Majorana Fermions and Odd-frequency Cooper Pairs

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Topological Quantum Phenomena in Condensed Matter with Broken Symmetries

Outline

1. Why odd-freq. pairs?

2. Detection of odd-freq. pairs

3. Anomalous proximity effect

4. Majorana fermions and odd-freq. pairs

5. Summary

Symmetry Classification

$$f_{\sigma,\sigma'}(r - r') = -\langle \psi_\sigma(\boldsymbol{r})\psi_{\sigma'}(\boldsymbol{r}')\rangle$$

Fourier trans. → $f_{\sigma,\sigma'}(p)$

Spin	Orbital
singlet	s, d (even-parity)
triplet	p, f (odd-parity)

spin X orbital = -1 Fermi-Dirac statistics

Spin-flip potential   mix spin-singlet and spin-triplet

Surface & interface   mix even- and odd-parity

Odd-freq. Pairs

General definition of pairing function

$$f_{\sigma,\sigma'}(r-r',\tau-\tau') = -\langle T_\tau \psi_\sigma(r,\tau)\psi_{\sigma'}(r'\tau')\rangle$$

$$f_{\sigma,\sigma'}(p,\omega_n)$$

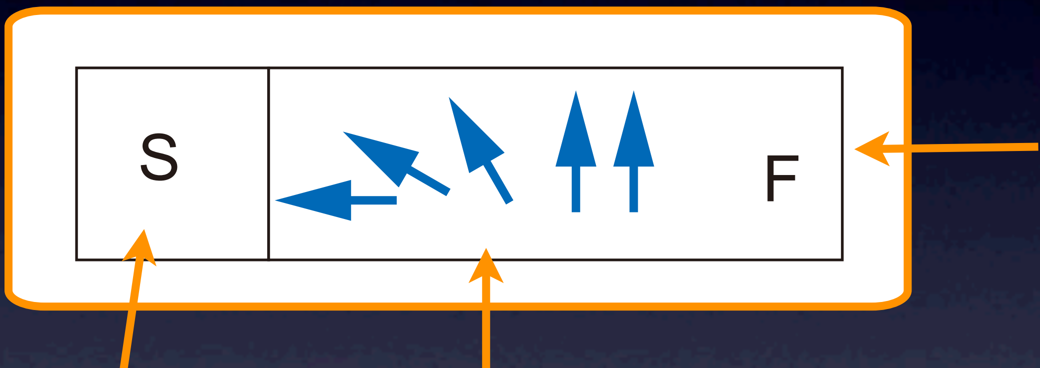
Fourier trans.

spin X orbital X frequency = -1

Recipes of pure odd-freq. pairs (I)

Inhomogeneous magnetic moment + metallic SC

Bergeret, Volkov, and Efetov, PRL 86, 4096 (2001)



triplet

$|\uparrow\uparrow\rangle$

$|\downarrow\downarrow\rangle$

If F is in dirty limit!

s-wave odd-freq.

singlet
s-wave
even-freq.

spin-flip scattering enables

$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

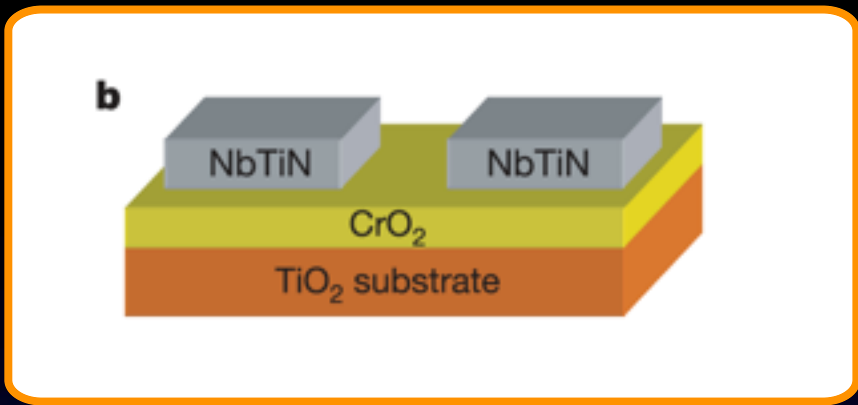
$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ short-range

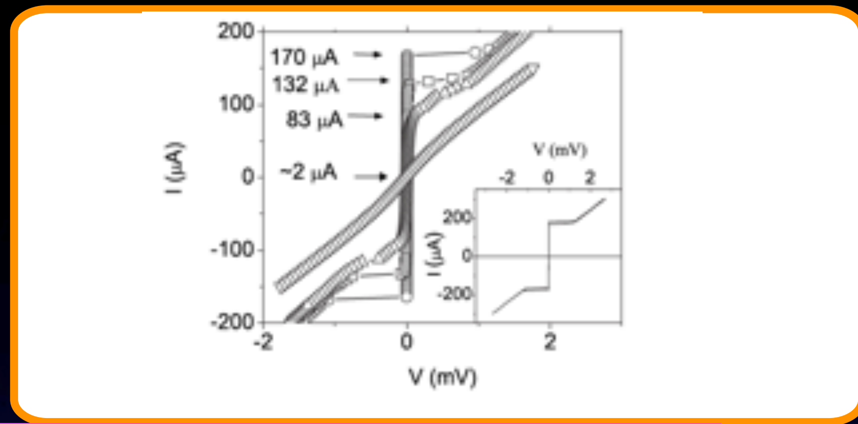
$|\uparrow\uparrow\rangle$

$|\downarrow\downarrow\rangle$ long-range

long-range proximity effect

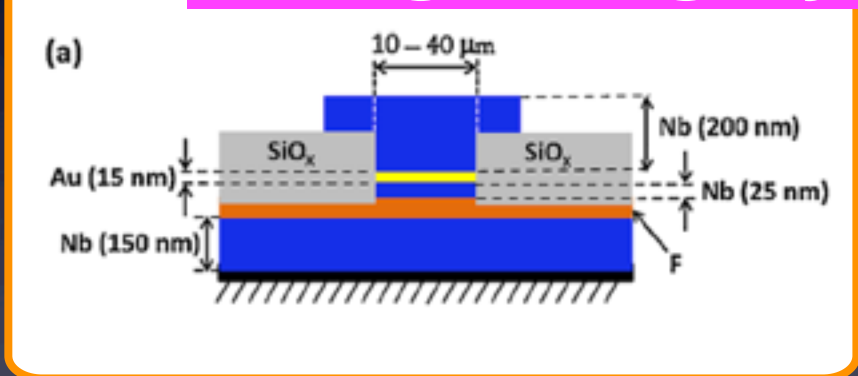
Experiments

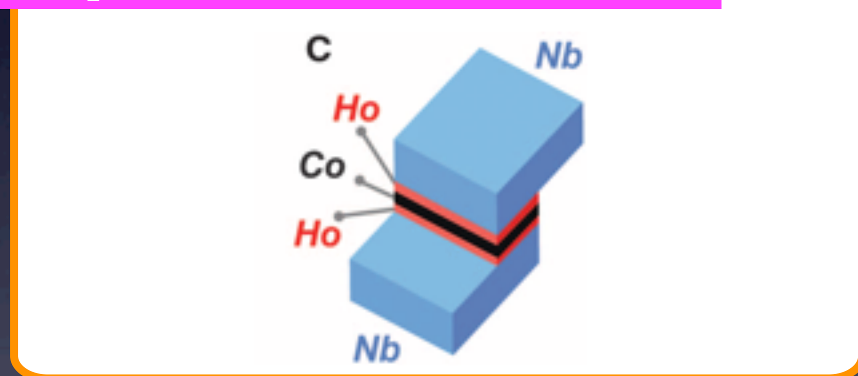




Keizer et … 01R (2010).

