Delocalization in Two-Dimensional Disordered Bose Systems and Depinning Transition in the Vortex State in Superconductors

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We investigate a two-dimensional (2D) Bose system with the long range interactions in the presence of disorder. Formation of the bound states at strong impurity sites gives rise to a depletion of the superfluid density. We predict the intermediate superfluid state where the condensate and localized bosons are present simultaneously. We find that interactions suppress localization and that with the increase of the boson density the system experiences a sharp *delocalization crossover* into a state where all bosons are delocalized. We map our results onto a 3D system of vortices in type II superconductors in the presence of columnar defects; the intermediate superfluid state maps to an intermediate vortex liquid where vortex liquid neighbors pinned vortices. We predict the *depinning crossover* within the vortex liquid and *depinning induced* vortex lattice-Bose glass melting.

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Effects of disorder on strongly correlated systems are central to contemporary condensed matter physics. Disordered Bose systems are of special interest [1–3]. Superfluid ^{4}He and ^{3}He remain the favorite and important experimental tools for studies of macroscopic quantum effects. Moreover, quantum mechanical mapping that relates the quantum Bose system to thermodynamics of superconducting vortices in the space of the one dimension up motivates research of Bose fluids in a random environment by the promise for further progress in vortex physics.

There has been a significant recent advance in understanding disordered Bose systems started by [4,5], who discussed a continuum model of the dilute interacting Bose gas in a random potential. The proposed model described the quasiparticle dissipation and depletion of superfluidity at zero temperature. Later we developed a systematic diagrammatic perturbation theory for the dilute Bose gas with weak disorder that enabled us to extend the description of superfluid thermodynamics on finite temperatures [6]. Disorder corrections to the thermodynamic potential and the disorder-induced shift of the condensation temperature resulting from disorder scattering of quasiparticles were found. In particular, we found that the superfluid density decreases monotonically with the temperature.

Yet many questions remain open. First, close inspection of models [4–6] shows that the described depletion of superfluid density takes account of only *scattering* of zero energy quasiparticles by the random potential neglecting the effect of the quasiparticle bound states. Further, the fact that some quasiparticles drop from the condensate and get localized suggests the existence of the *intermediate* state where both superfluid and localized components are present simultaneously. This poses the next question: If this intermediate state does exist, what part of the phase diagram does it cover? In other words, would even arbitrarily weak disorder localize the part of the condensate, or this localization effect *vanishes* if disorder is too weak or if the boson interactions are sufficiently strong?

In this Letter, we examine the behavior of a 2D interacting Bose system subject to strong disorder, having in mind the application of our results (via the quantum mechanical mapping) to the system of 3D superconducting vortices in type II superconductors containing columnar defects. At low temperatures, T, where the pristine vortex structure would form an Abrikosov lattice, columnar defects cause formation of strongly pinned Bose glass [7,8]. At the melting line $B_m(T)$, Bose glass melts into a vortex liquid. In the related quantum mechanical 2D Bose system, vortex liquid maps to the superfluid phase. The depletion of the superfluid density by disorder corresponds to enhancement of the average tilt modulus of the related vortex system [9,10]. Moreover, one can expect that this stiffening is accompanied by the change in transport properties: The part of the vortices remains pinned and does not participate in transport. This may be interpreted as the intermediate vortex phase where both vortex liquid and pinned vortices coexist.

We focus on the case where in the related vortex system the applied magnetic field is not too close to H_{c1} and vortex spacing is less than their interaction range, the magnetic field penetration length λ; this is the most common experimental situation. We show that, upon increasing the temperature (or, equivalently, the magnetic field), the vortex system undergoes a sharp crossover between the phase containing vortices pinned by the columnar defects and the phase where all vortices are delocalized. We find the expression for the crossover line:

$$\rho = \frac{c\kappa^4}{l_T^2 \ln(\lambda/L)} \exp\left[-\frac{\sqrt{8}\lambda^2}{\epsilon r_r l_T}\right], \qquad (1)$$

where ρ is the density of vortices, $\kappa = \lambda/\xi \gg 1$ is the ratio of the magnetic penetration and coherence lengths,

