajectories contribute that go almost parallel and close to the vortex axis. We distinguish two limits: (i) very low temperatures or relatively thin slabs, $T/T_c\ll\xi/d\ll 1$ and (ii) thick slabs or moderate temperatures, $1\gg T/T_c\gg\xi/d$. In the limit (i) of not very thick slabs, the trajectory deflection from the vortex axis is $x_0\sim\xi^2/d$ with $\theta\sim(\xi/d)^2$. Assuming $T\gg T_c/(k_F\xi)$, the heat current through one vortex is

$$I_{\mathcal{E}}\sim\nu_F\hbar^4v_F^5d^{-6}[\Delta'(0)]^{-4}(T_1^2-T_2^2).\qquad(16)$$

The thermal conductance of an isolated vortex is

$$\kappa\sim(T/\hbar)(k_F\xi)^2(\xi/d)^6.$$

This is equivalent to the number of channels $N\sim(k_F\xi)^2(\xi/d)^6$ open in the vortex core.

The d^{-6} power law changes for larger d. In the limit (ii) when $T/T_c\gg\xi/d$, Eq. (15) can be evaluated by counting the freely traversing trajectories with $D_e \sim 1$. Consider a point at the vortex-core cross section of an area x_ϵ^2 near the entrance, $z=0$. The exit area is visible from that point at a solid angle $\delta\Omega \sim x_\epsilon^2/d^2$. Therefore,

$$I_{\mathcal{E}} \sim \pi \nu_F v_F d^{-2} (\Delta')^{-4} \int_0^\infty \epsilon^5 (n_1 - n_2) d\epsilon. \tag{17}$$

This yields the single-vortex conductance

$$\kappa \sim (T/\hbar)(k_F \xi)^2 (\xi/d)^2 (T/T_c)^4$$

with the number of channels $N \sim (k_F\xi)^2(\xi/d)^2(T/T_c)^4$.

Our semiclassical approach requires either $k_r \lambda \gg 1$ or $k_r x_\epsilon \gg 1$, which restricts the number of channels to be $N \gg 1$. This puts an upper bound on the slab thickness: For $d \gtrsim d^*$ the conducting channels corresponding to freely traversing trajectories disappear and the single-particle transport is entirely due to the nonquasiclassical drift along the vortex core states described by the Landauer-type formula, Eq. (1). Let us define a temperature $T^* = T_c (k_F \xi)^{-1/3}$. For $T \gtrsim T^*$, the critical thickness is $d^* \sim \xi (k_F \xi)(T/T_c)^2$, while it is $d^* \sim \xi (k_F \xi)^{1/3}$ for $T \lesssim T^*$. In fact, relative contribution of freely traversing trajectories becomes small compared to that of the Landauer expression already for d essentially shorter than d^*, i.e., much before the semiclassical approach breaks down because the number of conducting channels corresponding to Eq. (1) is $N_{\mathrm{L}} \gg 1$.

III. LANDAUER FORMULA: BEYOND THE QUASICLASSICAL THEORY

In this section we show that for a very thick slab the single electron transport is exactly described by the Landauer formula (1). We establish first a simple identity for the localized states. Equations (2) and (3) have eigenvalues $\epsilon_\mu(k_z)$ for the CdGM bound states $u_{\mu k_z}$, $v_{\mu k_z}$ that belong to a given momentum k_z along the z axis. Calculating the derivative with respect to k_z from the both sides of Eqs. (2) and (3) and using the normalization of the wave functions, we find for the localized states

$$\int \left[u^*_{\mu k_z} \left(\hbar k_z - \frac{e}{c} A_z \right) u_{\mu k_z} - v^*_{\mu k_z} \left(\hbar k_z + \frac{e}{c} A_z \right) v_{\mu k_z} \right] d^2 r$$
$$= \frac{m}{\hbar} \frac{\partial \epsilon_\mu}{\partial k_z}. \tag{18}$$

This identity demonstrates a huge cancellation in the left-hand side: each term there is by a factor $k_F \xi$ larger than the right-hand side. Within the quasiclassical approximation the left-hand side vanishes for an infinite vortex line as a direct consequence of an approximate electron-hole symmetry. Note that the cancellation does not take place for a finite-thickness slab where the CdGM states are not truly localized.

