Reentrant localization of single-particle transport in disordered Andreev wires

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We study effects of disorder on the low energy single-particle transport in a normal wire surrounded by a superconductor. In addition to the Andreev diffusion that decreases with increase in the mean free path ℓ, the heat conductance is found to include the diffusive drift produced by a small particle-hole asymmetry, which increases with increasing ℓ. The conductance thus has a minimum as a function of ℓ leading to a peculiar reentrant localization as a function of the mean free path.

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I. INTRODUCTION

Transport phenomena in mesoscopic wires with dimensions much less than the dephasing length have been studied for several decades and are nowadays well understood (see Refs. 1–3 for review). Depending on the ratio between the wire length d and the mean free path of electrons ℓ one can separate different transport regimes. The ballistic regime holds for $d \ll \ell$; here the conductance is given by the Sharvin expression, $G=(e^2/\pi\hbar)N_{\mathrm{Sh}}$, where $N_{\mathrm{Sh}} \sim (k_F a)^2$ is the number of transverse modes, a is the wire radius, $a \ll d$, and k_F is the Fermi wave vector. For $\ell \ll d$, transport is diffusion controlled and exhibits an ohmic behavior $G \sim (e^2/\pi\hbar)N_{\mathrm{Sh}}\ell/d$. Upon the further decrease in the ratio ℓ/d, the ohmic dependence breaks down due to the localization effects: the conductance decays exponentially[4] when $d > N_{\mathrm{Sh}}\ell$.

This textbook picture holds if the transverse confinement of the electrons inside the wire is caused by an insulating gap in the surrounding material, which results in elastic scattering of electrons at the wire walls with large momentum transfer (normal reflections). In the present paper we consider another realization of a normal-metal wire conductor where the electrons are confined by a surrounding superconducting material. The superconducting gap Δ outside the normal wire suppresses to zero the density of states (DOS) of single-particle excitations for energies $\epsilon < \Delta$, thus localizing them in the transverse direction within the wire. These confined states are essentially determined by the particle-hole Andreev reflections with low momentum transfer at the superconducting/normal-metal (SN) boundaries. If the normal reflection processes at the SN boundaries can be ignored, we refer to such a normal-metal conducting region *inside* a superconducting environment as an "Andreev wire." An Andreev wire can be connected through bulk normal-metal leads to an external measuring circuit. Note that our definition of Andreev wire differs from that used, e.g., in Ref. 5 where it was applied for a normal conductor in an insulating environment, *connected* to superconducting leads. A simple way to create Andreev wires is to introduce vortex lines in a type II superconductor or to drive a type I superconductor into an intermediate filamentary state by applying a magnetic field. Andreev wires can be manufactured artificially in the form of normal channels in a superconducting matrix, using modern nano-fabrication techniques also employed for producing a wider class of hybrid SN structures such as Andreev interferometers[6] and billiards.[7] Experimentally, the main distinction between an Andreev wire and the usual conductor is that the single electron transport in the Andreev wire is to be probed rather by the measurements of thermal conductance because the single electron part of the charge current is short circuited by the supercurrent.

As shown in Refs. 8 and 9, in the ballistic limit $\ell \gg d$, Andreev processes suppress the single electron transport for all quasiparticle trajectories except for those which have momenta almost parallel to the wire thus avoiding Andreev reflection at the walls. The particles confined due to Andreev reflections also participate in the transport but through a slow drift along the transverse modes (Andreev states) with the group velocity $v_g=\hbar^{-1}\partial\varepsilon_{k_z}/\partial k_z \sim \epsilon/p_F$ much smaller than v_F. This Landauer-type drift contribution is the lower limit of conductance reached as the contribution of freely traversing trajectories decreases and nearly vanishes with increasing d. In total, the electronic heat conductance due to these two mechanisms is much lower than what could be derived from the Wiedemann-Franz law using the Sharvin conductance of a normal conductor: The effective number of conducting modes $N_{\mathrm{eff}}=\hbar\kappa/T$ is much smaller than $(k_F a)^2$. The conclusion of the suppression of the single electron transport in clean systems is in good agreement with experiments on the heat conductivity in type I and type II superconductors in the direction of magnetic field.[10,11]