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r_r is the characteristic size of the columnar defect, ϵ is the anisotropy parameter, the length l_T is defined as $l_T = (\Phi_0/4\pi)^2/T$, and c is a numerical factor of the order of 1. The localization length L entering the logarithmic factor is defined in Eq. (9). The corresponding phase diagram is shown in Fig. 1, where the delocalization line is drawn along with the vortex lattice melting line. In what follows, we formulate our model in terms of vortices, establish the existence of the delocalization crossover, and derive Eq. (1) for the crossover line.

Model.—A free energy of the vortex system can be written as a functional of the vortex displacements $\mathbf{r}_i(z)$ as

$$F = \int_0^L dz\left[\sum_i \frac{\epsilon_1}{2}\left(\frac{\partial \mathbf{r}_i}{\partial z}\right)^2 + U(\mathbf{r}_i) + \frac{1}{2}\sum_{i,j} v(\mathbf{r}_{ij})\right], \quad (2)$$

where the interaction between the vortices is

$$v(r) = \epsilon_0 K_0(r/\lambda), \quad (3)$$

with $\epsilon_0 = (\Phi_0/4\pi\lambda)^2$ and K_0 being the Bessel function. The vortex tension ϵ_1 is related to the anisotropy parameter ϵ and ϵ_0 as $\epsilon_1 = \epsilon^2\epsilon_0$, and the disorder potential $U(r)$ can be written as a sum of the potential wells representing the columnar defects $U(r) = \sum_i u(r - r_i)$, where r_i is the coordinate of the ith defect.

Statistical mechanics of vortices can be mapped onto the quantum mechanical problem of interacting two-dimensional bosons described by the Hamiltonian

$$\hat{H} = \int d^2r\, \hat{\psi}^\dagger(r)[\hat{p}^2/2m - \mu + U(r)]\hat{\psi}(r) + \frac{1}{2}\int d^2r_1\, d^2r_2\, \hat{n}(r_1)v(r_1 - r_2)\hat{n}(r_2), \quad (4)$$

where $\hat{\psi}^\dagger$, $\hat{\psi}$ are Bose creation and annihilation operators, μ is the chemical potential, and $\hat{n}(r) = \hat{\psi}^\dagger(r)\hat{\psi}(r)$ is the density operator. Results obtained in the Bose gas representation can be translated into the vortex language by the substitute $\hbar \to T$, $m \to \epsilon_1$. The Bose gas energy corresponds to the free energy per unit length of the vortex system. Keeping in mind the application to vortices, we use the interpolation formula which well describes the attractive potential of a single columnar defect at distances $\xi < r < \lambda$ and temperatures $T \le T_c$ [11]:

$$u(r) = -\frac{\epsilon_0}{2}\frac{r_r^2}{r^2 + 2\xi^2}, \quad (5)$$

where ξ is the coherence length and r_r is the scale that characterizes the size of the columnar defect. We introduce the dimensionless pinning strength

$$\beta = r_r^2\epsilon_1\epsilon_0/2T^2 \quad (6)$$

and consider the temperature interval where $\beta \ll 1$. In this limit, the solution of the Schrödinger equation

$$[-\nabla_r^2/2\epsilon_1 + u(r)]\psi(r) = E_1\psi(r), \quad (7)$$

results in the pinning energy

$$E_1 \sim (T^2/\epsilon_1\xi^2)e^{-1/\sqrt{\beta}}. \quad (8)$$

The bound state E_1 is localized within the length $L \sim \sqrt{\hbar^2/|E_1|m}$; translating to vortex language we write the lateral localization length of the pinned vortex as

$$L = \xi e^{1/(2\sqrt{\beta})}. \quad (9)$$

Further, we take the interaction between the bosons (vortices) be strong enough to suppress double occupancy of the pinning site. The interaction energy of the double occupied state is