Long-range Josephson effect





Khaire, et. al., PRL 104, 137002 (2010)

Robinson, et. al, , Science 239, 59 (2010)

Theories

Asano, Tanaka, and Golubov, PRL 98, 107002 (2007).
Braude and Nazarov, PRL 98, 077003 (2007).
Eschrig and Lofwander, Nature Phys. 4, 138 (2008).

Recipes of pure odd-freq. pairs (II)

spin-triplet SC + dirty normal metal

triplet
p-wave
even-freq.

triplet
s-wave
odd-freq.

Our theories

PRB 70, 012507 (2004) NS
PRB 72, 140503R(2005) NS Meissner
PRL 96, 097007 (2006) SNS
PRL 99, 067005 (2007) T-shape
PRL 98, 037003 (2007) odd freq.
PRL 107, 087001(2011) Impedance

Sr_2RuO_4
Maeno, Nature (94)

Theoretical Details

Solving the Usadel eq. in N

$$\hbar D\nabla^2\theta(\boldsymbol{r},\epsilon) + 2i\sin\theta(\boldsymbol{r},\epsilon) = 0$$

Boundary condition

$$\left.\frac{\partial\theta(x,\epsilon)}{\partial x}\right|_{x=0} = \frac{1}{L}\frac{R_D}{R_B}\frac{\langle F\rangle}{T_B}$$

contains full information of S

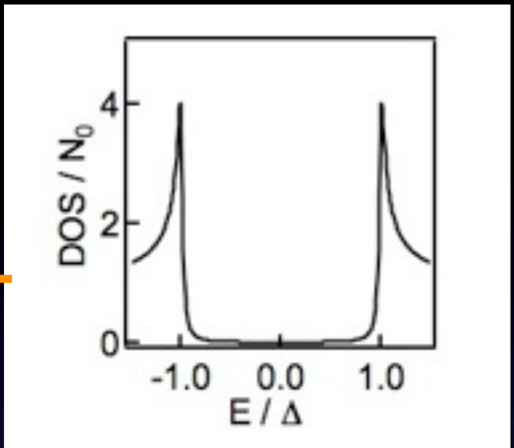
Green function

$$g(x,\epsilon) = \cos\theta(x,\epsilon), \quad f(x,\epsilon) = \sin\theta(x,\epsilon)$$

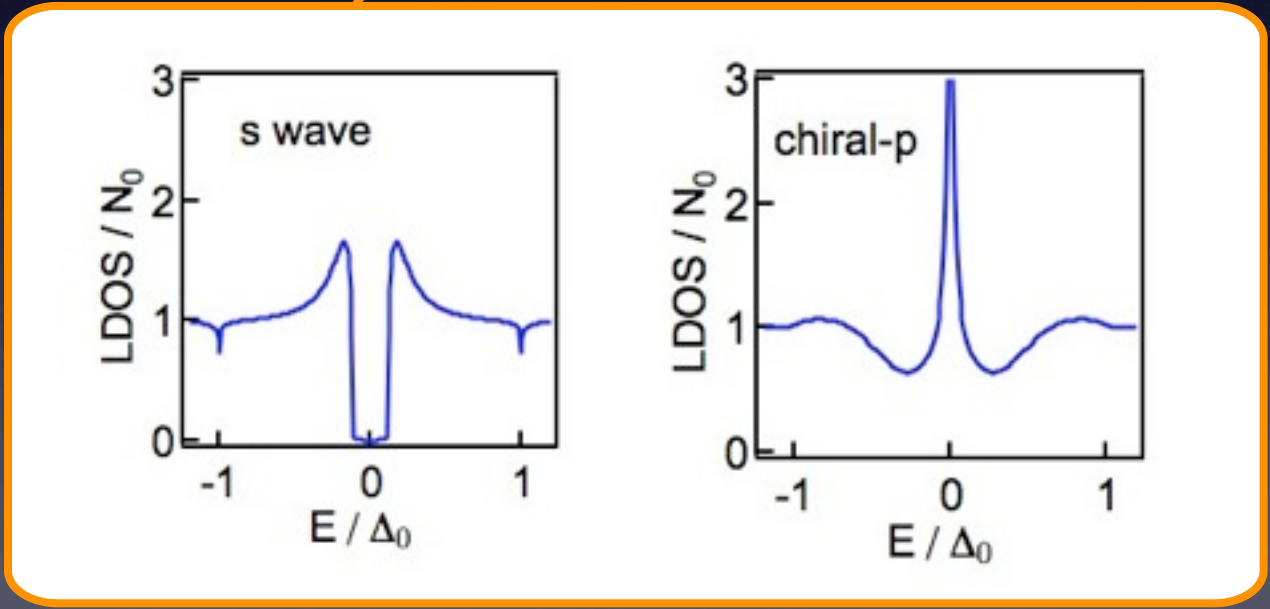
$$g^2 + f^2 = 1$$

$$\mathrm{DOS} = \mathrm{Re}[g(\epsilon)] \qquad \text{Pair Density} = \mathrm{Im}[f^2(\epsilon)]$$

