Detecting Majorana states by driving a topological Josephson junction out of equilibrium

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Motivation

Majorana states in hybrid superconducting junctions are actively searched

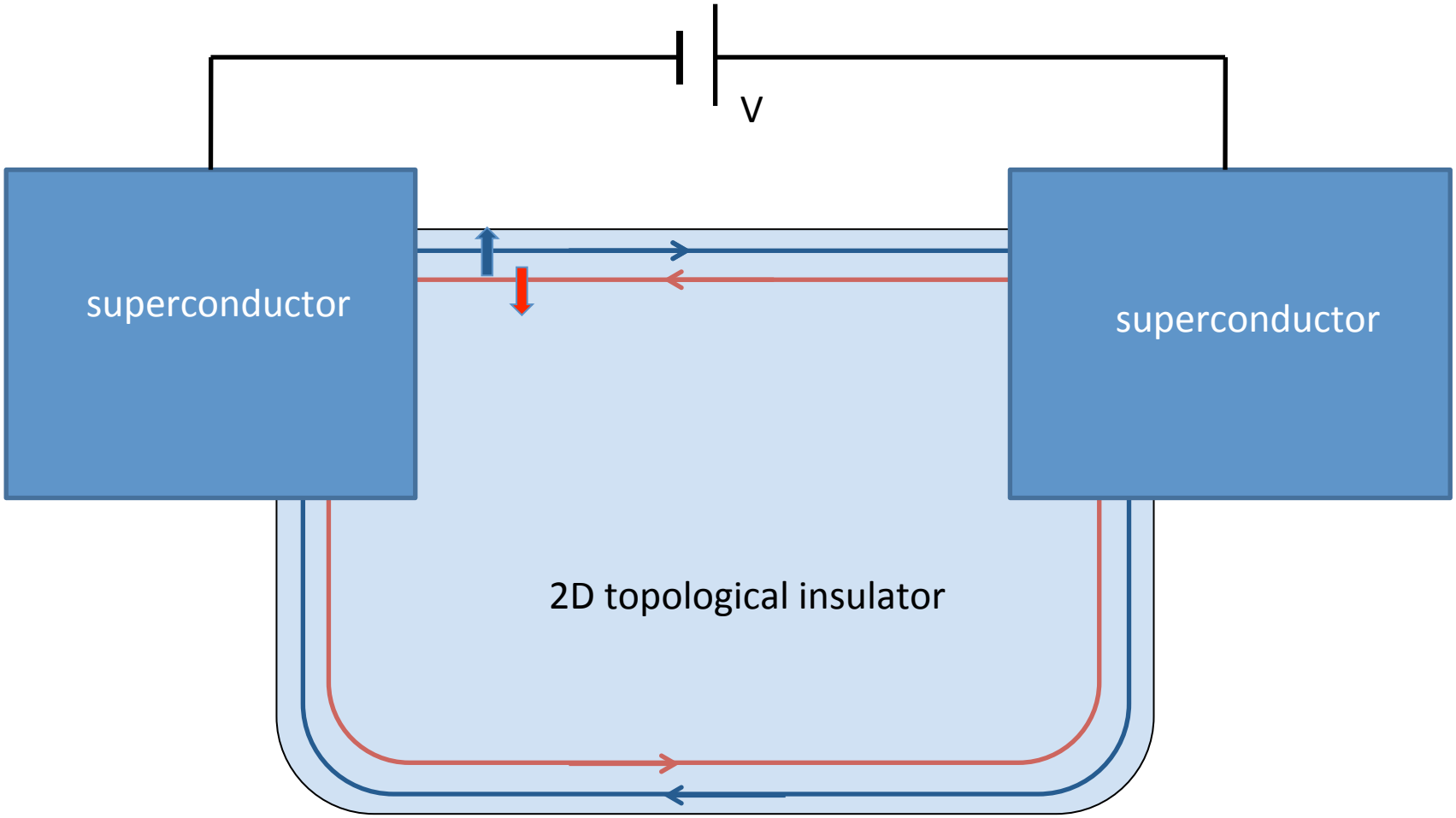
Proposed signatures:

- Zero-energy edge state
- **fractional AC Josephson effect?**
- **Even/odd asymmetry in Shapiro steps?**

Study non-equilibrium properties of a topological Josephson junction

[DC and DC + AC bias]

Setup



IV characteristics?