In the present paper we investigate the effects of a weak disorder introduced by impurity scattering on the low energy transport with $\epsilon \ll \Delta$ in an Andreev wire of a radius a much larger than the coherence length ξ. We consider clean wires $\ell \gg a$ and neglect inelastic processes assuming $\ell_\epsilon \gg \ell$ where ℓ_ε is the inelastic mean free path. To elucidate our main results we first outline briefly what is known for the transport along an Andreev wire. Let us consider a quasiparticle propagating within the wire along a trajectory that bounces from the NS walls at both its ends. Neglecting the slow drift, the distributions of particles and holes are equal at the wall due to the Andreev reflection. Without disorder-induced scattering, the distributions still remain equal throughout the wire, thus the single-particle transport associated with these trajectories vanishes. Disorder causes the distributions of particles and holes to deviate from each other by an amount proportional to the probability of scattering, a/ℓ, accumulated on their way in between the two walls. In the presence of a temperature difference at the ends of the wire, the driving force on the trajectory is proportional to $(a/d)(T_1-T_2)$, thus the thermal conductance becomes $\kappa_A=(T/\hbar)N_A$, where the effective number of modes is

$$N_A = AN_{\mathrm{Sh}}(a^2/\ell d). \tag{1}$$

The counterintuitive behavior of the single-particle conductance κ_A which *increases* with *decreasing* ℓ was first predicted by Andreev[12] (see also Ref. 13). The coefficient A in Eq. (1) appears to be a slow function of ℓ: $A\sim\ln(\ell/a)$. For such "Andreev diffusion," disorder with $a\ll\ell\ll d$ opens new single-particle conducting modes which were blocked by Andreev reflections in the ballistic limit and thus stimulates the single-particle transport. This differs from the disorder effects in normal-metal/insulator/superconductor systems where disorder opens two-particle tunneling processes for electrical conductance, see Ref. 3 for review. The conductance κ_A reaches its maximum when the mean free path decreases down to $\ell\sim a$; it further transforms into $\kappa_D=(T/\hbar)N_D$ for a dirty wire $\ell\ll a$ where[14]

$$N_D \sim \nu_F a^2 D/d \sim N_{\mathrm{Sh}}\ell/d. \tag{2}$$

Here ν_F is the normal-state DOS at the Fermi level, and $D=v_F\ell/3$ is the diffusion coefficient.

Upon increasing the mean free path, Eq. (1) transforms into the ballistic expression[8] $\kappa\propto d^{-2}$ with a number of modes

$$N_A \sim N_{\mathrm{Sh}}(a^2/d^2) \tag{3}$$

for $\ell\gg d$. The number of modes decreases for longer wires and becomes $N_A\sim 1$ for $d\gtrsim a(k_Fa)$. According to the criterion of Ref. 4 a decrease in N down to $N\sim 1$ leads to localization of the transport. However, the ballistic transport determined by Eq. (3) is shunted by the quasiparticle drift along the Andreev states with a group velocity v_g. The drift contribution in the limit of very long ℓ is determined by a Landauer-type expression[9]

$$N_L = N_L^{\mathrm{bal}} \sim N_{\mathrm{Sh}}(v_g/v_F). \tag{4}$$

For typical values $k_Fa\sim 10^3$ and $v_g/v_F\sim T/E_F\sim 10^{-3}$ we obtain a macroscopic number $N_L^{\mathrm{bal}}\sim 10^3$ of conducting modes. Therefore, the total number of modes N_L+N_A remains large: the transport is delocalized even for very long wires, $d\gg a(k_Fa)$, whose ballistic conductance is already switched off, $N_A\sim 1$. Localization of the transport for very clean wires may occur only for very low temperatures such that the Andreev states are not populated.