$$E_2 = \frac{1}{2}\int d^2r_1\, d^2r_2\, \phi_1^*(r_1)\phi_1(r_1)v(r_1 - r_2)\phi_1^*(r_2)\phi_1(r_2), \quad (10)$$

where ϕ_1 is the wave function of the bound sate. If $L \ll \lambda$, then $E_2 \sim \epsilon_0 \ln(\lambda/L)$, where we have used the asymptotic of the Bessel function $K_0(x) \approx \ln(1/x)$. The condition of no double occupation, $E_2 \gg |E_1|$, can be now written as

$$\epsilon_0\epsilon_1 \ln(\lambda/L) \gg (T/L)^2. \quad (11)$$

In case of low concentration of defects, the contributions from different potential wells can be treated separately; thus we arrive at the problem of the interacting Bose gas and single potential well (a single columnar defect immersed in the vortex liquid). We easily diagonalize the noninteracting Hamiltonian with a single potential well and formulate the problem in terms of the eigenfunctions ϕ_k of the noninteracting Hamiltonian

$$[\hat{p}^2/2m + u(r)]\phi_k(r) = E_k\phi_k(r), \quad (12)$$

where E_k is the energy of the kth state. The bound state corresponds to $k = 1$. Although this state has the lowest energy, the Bose condensation into it cannot occur since double occupation is forbidden. Thus, the condensation takes place to the extended state with the lowest energy (which we label with $k = 0$). In the absence of the current the basis can be chosen real $\phi^* = \phi$. The ψ operators that

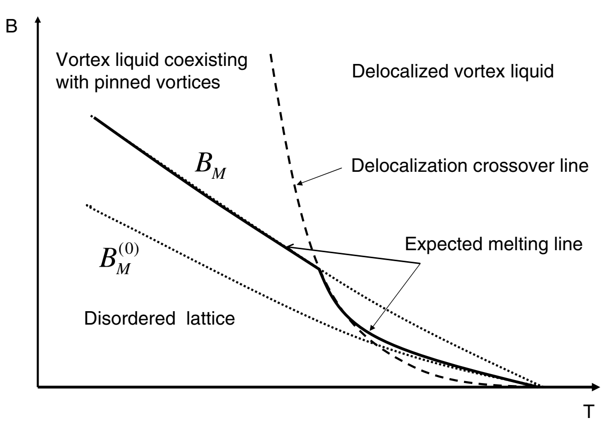

FIG. 1. Schematic phase diagram for the vortex system with columnar defects. The melting line under assumption of pinning is denoted by B_M, while the melting line of the pristine lattice is denoted by $B_M^{(0)}$.

enter the Hamiltonian (4) can be presented as a sum of three terms:

$$\hat{\psi} = \phi_0 \hat{b}_0 + \phi_1 \hat{b}_1 + \sum_{E_k>0} \phi_k \hat{b}_k, \tag{13}$$

representing the condensate, the bound state, and all other states having energy larger than the condensate energy. The effective temperature of the Bose gas $T^{(B)} \propto L_z^{-1}$, where L_z is length of the superconducting system in the z direction. A macroscopically large vortex system, thus, corresponds to the zero temperature limit for bosons. We address the most interesting case of the applied magnetic field not being too close to H_{c1} such that the average distance between vortices is smaller than the screening length λ. In this regime, the vortex system maps onto the problem of Bose gas with 2D Coulomb interaction that can be treated in the Bogoliubov approximation in the high density limit [12]. For the vortex system, the high density limit is satisfied above the melting line of the pristine lattice [13]. In this regime, the third term in Eq. (13) representing the out-of-the-condensate particles is small and can be neglected in the leading order.