How to detect odd-freq. pairs



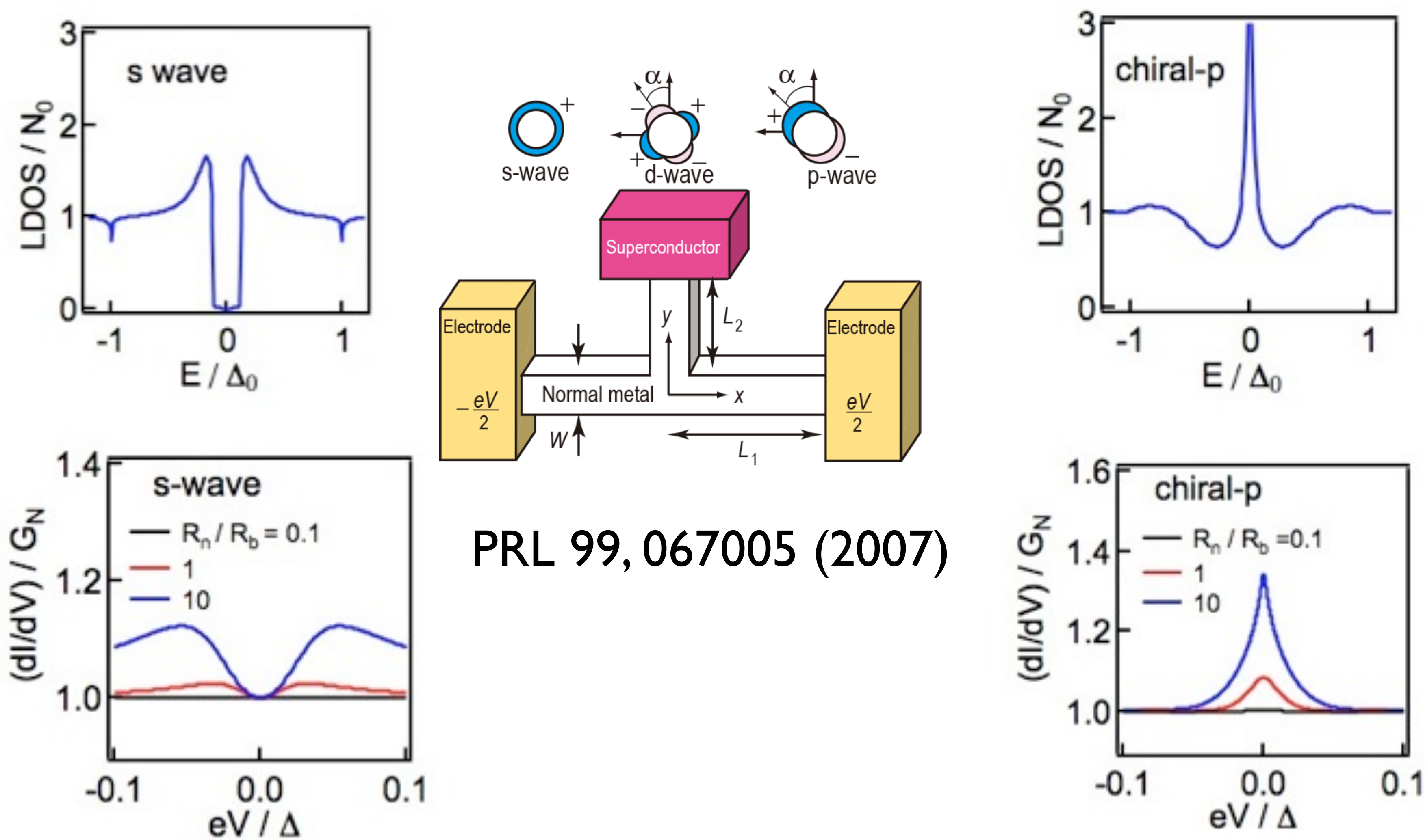
bulk DOS in S



S: singlet s-wave
even-freq. → gap

S: triplet chiral p
odd-freq. → peak

Conductance Spectroscopy



PRL 99, 067005 (2007)

zero-bias dip

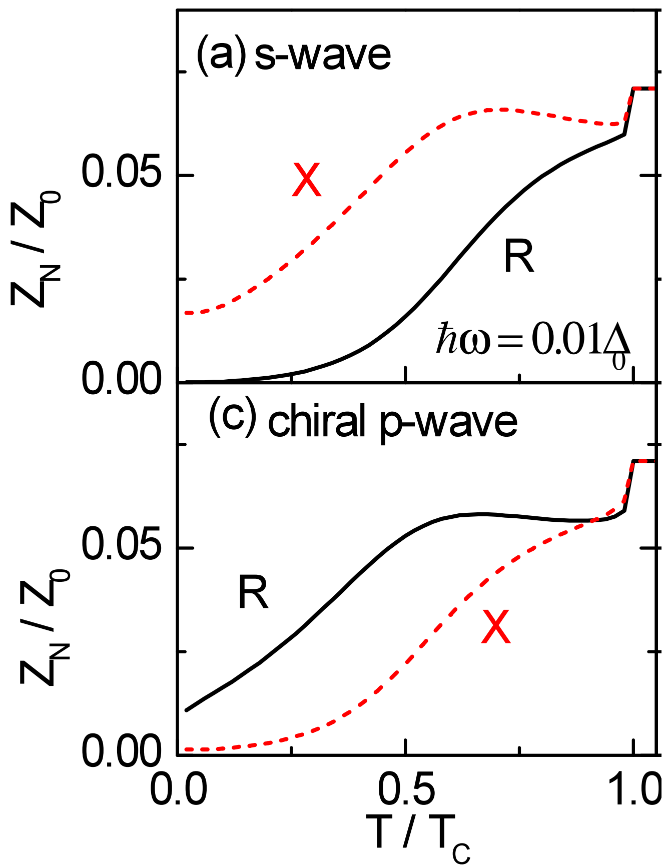
zero-bias peak!!

Copy of DOS (g-function) in normal metal

Detection of odd-freq. symmetry

Impedance at a surface of N

$$Z(-L,\omega) = R - iX \propto (\sigma_1 + i\sigma_2)^{-1/2}$$



singlet

even-freq.

$R < X$

triplet

odd-freq.

$R > X$

anomalous!

σ_2 reflect $f(\epsilon)$

$f(-\epsilon) = f^*(\epsilon)$

$\sigma_2 > 0$

$f(-\epsilon) = -f^*(\epsilon)$

$\sigma_2 < 0$

Asano, Golubov, Fominov, Tanaka, PRL 107, 087001(2011)

Anomalous Proximity Effect

spin-triplet p_x-wave junction

Conductance quantization in dirty NS junction

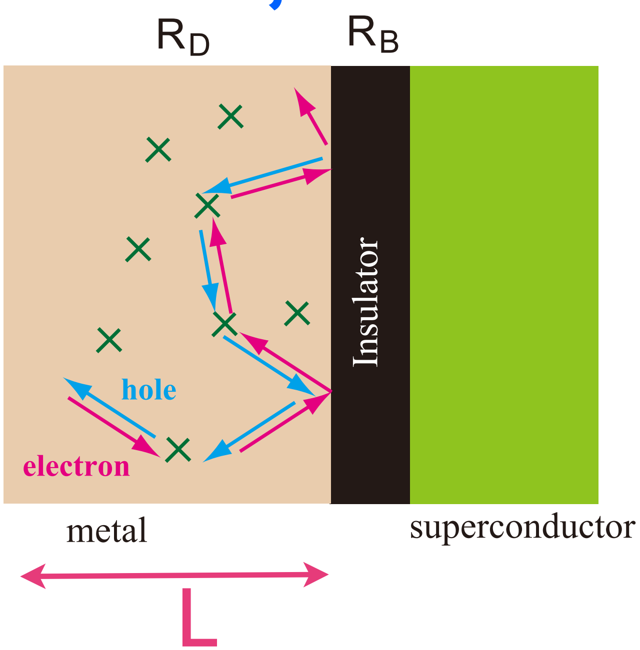
Fractional current-phase relation in dirty SNS junction