In this paper we find that disorder introduces new features into the transport by modifying the single-particle drift along the Andreev states. The effective mean free path that controls the drift appears to be $\ell_{\mathrm{eff}}=v_g\tau$, i.e, considerably shorter than the usual mean free path $\ell=v_F\tau$ where τ is the impurity mean free time: the drifting particles bounce many times from the wire sidewalls and experience considerably more collisions with impurities than those particles which fly freely through the wire. The thermal conductance $\kappa_L=(T/\hbar)N_L$ associated with the disorder-modified drift is found to be *proportional* to the mean free path for $\ell_{\mathrm{eff}}\ll d$

$$N_L \sim \nu_F a^2 D_{\mathrm{eff}}/d \sim N_{\mathrm{Sh}}(v_g/v_F)^2(\ell/d), \tag{5}$$

where $D_{\mathrm{eff}}=v_g\ell_{\mathrm{eff}}=v_g^2\tau$ is the effective diffusion coefficient much smaller than D that appears in Eq. (2). This drift saturates at the ballistic expression Eq. (4) only for very long ℓ when $\ell_{\mathrm{eff}}\gg d$.

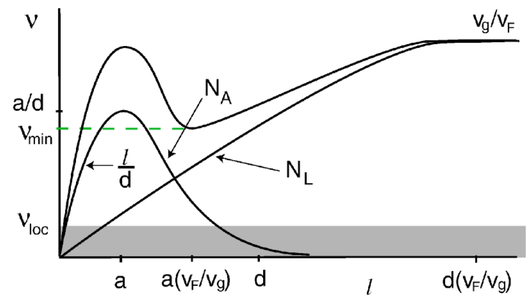

FIG. 1. Number of conducting modes (upper curve) normalized to that in a Sharvin contact, $\nu=N/N_{\mathrm{Sh}}$, as a function of ℓ interpolated according to Eqs. (1)–(4) for the wire length $d\gg a(v_F/v_g)$. The gray area at the bottom is the localization region, $N\sim 1$ corresponding to $\nu_{\mathrm{loc}}\sim(k_Fa)^{-2}$.

The total heat conductance $\kappa=(T/\hbar)(N_A+N_L)$ includes the Andreev diffusion decreasing as ℓ^{-1} and the diffusive drift that increases with increasing ℓ. Equations (1)–(4) are illustrated in Fig. 1 as functions of ℓ. If $d\gg a(v_F/v_g)$, the number of modes has a minimum

$$N_{\min} \sim N_{\mathrm{Sh}}(a/d)(v_g/v_F)$$

at $\ell_{\min}\sim a(v_F/v_g)$. Since the drift contribution is reduced by disorder, the minimum number of effective modes can now be substantially lower than that in the limit of long ℓ. For long wires, the effective number of modes can become $N_{\min}\sim 1$ which may lead to localization of the transport.[4] Thus, varying the mean free path around $\ell_{\min}$ we can obtain a peculiar *reentrant localization*: for a long enough Andreev wire the quasiparticles may become localized not only in a dirty limit $\ell\ll a$ but also for a quite long $\ell\gg a$ in such a way that the conduction opens again through either the quasiparticle drift for longer ℓ or through the Andreev diffusion for shorter ℓ. This occurs for a wire length $d>d_c$ where $d_c\sim aN_{\mathrm{Sh}}(v_g/v_F)\sim N_L^{\mathrm{bal}}a$. In contrast to the localization criterion $d>N_{\mathrm{Sh}}\ell$ for usual wires[4] mentioned earlier the new condition does not contain ℓ explicitly. The condition $d_c\gg a(v_F/v_g)$ required for self-consistency yields $(v_g/v_F)\times(k_Fa)\gg 1$ which is equivalent to the assumption that the Andreev states are well populated (see below).