Now we turn to the analysis of the population of the bound state in the presence of the condensate. Using the representation Eq. (13) in the Hamiltonian (4) and keeping only two first terms in the representation (13), we obtain the effective Hamiltonian as

$$\hat{H}_{eff} = b_1^\dagger (E_1 + \alpha - \mu) \hat{b}_1 + \gamma (\hat{b}_1^\dagger + \hat{b}_1), \tag{14}$$

where the terms $\hat{b}_1^\dagger \hat{b}_1^\dagger$ are omitted due to the no double occupancy condition. The coefficients α and γ in the Hamiltonian (14) are

$$\alpha = b_0^2 \int d^2 r_1\, d^2 r_2 [\phi_1^2(r_1) v(r_1 - r_2) \phi_0^2(r_2) + \phi_1(r_1)\phi_0(r_1) v(r_1 - r_2) \times \phi_1(r_2)\phi_0(r_2)], \tag{15}$$

$$\gamma = b_0^3 \int d^2 r_1\, d^2 r_2\, \phi_0^3(r_1) v(r_1 - r_2) \phi(r_2), \tag{16}$$

where the average value b_0 of the operator $\hat{b}_0$ is related to the condensate density ρ_0 as $b_0^2 = \rho_0$. The model (14) can be easily solved: Presenting the wave function in the form

$$\psi = a_0 |0\rangle + a_1 |1\rangle, \tag{17}$$

with a_0 and a_1 being the amplitudes of the zero and single occupied states, we find two eigenstates corresponding to the energy levels

$$E_\pm = \frac{E \pm \sqrt{E^2 + 4\gamma^2}}{2}, \tag{18}$$

where $E = E_1 + \alpha - \mu$. We see that $E_+ > 0$ and $E_- < 0$ for any sign of the energy E, thus the state with energy E_- is always occupied while E_+ is always empty. The occupation of the center is determined by the state with the lowest energy (E_-) and is given by

$$n = a_1^2 = \frac{2}{4 + (E/\gamma)^2 + (E/\gamma)\sqrt{(E/\gamma)^2 + 4}}. \tag{19}$$

One can easily see that $n \to 1$ when $E/\gamma \ll -1$ and $n \to 0$ when $E/\gamma \gg 1$. Thus, the quantity E/γ controls the occupation such that at $E = 0$ localization-delocalization crossover takes place. Making use of Eq. (15) and the relationship between the chemical potential, condensate density, and interaction $\mu = b_0^2 v_0$, where $v_0 = \int d^2 r\, v(r)$, for the parameter E we find $E = E_1 + \delta E$ with

$$\delta E = b_0^2 \int d^2 r_1\, d^2 r_2\, \phi_1(r_1) v(r_1 - r_2) \phi_1(r_2) \approx \rho L^2 \epsilon_0 \ln(\lambda/L). \tag{20}$$

Now, using the relationship between the bound state energy and the localization length, we derive the vortex density at the localization-delocalization crossover from the condition $E = 0$:

$$\rho = \frac{c\, T^2}{\epsilon_0 \epsilon_1 L^4 \ln(\lambda/L)}, \tag{21}$$

where $c \sim 1$ is a numerical constant. Plugging in L from Eq. (9) we arrive at the expression (1) for the delocalization crossover line which can be rewritten in terms of the *delocalization field* for vortices as

$$B_{dl} \simeq \Phi_0 \frac{cT^2}{\epsilon_0 \epsilon_1 \xi^4 \ln(\lambda/L)} \exp\left(-\frac{T}{T_0}\right), \tag{22}$$

where the effective temperature dependent depinning energy $T_0 = r_r \epsilon \epsilon_0 / \sqrt{8}$. One easily checks that the condition of no double occupancy (11) is always satisfied near the crossover provided $\rho L^2 < 1$.

As we see from Eq. (19), the occupation of the center varies continuously as long as "hybridization parameter" $\gamma > 0$. Therefore the boundary between the localized and delocalized phases should be viewed as a crossover rather than the true transition line. Note, however, that in case of finite concentration of defects delocalization may result in a significant change in the condensate density and thus induce the true superfluid-Bose glass transition as in [10].

