Effects of external radiation on biased Aharonov-Bohm rings

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We consider the currents flowing in a solid-state interferometer under the effect of both an Aharonov-Bohm phase and a bias potential. Expressions are obtained for these currents, allowing for electronic or electron-boson interactions, which may take place solely on a quantum dot placed on one of the interferometer arms. The boson system can be out of equilibrium. The results are used to obtain the transport current through the interferometer, and the current circulating around it under the effect of the Aharonov-Bohm flux. The modifications of both currents, brought about by coupling the quantum dot to an incoherent sonic or electromagnetic source, are then analyzed. By choosing the appropriate range of the boson source intensity and its frequency, the magnitude of the interference-related terms of both currents can be controlled.

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I. INTRODUCTION

Solid-state interferometers, restricted to the mesoscopic scale in order to retain the coherence of conduction electrons,[1] are constructed from narrow waveguides, possibly containing scatterers, for the electronic paths. An Aharonov-Bohm magnetic flux[2] between the two paths in such interferometers results in a periodic flux-dependence behavior, which stems from interference of the electronic wave functions. In recent experiments,[3–10] carried out on interferometers connected to several electronic reservoirs, the current passing through the system in response to a voltage difference has been used to investigate *coherent transport*. These experiments have revived interest in such systems, whose theoretical[11–13] and experimental[14] study was begun much earlier. The current experimental setups involve a quantum dot (or two[8,9]) embedded in the interferometer, aimed at the study of the transmission properties of the former. These experiments have been followed by many theoretical works, exploring the possibility of deducing the transmission phase of a quantum dot from the measured conductance of the interferometer,[15–26] and investigating its dependence on various interactions.

The interference of the electronic wave functions in an Aharonov-Bohm interferometer also creates a circulating current, which flows even at thermal equilibrium, and even when the ring is isolated (under these conditions it is usually called "persistent current"). This current was invoked as early as 1936 by Pauling,[27] to explain the large orbital magnetic response of π electrons moving on a ring in benzene-type molecules, and soon after was calculated[28] in terms of the tight-binding model. The analogy between persistent currents and the Josephson effect has been expounded upon in Refs. 29 and 30. Their discussion of the possible realization of a "normal Josephson current" in small metallic (or semiconductor) rings, in the presence of some disorder, has sparked much interest in this phenomenon and led to a considerable experimental effort to detect it, either by various magnetic response measurements[31–36] or by optical spectroscopy.[37–39] At thermal equilibrium, the persistent current is equivalent to the thermodynamic orbital magnetic moment of the electrons. Since it arises from the interference of the electronic wave functions, as long as the electrons are phase coherent it will survive the presence of moderate static disorder.[1,30] Recently, most of the theoretical interest in this phenomenon has shifted to studying charge- (or spin) fluctuation effects,[40–42] time-dependent properties and nonequilibrium situations,[43–46] or electronic interactions.[47–51] In addition, recently there have been several attempts[52–55] to relate the phenomenon of persistent current, which is intimately connected to electronic coherence, to the dephasing of electrons at equilibrium due to the coupling with a boson bath.

Here, we study the currents flowing *around and through* an "open" interferometer, connected to electronic reservoirs, with a quantum dot placed on one of its arms, when the latter is coupled to an external incoherent radiation source. The electronic reservoirs are held at slightly different chemical potentials, such that the voltages are small enough for the system to be in the *linear transport* regime. The external radiation source, on the other hand, will be taken as being, and possibly driving the system, out of equilibrium, so that its intensity can serve as a "control parameter" of the currents. In other words, we study the currents when the electrons are also coupled to an incoherent *out-of-equilibrium* boson source. We take the electronic system to be free of any interactions, except on the quantum dot, where the electrons are coupled to an external source of sonic (or electromagnetic) waves. The system is depicted in Fig. 1.

Although we use the term "electron-phonon interaction" throughout this article, our results apply equally, with minor modifications, to the case where the electrons are coupled to an electromagnetic source, that is, for the electron-photon interaction. In any event, in order to retain the coherence of the electrons, the systems we consider are necessarily confined to size scales small enough that the electrons stay coherent at the given temperature. At the same time, the strength of the acoustic source is assumed to be such that the additional decoherence due to it is not detrimental. The pre-

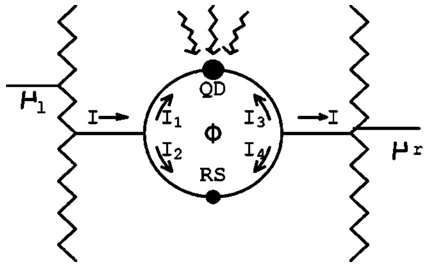

FIG. 1. An Aharonov-Bohm interferometer, containing a quantum dot (QD) on its upper arm and threaded by a magnetic flux Φ. The lower arm of the interferometer contains a "reference" site (RS). The ring is connected to two electronic reservoirs whose chemical potentials are either equal or have a small difference, allowing a current I to flow from the left to the right. The wavy vectors denote the external beam radiated on the dot.

cise parameter windows in which this can be achieved are sensitive to acoustic (or electromagnetic) mismatch, details of the sample geometries, etc.; hence, their calculations are not carried out here. Also, we do not discuss dephasing, but rather, as in Refs. 43 and 46, we concentrate on a *nonequilibrium* source of bosons.

When the electrons are coupled to a boson source, the naive expectation is that the coherent current decreases due to loss of coherence, caused by inelastic processes as well as by renormalization effects due to the "dressing" of the electrons by the bosons (the polaron effect[56]). The latter is manifested by an overall Debye-Waller exponent. However, it turns out that this is not the whole effect brought about by the radiation. In the case of *isolated* rings, it has been found[57] that when the electrons are coupled to phonons, the persistent current is not only diminished; rather, there appears an additional term, which originates from delicate resonance processes in which at least two phonons are involved (those were termed "doubly resonant processes"). The additional orbital magnetic moment appears at nonzero temperatures, and has a nonmonotonic temperature dependence at sufficiently low temperatures.[57] At thermal equilibrium, this new term has been found to further reduce the persistent current (beyond its value in the absence of the coupling to the bosons). However, at nonequilibrium situations, the magnitude of that "extra" contribution may be tuned by controlling the intensity of the radiation in a certain frequency range, which is experimentally feasible. Possibly related experiments with extremely interesting results have recently been reported in, e.g., Refs. 58 and 59. Here, we examine the effect of the electron-phonon coupling on a biased Aharonov-Bohm interferometer, which consists of an "open" ring, connected to two reservoirs. Then, in addition to the circulating current induced by the magnetic phase, there appears a transport current. We find that, in a certain sense, the open ring is more amenable to manipulations by an external radiation source. We show that both the circulating and transport currents are affected by a radiation source in a similar manner: Besides the overall Debye-Waller factor, they each acquire an additional contribution. In the case of an open ring, that additional term does not necessitate the existence of real resonant transitions between the initial and the final state; it appears at a lower order in the electron-phonon interaction (as compared to the situation in isolated rings) and it exists even at zero temperature. The magnitude of that contribution can again be tuned by controlling the intensity of the radiation in a certain frequency range. In other words, by coupling the electrons to an *out-of-equilibrium* radiation source, one may control both the circulating and transport currents. Such a relation between the radiation intensity and the orbital magnetic moment may open interesting possibilities for future nano-devices.

Our method of calculation is to express all partial currents flowing in the system (i.e., I_1, I_2, I_3, and I_4; see Fig. 1) in terms of the exact (and generally, unknown) Green function on the dot, which includes all the effects of the coupling to the interferometer, the external reservoirs, and the interactions taking place on the dot. These expressions do not necessitate a near-equilibrium situation. Thus, we derive general expressions for the current passing through the interferometer, I, and the current circulating around it which is induced by the Aharonov-Bohm flux, in terms of the exact Green function on the dot. We then use these results to investigate the effect of coupling to a boson source on both currents.

We begin in Sec. II with the derivation of the partial currents, the transport current, and the circulating current. The expressions we obtain are valid also for the case in which the electrons experience electronic interactions on the dot. In particular, our result for the transport current generalizes the ones reported previously,[22,23] which were derived under the assumption that there is no scattering on the reference arm. Within that approximation, the flux dependence of the linewidth on the quantum dot level is lost. This flux dependence, as we will show, turns out to be crucial in determining the circulating current. Section II is supplemented by an Appendix, detailing the computation of the partial currents. In Sec. III we employ the general result for the transport current to study the effect of the coupling to a boson source. To this end, we use an approximate expression for that dot Green function,[60–62] for the case in which the electrons on the dot are coupled linearly to a sonic source. Section IV is devoted to the analysis of the circulating current under irradiation. For the sake of completeness, we include in that section a discussion of the effect of electron-phonon coupling on electrons moving on electronically isolated rings (which are decoupled from the leads). Finally, we summarize our findings in Sec. V.