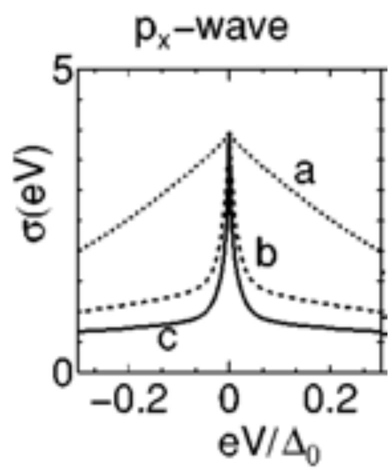
↓

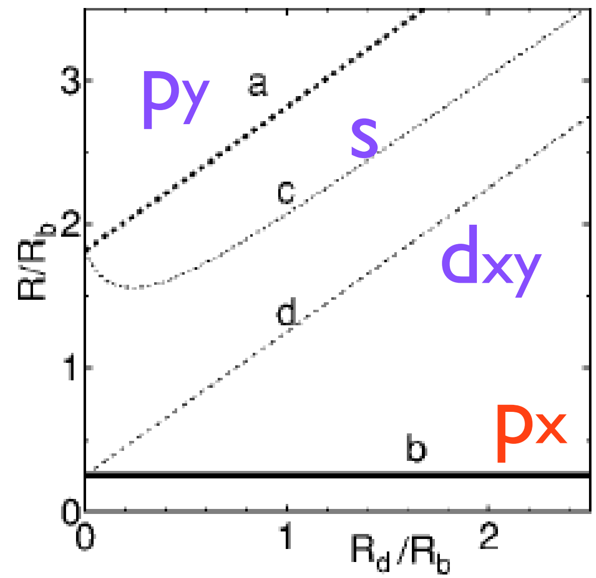
perfect charge propagation through dirty metals

Anomalous transport I

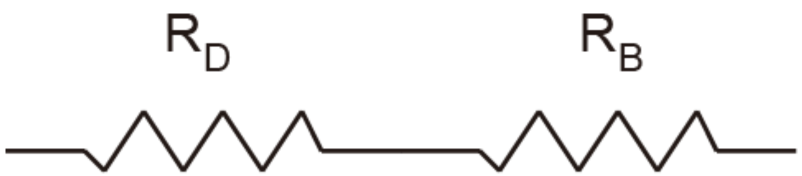
NS junction







L



$R = R_D + R_B$?

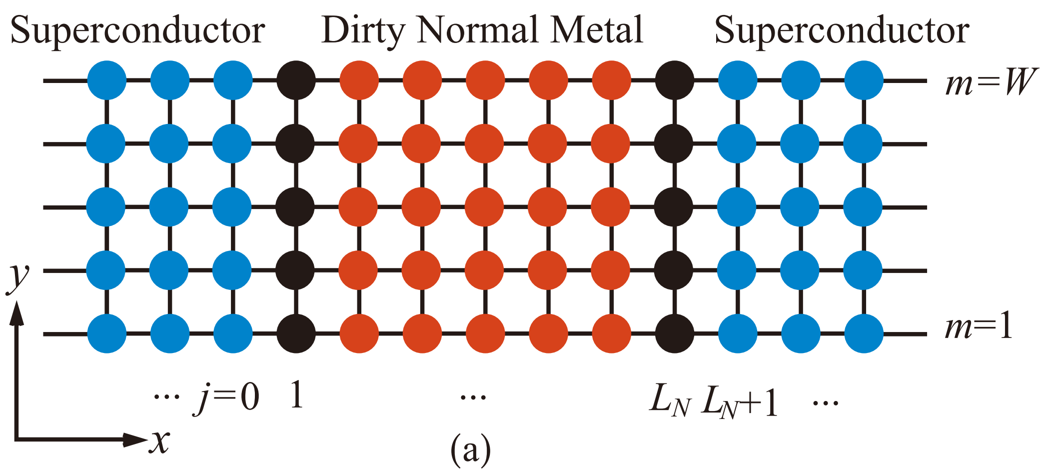
zero-bias conductance is quantized $\frac{2e^2}{h} N_c$

Ballistic transport?

Tanaka and Kashiwaya, PRB 70, 012507 (2004)

Anomalous transport II

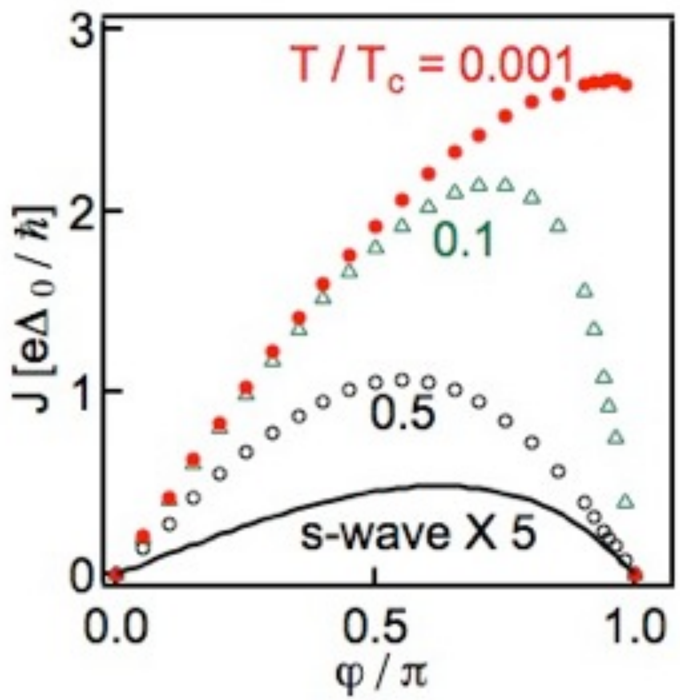
SNS junction



Ballistic transport?