II. MODEL

To develop a more quantitative description we use a quasiclassical approach modified to account for the trajectory drift along the Andreev wire. The spectrum of transverse motion in a superconductor-normal-superconductor structure is[15]

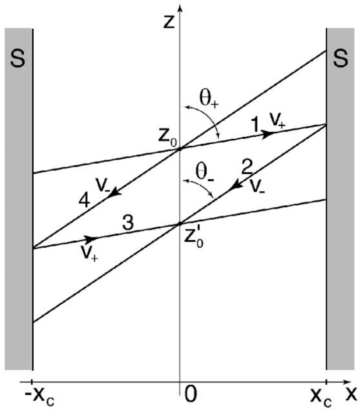

FIG. 2. Trajectories for particles (1), (3) and holes (2), (4).

$$\epsilon_n = \frac{\hbar v_F}{2x_c}\left(n+\frac{1}{2}\right)\sqrt{1-\frac{p_z^2}{p_F^2}}. \tag{6}$$

Here $p_z = p_F\cos\theta$ and $2x_c \sim a$ is the length of the projection of the trajectory section between two superconducting walls onto the plane perpendicular to the wire axis (the z axis), see Fig. 2. The particular gap profile near the wire wall is not relevant as long as $a \gg \xi$. Due to a p_z dependence of the energy, particles perform a slow drift along z with a group velocity

$$v_g = \frac{\partial\epsilon_n}{\partial p_z} = -v_F\frac{\epsilon_n}{2E_F}\frac{\cos\theta}{\sin^2\theta}. \tag{7}$$

The group velocity can also be obtained[16] by considering particle and hole trajectories, Fig. 2. The momentum along x is $p_x^{\pm} = p_\perp \pm m\epsilon/p_\perp$ for particles and holes, where $p_\perp = \sqrt{p_F^2 - p_z^2} = p_F\sin\theta$. These trajectories have the direction angles $\tan\theta_\pm = p_x^\pm/p_z$ with respect to the z axis such that $\sin(\theta_+ - \theta_-) = (\epsilon/\epsilon_F)\cot\theta$. The velocity along the trajectories is

$$v_\pm = m^{-1}\sqrt{(p_x^\pm)^2 + p_z^2} = v_F \pm \epsilon/p_F. \tag{8}$$

A particle on trajectory (1) in Fig. 2 with the coordinates $x = s_+\sin\theta_+$, $z = z_0 + s_+\cos\theta_+$, where s_+ is the distance along the trajectory, is Andreev reflected at the wall and returns back along the hole trajectory (2) with the coordinates $x = s_-\sin\theta_-$, $z = z_0' + s_-\cos\theta_-$. Both $s_\pm$ are measured from the wire axis. The intersection with the wall has the coordinates $x = x_c$ and $z = z_0 + x_c\cot\theta_+ = z_0' + x_c\cot\theta_-$. The drift velocity defined as $v_g = v_F\sin\theta(z_0' - z_0)/2x_c$ coincides with Eq. (7). This approach is valid for wide wires $a \gg \xi$ where the drift z_c is much larger than the width k_F^{-1} of the wave packet. Since $z_c \sim a(v_g/v_F)$, this requires $(v_g/v_F)(k_F a) \gg 1$ which is equivalent to the condition $\epsilon \gg \epsilon_0$, where ϵ_0 is the lowest level energy in Eq. (6).

III. KINETIC EQUATIONS

For $\epsilon \gg \epsilon_0$ the DOS in the normal region coincides with that in the normal state, ν_F. For particle and hole distributions

$$f_+(\epsilon,\mathbf{p}) = n_{\mathbf{p},\epsilon},\ f_-(\epsilon,\mathbf{p}) = 1 - n_{-\mathbf{p},-\epsilon},$$

respectively, the Boltzmann equation takes the form

$$\pm v_\pm\frac{\partial f_\pm}{\partial s} = -\frac{1}{\tau}(f_\pm - \langle f_\pm\rangle). \tag{9}$$

Here $\langle\cdots\rangle$ denotes averaging over the momentum directions. In both the upper-sign and lower-sign equations, the distance s is measured in the direction of $+\mathbf{p}$.