II. THE CURRENTS IN A BIASED INTERFEROMETER

Figure 1 portrays an Aharonov-Bohm interferometer, with a quantum dot placed on its upper arm, and a second electronic site placed on the other arm, serving as a "reference" site. All interactions (among the electrons, or electron-boson interactions) are taking place only on the quantum dot. The interferometer is connected at the left and at the right to electronic reservoirs, kept at slightly different (or equal) chemical potentials. The connection is via single-channel leads. The model Hamiltonian describing this system consists of four terms

$$\mathcal{H} = \mathcal{H}_{\text{leads}} + \mathcal{H}_{\text{ref}} + \mathcal{H}_{\text{d}} + \mathcal{H}_{\text{tun}}, \tag{1}$$

in which the first term describes the leads, which are assumed to be two free-electron systems

$$\mathcal{H}_{\text{leads}} = \sum_k \epsilon_k c_k^\dagger c_k + \sum_p \epsilon_p c_p^\dagger c_p. \tag{2}$$

(We omit spin indices when they are not necessary.) The left lead states are denoted by k, and the right ones by p, with c_k (c_p) being the destruction operator for states on the left (right) lead. For one-dimensional leads, described by a tight-binding model with a nearest-neighbor hopping matrix element J, one has $\epsilon_k = -2J\cos k$, and similarly $\epsilon_p = -2J\cos p$. The chemical potential in the reservoir connected to the left lead, μ_ℓ, can differ from that on the right reservoir, μ_r. Otherwise, the two leads are taken as identical, i.e., they have the same large bandwidth, $2J$. The reference site is taken for simplicity as having a single localized level of energy ϵ_0; hence

$$\mathcal{H}_{\text{ref}} = \epsilon_0 c_0^\dagger c_0. \tag{3}$$

The dot Hamiltonian $\mathcal{H}_{\text{d}}$ is not specified at the moment; it may include electron-electron interactions or electron-phonon interactions. For simplicity, we assume that only one of the dot single-energy levels is effectively connected to the leads. It is possible to carry out a more general calculation; however, the algebra then becomes complicated and may obscure the physical effects we wish to explore. Hence, we write for the tunneling Hamiltonian

$$\mathcal{H}_{\text{tun}} = \sum_k V_k c_k^\dagger d + \sum_p V_p c_p^\dagger d + \sum_k \upsilon_k c_k^\dagger c_0 + \sum_p \upsilon_p c_p^\dagger c_0 + \text{hc}, \tag{4}$$

where d is the destruction operator for the electron on the dot. The tunneling matrix elements for a one-dimensional tight-binding model read

$$V_k = -\sqrt{\frac{2}{N}} j_\ell \sin k, \quad V_p = -\sqrt{\frac{2}{N}} j_r \sin p,$$

$$\upsilon_k = -\sqrt{\frac{2}{N}} i_\ell e^{i\phi_\ell} \sin k, \quad \upsilon_p = -\sqrt{\frac{2}{N}} i_r e^{-i\phi_r} \sin p, \tag{5}$$

where N is the number of sites on each of the leads, and gauge invariance allows one to assign the flux dependence to the reference arm, such that the total flux (which is the magnetic flux threading the ring, measured in units of the flux quantum) is

$$\Phi = \phi_\ell + \phi_r. \tag{6}$$

In Eq. (5), j_ℓ and j_r are the matrix elements coupling the dot to the left and right point contacts, and i_ℓ and i_r are those connecting the reference site to the same points. We emphasize that the model considered here does *not allow*[25] *for any electron losses*. This is often referred to as a "closed interferometer."

Under the circumstances described above, a transport current I is passed through the ring, say from left to right. This current splits into the currents moving in the upper and lower arms of the ring

$$I = I_1 + I_2. \tag{7}$$

When all electrons entering the interferometer from the left reservoir leave it for the right reservoir, and are not lost to the surrounding (as sometimes happens in the experiments), one has $I_1 + I_3 = I_2 + I_4 = 0$. For reasons related to the detailed calculations below, we keep the four partial currents separate. The current *circulating* the ring under the effect of the Aharonov-Bohm flux, I_{cir}, is conveniently defined as

$$2I_{\text{cir}} = \left.\frac{1}{2}(I_1 - I_2)\right|_{\Phi} - \left.\frac{1}{2}(I_1 - I_2)\right|_{-\Phi}, \tag{8}$$

in order to avoid spurious currents caused by geometrical asymmetries. It is therefore seen that the calculation of both the transport current and the circulating one requires the knowledge of the partial currents in the interferometer.

An efficient way to find those currents is to employ the Keldysh technique, which is particularly suitable to handle nonequilibrium situations.[63] Using the Keldysh notations, the partial currents I_1 and I_2 are given by[62] (in units in which $\hbar = 1$)

$$I_1 = e\int \frac{d\omega}{2\pi} \sum_k V_k \left(G_{kd}^<(\omega) - G_{dk}^<(\omega)\right),$$

$$I_2 = e\int \frac{d\omega}{2\pi} \sum_k \left(\upsilon_k^* G_{k0}^<(\omega) - \upsilon_k G_{0k}^<(\omega)\right), \tag{9}$$

where

$$G_{ab}^<(\omega) = \int dt\, e^{i\omega t} i\langle b^\dagger a(t)\rangle, \tag{10}$$

and the operators a and b stand for c_k, c_p, c_0, or d. The other two partial currents, I_3 and I_4, are derived from Eqs. (9) by changing the lead index k into the second lead index, p.

The computation of all four partial currents is detailed in the Appendix. Here, we summarize the results. The first step taken there is to obtain explicit expressions [see Eqs. (A19) and (A25)] for the partial currents in terms of the various parameters, and the *exact* Green function on the dot, which includes all effects of interactions, as well as the couplings to the interferometer, to the electronic reservoirs, and to the phonon source. In the Keldysh technique this means that the above-mentioned expressions include the Keldysh function $G_{dd}^<$ [see Eq. (10)], and the usual retarded (G_{dd}^R) and advanced [$G_{dd}^A = (G_{dd}^R)*$] dot Green functions. The frequency (ω) integration of the former, $\int d\omega G_{dd}^<$, has a very clear physical meaning: It gives the occupation number of the electrons on the dot, n_d.

When the interferometer is biased, the Keldysh Green function $G_{dd}^<$ and the occupation n_d are affected by the voltage difference such that current conservation, $I_1 + I_3 = 0$, is ensured (see Fig. 1). In practice, however, the calculation of the Keldysh function is not simple (except for the interaction-free system). We therefore resort to an approximation, which gives it in terms of G_{dd}^R and G_{dd}^A. Explicitly, one finds (see the Appendix, Sec. 2 for details)

$$I_1 + I_3 = e \int \frac{d\omega}{2\pi}[(\Sigma^R_{\rm ext} - \Sigma^A_{\rm ext})G^<_{dd} + \Sigma^<_{\rm ext}(G^A_{dd} - G^R_{dd})], \tag{11}$$

where the frequency dependence of the various functions is suppressed for brevity. Here, $\Sigma^R_{\rm ext}$ is that part of the retarded self-energy on the dot which comes solely from the couplings to the interferometer and to the leads. Namely, it is the self-energy part for the interaction-free system. Similarly, $\Sigma^A_{\rm ext} = (\Sigma^R_{\rm ext})^*$ is the advanced self-energy coming from those couplings, and $\Sigma^<_{\rm ext}$ is the corresponding Keldysh function. All three of the above self-energies can be found quite straightforwardly, as they pertain to the noninteracting parts of the Hamiltonian [see Eqs. (A27) and (28)]. When the system is free of interactions, or when it is unbiased, namely, $\mu_\ell = \mu_r$ (see the Appendix, Sec. 2), the integrand in Eq. (11) vanishes. When the (interacting) system is slightly biased, the bias causes only very small changes in the Fermi functions f_ℓ and f_r, of the left and of the right reservoirs, except in the range $\mu_\ell - \mu_r$ around the Fermi energy. Here

$$f_\ell(\omega) = \frac{1}{e^{\beta(\omega-\mu_\ell)} + 1}, \quad f_r(\omega) = \frac{1}{e^{\beta(\omega-\mu_r)} + 1}. \tag{12}$$

Hence, we expect the integrand in Eq. (11) to be dominated by contributions from that vicinity of the Fermi energy. If the integrand in Eq. (11) varies slowly with the frequency there, then the vanishing of the integral would also imply the vanishing of the integrand, namely

$$G^<_{dd} = \Sigma^<_{\rm ext}\frac{G^R_{dd} - G^A_{dd}}{\Sigma^R_{\rm ext} - \Sigma^A_{\rm ext}}. \tag{13}$$

In some cases,[62] this equation follows from the "wideband approximation," which neglects the ω dependence of the resonance width $\Im\Sigma^A_{\rm ext}$. Equation (13) is used to eliminate the dot Keldysh Green function from the expressions for the currents. It should be emphasized (see the Appendix, Sec. 4) that the relationship Eq. (13) is *exact* for the unbiased system. This point is important for the calculation of the persistent current, for which one has to keep the contributions of all frequencies. We note in passing that the sum $I_2 + I_4$ vanishes identically as checked by an explicit calculation.

The next step taken in the Appendix is to employ the partial currents in order to obtain the transport current [Eq. (7)] and the persistent current [Eq. (8)]. The former is obtained using the wideband approximation, in which the frequency dependence of the self-energies is suppressed (see the Appendix, Sec. 3 for details)

$$\begin{aligned} I = e \int \frac{d\omega}{2\pi}(f_r - f_\ell)\Bigg\{ & T_B\Bigg(1 + G^R_{dd}\Sigma^R_{\rm ext} + G^A_{dd}\Sigma^A_{\rm ext} + \Sigma^R_{\rm ext}\Sigma^A_{\rm ext}\frac{G^R_{dd} - G^A_{dd}}{\Sigma^R_{\rm ext} - \Sigma^A_{\rm ext}}\Bigg) \\ & + 4\Gamma_\ell\Gamma_r X_B \frac{G^R_{dd} - G^A_{dd}}{\Sigma^R_{\rm ext} - \Sigma^A_{\rm ext}} + \sqrt{T_B\Gamma_\ell\Gamma_r X_B}2\cos\Phi \\ & \times\Bigg(G^R_{dd} + G^A_{dd} + (\Sigma^R_{\rm ext} + \Sigma^A_{\rm ext})\frac{G^R_{dd} - G^A_{dd}}{\Sigma^R_{\rm ext} - \Sigma^A_{\rm ext}}\Bigg)\Bigg\}. \end{aligned} \tag{14}$$

The transport current consists of three parts: The first term in the curly brackets of Eq. (14) is the current flowing through the reference arm (the lower arm of the interferometer in Fig. 1), "dressed" by the processes in which the electrons travel around the ring, as is manifested by the appearance of the dot Green functions. Here, T_B is the transmission coefficient of the reference arm of the interferometer (when decoupled from the quantum dot). In the wideband approximation (in which the energy is taken to be at the middle of the band)

$$T_B = \frac{4\gamma_\ell\gamma_r}{\epsilon_0^2 + (\gamma_\ell + \gamma_r)^2}, \tag{15}$$

where γ_ℓ and γ_r [see Eq. (A38)] are the partial linewidths on the reference site, caused by the couplings to the leads. The second term in the curly brackets of Eq. (14) is the current flowing through the interferometer arm containing the quantum dot; here, Γ_ℓ and Γ_r [see Eq. (A39)] are the partial linewidths on the dot, caused by the couplings to the leads, and

$$X_B = 1 - T_B\frac{(\gamma_\ell + \gamma_r)^2}{4\gamma_\ell\gamma_r}. \tag{16}$$

(Note that when $\gamma_\ell = \gamma_r$, X_B becomes equal to the reflection coefficient of the reference branch, $R_B = 1 - T_B$.) The last term in Eq. (14) results directly from interference, since it necessitates transmission through both arms of the interferometer, as is manifested by the product $\sqrt{T_B\Gamma_\ell\Gamma_r}$ there.