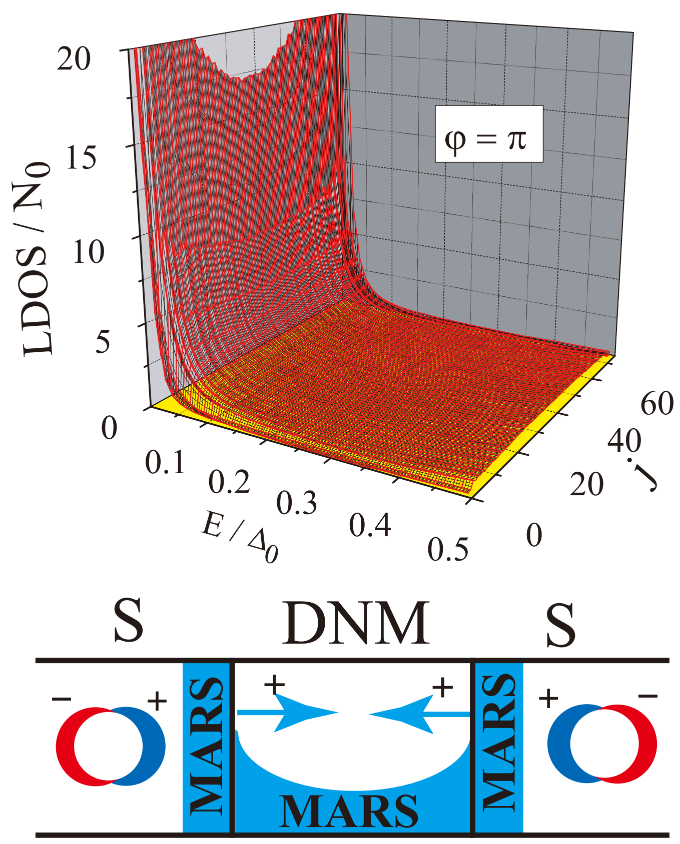
Fractional CPR

$$J \propto \sin(\varphi/2)$$



Asano, Tanaka, and Kashiwaya, PRL 96, 097007 (2006)

DOS anomaly at E=0



(d) p_x-wave: $\varphi = \pi$

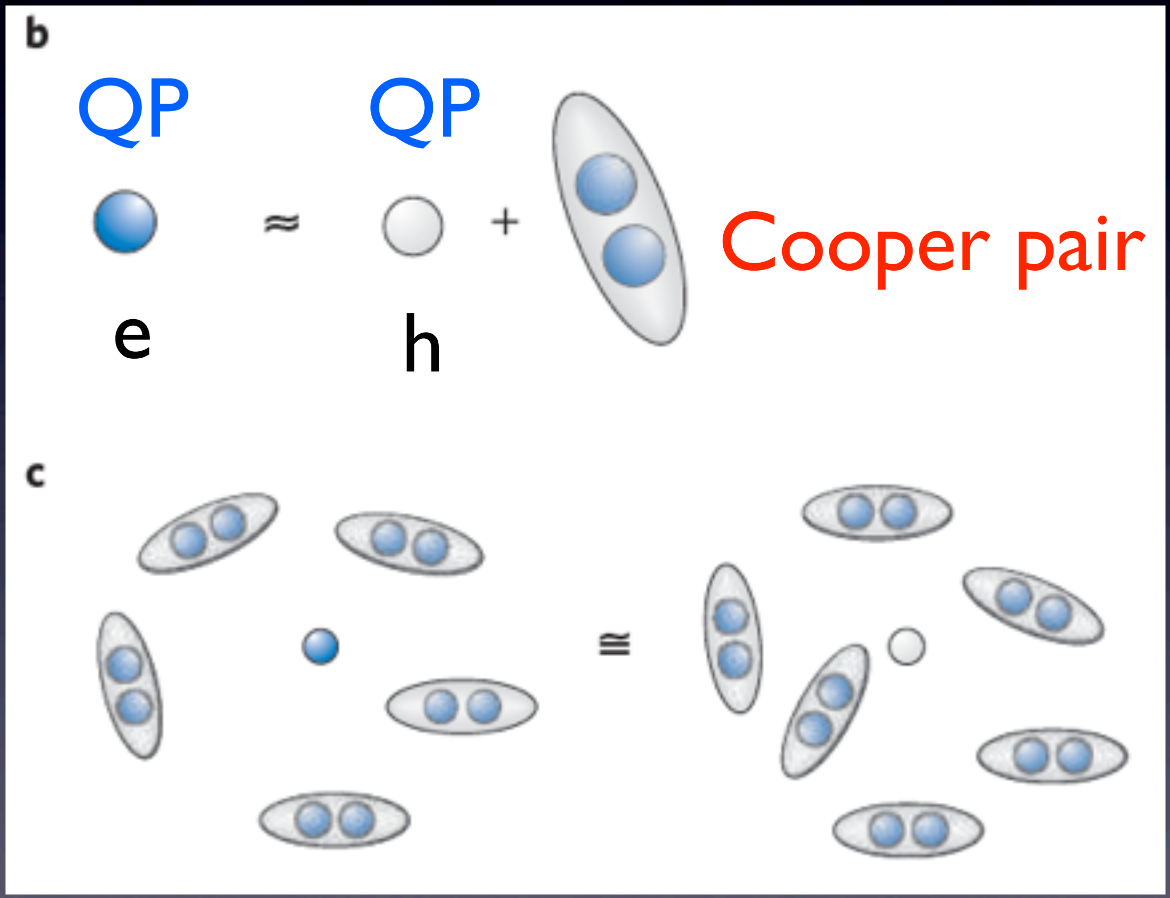
Penetration of odd-freq. pairs into dirty metals

Majorana fermion bound state

Majorana Fermions
&
Odd-freq. pairs

Majorana Fermions (MF)

$$\gamma = \gamma^\dagger$$



But, it always hold, does't it?

What are extra conditions?

F. Wilczek, Nat. Phys. 5, 614(2009)

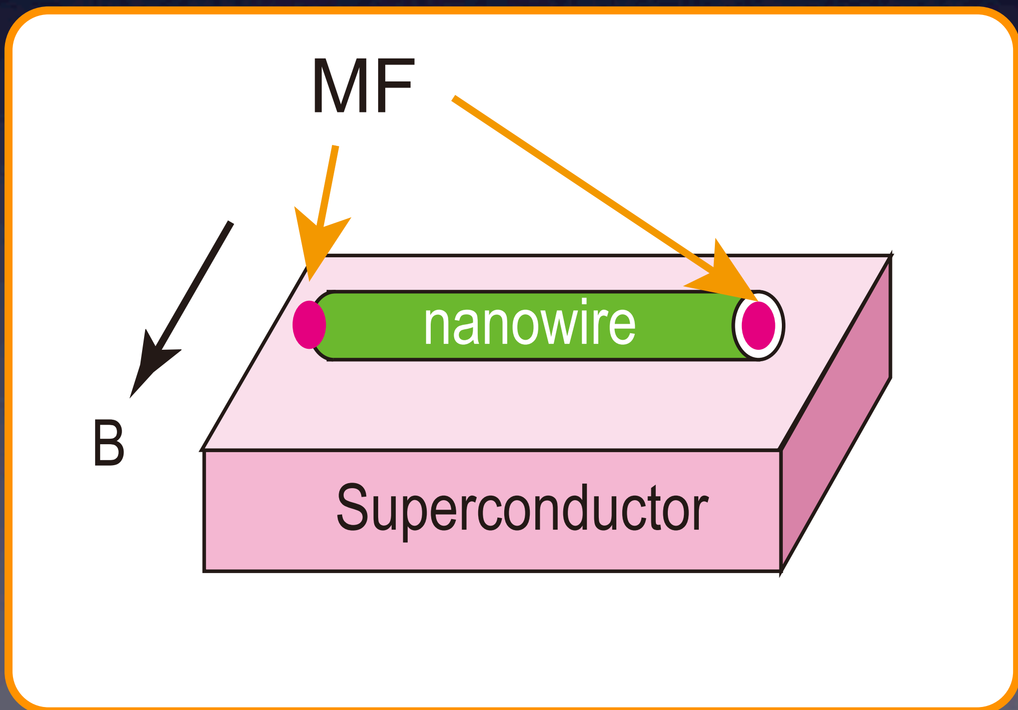
MF in a nanowire

Lutchyn, Sau, and Das Sarma, PRL 105, 077001 (2010)
Oreg, Refael, and von Oppen, PRL 105, 177002 (2010).