Outline

1. Andreev bound states: conventional vs. topological Josephson junctions

   - 4π-periodic phase dependence of the ABS energy

2. Scattering theory for voltage-biased Josephson junctions

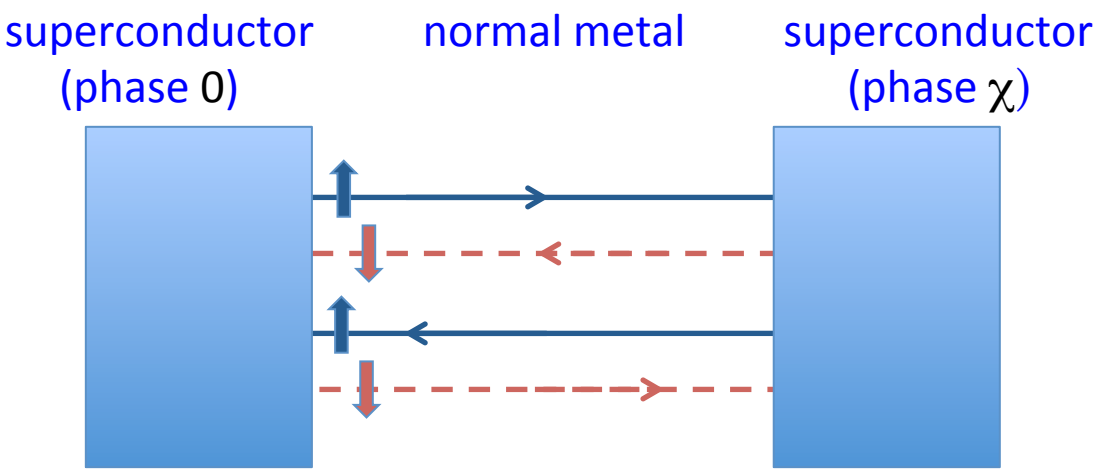
   - Signatures of Majorana states in the DC current
   - No signature of the 4π-periodicity in the AC current
   - peak in the finite-frequency noise

3. Effective model: bound state dynamics

   - Characteristic time scales
   - DC bias
   - DC + AC bias $\rightarrow$ Shapiro steps

1. Andreev bound states:
   conventional vs. topological junctions

Andreev bound state



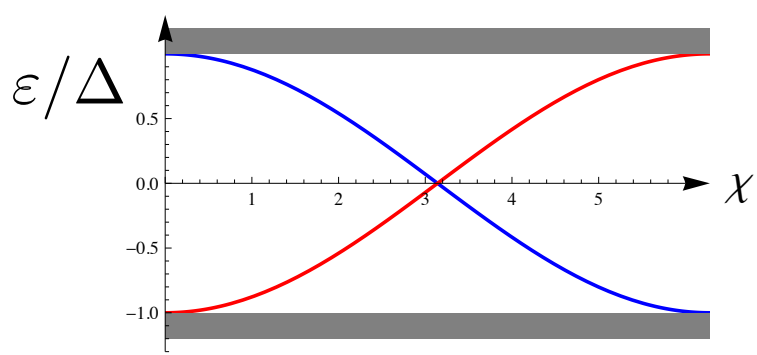
Andreev reflection $|\varepsilon| < \Delta$