Several comments on the result (14) are called for. (1) The transport current I is *even* in the Aharonov-Bohm flux as it should be, obeying the Onsager symmetry.[64] This happens because[25] the dot Green functions G^R_{dd} and $\Sigma^R_{\rm ext}$ are even functions of Φ, due to additive contributions (with equal amplitudes) from clockwise and counterclockwise motions of the electron around the ring. (2) When the electronic system is free of interactions, the "external" self-energy part $\Sigma_{\rm ext}$ constitutes the entire self-energy of the dot Green function, namely

$$G^{R0}_{dd} = \frac{1}{\omega - \epsilon_d - \Sigma^R_{\rm ext}}, \tag{17}$$

where ϵ_d is the energy of the localized level on the dot, and the superscript "0" denotes the absence of interactions. In that case $(G^{R0}_{dd} - G^{A0}_{dd})/(\Sigma^R_{\rm ext} - \Sigma^A_{\rm ext}) = G^{R0}_{dd}G^{A0}_{dd}$, and the transport current becomes

$$I^0 = e \int \frac{d\omega}{2\pi}(f_r(\omega) - f_\ell(\omega))T^0(\omega) \simeq \frac{e^2}{2\pi}T^0(0)V, \tag{18}$$

where V is the potential difference on the interferometer, and T^0 is the transmission coefficient of the noninteracting ring

$$T^0(\omega) = |G^{R0}_{dd}(\omega)|^2|\sqrt{T_B}(\omega - \epsilon_d)e^{i\Phi} + 2\sqrt{\Gamma_\ell\Gamma_r X_B}|^2. \tag{19}$$

Equation (19) resembles the *two-slit* formula, as it consists of the absolute value squared of the sum of two terms: The one related to the transmission amplitude of the reference arm (having the factor $\sqrt{T_B}$) and the other which is related to the transmission amplitude through the dot (as expressed by $\sqrt{\Gamma_\ell \Gamma_r}$), with the Aharonov-Bohm phase factor multiplying one of those. However, in contrast to the two-slit formula, here both terms are *real*, resulting in an expression which is even in the flux. This aspect of the transmission has been discussed in great detail in Refs. 25 and 26. In the next section, we find that it persists also when the electrons on the dot are exposed to external radiation. (3) For general values of the flux Φ, the interaction-free transmission (19) *does not* show the Fano antiresonances, at which the transmission vanishes (although the line shape will be asymmetric). The reason is that when $\Phi \neq 0$ or $\Phi \neq \pi$, the interference between the two arms of the interferometer can never be made completely destructive, as was noted in Ref. 65. On the other hand, finite values of the flux do not prevent the transmission from achieving the unitary limit. Inspection of Eq. (19) in conjunction with the explicit expressions for the external self-energy, Eqs. (A40), shows that the maximal value for the transmission, $T^0=1$, is reached when the interferometer is symmetric, i.e., when $\Gamma_\ell=\Gamma_r$ and $\gamma_\ell=\gamma_r$, the local level on the dot becomes a resonance, i.e., $\omega-\epsilon_d-\Re\Sigma^R_{\rm ext}=0$, and the Aharonov-Bohm flux takes the particular value $\cos\Phi = -T_B/(1+R_B)$.

We next turn to the computation of the circulating current in an open ring, Eq. (8). In the case of noninteracting electrons, that current has been the subject of several studies.[13,66–68] Here, we generalize those calculations to the case where the electrons experience interactions on the quantum dot.

Inserting the expressions for the partial currents into Eq. (8) (see the Appendix, Sec. 4 for details), we find that the circulating current consists of two contributions. The first one is related to the *sum* of the two electronic distributions, $f_\ell+f_r$. It therefore flows even when the interferometer is unbiased, and $f_\ell=f_r$. The second contribution [see Eq. (A43)] arises only when the system is biased, being related to the difference $f_\ell-f_r$, and only when, in addition, the couplings of the dot and/or the couplings of the reference site to the interferometer are not equal, namely, when $i_\ell \neq i_r$ and/or $j_\ell \neq j_r$ [see Eqs. (5)]. Both contributions are induced by the Aharonov-Bohm flux and hence are proportional to $\sin\Phi$. However, the second term seems to be not as interesting as the first. We therefore omit any further consideration of that part of the circulating current, and focus only the first contribution, which reads[69]

$$I_{pc} = e\int \frac{d\omega}{i\pi}\frac{f_\ell+f_r}{4}\left[\frac{\partial\Sigma^R_{\rm ext}}{\partial\Phi}G^R_{dd} - {\rm cc}\right]. \tag{20}$$

It is interesting to note that Eq. (20) averages the flux derivative of the external self-energy over energy, with weights containing the densities of electrons and single-particle states with that energy (which are contained in G_{dd}). The flux derivative of G_{dd} does *not* appear. This is reminiscent of the equilibrium case, where the persistent current is given by the flux derivative of the energies, weighed by the electronic populations, without the appearance of the flux derivative of those (see, for example, Ref. 57). Since both $\Sigma^R_{\rm ext}$ and G^R_{dd} are even in Φ, $I_{\rm pc}$ is odd in Φ, as it should be.

It is sometimes useful to discuss the properties of an open electronic system in the language of scattering theory,[66,68] employing the concept of "transmission phases," or the Friedel phase. Such a description is particularly useful in the case of interaction-free electrons. Indeed, by manipulating Eqs. (17) and (20), one obtains that the persistent current of such a system, I^0_{pc}, is given by

$$I^0_{pc} = e\int \frac{d\omega}{\pi}\frac{f_\ell(\omega)+f_r(\omega)}{2}\frac{\partial\delta^0(\omega)}{\partial\Phi}, \tag{21}$$

where δ^0 is the phase of the retarded Green function $G^{R0} = (G^{A0})*$

$$\tan\delta^0(\omega) = -\frac{\Im\Sigma^R_{\rm ext}}{\omega-\epsilon_d-\Re\Sigma^R_{\rm ext}}. \tag{22}$$

Hence, in a steady-state situation, the persistent current of noninteracting electrons is related to the variation of the transmission phase with the Aharonov-Bohm flux. (See Ref. 67 for a different derivation of this result.) This variation replaces the variation of the eigenenergies with the flux in the equilibrium situation as the origin of the persistent current.[57]

III. RADIATION EFFECTS ON THE TRANSPORT CURRENT

The coupling between the electrons residing on the dot and a sonic source may be described by a linear, local, electron-phonon interaction[56]

$$\mathcal{H}_{\rm el-ph} = \sum_{\mathbf{q}} \alpha_{\mathbf{q}}(b^\dagger_{\mathbf{q}} - b_{\mathbf{q}})d^\dagger d, \tag{23}$$

in which $\alpha_{\mathbf{q}}=-\alpha_{-\mathbf{q}}=-\alpha^*_{\mathbf{q}}$ is the electron-phonon coupling and $b^\dagger_{\mathbf{q}}$ is the creation operator for the boson of wave vector $\mathbf{q}$. To study the transport current in the presence of such an interaction, one has to add $\mathcal{H}_{\rm el-ph}$ to the Hamiltonian Eq. (1), together with the free Hamiltonian of the boson excitations, compute the dot Green function, and then use it in Eq. (14). In the case of a linear electron-phonon interaction, one is able to obtain an approximate form for the Green function, G_{dd}, by assuming that the external self-energy does not depend on the frequency.[60,61] (For a numerical solution in the presence of an *equilibrium* phonon source, see Ref. 70.) This is a valid approximation, since the small potential difference, temperature, etc., restrict the frequency integration in Eq. (14) to a narrow range around the common Fermi energy of the two reservoirs. The explicit expression for $\Sigma^R_{\rm ext}$, valid for the case of an Aharonov-Bohm interferometer, is given in Eq. (A40).

The Green function of the dot, which takes into account the electron-phonon coupling (23), was found in Refs. 60–62. Here, we extend their result to include the effect of the reference arm and to allow for a finite electronic occupation, n_d, on the dot. The resulting form is then

$$G^R_{dd}(\omega) = -iK\Big[(1-n_d)\int_0^\infty dt\, e^{i(\omega-\epsilon_d-\Sigma^R_{\rm ext})t}e^{\Psi(t)} + n_d\int_0^\infty dt\, e^{i(\omega-\epsilon_d-\Sigma^R_{\rm ext})t}e^{\Psi(-t)}\Big]. \quad (24)$$

In the nonequilibrium case, n_d is determined by both the acoustic intensity and the relaxation processes. The on-site energy on the dot, ϵ_d, is now renormalized by the polaron shift, $\epsilon_P=\Sigma_{\mathbf{q}}|\alpha_{\mathbf{q}}|^2/\omega_q$, where ω_q denotes the phonon frequency. Since this renormalization is temperature- and flux independent, it will be omitted. The other phonon variables are contained in K, the Debye-Waller factor, and in $\Psi(t)$. Explicitly

$$K = \exp\left[-\sum_{\mathbf{q}}\frac{|\alpha_{\mathbf{q}}|^2}{\omega_q^2}(1+2N_q)\right],$$

$$\Psi(t) = \sum_{\mathbf{q}}\frac{|\alpha_{\mathbf{q}}|^2}{\omega_q^2}[N_q e^{i\omega_q t} + (1+N_q)e^{-i\omega_q t}], \quad (25)$$

where $N_q=\langle b^\dagger_{\mathbf{q}} b_{\mathbf{q}}\rangle$ is the phonon occupation of the $\mathbf{q}$ mode, which is not necessarily the thermal equilibrium one, but may be tuned externally.