Decrease degree of freedom

One-dimension + Spin-orbit + Zeeman field

$$V_{ex} > V_c = \sqrt{\Delta^2 + \mu_F^2}$$



+ Pair potential

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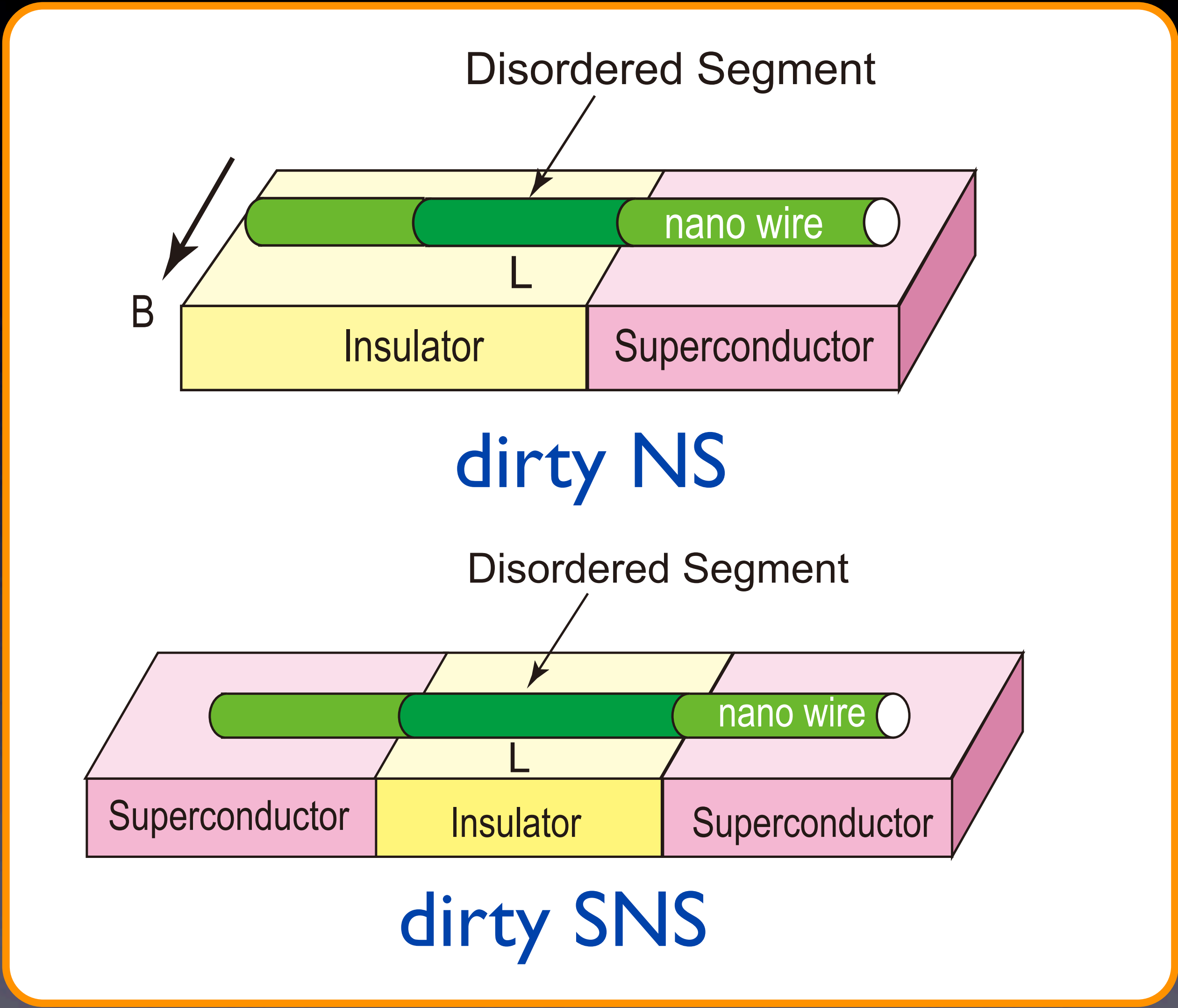
Topologically nontrivial superconducting nano wire