While the ultimate type of the crossover remains an open question, experimentally it can appear very sharp and undistinguishable from the true first order transition. Indeed, the sharpness of the crossover is controlled by the ratio of γ and δE. Formula (16) determining γ should be corrected to include the screening of the potential by the surrounding Bose gas. In the dense limit, the screening length is $l_{sc} = \lambda(1/2\pi\rho\epsilon^2 l_T^2)^{1/4}$, and γ is estimated as

$$\gamma \approx \frac{\rho L \lambda^2 \epsilon_0}{l_T \epsilon \sqrt{2\pi}}, \tag{23}$$

such that the ratio $\gamma/\delta E = \gamma/\delta E \sim \lambda^2/[l_T L \epsilon \ln(\lambda/L)]$ is small due to the no double occupancy condition (11).

We thus conclude that, in the regime under consideration where (i) the average vortex spacing is much less than the screening length, (ii) the density of defects is much less than the vortex density, and (iii) the vortex-vortex interactions are so strong that the double occupancy of the defect never happens, there is a sharp crossover between the localized and the nonlocalized phases.

We are now in a position to construct a phase diagram for the disordered Bose system. We use the corresponding "superconductor vortex language"; the schematic vortex phase diagram for the system with columnar defects is presented in Fig. 1. Shown are the melting line of the pristine sample $B_M^{(0)}$, and melting line B_M in the sample with the columnar defects shifted, as it should [7,8], upwards as compared to $B_M^{(0)}$. Note that because of the exponential dependence on temperature, B_{dl} drops much faster than B_M as temperature approaches T_c. Thus, the delocalization line, which at high enough fields lies in the vortex liquid domain, hits and then crosses the melting line B_M, traverses the strip between B_M and $B_M^{(0)}$, and on its way to T_c crosses the pristine melting line.

In the liquid phase, the delocalization line marks the crossover separating the intermediate state where vortex liquid coexists with the pinned vortices, and the phase where all vortices are delocalized. In the strip confined between the $B_M^{(0)}$ and B_M lines, B_{dl} marks melting of the vortex lattice. Indeed, at $B_M^{(0)} < B < B_M$ the lattice is stabilized by vortices pinned by columnar defects. As soon as depinning at B_{dl} occurs, columnar defects become inessential, and the lattice loses its stability since in the absence of defects vortex liquid is a stable thermodynamic state above $B_M^{(0)}$. This is a novel type of the vortex lattice melting, the *depinning induced melting*, which occurs in the interval $B_M^{(0)} < B < B_M$. At lower fields, the melting line almost coincides with B_M^0 since the effect of pinning near the pristine melting line becomes exponentially weak.

Depinning induced melting may well explain the origin of the low-field kink in the melting line that clearly indicates a switch between the different melting mechanisms. This kink has been observed in almost all experiments on the Bose glass melting [8], and was addressed specifically in the recent study of the melting of "porous" vortex matter [14]. Although the application of our results to highly anisotropic layered systems such as Bi-based compounds (BSCCO) requires caution and many reservations, it is interesting to note that at temperatures around 80 K the characteristic energy T_0 for BSCCO parameters can be estimated as $T_0 \simeq 5$ K, close enough to that observed in [14]. The line separating the intermediate liquid and fully molten homogeneous liquid states was recently observed in $YBa_2Cu_3O_{7-\delta}$ [15].

In conclusion, we have investigated the phase diagram of a disordered Bose system with the long range interactions and established the existence of an intermediate phase, where both superfluid and localized bosons are present simultaneously. We have demonstrated that a sharp delocalization crossover occurs with the increase of boson density. We have applied our results to the vortex system in type II superconductors in the presence of columnar defects and found that this delocalization crossover describes the depinning line separating two liquid vortex phases, the intermediate phase containing both pinned vortices and liquid, and "fully molten" homogeneous liquid. We have predicted a new kind of the vortex lattice melting, the depinning induced melting, at low fields.

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