Perhaps the simplest way to access the effect of the acoustic coupling is by expanding G_{dd} in the electron-phonon coupling $|\alpha_{\mathbf{q}}|^2$

$$G^R_{dd}(\omega) = KG^{R0}_{dd}(\omega) + \sum_{s=\pm}\sum_{\mathbf{q}} C^s_{\mathbf{q}} G^{R0}_{dd}(\omega+s\omega_q), \quad (26)$$

where the interaction-free Green function G^{R0}_{dd} is given in Eq. (17). Here, $s=\pm$, and

$$C^+_{\mathbf{q}} = \frac{|\alpha_{\mathbf{q}}|^2}{\omega_q^2}(N_q+n_d), \quad C^-_{\mathbf{q}} = \frac{|\alpha_{\mathbf{q}}|^2}{\omega_q^2}(1+N_q-n_d). \quad (27)$$

For a weak electron-phonon coupling, the Debye-Waller factor is

$$K \simeq 1 - \sum_{s=\pm}\sum_{\mathbf{q}} C^s_{\mathbf{q}}. \quad (28)$$

However, it is instructive to keep the Debye-Waller factor K, which multiplies the zero-order term in the expansion (26) (and, in principle, all other terms in the expansion) in its implicit form, in order to demonstrate its role in diminishing *all* contributions to the current, and not only those arising from interference.[71]

It is thus seen that the dot Green function in the presence of the electron-phonon coupling may be written as a series of terms in which there appear the interaction-free Green functions, with their frequency argument shifted by the phonon frequencies,[72,73] each multiplied by the relevant phonon occupation numbers. Hence, it is quite obvious that the transport current will have a similar form. Indeed, upon inserting the result (26) into the expression for the transport current, Eq. (14), one finds

$$I = e\int\frac{d\omega}{2\pi}(f_r(\omega)-f_\ell(\omega))T^{\rm rad}(\omega), \quad (29)$$

in which the transmission of the irradiated interferometer, $T^{\rm rad}$, is

$$T^{\rm rad}(\omega) = KT^0(\omega) + \sum_{s=\pm}\sum_{\mathbf{q}} C^s_{\mathbf{q}} T^0(\omega+s\omega_q), \quad (30)$$

and the interaction-free transmission is given in Eq. (19). It is seen that the processes contained in $T^0(\omega+s\omega_q)$ compensate partially for the detrimental effect of the Debye-Waller factor, K. We will encounter a similar situation in the discussion of the circulating current. Since we are operating in the linear response regime, it suffices to study the result (30) at the Fermi energy, namely, at zero frequency in our notations.

Let us first consider the radiation effect on the transport through the ring in the unitary limit, namely when $T^0(0)=1$. This situation, as mentioned above, occurs for a symmetric ring, when $\epsilon_d+\Re\Sigma^R_{\rm ext}=0$ and $\cos\Phi=-T_B/(1+R_B)$. Under these conditions

$$T^0(s\omega_q)|_{\rm res} = 1 - R_B\frac{\omega_q^2}{(\Im\Sigma^R_{\rm ext})^2+\omega_q^2}. \quad (31)$$

Inserting this into Eq. (30), and using Eq. (28), yields

$$T^{\rm rad}(0)|_{\rm res} = 1 - R_B\sum_{s=\pm}\sum_{\mathbf{q}} C^s_{\mathbf{q}}\frac{\omega_q^2}{(\Im\Sigma^R_{\rm ext})^2+\omega_q^2} = 1 - R_B\sum_{\mathbf{q}}\frac{|\alpha_{\mathbf{q}}|^2}{\omega_q^2}(1+2N_q)\frac{\omega_q^2}{(\Im\Sigma^R_{\rm ext})^2+\omega_q^2}. \quad (32)$$

At resonance, the transmission is independent of the electronic occupation on the dot. The coupling with the bosons reduces the transmission at resonance, the more so as the intensity of the boson source in a certain frequency range increases. It is interesting to note, however, that this effect becomes smaller as the reflection coefficient of the reference arm decreases (and therefore the current tends to go mainly through that arm).

In the general case, the transmission $T^{\rm rad}$ takes the form

$$T^{\rm rad}(0) = T^0(0) + \frac{1}{2}\sum_{\mathbf{q}} A^-_{\mathbf{q}}[T^0(\omega_q) - T^0(-\omega_q)] + \frac{1}{2}\sum_{\mathbf{q}} A^+_{\mathbf{q}}[T^0(\omega_q) + T^0(-\omega_q) - 2T^0(0)], \quad (33)$$

where $-2T^0(0)$ comes from the Debye-Waller factor. Here, $A^+_{\mathbf{q}}$ is directly proportional to the radiation intensity, while $A^-_{\mathbf{q}}$ does not depend on it. Explicitly

$$A^+_{\mathbf{q}} = C^+_{\mathbf{q}} + C^-_{\mathbf{q}} = \frac{|\alpha_{\mathbf{q}}|^2}{\omega_q^2}(1+2N_q),$$

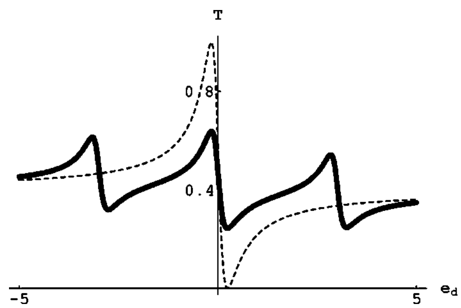

FIG. 2. The total transmission of the interferometer, under the effect of the radiation (thick line), and in the interaction-free case (dashed line), as function of the localized energy on the dot, at zero flux and for $T_B=0.4$, $\Gamma_0=0.3$, and radiation intensity scaled by 0.3 (see the text). The phonon frequency is $\Omega=3$ (energies are measured in meV).

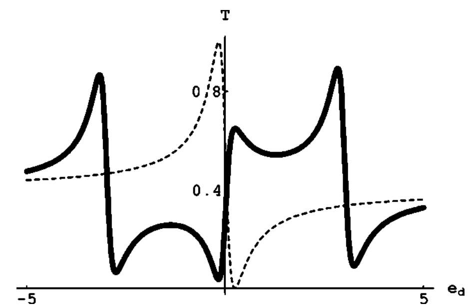

FIG. 3. The same as in Fig. 2, when the radiation intensity is doubled.

$$A_{\mathbf{q}}^{-}=C_{\mathbf{q}}^{+}-C_{\mathbf{q}}^{-}=\frac{|\alpha_{\mathbf{q}}|^2}{\omega_q^2}(2n_d-1). \tag{34}$$

Equation (33) shows that upon shining a beam of bosons at a certain frequency range, the transport current varies linearly with the intensity of the beam, as long as the latter is not too large. For example, when the interferometer is far from resonance, namely when $|\epsilon_d| \gg \Gamma_0$, where $\Gamma_0=\Gamma_\ell+\Gamma_r$ (Γ_0 is the width of the resonance level of the quantum dot itself, when it is disconnected from the reference arm), we find that the transmission, to lowest order in $\Gamma_0/|\epsilon_d|$, becomes

$$\begin{aligned} T^{\mathrm{rad}}(0)|_{\mathrm{off\ res}} &= T_B-\sqrt{T_BR_B}(T_B+(1+R_B)\cos\Phi) \\ &\times\left[\frac{\Gamma_0}{\epsilon_d}\left(1+\sum_{\mathbf{q}}A_{\mathbf{q}}^{+}\frac{\omega_q^2}{\epsilon_d^2-\omega_q^2}\right)\right. \\ &\left.+\Gamma_0\sum_{\mathbf{q}}A_{\mathbf{q}}^{-}\frac{\omega_q}{\epsilon_d^2-\omega_q^2}\right]. \end{aligned} \tag{35}$$

Of particular interest is the point where the magnitude of the interference term can be controlled by coupling the dot to a sonic source. The other factor, $A_{\mathbf{q}}^{-}$, may change sign depending on the relative location of the on-site energy on the dot and the Fermi level, but its magnitude cannot vary much, $-1 \leqslant 2n_d-1 \leqslant 1$.

In order to exemplify these results, we portray the transmission, Eq. (33), as a function of the energy of the local level on the dot (which may be controlled by a gate voltage), in Figs. 2 and 3. In drawing these curves, we have assumed a single phonon frequency, Ω, and scaled the beam intensity by $\Sigma_{\mathbf{q}}(|\alpha_{\mathbf{q}}|^2/\omega_q^2)(N_q+1/2)$.[72] The parameters used in these figures (and following ones) are meant as representative examples; clearly, the current (as well as the persistent current in the next section) is determined by the ratios of the relevant energies.

Figures 2 and 3 show the transmission at zero flux. Hence, in the absence of the radiation, the transmission exhibits the asymmetric Fano line shape. The coupling to the radiation source modifies this line shape (in particular, the transmission does not exactly vanish). It also produces additional sharp structures at $\pm\Omega$, which have still the asymmetric Fano line shape.

The dependence of the transmission on the Aharonov-Bohm flux is shown in Fig. 4. It is interesting to watch the oscillation magnitude as a function of the beam intensity. At very small values of the latter (not shown) the curve follows closely the transmission in the absence of the coupling to the bosons. Then, as the intensity is increased, the oscillation amplitude becomes *smaller* (the thick line in Fig. 4), until the curve looks almost flat. Upon further increase of the intensity, the amplitude grows again, but in the opposite direction to the one in the absence of the boson source (dashed thick line in Fig. 4).