Equation (18) shows that in contrast to the electrical current the energy flow is determined by the group velocity of excitations and this is why the conductance in Eq. (1) is much smaller than κ_{Sh}. Now we can use Eq. (18) to derive the thermal conductance for a thick slab beyond the quasiclassical approximation. The energy current Eq. (14) between the two electrodes becomes

$$I_{\mathcal{E}} = \sum_\mu \int \epsilon_\mu n(\epsilon_\mu) \frac{\partial \epsilon_\mu}{\partial k_z} \frac{dk_z}{2\pi\hbar}.$$

Excitations with positive group velocity $v_g = \partial \epsilon_\mu / \hbar \partial k_z$ have a distribution n_1 as in the electrode 1. For those with negative group velocity the distribution is n_2 as in the electrode 2. Therefore

$$I_{\mathcal{E}} = \sum_\mu \int_{k_z > 0} \epsilon_\mu [n_1(\epsilon_\mu) - n_2(\epsilon_\mu)] \left| \frac{\partial \epsilon_\mu}{\partial k_z} \right| \frac{dk_z}{2\pi\hbar}. \tag{19}$$

By the order of magnitude, the heat current through a single vortex is $I_{\mathcal{E}} \sim (T/\hbar)(k_F\xi)(T/T_c)(T_1 - T_2)$ with the thermal conductance given by Eq. (1).

The electron transport in Eq. (19) is determined by incident angles $\theta \sim 1$: Particles that penetrate into the core at large angles get trapped by Andreev reflections and drift slowly along the vortex. This process yields a small but thickness-independent transport which dominates for very thick slabs. Comparing Eqs. (17) and (19) one notices that $D \sim v_g / v_F$ is the effective transmission coefficients for the Andreev trajectories in the vortex core.

As we mentioned already, the single-particle conductance saturates at the Landauer expression with increasing d. It is also required that the inelastic relaxation length is larger than d. For electron-electron interactions, $\ell_{\mathrm{inel}} \sim v_F \hbar E_F / T^2 \sim \xi (k_F \xi)(T_c/T)^2$, which well exceeds d^*. The electron-phonon relaxation length is proportional to $(T_c/T)^3$ and can also exceed d^* for low temperatures. However, both inelastic and elastic scattering may affect the drift along vortex-core states when $\ell \lesssim d_{\mathrm{eff}}$ where the effective length d_{eff} can be considerably longer than d since the drifting particles traverse much longer distance on their way from one end of the channel to another due to Andreev reflections. The effects of scattering on the particle drift along the core states will be studied elsewhere.

IV. CONCLUSIONS

In conclusion, we show that the low-energy single-electron transport along the vortex core in a clean superconductor is similar to that in a mesoscopic channel where the conductance is given by the Landauer formula with CdGM states playing the role of transverse modes. For a finite slab thickness d, the vortex core behaves like an Andreev wire: the thermal conductivity drops off as d^{-6} for not very thick slabs and as d^{-2} for larger d. These results allow us to conclude that single-electron transport parallel to vortices in clean systems is strongly suppressed as compared to the dirty limit, which is in a good agreement with the experimentally observed suppression of the thermal conductance in clean superconductors.[9,10]

Equations (16) and (17) apply to ideally rectilinear vortices. The Andreev-type thermal conductance is very sensitive to a vortex curvature: It is blocked if vortices are bent by an angle exceeding $\theta_c \sim (\xi/d)^2$ or $\theta_c \sim (\xi/d)(T/T_c)$ for the limits $T/T_c \ll \xi/d$ or $T/T_c \gg \xi/d$, respectively. This is distinct from the Landauer expression, Eq. (19), which is expected to hold for curved vortices, as well. If vortices are pinned in the slab, one can bend them, for example, by applying a transport current, which thus provides an efficient mechanism for control of the heat transport through the *NSN* hybrid structure.

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