

**Universal description of granular metals at low temperatures: Granular Fermi liquid**

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We present a unified description of the low-temperature phase of granular metals that reveals a striking generality of the low-temperature behaviors. Our model explains the universality of the low-temperature conductivity that coincides exactly with that of the homogeneously disordered systems and enables a straightforward derivation of low-temperature characteristics of disordered conductors.

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Granular metals exhibit a wealth of behaviors generic to strongly interacting disordered electronic systems and offer a unique experimental tool for studying the interplay between the effects of disorder and interactions. Depending on the strength of coupling between the grains these systems can assume either insulating or metallic phases. Remarkably, metallic samples exhibit qualitatively different transport properties in different temperature regimes; in particular, the low-temperature phase appears to be similar to disordered Fermi liquids.

The electronic transport in granular metals is governed by the nontrivial interplay between the diffusive intragrain electron motion and grain-to-grain tunneling, which is accompanied by sequential charging of the grains involved in the particular electron transfer process. This brings the notion of the Coulomb blockade, and one expects that it is the competition of intergrain coupling and electron-electron Coulomb interactions that eventually controls transport properties of granular metals. The basic parameter that characterizes transport properties is the dimensionless tunneling conductance,  $g_T$ . Depending on the bare tunneling conductance  $g_T^{(0)}$ , the conductivity can demonstrate either exponential (insulating), at  $g_T^{(0)} \ll g_T^C$ , or logarithmic (metallic), at  $g_T^{(0)} \gg g_T^C$ , temperature dependencies,<sup>1-4</sup> experiencing a metal-insulator transition at  $g_T^{(0)} = g_T^C$ .

The metallic phase was recently studied in Ref. 1 where it was shown that the low-temperature dependence of the conductivity of granular metals coincides exactly with the corresponding result for the conductivity of the homogeneously disordered samples. A question immediately arises: is it a coincidence that two different physical systems exhibit identical low-temperature transport behaviors, or there is an underlying deep connection between the two? Furthermore, do all the other physical quantities (specific heat, tunneling density of states, etc.) possess the same universality? The main result of our paper is the answer to these fundamental questions.

Generally speaking, all the universalities that one observes in nature can usually be attributed to one or another kind of the fundamental symmetry inherent to the physical system in question. For example, all critical phenomena are described in terms of universal models (Ginzburg-Landau Hamiltonian) that essentially include only the information about the large scale symmetry of the order parameter corre-

sponding to the relevant degrees of freedom. Thus in order to uncover the universality of inhomogeneous metals one has to seek for a universal description in terms of the appropriate large scale degrees of freedom that characterize disordered conductors.

We construct such a universal description of low-temperature physical properties of granular metals building on the  $\sigma$  model introduced first for the noninteracting dirty metals in Refs. 5 and 6 and generalized in Ref. 7 to include the interaction effects. Our approach applies at temperatures  $T < g_T \delta$ , where  $\delta$  is the mean energy level spacing in a single grain. The energy scale  $g_T \delta$  appears naturally as the upper energy cutoff of the effective model, since  $\hbar/g_T \delta$  is the mean time for the electron to escape from the granule.<sup>8</sup>

The main results of our work are as follows: Making use of the effective description of the granular metals in terms of the  $\sigma$  model, Eq. (9a), we show the following: (i) All the phenomena that are described in terms of the  $\sigma$  model including interaction and localization effects, and all the thermodynamic quantities are universal for granular metals. (ii) There are several physical quantities, which, although not directly related to the charges in the  $\sigma$  model can, nevertheless, be found from the renormalization group equations describing the flow of charges of the  $\sigma$  model. The important example of such a quantity is the tunneling density of states. We summarized our result in Fig. 1 where three distinct phases that one can identify on the basis of our approach are presented: (i) The universal low-temperature phase, which we refer to as to the granular Fermi liquid, generalizes naturally the Fermi-liquid phase of homogeneously disordered metals. The granular Fermi-liquid phase neighbors (ii) the high temperature,  $T > g_T \delta$ , metallic phase governed by the local single-grain physics, and (iii) the insulating phase, where  $g_T^{(0)} < g_T^C$ , characterized by the activation behavior of the conductivity.