IV. RADIATION EFFECTS ON THE CIRCULATING CURRENT

The subtle effect that electron-phonon interactions may have on interference-related properties of electrons was invoked a long time ago by Holstein,[74] in his theory of the Hall effect in hopping conduction. Holstein proposed that in order to capture the Hall effect, it is necessary to consider processes where the amplitude of the direct electron tunneling between two "sites" around which the electronic wave functions are localized interferes with an indirect tunneling amplitude, through an intermediate third site. Moreover, that interference must involve energy-conserving electron transitions to and from the intermediate site, which are assisted by

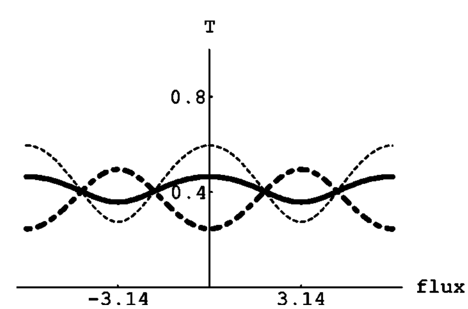

FIG. 4. The transmission as function of the magnetic flux (in units of the flux quantum). The thin dashed line presents the interaction-free case, the thick line pertains to the same beam intensity as in Fig. 2, while the thick dashed line is for the intensity as in Fig. 3. The parameters are as in those figures, with $\epsilon_d=-1$.

phonons. It turns out that this "Holstein process" has intriguing consequences for the persistent current in *electronically isolated* interferometers.[57] Since it is of interest to compare the radiation effect on persistent currents in isolated and in open rings, we begin this section with a brief summary of the Holstein process and its consequences for the isolated system, and then analyze the situation in an open ring.

The Holstein mechanism can be explained in a somewhat technical language as follows. Under hopping conduction conditions, transport can be related to *transition probabilities*. Imagine now the transition probability per unit time, P_{ij}, to tunnel from the electronic state localized at i to that localized at j. When the system is subject to a constant magnetic field, the tunneling amplitude between i and j is multiplied by the magnetic phase factor acquired from the field along the path $i-j$. Upon taking the absolute value squared of such an amplitude to obtain P_{ij} due to direct hopping alone, the result is independent of the magnetic field. Now, let us add to the direct tunneling amplitude between i and j the amplitude for indirect tunneling, for example, the path $i-\ell-j$, where ℓ denotes an intermediate site. The transition probability now depends on the total, gauge-invariant, magnetic flux enclosed by the two paths (i.e., the Aharonov-Bohm phase). However, it is an *even* function of the magnetic phase, as the tunneling amplitudes themselves can always be chosen to be real. As such, this transition probability cannot lead to a dc Hall conduction, which is *odd* in the field. This line of argument shows that, technically speaking, an imaginary contribution to at least one of the transition amplitudes is required in order to render a term odd in the magnetic phase in the transition probability.

Where can this imaginary part come from? Holstein[74] argued that when electron-phonon processes are taken into account, the intermediate state becomes in fact a continuum of energy states, consisting of the intermediate electronic energy and the continuum of phonon energies. This continuum suffices to supply the required imaginary contribution. Roughly speaking, when electron-phonon interactions are accounted for, the tunneling amplitude for the indirect path acquires, for $\epsilon_\ell > \epsilon_i$, terms such as

$$J_{i-\ell-j} \sim \sum_{\substack{n_{\mathbf{q}},\mathbf{q} \\ n_{\mathbf{q}'},\mathbf{q}'}} \frac{\langle \ell, n_{\mathbf{q}}-1, n_{\mathbf{q}'} | V | j, n_{\mathbf{q}}, n_{\mathbf{q}'} \pm 1\rangle}{\epsilon_i - \epsilon_j \mp \omega_{q'} + i\eta} \times \langle j, n_{\mathbf{q}}, n_{\mathbf{q}'} \pm 1 | V | i, n_{\mathbf{q}}, n_{\mathbf{q}'}\rangle. \tag{36}$$

Here, ϵ_i, etc. denotes electronic site energies, $\eta \to 0^+$, ω_q and $\omega_{q'}$ are boson energies, and $n_{\mathbf{q}}$ and $n_{\mathbf{q}'}$ are the quantum numbers of the $\mathbf{q}-$ and $\mathbf{q}'-$ mode occupations, respectively. In Eq. (36), V is the operator that transfers the electron between sites, and at the same time may cause the phonon states to change, obtained after an appropriate[74] unitary transformation on the electron-boson Hamiltonian, Eq. (23). Since the intermediate state now lies in a *continuum* of energies, the infinitesimal part η leads to a *finite* imaginary contribution, provided that the sum of energies in the denominator vanishes, namely, when there is an exact energy conservation, as would be needed to make a *real* transition[75] between the initial and intermediate states of the process. We emphasize, however, that the boson created/destroyed in going from i to j is only virtual; exactly *the same* boson is destroyed/created in going from j to ℓ. This exact identity is necessary for *incoherent* phonons in order to retain phase coherence[76] with the direct process from i to ℓ. More technically, one uses the relation $1/(x+i\eta)=\mathcal{P}/x - i\pi\delta(x)$, where $\mathcal{P}$ denotes the principal part. The delta-function term within the infinite sum over the phonon modes gives rise to the required finite imaginary contribution. The resulting imaginary part in $J_{i-\ell-j}$ yields a term odd in the flux in the transition probability. It is worth noting that the energy-conserving process occurs here in the intermediate state of the perturbation theory [of which Eq. (36) is the lowest term] for the combined amplitudes. Recently, this unique process has been proposed as the origin of the anomalous Hall effect in ferromagnetic semiconductors.[77]

The argument above exemplifies the necessity for one resonant process. However, in fact the Holstein process requires at least two resonant electron-phonon processes. This can be explained as follows: The three electronic energies involved in the indirect tunneling and their differences are in general all different. Hence, at least one phonon (the one denoted above by q) is needed to supply the energy difference $\epsilon_i - \epsilon_\ell$ between the initial and final electronic states. The second phonon (q' above) appears in the intermediate process, as explained above. The phonon-assisted indirect amplitude, Eq. (36), gives rise also to a contribution which is *even* in the field (coming from the principal part). That contribution does not require exact energy conservation within the intermediate state of the perturbation energy (it does, however, require the phonon supplying the energy difference between the initial and final electronic states).

The fact that the transition probability per unit time for an electron to hop between two sites may include a term which is odd in the Aharonov-Bohm flux (in addition to the term even in the flux) has an immediate result: detailed balance is broken even at thermal equilibrium. Stated in terms of transition probabilities, $P_{ij} - P_{ji} \neq 0$, and the difference is *odd* in the magnetic flux. To appreciate the outcome of this observation, let us focus our attention on a triad of three sites, i, j, and ℓ, the smallest cluster in which the doubly resonant transitions can take place. The transition probability to go from site i to site j, P_{ij} (which includes also the indirect processes via site ℓ), and the transition probability to go from that site to site ℓ (now also through the intermediate site j), $P_{i\ell}$, are such that

$$P_{ij} + P_{i\ell} = P_{ji} + P_{\ell i}, \tag{37}$$

so that charge balance is maintained at the electronic site i. However, since $P_{ij} \neq P_{ji}$, there is a net current circulating around the triad, proportional to $P_{ij} - P_{ji}$, and therefore arising from the Holstein process. That current is additional to the persistent current flowing in this system in the absence of the coupling to the phonon source. In fact, it has been found[57] that it is always flowing in the reverse direction! (The direction of the current in the triad is determined by delicate effects related to the location of the Fermi level with respect to the site energies, etc.) This current has therefore been termed "counter current." When the full transition probabilities, including the terms even and odd in the magnetic flux, are used in the proper rate equations to find the current, the resulting conductivity tensor satisfies the Onsager relations.[78]

Having related the doubly resonant processes of Holstein to the persistent current, it is worthwhile to re-examine the resonance conditions from the point of view of coherence. As we have pointed above, and as is borne out by the full calculation,[57] one of the two phonons is common to both interfering tunneling paths, thus retaining their coherence,[76] while the other is, as explained above, absorbed and reemitted by one of the paths, again retaining coherence with the other path. Hence, despite the fact that the Holstein mechanism also involves a real, energy-conserving, electron-phonon transition, it still contributes in a nontrivial way to the persistent current. However, since this contribution arises from "real" processes, it requires real phonon modes, namely, nonzero temperatures. One therefore expects that the counter current will *increase* with the temperature. On the other hand, the counter current is also multiplied by the overall damping Debye-Waller factor. Hence, the resulting temperature dependence of the counter current is nonmonotonic.[57]

We have not emphasized here the contribution of non-Holstein processes [i.e., those arising from the principal part in Eq. (36)]. Such processes are not specific to a definite phonon frequency, and therefore cannot be increased without heating/decohering the system.

When the interferometer is connected by leads to external electronic reservoirs, the energy levels on the ring acquire finite widths, given by the imaginary part of the external self-energies. Then, the effect of the coupling to the sonic source is modified. While for discrete states it required exact energy conservation (up to the width introduced by the coupling to the phonons), here it operates in a finite energy band. Nonetheless, the radiation introduces again a unique effect, which goes beyond that of the Debye-Waller exponent. In the present situation the sonic effect is of a lower order in the electron-phonon coupling, and may exist even in the $T\to 0$ limit, as will be discussed later. (For a concise summary of this result see Ref. 69.)

Indeed, inserting the expansion of the dot Green function at small electron-phonon coupling, Eq. (26), into our general result for the circulating current, Eq. (8), yields

$$I_{pc}=I_{pc}^{0}+\Delta I_{pc}, \tag{38}$$

where I_{pc}^{0} is the persistent current of the noninteracting interferometer, given by Eq. (21) above, and ΔI_{pc} is the acousto-persistent current, given, within our approximation, by

$$\Delta I_{pc}=\int\frac{d\omega}{\pi}\frac{f_\ell(\omega)+f_r(\omega)}{4}\sum_{\mathbf{q}}\Bigg[A_q^{-}\frac{\partial}{\partial\Phi}(\delta^0(\omega+\omega_q) - \delta^0(\omega-\omega_q))+A_q^{+}\frac{\partial}{\partial\Phi}(\delta^0(\omega+\omega_q)+\delta^0(\omega-\omega_q) - 2\delta^0(\omega))\Bigg], \tag{39}$$

where δ^0, the Friedel phase of the noninteracting system, is given in Eq. (22), and $A_{\mathrm{q}}^{\pm}$ are defined in Eqs. (34). The acousto-induced persistent current, ΔI_{pc}, consists of two parts: The first term depends only on the dot's occupation, n_d, and its sign may change according to the relative location of ϵ_d with respect to the Fermi energy. The second term in Eq. (39) is dominated by the phonon occupations [see Eq. (34)], via $A_{\mathbf{q}}^{+}$. [Note that the term $-2\delta^0(\omega)$ there comes from the expansion of the Debye-Waller exponent.] Examining this contribution shows that by shining a beam of phonons of a specific frequency, the magnitude of that term can be controlled experimentally, as long as the temperature of the electronic system and the intensity of the phonon source N_q are low enough to retain coherent motion of the electrons. (The intensity is also limited in the present calculation by the assumption of weak electron-phonon coupling; however, there is no conceptual difficulty to extend the calculation to stronger values.) Similar considerations apply to photons. Both the precise magnitude of these effects and the above bounds depend on the detailed geometry of the dot and on the acoustic (or electromagnetic) mismatch.