Since all particles are Andreev reflected as holes, the single-particle current through the wire sidewalls vanishes. Using $mv_\pm\sin\theta_\pm = p_x^\pm$ we put at the walls

$$v_+\sin\theta_+ f_+ = v_-\sin\theta_- f_-. \tag{10}$$

The kinetic Eq. (9) on the trajectory (1) gives

$$f_+(s_+) = f_+(0)e^{-s_+/\ell_+} + \ell_+^{-1}e^{-s_+/\ell_+}\int_0^{s_+}\langle f_+(s_+')\rangle e^{s_+'/\ell_+}ds_+'. \tag{11}$$

The function $f_+(0)$ is taken at $x=0$, $z=z_0$. To get the corresponding expression on trajectory (2) one substitutes f_+, s_+, ℓ_+, and z_0 with f_-, s_-, ℓ_-, and z_0', respectively. Putting $s_+ = x_c/\sin\theta_+$ and $s_- = x_c/\sin\theta_-$ for f_+ and f_-, respectively, we insert the result into the boundary condition Eq. (10). The distributions at the trajectories (3) and (4) for $s<0$ are found by replacing $z_0 \leftrightarrow z_0'$. The boundary condition for them is applied at $s_+ = -x_c/\sin\theta_+$ for f_+ and at $s_- = -x_c/\sin\theta_-$ for f_-.

To solve Eq. (11) we assume that $\langle f(x,z)\rangle$ depends only on z

$$\langle f(s)\rangle = \langle f(0)\rangle + s\cos\theta\frac{\partial\langle f\rangle}{\partial z}.$$

Within the leading terms in v_g/v_F the equations for the distribution functions at the wire axis have the form

$$f_2 = \frac{v_g}{v_F\cos\theta\cosh^2(s_c/\ell)}(f_1 - \langle f_1\rangle) + \frac{v_g}{v_F\cos\theta\cosh(s_c/\ell)}\langle f_1\rangle - \frac{1-\cosh(s_c/\ell)}{\cosh(s_c/\ell)}\left[\langle f_2\rangle - \ell\cos\theta\frac{\partial\langle f_1\rangle}{\partial z}\right] \tag{12}$$

and

$$\frac{v_g|s_c|}{v_F}\left[\frac{\partial f_1}{\partial z} + \frac{1-\cosh(s_c/\ell)}{\cosh(s_c/\ell)}\frac{\partial\langle f_1\rangle}{\partial z}\right] = -\tanh\left(\frac{|s_c|}{\ell}\right)(f_1 - \langle f_1\rangle). \tag{13}$$

We denote $s_c = x_c/\sin\theta$ and introduce

$$f_1 = -(f_+ + f_-),\quad f_2 = -(f_+ - f_-) \tag{14}$$

in accordance with the definitions used in the theory of superconductivity.[17] The functions f_1 and f_2 are nearly constant along the trajectory within the wire.

For angles $\theta \gg \theta_\ell$ where $\theta_\ell \sim a/\ell \ll 1$ these equations reduce to

$$f_2 = \frac{v_g}{v_F\cos\theta}f_1 - \cos\theta\frac{s_c^2}{2\ell}\frac{\partial\langle f_1\rangle}{\partial z}, \tag{15}$$

$$\ell \frac{v_g}{v_F}\frac{\partial f_1}{\partial z} = -(f_1 - \langle f_1\rangle). \tag{16}$$

Analysis of Eq. (12) shows that for angles $\theta \ll \theta_\ell$, the distinction between the usual and the Andreev diffusion disappears, and $f_2 = \langle f_2\rangle - \ell\cos\theta(\partial\langle f_1\rangle/\partial z)$ while the counterpart of the first term in Eq. (15) proportional to $(v_g/v_F)f_1$ decreases exponentially as $e^{-2\theta_\ell/\theta}$.