Now we turn to the description of our model and the derivation the phase diagram in Fig. 1: We consider a  $d$ -dimensional array of metallic grains with the Coulomb interaction between electrons. The motion of electrons inside the grains is diffusive, and they can tunnel from grain to grain. We assume that in the absence of the Coulomb interaction, the sample would be a good metal. The system of weakly coupled metallic grains is described by the Hamiltonian

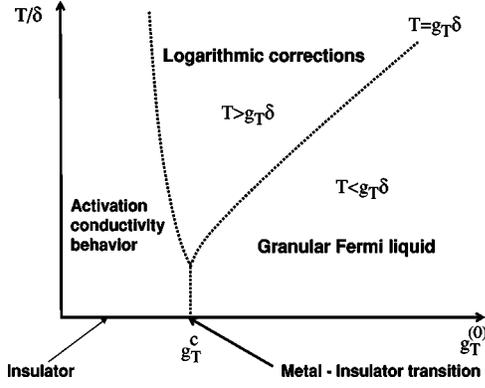


FIG. 1. Schematic phase diagram for granular metals showing crossovers between three distinct phases. At small bare tunneling conductances,  $g_T^{(0)} < g_T^C$ , a granular metal is in an insulating phase at zero temperature. The metal-insulator transition in three dimensions at  $T=0$  occurs at  $g_T^C = (1/6\pi)\ln(E_C/\delta)$ , where  $E_C$  and  $\delta$  are the charging energy and the mean energy level spacing in a single grain respectively. At large tunneling conductance,  $g_T^{(0)} > g_T^C$ , the two different types of conductivity behavior are possible: (i) the high-temperature phase,  $T > g_T\delta$ , is characterized by the logarithmic temperature dependence of the conductivity in all dimensions; (ii) the low-temperature phase,  $T < g_T\delta$ , is the universal granular Fermi-liquid phase described in terms of low-energy interacting diffusion modes.

$$\hat{H} = \hat{H}_0 + \hat{H}_{int} + \sum_{ij} \hat{\psi}^\dagger(\mathbf{r}_i) t_{ij} \hat{\psi}(\mathbf{r}_j), \quad (1a)$$

where  $t_{ij}$  is the tunneling matrix elements ( $t_{ij} = t_{ji}$ ) corresponding to the points of contact  $\mathbf{r}_i$  and  $\mathbf{r}_j$  of  $i$ th and  $j$ th grains. The Hamiltonian  $\hat{H}_0$  in Eq. (1a) is

$$\hat{H}_0 = \sum_i \int d^3r_i \hat{\psi}_i^\dagger [\hat{\mathbf{p}}^2/2m - \mu + u(\mathbf{r}_i)] \hat{\psi}_i, \quad (1b)$$

with  $\mu$  being the chemical potential; it describes noninteracting electrons scattered by the random impurity potential  $u(\mathbf{r}_i)$ . The second term in the right-hand side of Eq. (1a) describes the electron-electron interaction

$$\hat{H}_{int} = \frac{e^2}{2} \sum_{ij} \hat{n}_i C_{ij}^{-1} \hat{n}_j, \quad (1c)$$

where  $C_{ij}$  is the capacitance matrix and  $\hat{n}_i = \int d^3r_i \hat{\psi}_i^\dagger \hat{\psi}_i$  is the electron number operator in the  $i$ th grain.

Using Eqs. (1a)–(1c) the  $\sigma$  model for granular systems can be derived in the usual way:<sup>7–9</sup> we decouple the Coulomb interaction term in Eq. (1c) using the auxiliary fields  $V$ , average over disorder introducing  $Q$ -matrix field,<sup>6,7</sup> and expand around the diffusive saddle point. The final expression for the effective low-energy action reads