It is important to appreciate the difference between this result and the corresponding one found for the isolated ring. In the isolated ring, the Holstein process[56] required the emission (absorption) of a specific phonon, with the exact excitation energy of the electron on the ring. In the present case, the coupling to the leads turns the bound state into a resonance, with a width Γ_0 which decreases when the ring is decoupled from the leads. As a result, there is always some overlap between the tail of the Green function $G_{dd}^{R0}(\omega)$ and the Fermi distribution $f(\omega)$, yielding contributions from Holstein-type processes via phonons with many (including very low) energies. Indeed, each contribution to ΔI_{pc} contains the phase $\delta^0(\omega)$, which vanishes with Γ_0 ($\delta^0\sim\Gamma_0/|\epsilon_d|$ far from the resonance). In particular, this results in a nonzero ΔI_{pc} even at zero temperature: In that limit, if $\epsilon_d<\mu_\ell=\mu_r=0$, then $n_d=1$. Even with no phonons, $N_q=0$, the square brackets in Eq. (39) become proportional to $\partial[\delta^0(\omega+\omega_q)-\delta^0(\omega)]/\partial\Phi$, reflecting processes which begin by an emission of phonons. None of this remains for the isolated ring, when $\Gamma_0=0$.

To obtain explicit expressions, we now evaluate the frequency integration appearing in Eq. (39). Since we are operating within the linear response regime, the voltage is not essential to our effect and we may safely assume $f_\ell(\omega)=f_r(\omega)\equiv f(\omega)$. Furthermore, we take the electronic temperature to be low compared to all other energies, so that $f(\omega)\approx\Theta(-\omega)$. We also take the typical phonon frequency to be much smaller than the large bandwidth in the leads. With these approximations the frequency integration in Eq. (39) is easily performed, yielding

$$\Delta I_{pc}=\frac{\Gamma_0}{4\pi}\sin\Phi\sum_{\mathbf{q}}[A_{\mathbf{q}}^{+}(F(\omega_q)+F(-\omega_q)-2F(0)) + A_{\mathbf{q}}^{-}(F(\omega_q)-F(-\omega_q))], \tag{40}$$

where $F(\omega)$ is given in terms of $\delta^0(\omega)$, Eq. (22)

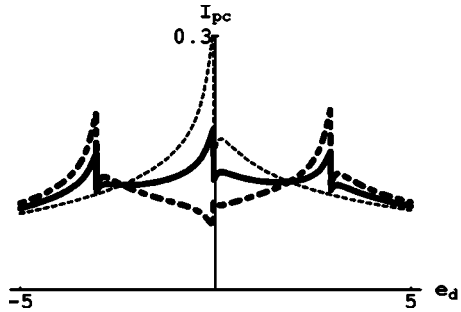

FIG. 5. The persistent current as function of ϵ_d. The thin dashed line shows I^0_{pc}, the thick line is for beam intensity 0.3, and the thick dashed line for 0.6. The flux is taken at $\pi/2$, and the other parameters are as in Fig. 2.

$$F(\omega) = -\sqrt{T_B R_B}\delta^0(\omega) - T_B \ln|\sin \delta^0(\omega)|. \tag{41}$$

The acousto-persistent current contains two types of contributions: the part associated with $F(0)$, which simply represents the "trivial" Debye-Waller renormalization of the current, and the frequency-dependent parts, which reflect the change in the persistent current due to Holstein-type processes.

As is the case in the absence of the coupling to the boson source, the acousto-persistent current is closely related to the Friedel phase of the dot, at resonance frequencies.[69] This leads to a sharp structure of I_{pc} as function of, e.g., ϵ_d, occurring whenever ϵ_d coincides with the resonance energies of the system; see Fig. 5.

On the other hand, the dependence of the persistent current on the flux is rather smooth. We portray that dependence in Fig. 6. Inspecting the behavior of the oscillation magnitude as a function of the beam intensity, we observe a similar phenomenon as has been found above for the transport current (see Fig. 4). Namely, upon increasing the intensity, the magnitude of the oscillations first decreases, then increases in the opposite direction. In order to exemplify this behavior, the thick dashed line in Fig. 6 is drawn for a rather high intensity (in our unit scheme), which is not necessarily compatible with our assumption of weak electron-phonon interaction. This sign reversal of the persistent current at a certain flux is reminiscent of the counter current alluded to above.[57]

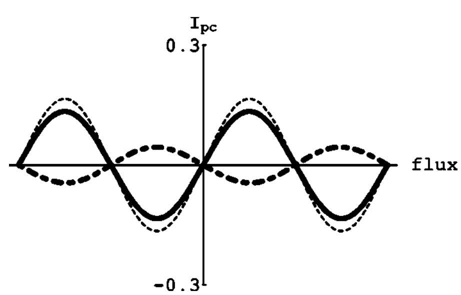

FIG. 6. The persistent current as function of the Aharonov-Bohm flux. Parameters are as in the previous figures, with $\epsilon_d = -1$. The thin dashed line portrays I^0_{pc}, the thick line is for beam intensity 0.3, and the thick dashed line is for beam intensity 2; see the text.

V. SUMMARY

We have considered the effect of coupling the electrons to a boson source on their interference pattern in an Aharonov-Bohm interferometer, and in particular focused our attention on the modifications in the transport current and in the circulating current. In both cases the overall Debye-Waller exponent appears, which reduces the interference term (as well as the "classical" term), and hence the currents, as the temperature is raised. This outcome of the coupling to the boson source is not surprising. However, in both cases there is an additional contribution, which is confined to a bounded range of phonon energies, dictated by the electronic energies.

In the case of hopping conduction, which involves transitions between discrete localized electronic states that in general differ in energy, a phonon (common to the two paths) is necessary to conserve energy in the overall hopping process.[75] In the case of an open interferometer that phonon is not necessary, since the electronic states on the two leads form continua and overlap in energy. To obtain a term odd in the magnetic field in the hopping regime, another, "second," phonon is needed, which has *to conserve the total energy between the initial and intermediate states*.[74] The reverse phonon process (namely, restoring the phonon system back to its original state) then occurs between the intermediate and final states, thus retaining phase memory in the overall process (which can then interfere with another phonon-less path). The conservation of energy in the intermediate state is a rather unusual feature, which introduces an imaginary part to the hopping amplitude for that path, and hence a nontrivial phase. That phase was crucial for the theory of the Hall effect in the hopping regime. When the ring is coupled to external leads, the effect of the radiation appears at a lower order in the electron-phonon interaction, as compared to the situation in isolated rings with localized electronic states.[57] In addition, the intermediate electronic state acquires a width via coupling to the leads. Therefore, the process may exist even at zero temperature. This is due to the finite overlap of the intermediate electronic state with the "band."

Because this contribution to the currents comes from a confined range of boson frequencies, it is expected that by modulating the intensity of the radiation in that frequency range it will be possible to manipulate the magnitude of the currents. This will require boson intensities low enough to retain the coherent motion of the electrons. However, the fact that this unique effect is confined to a rather narrow region of boson frequencies (while the detrimental Debye-Waller factor comprises all boson frequencies) gives some hope that such an acousto-magnetic effect is feasible in experiments.

ACKNOWLEDGMENTS

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APPENDIX: DETAILS OF THE CURRENT CALCULATION

As is clearly explained in Ref. 63 (see also Ref. 62), the Green functions required in the Keldysh technique can be found by considering the time-ordered Green functions, G^T. The latter satisfy the frequency-dependent Dyson equation for $G^T(\omega)$

$$G^T = G^{0T} + G^{0T}\Sigma^T G^T. \tag{A1}$$

Then, the retarded (G^R) and the advanced (G^A) Green functions are obtained by replacing T above by R or A, while $G^<$ is found according to the rule[63]

$$(\Sigma G)^< = \Sigma^R G^< + \Sigma^< G^A, \tag{A2}$$

and similarly for any other product. In the following, we omit for brevity the notation T from the time-ordered Green functions.