Bohr-Sommerfeld quantization: bound state if $-2\arccos\frac{\varepsilon}{\Delta} + \chi = 2\pi n$

$$\boxed{\varepsilon_{A\pm} = \pm\Delta\cos\frac{\chi}{2}}$$

"short" junction

$L \ll v_F/\Delta$



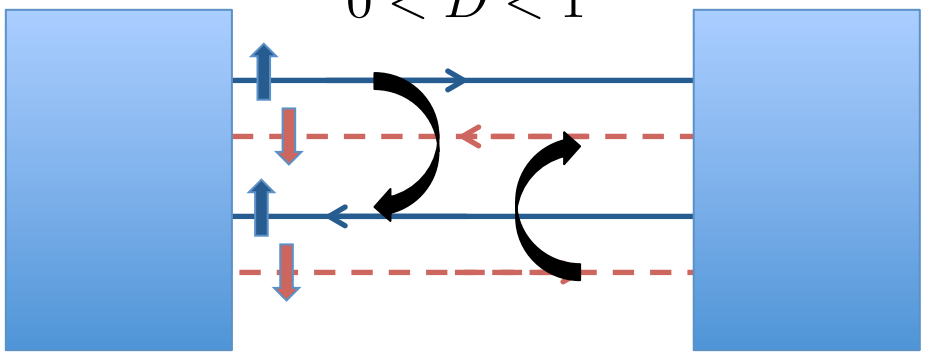
(doubly degenerate)

Beenakker 1991

Conventional Josephson junction

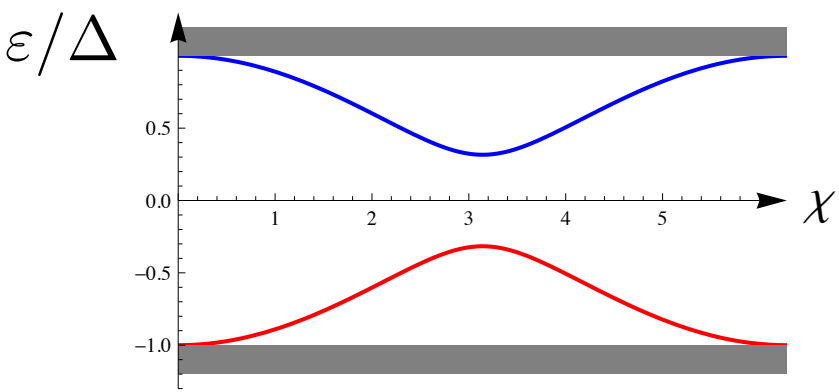
normal backscattering

$$0 < D < 1$$



$$\boxed{\varepsilon_{A\pm} = \pm\Delta\sqrt{1 - D\sin^2\frac{\chi}{2}}}$$

avoided crossing



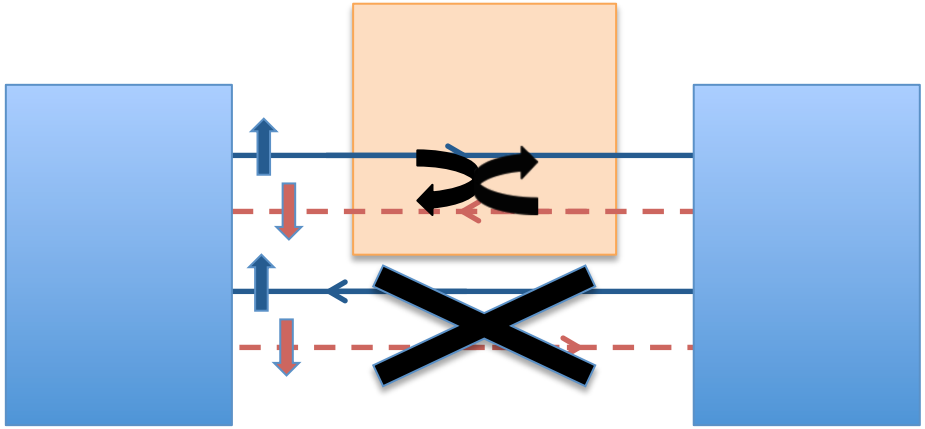
$$I = \frac{2e}{\hbar}\frac{\partial\varepsilon_{A-}}{\partial\chi}$$