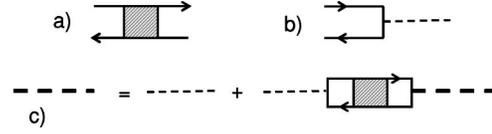


FIG. 2. Diagram (a) defines the bare propagator of fields  $B$  given by Eq. (4). Diagram (b) represents the interaction vertex that appears from the expansion of terms  $\text{Tr}[V_i Q_i]$  in Eq. (2) to first order in the field  $B$ . Diagrams (c) represent the Dyson equation that defines the screened Coulomb interaction (thick dashed line). Thin dashed lines denote the bare Coulomb interaction.

$$S = -\frac{\pi}{2\delta} \sum_i \text{Tr}[(\hat{\varepsilon} + V_i) Q_i] - \frac{\pi g_T}{8} \sum_{\langle i,j \rangle} \text{Tr}[Q_i Q_j] + \frac{1}{T} \sum_{i,j} V_i^* \frac{C_{ij}}{2e^2} V_j. \quad (2)$$

Here the sums are performed over the grain indices, the symbol  $\langle \dots \rangle$  means summation over the nearest neighbors, the trace is taken over spin, replica, and Matsubara indices, and  $\hat{\varepsilon} = i\partial_\tau$ . The field  $V$  in Eq. (2) is a vector in the frequency and replica spaces and the corresponding contraction is assumed:  $V^* V = \sum_{\omega_n, \alpha} V_{\omega_n, \alpha}^* V_{\omega_n, \alpha}$ , where  $\alpha$  is the replica index and  $\omega_n = 2\pi T n$  are the bosonic Matsubara frequencies. The  $Q$  matrix in Eq. (2) is the matrix in the Matsubara, spin, and replica spaces  $Q \rightarrow Q_{\omega_{n_1}, \omega_{n_2}; \alpha, \beta}^{a,b}$  subject to the constraint  $Q^2 = 1$ . In addition, each element of the  $Q$  matrix is a quaternion, i.e., it can be presented as  $Q = q^i \tau_i$ , where  $q_i$  is the real vector and  $\tau_i$  are the quaternion matrices.<sup>6</sup> For energies  $\varepsilon \ll E_T$ , where  $E_T$  is the Thouless energy, the  $Q$  matrices in Eq. (2) are coordinate independent within each grain. In what follows we use the Hikami parametrization for the  $Q$  matrix,<sup>10</sup>

$$Q = \begin{pmatrix} \sqrt{1 - BB^\dagger} & B \\ B^\dagger & -\sqrt{1 - B^\dagger B} \end{pmatrix}, \quad (3)$$

where the matrix  $B_{\omega_{n_1}, \omega_{n_2}}$  has nonzero elements only for frequencies  $\omega_{n_1} > 0$ ,  $\omega_{n_2} < 0$ . Expansion of the  $Q$  matrix in powers of the field  $B$  in Eq. (2) provides a systematic way to take into account  $1/g_T$  corrections.

First we consider the action (2) in the quadratic approximation in the field  $B$ , which defines the bare propagator  $D = \langle BB^\dagger \rangle$  shown in Fig. 2(a),

$$D(\omega_n, \mathbf{q}) = (2\delta/\pi)(g_T \varepsilon_{\mathbf{q}} \delta + |\omega_n|)^{-1}, \quad (4)$$

where  $\mathbf{q}$  is the quasimomentum and  $\varepsilon_{\mathbf{q}} = 2\sum_{\mathbf{a}} (1 - \cos \mathbf{q} \cdot \mathbf{a})$  with  $\{\mathbf{a}\}$  being the lattice vectors.

The Coulomb interaction, represented by the propagator of the  $V$  fields, is significantly modified by the electron screening. To include this effect in the  $\sigma$ -model approach one needs to consider the interaction vertex shown in Fig. 2(b) that appears from the expansion of the term  $\text{Tr}[V_i Q_i]$  in Eq. (2) to first order in field  $B$ . The propagator  $\Phi = \langle VV^* \rangle$  that describes the screened Coulomb interaction is given by the diagrams shown in Fig. 2(c). For a strong Coulomb interaction,  $E_C \gg (|\omega_n| + g_T \varepsilon_{\mathbf{q}} \delta)/g_T \varepsilon_{\mathbf{q}}$  the resulting propagator is given by the following expression:

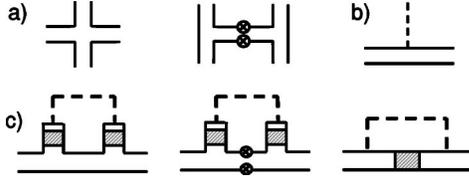


FIG. 3. Diagrams (a) appear from the expansion of terms  $\text{Tr}[\varepsilon Q_i]$  and  $\text{Tr}[Q_i Q_j]$  in Eq. (2) to the fourth order in the field  $B$ . Diagram (b) appears from the expansion of interaction term  $\text{Tr}[V_i Q_i]$  in Eq. (2) to the second order in the field  $B$ . Diagrams (c) represent the renormalization of the diffusion (Cooperon) self-energy due to Coulomb repulsion leading to Eq. (6a). The thick dashed lines denote the screened Coulomb interaction. The tunneling vertices are represented by circles.

$$\Phi(\omega_n, \mathbf{q}) = (g_T \varepsilon_q \delta + |\omega_n|) / 2g_T \varepsilon_q. \quad (5)$$

Now we turn to the description of the renormalization group (RG) procedure that we apply to the action (2) to derive the low-energy  $\sigma$  model that provides the universal description of the low-temperature phase,  $T < g_T \delta$ . We divide the field  $B$  into the slow and fast parts  $B = B_s + B_f$  and define the fast part of the field  $B_{\omega_{n_1}, \omega_{n_2}}$  in such a way that it exists either for  $\Lambda < \omega_{n_1} < \Lambda + d\Lambda$  or  $-\Lambda - d\Lambda < \omega_{n_2} < -\Lambda$ . The renormalization of the diffusion propagator  $D(\omega_n, \mathbf{q})$  in Eq. (4) due to the Coulomb repulsion after integration over the frequency shell  $d\Lambda$  is given by the sum of the diagrams shown in Fig. 3(c), resulting in

$$D(\omega_n, \mathbf{q}) = (2\delta/\pi) \xi / (\tilde{g}_T \varepsilon_q \delta + |\omega_n|). \quad (6a)$$

Here the wave function correction  $\xi$  and the renormalized intergranular tunneling conductance  $\tilde{g}_T$  are given by the following expressions:

$$\xi = 1 - \frac{1}{\pi g_T} \sum_q \frac{1}{\varepsilon_q} \frac{d\Lambda}{\Lambda}, \quad \tilde{g}_T = g_T - \frac{1}{2\pi d} \frac{d\Lambda}{\Lambda}, \quad (6b)$$

where  $d$  is the dimensionality of the granular array. The wave function correction  $\xi$  in Eq. (6a) is directly related to the correction to the single-particle density of states derived in Refs. 2 and 11. One can see from Eq. (6b) that this correction diverges in two dimensions. To avoid this problem the RG procedure in two dimensions should be modified in the way it was done in Ref. 11 where the tunneling density of states was derived on the basis of the effective phase functional. However, for the purpose of the derivation of the low-energy  $\sigma$  model, this complication can be avoided since the wave function renormalization appears only in an intermediate step of the derivation and does not contribute to the final result for the low-temperature action, as happens in two-dimensional homogeneously disordered metals.<sup>7</sup>

To find the propagator of the renormalized effective Coulomb interaction  $\Phi(\omega_n, \mathbf{q})$  in Eq. (5) one needs to consider corrections to the polarization operator shown in Fig. 4. Apart from taking into account the proper renormalized expression for the diffusion propagator  $D(\omega_n, \mathbf{q})$  in Eq. (6a), one needs to take into account the vertex correction shown in Fig. 4(b). This vertex correction cancels the wave function

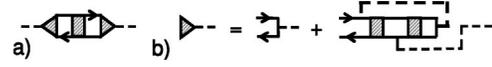


FIG. 4. Diagram (a) represents the renormalized polarization operator that includes (i) the renormalized expression for the diffusion propagator  $D(\omega_n, \mathbf{q})$  in Eq. (6a) (black rectangle) and (ii) the renormalized vertex (black triangle) shown in diagram (b). The thick dashed line denotes the screened Coulomb interaction.