1. The calculation of the partial currents

The Green functions required for I_1 are $G^<_{kd}$ and $G^<_{dk}$. We present the detailed derivation of the first one. The equation of motion for the temporal Fourier transform of the time-ordered counterpart reads

$$G_{kd} = V_k g_k G_{dd} + v_k g_k G_{0d}, \tag{A3}$$

in which g_k is the free Green function of the left lead, namely

$$g_k^{R,A} = \frac{1}{\omega \pm i\eta - \epsilon_k}, \quad g_k^< = f_\ell(\omega)(g_k^A - g_k^R). \tag{A4}$$

Here, $\eta \to 0^+$, f_ℓ [Eq. (12)] is the electron distribution in the left electronic reservoir, and Eq. (10) has been employed to obtain $g_k^<$. Since we assume that the two leads in Fig. 1 are identical except for being connected to reservoirs of different chemical potentials, the free Green functions of the right lead are given by Eqs. (A4), with f_ℓ replaced by f_r. For brevity, the dependence on the frequency ω will be suppressed in most of the equations. Using Eq. (A2), we find

$$G^<_{kd} = V_k(g_k^R G^<_{dd} + g_k^< G^A_{dd}) + v_k(g_k^R G^<_{0d} + g_k^< G^A_{0d}). \tag{A5}$$

Writing explicitly the couplings V_k and v_k [see Eqs. (5)], it turns out that it is useful to define

$$\alpha^{R,A} = \frac{2}{N}\sum_k g_k^{R,A}\sin^2 k, \tag{A6}$$

and

$$\Delta = \alpha^A - \alpha^R \equiv \frac{4\pi i}{N}\sum_k \delta(\omega - \epsilon_k)\sin^2 k. \tag{A7}$$

With these notations, the partial current I_1 becomes

$$I_1 = e\int\frac{d\omega}{2\pi}(-\Delta j_\ell^2[G^<_{dd} + f_\ell(G^R_{dd} - G^A_{dd})] + j_\ell i_\ell e^{i\phi_\ell}[f_\ell \Delta G^A_{0d} + \alpha^R G^<_{0d}] - j_\ell i_\ell e^{-i\phi_\ell}[\alpha^A G^<_{d0} + f_\ell \Delta G^R_{d0}]). \tag{A8}$$

The equation of motion for the time-ordered Green function G_{0d} reads

$$G_{0d} = g_0\left(\sum_k v_k^* G_{kd} + \{k \to p\}\right), \tag{A9}$$

in which the notations $\{k \to p\}$ stand for the analogous sum on the right lead, and g_0 is the free Green function on the reference site, with

$$g_0^{R,A} = \frac{1}{\omega \pm i\eta - \epsilon_0}. \tag{A10}$$

Since the bare reference site is not coupled to any electronic reservoir, the free Keldysh Green function for that site vanishes. Making use of Eqs. (A3), we have

$$G^A_{0d} = Y D_0^A \alpha^A G^A_{dd}, \tag{A11}$$

where D_0^A is the reference site Green function when the upper arm of the ring is cut off

$$D_0^A = \frac{1}{\omega - i\eta - \epsilon_0 - \alpha^A(i_\ell^2 + i_r^2)}, \tag{A12}$$

and Y denotes the interference coupling

$$Y = i_\ell j_\ell e^{-i\phi_\ell} + i_r j_r e^{i\phi_r}. \tag{A13}$$

A similar calculation yields

$$G^A_{d0} = Y^* D_0^A \alpha^A G^A_{dd}. \tag{A14}$$

Applying Eq. (A2) to Eq. (A9) yields

$$G^<_{0d} = \alpha^R D_0^R Y(G^<_{dd} - f_\ell G^A_{dd}) + f_\ell \alpha^A D_0^A Y G^A_{dd} + \Delta D_0^R G^A_{dd} \times (f_r - f_\ell) i_r (J_r^R(\Phi))^* e^{i\phi_r}, \tag{A15}$$

and similarly

$$G^<_{d0} = \alpha^A D_0^A Y^*(G^<_{dd} + f_\ell G^R_{dd}) - f_\ell \alpha^R D_0^R Y^* G^R_{dd} + \Delta D_0^A G^R_{dd} \times (f_r - f_\ell) i_r J_r^R(\Phi) e^{-i\phi_r}. \tag{A16}$$

Here, we have introduced the effective couplings connecting the quantum dot to the right part of the ring

$$J_r^R(\Phi) = j_r + i_r \alpha^R D_0^R(i_\ell j_\ell e^{i\Phi} + i_r j_r), \tag{A17}$$

and to the left side

$$J_\ell^R(\Phi) = j_\ell + i_\ell \alpha^R D_0^R(i_\ell j_\ell + i_r j_r e^{-i\Phi}), \tag{A18}$$

and used the relation $D_0^R - D_0^A = -\Delta D_0^R D_0^A(i_\ell^2 + i_r^2)$.

Introducing all these results into I_1, Eq. (A8) gives that partial current in terms of the dot Green function

$$I_1 = e\int\frac{d\omega}{2\pi}\{2i\sin\Phi(i_\ell j_\ell i_r j_r)[(\alpha^A)^2 D_0^A G^A_{dd} - \mathrm{cc}]f_\ell + [\alpha^R j_\ell J_\ell^R(-\Phi) - \mathrm{cc}][G^<_{dd} + f_\ell(G^R_{dd} - G^A_{dd})] + [(J_r^R(\Phi))^* j_\ell i_\ell i_r e^{i\Phi}\alpha^R D_0^R G^A_{dd} - \mathrm{cc}]\Delta(f_r - f_\ell)\}. \tag{A19}$$

(Note that $\Delta^* = -\Delta$.) Examining Eq. (9), it is seen that the partial current I_3 is obtained from I_1, upon the replacements $\ell \leftrightarrow r$ with $k \leftrightarrow p$, and $\phi_\ell \leftrightarrow -\phi_r$, namely $\Phi \to -\Phi$. Then [see Eqs. (A17) and (A18)], $J_\ell^R(-\Phi) \leftrightarrow J_r^R(\Phi)$.

Next, we consider the partial current I_2. A similar calculation to the one leading to Eq. (A8) yields

$$I_2 = e\int \frac{d\omega}{2\pi}(-\Delta i_\ell^2[G_{00}^< + f_\ell(G_{00}^R - G_{00}^A)] + j_\ell i_\ell e^{-i\phi_\ell}[\alpha^R G_{d0}^< + \Delta f_\ell G_{d0}^A] - j_\ell i_\ell e^{i\phi_\ell}[\Delta f_\ell G_{0d}^R + \alpha^A G_{0d}^<]). \quad \text{(A20)}$$

The equation of motion for the time-ordered G_{00} gives

$$G_{00} = g_0 + g_0\left[\sum_k v_k^* G_{k0} + \{k \leftrightarrow p\}\right], \quad \text{(A21)}$$

with

$$G_{k0} = V_k g_k G_{d0} + v_k g_k G_{00}. \quad \text{(A22)}$$

Making use of Eqs. (A12) and (A14), we find

$$G_{00}^R = D_0^R + (D_0^R \alpha^R)^2 |Y|^2 G_{dd}^R, \quad \text{(A23)}$$

and

$$G_{00}^< = \Delta D_0^R D_0^A (i_\ell^2 f_\ell + i_r^2 f_r) + |Y\alpha^R D_0^R|^2 [G_{dd}^< + f_\ell(G_{dd}^R - G_{dd}^A)] + [\Delta \alpha^R D_0^R D_0^A Y i_r J_r^R(\Phi) e^{-i\phi_r}(f_r - f_\ell) G_{dd}^R - \text{cc}] + |Y|^2 f_\ell [(\alpha^A D_0^A)^2 G_{dd}^A - \text{cc}]. \quad \text{(A24)}$$

The first term here is the contribution of the lower arm of the ring alone; the other terms arise from interference.

Introducing these results into Eq. (A20) for I_2, we find

$$I_2 = e\int \frac{d\omega}{2\pi}\{D_0^R D_0^A i_\ell^2 i_r^2 \Delta^2 (f_\ell - f_r) - 2i \sin\Phi (i_\ell j_\ell i_r j_r) \times [(\alpha^A)^2 D_0^A G_{dd}^A - \text{cc}] f_\ell + [\alpha^R J_\ell^R(-\Phi)((J_\ell^R(-\Phi))*-j_\ell) - \text{cc}][G_{dd}^< + f_\ell(G_{dd}^R - G_{dd}^A)] + [\alpha^R D_0^A i_\ell i_r e^{-i\Phi} G_{dd}^R J_r^R(\Phi) \times (j_\ell - \Delta D_0^R i_\ell e^{i\phi_\ell} Y) - \text{cc}]\Delta(f_r - f_\ell)\}. \quad \text{(A25)}$$

Examining Eq. (9), it is seen that the partial current I_4 is obtained from I_2, upon the replacements $\ell \leftrightarrow r$ and $\phi_\ell \leftrightarrow -\phi_r$.

2. Current conservation

Having obtained the partial currents in terms of the dot Green functions, we now examine the consequences. The important point to bear in mind is that in the presence of interactions (confined to the quantum dot alone), those Green functions are not known and may be found only approximately. Therefore, imposing current conservation will yield general relations which the G_{dd}'s have to satisfy.

Current conservation means (see Fig. 1) that $I_1 + I_3 = I_2 + I_4 = 0$. A lengthy calculation of the sum $I_2 + I_4$ shows that it indeed vanishes. In contrast, the sum of the currents on the interferometer arm containing the dot gives

$$I_1 + I_3 = e\int \frac{d\omega}{2\pi}[(\Sigma_{\text{ext}}^R - \Sigma_{\text{ext}}^A) G_{dd}^< + \Sigma_{\text{ext}}^< (G_{dd}^A - G_{dd}^R)], \quad \text{(A26)}$$

in which Σ_{ext} denotes the self-energy of the dot Green function, which arises from the connection of the dot to the interferometer and the leads. This quantity is found from the Dyson equation (A1), using only the noninteracting parts of the Hamiltonian

$$\Sigma_{\text{ext}}^R = \alpha^R (j_\ell^2 + j_r^2 + \alpha^R D_0^R |Y|^2), \quad \text{(A27)}$$

with an analogous expression for Σ_{ext}^A, and

$$\Sigma_{\text{ext}}^< = \Delta[f_\ell |J_\ell^R(\Phi)|^2 + f_r |J_r^R(\Phi)|^2]. \quad \text{(A28)}$$

When the electronic system is unbiased, namely, when the chemical potentials on both reservoirs are identical

$$f_\ell = f_r \equiv f_{th}, \quad \text{(A29)}$$

then

$$\Sigma_{\text{ext}}^< = f_{th}(\Sigma_{\text{ext}}^A - \Sigma_{\text{ext}}^R), \quad \text{(A30)}$$

and also[63]

$$G_{dd}^< = f_{th}(G_{dd}^A - G_{dd}^R). \quad \text{(A31)}$$

It follows that, without a bias, the integrand in Eq. (A26) vanishes. In other words, when the ring is not biased, current conservation is trivially satisfied.