The first term in Eq. (15) describes the drift along the Andreev states with the velocity v_g. The second term is the Andreev diffusion.[12] Equation (16) introduces an effective mean free path $\ell_{\rm eff} = v_g\tau$ much shorter than the usual ℓ. In the very clean limit $\ell_{\rm eff} \gg d$, the distribution f_1 is constant along the wire and

$$f_2 = \frac{v_g}{v_F\cos\theta} f_1. \tag{17}$$

In the most practical limit when $\ell_{\rm eff} \ll d$,

$$f_1 = \langle f_1\rangle - v_g\tau\frac{\partial\langle f_1\rangle}{\partial z} \tag{18}$$

and

$$f_2 = -\left[\frac{v_g^2\ell}{v_F^2\cos\theta} + \cos\theta\frac{s_c^2}{2\ell}\right]\frac{\partial\langle f_1\rangle}{\partial z} + \frac{v_g}{v_F\cos\theta}\langle f_1\rangle. \tag{19}$$

The first term in brackets describes the disorder-modified drift. Its relative magnitude with respect to the Andreev diffusion (the second term) is of the order of $(v_g/v_F)^2(\ell/s_c)^2$, i.e., much larger than nonquasiclassical corrections of the order $(\epsilon/E_F)^2$ to the usual diffusion. We neglect the latter corrections in what follows.

IV. ENERGY CURRENT

The energy current has the form[17]

$$I_{\mathcal{E}} = -\nu_F v_F \int d^2r \int \frac{d\Omega_{\mathbf{p}}}{4\pi}\int_{-\infty}^{+\infty}\cos\theta\,\epsilon f_2\, d\epsilon. \tag{20}$$

For very long mean free path $\ell_{\rm eff} \gg d$ the distribution is determined by Eq. (17), and we recover the Landauer formula derived in Ref. 9

$$I_{\mathcal{E}} = -\nu_F \int d^2r \int \frac{d\Omega_{\mathbf{p}}}{4\pi}\int_{-\infty}^{+\infty}\varepsilon v_g f_1\, d\varepsilon. \tag{21}$$

With $f_1 = \tanh(\epsilon/2T_1) - 1$ for $v_g > 0$ and $f_1 = \tanh(\epsilon/2T_2) - 1$ for $v_g < 0$ we arrive at Eq. (4) for the heat conductance.

Assuming that the group velocity is independent of θ we obtain a qualitative behavior described in the introduction. Indeed, for $\ell_{\rm eff} \ll d$, the distribution obeys Eq. (19) where the last term does not contribute to the current. The current becomes $I_\varepsilon = \kappa(T_1 - T_2)$ with the total thermal conductance $\kappa = (T/\hbar)N = (T/\hbar)(N_A + N_L)$ given by Eqs. (1) and (5).

This simplified picture has to be modified, however, due to a rapid divergence of v_g at small angles. This leads to a more complicated behavior of the drift contribution N_L to the heat conduction as a function of temperature and of the mean free path characterized by different power laws in different regions of ℓ and T. To take into account the singular behavior of v_g at small angles it is useful to introduce two cutoff angles. We define the angle $\theta_{\min}$ such that $v_g(\theta_{\min}) \sim v_F$, i.e., $\theta_{\min} \sim \sqrt{T/E_F}$. It marks the absolute minimum angle since the group velocity v_g can under no circumstances exceed v_F. Another angle $\theta_d \sim a/d$ is the one below which the trajectory traverses freely through the wire and is not Andreev reflected from the wall. These trajectories contribute to the d^{-2} dependence of the conductance. We denote $\tilde{\theta} = \max\{\theta_{\min}, \theta_d\}$. In addition we define the angle θ_c such that $\ell v_g(\theta_c) \sim v_F d$, i.e.

$$\sin^2\theta_c \sim \frac{\epsilon}{E_F}\frac{\ell}{d}.$$

For $\theta \ll \theta_c$ the term with the derivative in Eq. (16) dominates, and the distribution corresponds to that for a ballistic drift, Eq. (17). For $\theta \gg \theta_c$ the distribution is determined by Eq. (19).