renormalization  $\xi$ , as happens in the case of homogeneous disordered metals,<sup>7,13-15</sup> such that the final expression for the renormalized Coulomb potential has the following form:

$$\Phi(\omega_n, \mathbf{q}) = (\tilde{g}_T \varepsilon_q \delta + |\omega_n|) / 2\tilde{g}_T \varepsilon_q. \quad (7)$$

Here the renormalized tunneling conductance  $\tilde{g}_T$  is defined by Eq. (6b). Thus the renormalization of the Coulomb interaction is reduced to the renormalization of the intergranular tunneling conductance. This fact is very important for the formulation of the RG scheme.

Integration of Eq. (6b) in the energy interval  $(g_T \delta, E_C)$ , where  $E_C$  is the charging energy for a single grain, results in the following tunneling conductance:

$$g_T = g_T^{(0)} - (1/2\pi d) \ln(E_C/\delta). \quad (8)$$

For energies lower than  $g_T \delta$  the physics is dominated by the distances that are much larger than the size of a single grain. This allows us to consider the continuum limit of the  $\sigma$  model, and arrive at the corresponding final expression for the action,

$$S = -\frac{\pi}{2\delta} \int \text{Tr} \left[ (\varepsilon + V) Q - \frac{D}{4} (\nabla Q)^2 \right] \frac{d\mathbf{r}}{a^d} + \frac{1}{T} \int \frac{d\mathbf{r} d\mathbf{r}'}{a^{2d}} \text{Tr} \left[ V_{\mathbf{r}}^* \frac{C_{\mathbf{r}\mathbf{r}'}}{2e^2} V_{\mathbf{r}'} \right], \quad (9a)$$

where  $a$  is the grain size. The renormalized diffusion coefficient  $D$  in Eq. (9a) is given by

$$D = g_T a^2 \delta, \quad (9b)$$

with  $g_T$  being the renormalized tunneling conductance defined by Eq. (8). Since the effective model (9a) operates with the  $Q$  matrices that have only long-range degrees of freedom, it applies, with the appropriate charges and upon the high-energy renormalization, to any disordered metal, including a homogeneously disordered one. Thus, all the information about the granularity of the sample is hidden in the temperature-independent renormalization of coefficients of the effective model (9a). The conductivity of the sample is related to the effective diffusion coefficient  $D$  through the usual Einstein relation

$$\sigma = 2e^2 D (a^d \delta)^{-1}. \quad (10a)$$

The effective model (9a) together with Eq. (8) for the renormalized conductance naturally explains the result for the low-temperature,  $T < g_T \delta$ , conductivity obtained in Ref. 1. The interaction correction to conductivity has two contributions. The first contribution is temperature independent and is given by

$$\delta\sigma_1 = -\sigma_0 \frac{1}{2\pi d g_T} \ln \left[ \frac{E_C}{\delta} \right], \quad (10b)$$

where  $\sigma_0 = 2e^2 g_T a^{2-d}$  is the classical Drude conductivity of granular metals. Equation (10b) follows immediately from the renormalization of the tunneling conductance  $g_T$  in Eq. (8) and is specific to granular metals. The second contribution to conductivity is temperature dependent and comes from the low-energy renormalization of the diffusion coefficient  $D$  in the effective model (9a). It coincides with the corresponding correction to conductivity obtained for homogeneously disordered metals in Ref. 12,

$$\delta\sigma_2 = \sigma_0 \times \begin{cases} \frac{\alpha}{12\pi^2 g_T} \sqrt{\frac{T}{g_T \delta}}, & d=3 \\ -\frac{1}{4\pi^2 g_T} \ln \frac{g_T \delta}{T}, & d=2 \\ -\frac{\beta}{4\pi} \sqrt{\frac{\delta}{T g_T}}, & d=1 \end{cases} \quad (10c)$$

where  $\alpha \approx 1.83$  and  $\beta \approx 3.13$  are the numerical constants.