Another case in which Eq. (A26) is trivially satisfied is when the dot is free of any interactions. Then, the dot Green function G_{dd}^{R0} is given in Eq. (17), and obeys

$$G_{dd}^{R0} - G_{dd}^{A0} = G_{dd}^{A0}[\Sigma_{\text{ext}}^R - \Sigma_{\text{ext}}^A] G_{dd}^{R0}, \quad \text{(A32)}$$

where for simplicity it has been assumed that there is only a single electronic level on the dot, denoted ϵ_d. For the noninteracting system, one also has[63]

$$G_{dd}^{<0} = G_{dd}^{A0} \Sigma_{\text{ext}}^< G_{dd}^{R0}, \quad \text{(A33)}$$

and therefore, again, the integrand in Eq. (A26) vanishes.

Had we known the exact forms of $G_{dd}^{R,A}$ and $G_{dd}^<$ for the interacting electronic system, we would have found that current conservation is also satisfied when the ring is biased. However, as mentioned above, the dot Green function is not known exactly. Therefore, we may regard the relation Eq. (A26) as a condition imposed on $G_{dd}^{R,A}$ and $G_{dd}^<$. In order to make practical use of this condition, we assume that the main contribution to the ω integration in Eq. (A26) comes from frequencies at about the Fermi level of the electrons, and require that the *integrand* in Eq. (A26) will vanish, yielding Eq. (13) above. This approximation (sometimes referred to as the "wideband" approximation) is insufficient to determine the dot Green function, but at least it eliminates the necessity to calculate the Keldysh Green function $G_{dd}^<$, and ensures that the current through the ring is conserved. In particular, this yields the charge in the dot, which is equal to the expectation value of the dot occupation, $n_d = -i\int (d\omega/2\pi) G_{dd}^<(\omega)$ [see Eq. (10)]. With a finite bias, Eq. (13) is only approximate. However, the implied dependence of n_d on the bias voltage will still obey current conservation. We emphasize again that when the ring is not biased, or when the dot is free of any interactions, the relation (13) is always satisfied.

3. The current through the ring

A glance at Fig. 1 shows that the current through the ring, I, is given by $I = I_1 + I_2 = -I_3 - I_4$. This current is conveniently found by calculating $(I_1 + I_2 - I_3 - I_4)/2$. The terms proportional to $\sin\Phi$ are then canceled. Making use of the approximation (13), the current through the ring takes the form

$$I = I_{\rm ref} + I_{\rm dot} + I_{\rm int}, \tag{A34}$$

where the first term here, $I_{\rm ref}$, reduces to the current through the reference arm when the other arm is disconnected

$$I_{\rm ref} = e\int \frac{d\omega}{2\pi}(f_\ell - f_r)\Delta^2 i_\ell^2 i_r^2 |D_0^R|^2 \times\left(1 + G_{dd}^R\Sigma_{\rm ext}^R + G_{dd}^A\Sigma_{\rm ext}^A + \Sigma_{\rm ext}^R\Sigma_{\rm ext}^A\frac{G_{dd}^R - G_{dd}^A}{\Sigma_{\rm ext}^R - \Sigma_{\rm ext}^A}\right). \tag{A35}$$

Similarly, the current $I_{\rm dot}$, which reduces to the one flowing in the absence of the reference arm, is

$$I_{\rm dot} = e\int \frac{d\omega}{2\pi}(f_\ell - f_r)\Delta^2 j_\ell^2 j_r^2 |1 + \alpha^R D_0^R(i_\ell^2 + i_r^2)|^2 \frac{G_{dd}^R - G_{dd}^A}{\Sigma_{\rm ext}^R - \Sigma_{\rm ext}^A}. \tag{A36}$$

Each of these currents is "dressed" by processes in which the electrons travel through the other branch. As might be expected, the interference between the two branches always appears via the product $j_\ell j_r i_\ell i_r \cos\Phi$. In addition to appearing implicitly, via $\Sigma_{\rm ext}$, in $I_{\rm dot}$ and $I_{\rm ref}$, this product appears explicitly in the last member in Eq. (A34)

$$I_{\rm int} = e\int \frac{d\omega}{2\pi}(f_\ell - f_r)\Delta^2 i_\ell i_r j_\ell j_r D_0^R[1 + \alpha^A D_0^A(i_\ell^2 + i_r^2)] \times \cos\Phi\left(G_{dd}^R + G_{dd}^A + (\Sigma_{\rm ext}^R + \Sigma_{\rm ext}^A)\frac{G_{dd}^R - G_{dd}^A}{\Sigma_{\rm ext}^R - \Sigma_{\rm ext}^A}\right). \tag{A37}$$

An important aspect of the result (A34) is that it is an *even* function of the flux Φ (since both $G_{dd}^{R,A}$ and $\sigma_{\rm ext}^{R,A}$ are even functions of Φ). Namely, the current through the interferometer obeys the Onsager relations.[64] It is interesting to note that this property is not apparent from the formal expression for the current; however, once we use the relation (13), which ensures current conservation, then the flux parity of I becomes clear.

To present the current in a more transparent manner, we write the couplings i_ℓ, i_r, j_ℓ, and j_r, in terms of the partial widths they induce on the localized levels of the interferometer (the one on the reference arm and the one on the dot). Making use of the matrix elements, Eqs. (5), in conjunction with Eq. (A7), we define

$$\gamma_\ell = i_\ell^2\frac{\Delta}{2i}, \quad \gamma_r = i_r^2\frac{\Delta}{2i}, \tag{A38}$$

for the partial widths on the reference site, and

$$\Gamma_\ell = j_\ell^2\frac{\Delta}{2i}, \quad \Gamma_r = j_r^2\frac{\Delta}{2i}, \tag{A39}$$

for the partial widths on the quantum dot. In accordance with the approximation used to obtain the current Eq. (A34), we also neglect the frequency dependence of those widths. One then finds

$$\Sigma_{\rm ext}^R = -i\left(\Gamma_\ell + \Gamma_r - \frac{T_B(\gamma_\ell + \gamma_r)}{4}Z_B\right) + \mathrm{sgn}(\epsilon_0)\frac{Z_B}{2}\sqrt{T_B\gamma_\ell\gamma_r X_B}, \tag{A40}$$

where

$$Z_B = \frac{\Gamma_\ell}{\gamma_r} + \frac{\Gamma_r}{\gamma_\ell} - 2\cos\Phi\sqrt{\frac{\Gamma_\ell\Gamma_r}{\gamma_\ell\gamma_r}}, \tag{A41}$$

and T_B and X_B are given in Eqs. (15) and (16), respectively. It is thus seen that both the imaginary and the real parts of $\Sigma_{\rm ext}$ depend on the flux threading the interferometer, through the interference term Z_B. This expression for the external self-energy differs from the one reported in Ref. 23, in which the imaginary part of $\Sigma_{\rm ext}^R$ is independent of the flux, while its real part vanishes for $\Phi = \pi/2$. Although the details of $\Sigma_{\rm ext}^R$ are necessarily model dependent, the result given there, which apparently neglects any scattering on the reference arm, is obviously rather restricted to a very specific situation. Using these results yields our final result for the current through the interferometer, Eq. (14), where for simplicity we have chosen the sign of the on-site energy on the reference site to be positive. We note that this result is not the same as the ones given in Refs. 22 and 23, which neglected the scattering on the reference site. On the other hand, our expression reduces to the result obtained from a straightforward calculation (that does not employ the Keldysh technique), for an interaction-free system, as discussed above; see Eqs. (18) and (19).

4. The circulating current

In order to calculate the current circulating *around* the interferometer, we consider the quantity $(I_1 - I_2 - I_3 + I_4)/2$ employing Eqs. (A19) and (A25), and then take its antisymmetric part with respect to the flux. Clearly the first term in Eq. (A25) will eventually disappear, since it is independent of Φ. A detailed calculation shows that once the interferometer is biased, there appear terms in the circulating current resulting from asymmetries in the couplings. Accordingly, we separate the circulating current into two parts

$$I_{\rm cir} = I_{pc} + I_a, \tag{A42}$$

where I_{pc} denotes the part of the circulating current which survives even when the system is unbiased ("persistent current"), and is given in Eq. (20). The additional circulating current, which arises only when the system is biased and when there are asymmetries in the couplings, is denoted I_a

$$I_a = e i_\ell i_r j_\ell j_r (i\sin\Phi)\int\frac{d\omega}{4\pi}(f_r - f_\ell)\Delta\left[(i_r^2 - i_\ell^2)|\alpha^R D_0^R|^2\left(G_{dd}^R + G_{dd}^R + (\Sigma_{\rm ext}^R + \Sigma_{\rm ext}^A)\frac{G_{dd}^R - G_{dd}^A}{\Sigma_{\rm ext}^R - \Sigma_{\rm ext}^A}\right) + 2(j_r^2 - j_\ell^2)|\alpha^R|^2 \times D_0^A(1 + \alpha^R D_0^R(i_\ell^2 + i_r^2))\frac{G_{dd}^R - G_{dd}^A}{\Sigma_{\rm ext}^R - \Sigma_{\rm ext}^A}\right]. \tag{A43}$$

In particular, when the system is free of interactions, the "asymmetric" part of the circulating current is given by

$$I_a^0 = e i_\ell i_r j_\ell j_r (2i \sin \Phi) \int \frac{d\omega}{4\pi}(f_r - f_\ell)\Delta|\alpha^R D_0^R G_{dd}^{R0}|^2 \times [(i_r^2 - i_\ell^2)(\omega - \epsilon_d) + (j_r^2 - j_\ell^2)(\omega - \epsilon_0)]. \tag{A44}$$

In the main text we omit this part of the circulating current, which arises from the coupling asymmetries, and consider only the term I_{pc}. Moreover, since the potential difference across the interferometer is small (namely, the system is in the linear response regime), one may neglect this difference altogether in the sum $f_\ell + f_r$, and replace the electron distributions by the thermal distribution one, Eq. (A29). Note that then, the relation (13) becomes *exact*, and therefore the result (20) for I_{pc} does not rely on the wideband approximation. This is quite fortunate, since the persistent current, as opposed to the transport current, requires integration over the entire band. Hence, using for it an approximation which is valid at a narrow range around the common Fermi energy is not easily justifiable.

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