The integral over angles in Eq. (20) can be split into two regions: one is for trajectories that are almost parallel to the wire axis $\tilde{\theta} < \theta < \theta_c$, $\pi - \theta_c < \theta < \pi - \tilde{\theta}$ with the distribution as in Eq. (17), and the other is for large angles $\theta_c < \theta < \pi - \theta_c$ with the distribution as in Eq. (19). Therefore, calculating the contribution of the first term in Eq. (15) to the energy current we get by the order of magnitude

$$I_{\rm L} \sim -N_{\rm Sh}\int \delta f_1 \epsilon\, d\epsilon\left[\int_{\tilde{\theta}}^{\theta_c} e^{-2\theta_\ell/\theta}\frac{|v_g|}{v_F}\theta\, d\theta + \int_{\theta_c}^{\pi/2} e^{-2\theta_\ell/\theta}\frac{\ell v_g^2}{v_F^2 d}\theta\, d\theta\right], \tag{22}$$

where $\delta f_1 = \tanh(\epsilon/2T_1) - \tanh(\epsilon/2T_2)$. The factors $e^{-2\theta_\ell/\theta}$ account for the exponential decay of the drift contribution for small angles $\theta \ll \theta_\ell$. This equation holds if $\theta_c > \tilde{\theta}$. In the opposite limit, $\theta_c < \tilde{\theta}$, the first term disappears while the angle θ_c in the second term is replaced with $\tilde{\theta}$. Consider the total number of channels N as a function of increasing ℓ.

I. Intermediate regime: $a < \ell < d$. Here $\theta_c < \theta_{\min}$ thus there are no trajectories with ballistic drift. We find for the ratio $\nu_L = N_L/N_{\rm Sh}$

$$\nu_L \sim \frac{T^2\ell}{E_F^2 d}\frac{1}{\max\{\theta_{\min}^2, \theta_\ell^2\}}. \tag{23}$$

II. Clean limit: $d \ll \ell \ll d(E_F/T)$, such that $\theta_c \ll 1$. One can separate two cases: (i) long wires, $d \gg a\sqrt{E_F/T}$, such that $\theta_c > \theta_{\min} > \theta_d$

$$\nu_L \sim \frac{T}{E_F}\ln\left(\frac{\theta_c}{\theta_{\min}}\right) \sim \frac{T}{E_F}\ln\left(\frac{\ell}{d}\right)$$

and (ii) short wires, $d \ll a\sqrt{E_F/T}$, such that $\theta_d > \theta_{\min}$

$$\nu_L \sim \frac{T}{E_F}\ln\left[1 + \frac{\theta_c}{\theta_d}\right] \sim \frac{T}{E_F}\ln\left[1 + \frac{T\ell d}{E_F a^2}\right].$$

III. Superclean limit: $\ell \gg d(E_F/T)$, $\theta_c \to \pi/2$. The Landauer drift saturates at its ballistic limit

$$\nu_L \sim \frac{T}{E_F} \ln[\min\{E_F/T, d^2/a^2\}]. \tag{24}$$

V. DISCUSSION

Using these estimates, two regimes of long and short wires can be distinguished.

A. Long wires $d \gg a\sqrt{E_F/T}$.

$$a < \ell < a\sqrt{E_F/T}:\ \nu_L \sim \frac{T^2}{E_F^2}\frac{\ell^3}{a^2 d}, \tag{25}$$

$$a\sqrt{E_F/T} < \ell < d:\ \nu_L \sim \frac{T}{E_F}\frac{\ell}{d}, \tag{26}$$

$$d < \ell < d(E_F/T):\ \nu_L \sim \frac{T}{E_F}\ln\frac{\ell}{d}, \tag{27}$$

$$d(E_F/T) < \ell:\ \nu_L \sim \frac{T}{E_F}\ln\frac{E_F}{T}. \tag{28}$$

Therefore, the full saturation at the ℓ-independent Landauer-type drift conduction Eq. (21) occurs when $\ell_{\mathrm{eff}} \sim (T/E_F)\ell$ becomes larger than d.