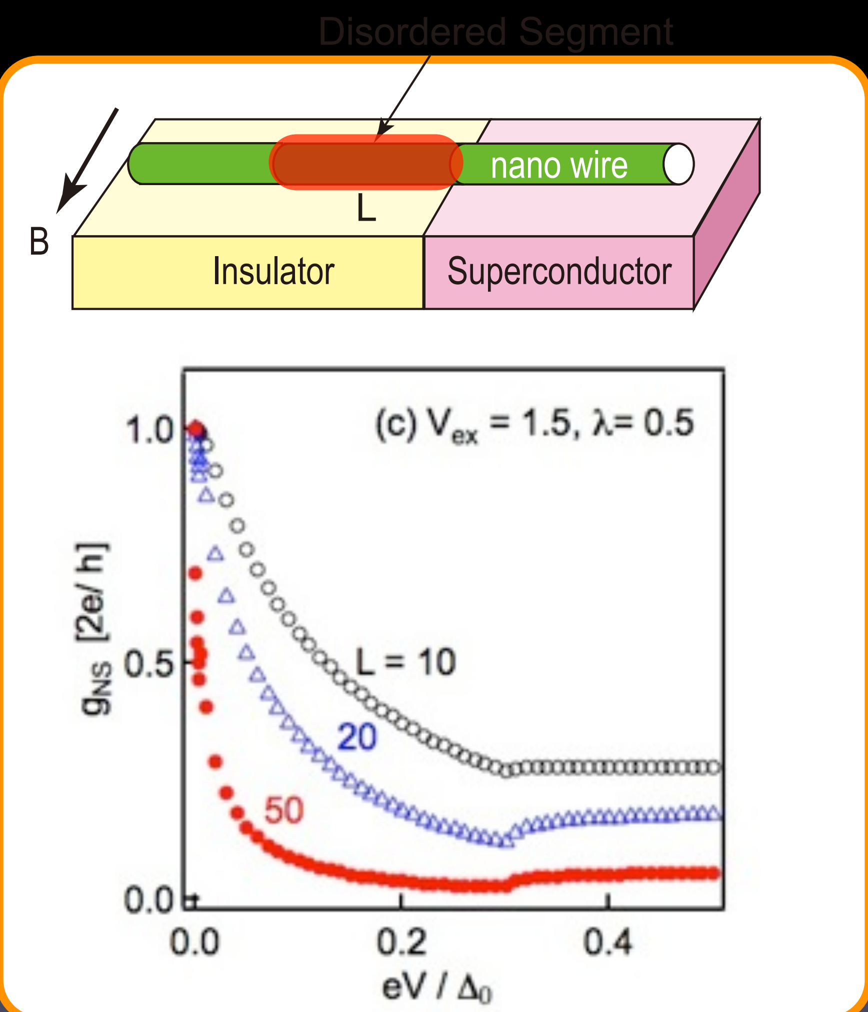
hosting MF

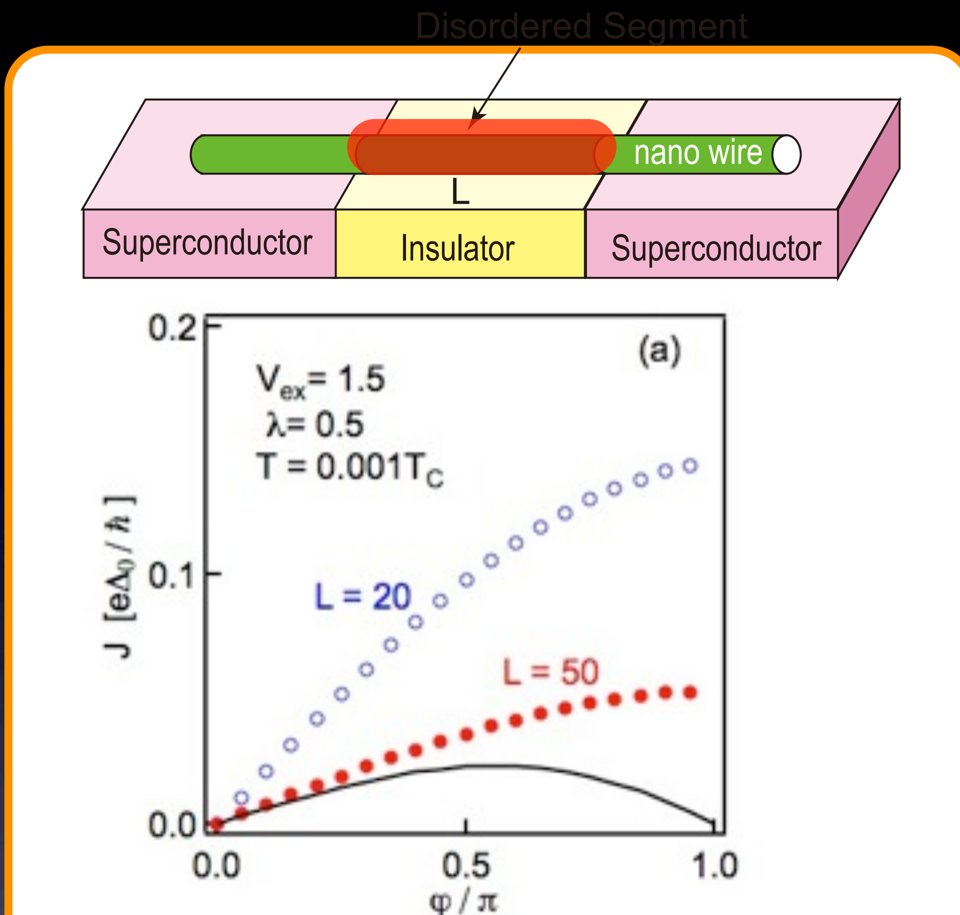
Proximity Effect of nanowires



Disorder in N → Strong localization regime

Proximity Effect of nano wire





$$J \propto \sin(\varphi/2)$$

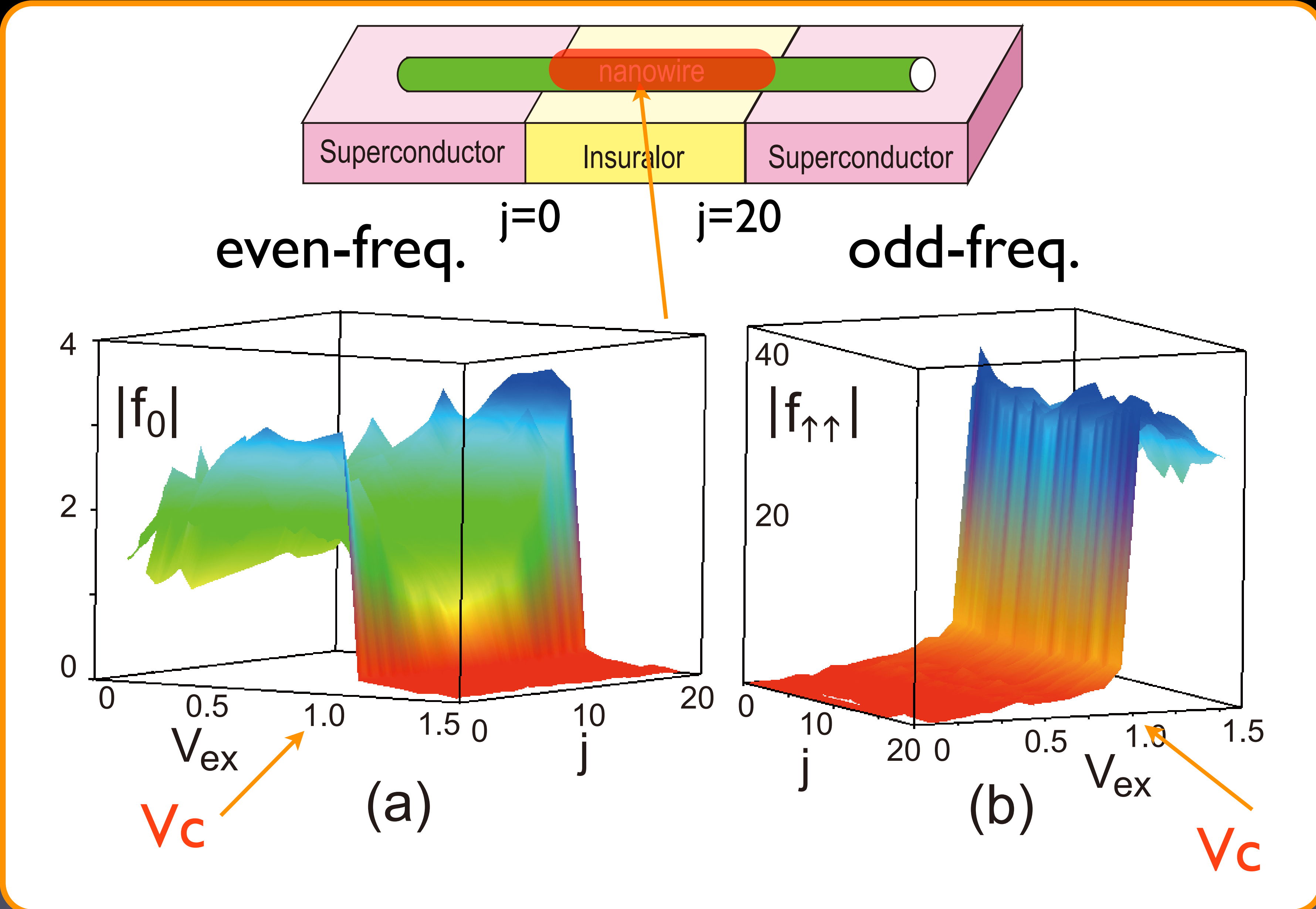
Quantized condactance

Fractional CPR

The anomalous PE due to odd-freq. pair

Asano, Tanaka, Kashiwaya, PRL 96, 097007 (2006)

Tanaka, Kahiwaya, PRB 70, 012507 (2004)



$V_{ex} > V_c$ Pure odd-freq. pair

Odd-freq. pairs host MF in solids!

Ballistic transmission?

$$H_{imp} = \int dx V(x) \gamma^\dagger(x) \gamma(x)$$

$$\gamma^\dagger(x) = \gamma(x)$$

$$H_{imp} = \int dx V(x) \gamma(x) \gamma(x) = \mathrm{Const.}$$

MF does not suffer from any potential scatterings

Nanowire & px-wave

BdG Hamiltonian of nanowire in 4X4

$$H(k) = \begin{bmatrix} \xi_k - V_{ex}\sigma_3 + \boldsymbol{g}_k \cdot \boldsymbol{\sigma} & i\Delta\sigma_2 \\ -i\Delta\sigma_2 & -\xi_k + V_{ex}\sigma_3 - \boldsymbol{g}_{-k} \cdot \boldsymbol{\sigma}^* \end{bmatrix}$$