Although the tunneling density of states is not directly related to the charges in the  $\sigma$  model, it can, nevertheless, be found from the renormalization-group equations describing the flow of charges of the  $\sigma$  model. Since at low temperatures the flow of coupling constants of the  $\sigma$  model of granular metals is determined by the same renormalization-group equations as in the case of homogeneously disordered metals, one arrives at the important conclusion that the tunneling density of states has a multiplicative structure:

$$\nu/\nu_0 = \nu_h \nu_l, \quad (11)$$

where  $\nu_0$  is the density of states of noninteracting electrons,  $\nu_h$  is the contribution to the density of states that comes from high energies,  $\varepsilon > g_T \delta$ , while  $\nu_l$  is the contribution from low energies,  $\varepsilon < g_T \delta$ , which up to the proper renormalization of all constants coincides with the corresponding result for the density of states of disordered homogeneous metals. As an application of Eq. (11) let us consider the density of states of granular films. The low-energy contribution,  $\nu_l$  in Eq. (11), is

$$\nu_l = \exp \left[ -\frac{1}{16g_T \pi^2} \ln \frac{g_T \delta}{T} \ln \frac{g_T E_C^4}{T \delta^3} \right], \quad (12a)$$

whereas the ‘‘high-energy’’ part of the tunneling density of states,  $\nu_h$  in Eq. (11), is temperature independent at  $T < g_T \delta$  and for granular films is given by

$$\nu_h = \left[ \frac{E_C}{\delta} \right]^{1/\pi} \left[ 1 - \frac{\ln(E_C/\delta)}{4\pi g_T^{(0)}} \right]^{4g_T^{(0)}}. \quad (12b)$$

At large tunneling conductance,  $g_T \gg 1$ , the result for the low-energy contribution,  $\nu_l$ , in Eq. (12a) coincides with the

perturbative result for the density of states of granular metals obtained in Ref. 1.

Having described the effects of the electron-electron interaction on the transport properties of granular metals, we now turn to quantum (or weak localization) corrections to the conductivity.<sup>16</sup> To leading order in the inverse tunneling conductance,  $1/g_T$ , interaction and weak localization corrections can be considered independently. As usual, the weak localization correction  $\delta\sigma_{WL}$  is defined by the following expression:

$$\delta\sigma_{WL} = -\frac{2}{\pi} e^2 g_T \int C(0, \mathbf{q}) \frac{d^d q}{(2\pi)^d}, \quad (13)$$

where  $C(0, \mathbf{q})$  is the Cooperon propagator, which in the absence of electron-electron interactions is given by  $C(\omega, \mathbf{q}) = (D\mathbf{q}^2 - i\omega)^{-1}$  with  $D$  being the effective diffusion coefficient in Eq. (9b). For two- and one-dimensional samples it is important to take into account dephasing effects since Eq. (13) diverges. The dephasing time  $\tau_\phi$  may be obtained from the effective model (9a) straightforwardly using the corresponding results for homogeneously disordered metals<sup>17</sup> with the proper effective diffusion coefficient,  $D = g_T a^2 \delta$ . The final result for the weak localization corrections reads

$$\frac{\delta\sigma_{WL}}{\sigma_0} = -\frac{1}{4\pi^2 g_T} \ln \left( \frac{g_T^2 \delta}{T} \right) \quad (14a)$$

for granular films, and

$$\frac{\delta\sigma_{WL}}{\sigma_0} = -\frac{1}{2\pi g_T} \left( \frac{g_T^2 \delta}{T} \right)^{1/3} \quad (14b)$$

for granular wires. We notice that the quantum interference corrections  $\delta\sigma_{WL}$  may be easily suppressed by applying a relatively weak magnetic field such that the main temperature dependence of the conductivity comes from electron-electron interaction effects, Eqs. (10b) and (10c).

In conclusion, we have derived a low-energy  $\sigma$  model that provides a universal description of the low-temperature phase of granular metals and, more generally, of any disordered conducting medium. This model explains a striking similarity of the low-temperature transport behaviors of different disordered conductors as being governed by the same long-wavelength electronic diffusion modes. The proposed model enables one to derive the low-energy properties of granular metals from the corresponding characteristics of disordered homogeneous metals. We demonstrated the power of the developed approach by finding the density of states, the interaction, and localization corrections to the conductivity of granular metals.

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