B. Short wires $d \ll a\sqrt{E_F/T}$.

$$a < \ell < d:\ \nu_L \sim \frac{T^2}{E_F^2}\frac{\ell^3}{a^2 d}, \tag{29}$$

$$d < \ell < d\frac{E_F}{T}:\ \nu_L \sim \frac{T}{E_F}\ln\left[1 + \frac{T\ell d}{E_F a^2}\right], \tag{30}$$

$$d\frac{E_F}{T} < \ell:\ \nu_L \sim \frac{T}{E_F}\ln\frac{d}{a}. \tag{31}$$

To summarize: As ℓ decreases, the breakdown of the Landauer formula for an Andreev wire Eq. (24) begins at $\ell \lesssim d(E_F/T)$ with a decrease in the logarithm. However, since the group velocity diverges at small angles, the Landauer-like formula survives down to shorter ℓ. If $d \ll a(E_F/T)^{1/2}$, the logarithm is decreased from Eq. (31) to Eq. (30). For $\ell \lesssim (a^2/d)(E_F/T)$, the logarithmic term disappears completely, and the Landauer behavior is replaced with a diffusive drift along the bound states followed by Eq. (29) for yet shorter $\ell \ll d$. If $d \gg a(E_F/T)^{1/2}$, the transition occurs from Eq. (27) or (28) through Eq. (26) to Eq. (25).

For long wires $d \gg a\sqrt{E_F/T}$ we can get a *reentrant localization* for $a < \ell < d$. Indeed, the total number of modes including the Andreev diffusion is in this case

$$N \sim N_{\mathrm{Sh}}\left[\frac{a^2}{\ell d}\ln\frac{\ell}{a} + \frac{T\ell}{E_F d}\min\left\{\frac{T\ell^2}{E_F a^2}, 1\right\}\right].$$

The total conduction reaches its minimum

$$N_{\min} \sim N_{\mathrm{Sh}}(a/d)\sqrt{T/E_F}$$

at $\ell_{\min} \sim a\sqrt{E_F/T}$. The reentrant localization is possible for wire lengths longer than $d_c \sim a(k_F a)^2\sqrt{T/E_F}$. The condition $d_c \gg a\sqrt{E_F/T}$ requires $(T/E_F)(k_F a)^2 \gg 1$ which is always satisfied for $T \gtrsim \epsilon_0$. Since ℓ_{eff} and d_c are T dependent, the predicted localization regime in a wire with a given length and disorder can be observed by varying the temperature.

For short wires, $d \ll a\sqrt{E_F/T}$ the number of conducting channels has a minimum for $\ell \sim d$. This regime is similar to one discussed in the Introduction for ballistic wires: The minimum is determined by free traversing trajectories, Eq. (3). The localization can appear if $d > d_c$ where $d_c \sim a(k_F a)$. This length should be smaller than $a(E_F/T)^{1/2}$ which requires very low temperatures $(T/E_F)(k_F a)^2 \ll 1$. The corresponding temperatures are much lower than the distance between the energy levels in the wire $\hbar v_F/a$. At these temperatures the Landauer drift is already localized.

Note that the localization effects discussed above for artificially fabricated Andreev wires can be reduced by normal scattering at the wire boundaries if there is a mismatch of Fermi velocities or possible potential barriers. However, we expect that the nonmonotonic ℓ dependence of the conductance should hold as long as the normal reflection is small.

To conclude, we have developed a theory of single electron transport and reentrant localization in clean Andreev wires. Our results could stimulate experimental research of these phenomena in the mixed or intermediate state, as well as in hybrid SN structures.

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