Unitary transformation $D = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i\sigma_2 \\ i\sigma_2 & 1 \end{bmatrix}$

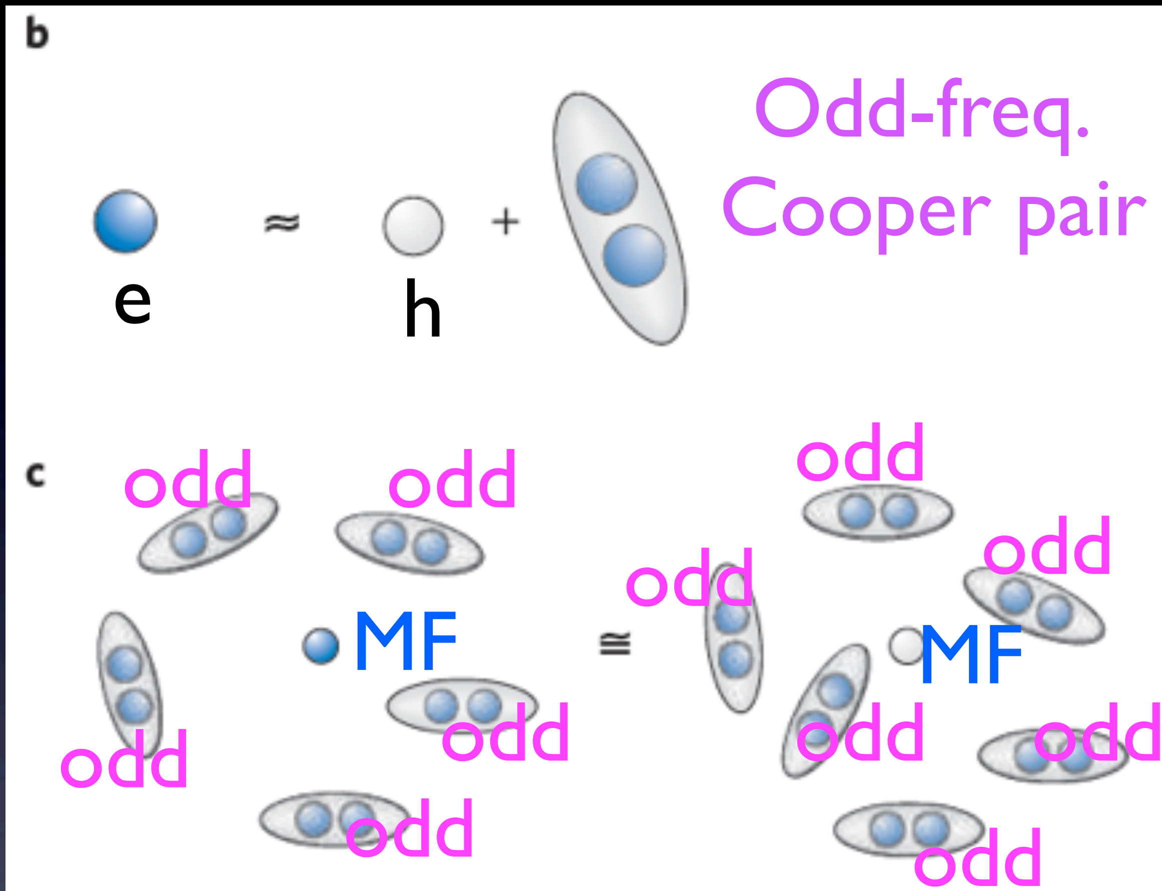
$$H'(k) = DH(k)D^\dagger = \begin{bmatrix} \Delta - V_{ex}\sigma_3 & -i\xi_k - i\boldsymbol{g}_k \cdot \boldsymbol{\sigma}\sigma_2 \\ i\xi_k + i\sigma_2 \boldsymbol{g}_k \cdot \boldsymbol{\sigma} & -\Delta + V_{ex}\sigma_3 \end{bmatrix}$$

Assume 1D & $\xi_k \approx 0$

$$H(k) = \begin{bmatrix} \Delta - V_{ex} & gk_x \\ gk_x & -(\Delta - V_{ex}) \end{bmatrix}$$

BdG Hamiltonian of px-wave like S in 2X2

The physical picture is true



$$\gamma = \gamma^\dagger$$

only when Cooper pairs have the odd-freq. symmetry

Asano and Tanaka, arXiv:1204.4226

Summary

Two recipes for odd-frequency Cooper pairs

Magnetic inhomogeniety : SF Hybrids
Spatial inhonogeniety : Triplet SC/ N

Detection of odd-freq. pairs

Zero-energy peak in N
zero-bias anomaly in nonlocal conductance
Anomalous surface impedance

Strong relation to Majorana physics

MFs = odd-freq. pairs