$$I \propto \sin\chi \qquad D \ll 1$$

Beenakker 1991

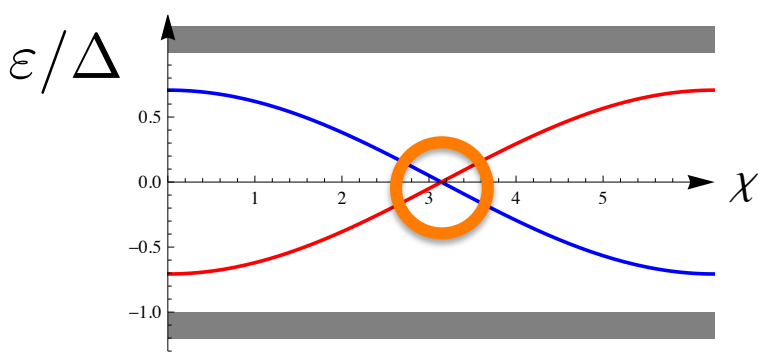
Topological Josephson junction

magnetic barrier $\mathcal{H}_M = M(x)\sigma_x$ (spin-flip scattering)

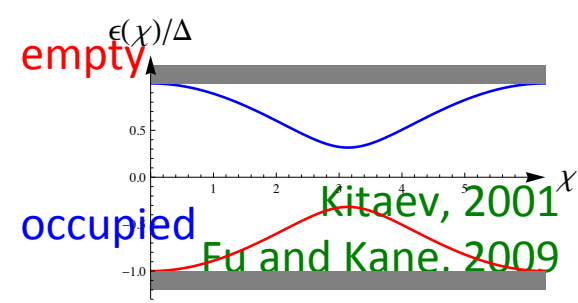


$$\boxed{\varepsilon_{A\pm} = \pm\Delta\sqrt{D}\cos\frac{\chi}{2}}$$

non degenerate ABS state



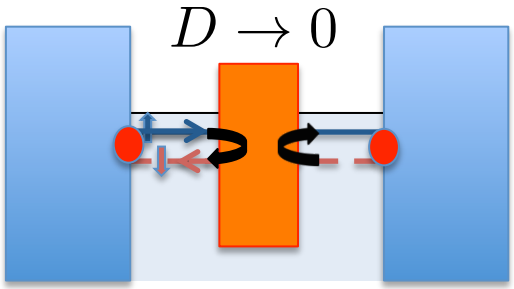
SNS for comparison:



Kitaev, 2001

Fu and Kane, 2009

Signatures



$$\varepsilon_{A\pm} = \pm\Delta\sqrt{D}\cos\frac{\chi}{2}$$

Majorana fermion:

$$\gamma_\varepsilon = \gamma^\dagger_{-\varepsilon}$$
$$\varepsilon = 0$$

- Zero-energy state: **ZBA in tunneling spectroscopy**

  $D \to 0$

Recent experiments from Kouwenhoven, Xu, and Heiblum's groups (2012)

- 4π-periodic $\varepsilon_A(\chi)$ vs. χ

$$I = \frac{2e}{\hbar}\frac{\partial\varepsilon_{A-}}{\partial\chi}$$

$$\boxed{\dot{\chi} = \frac{2e}{\hbar}V}$$

$$I \propto \sin(\chi/2)$$

→ **Fractional AC Josephon effect?** $I(t) \propto \sin\left(\frac{eV}{\hbar}t\right)$ at half the Josephson frequency:

$$\omega_J = \frac{2eV}{\hbar}$$

→ **Large even Shapiro steps?** $\overline{\sin[eVt + \frac{eV_{ac}}{\omega}\cos\omega t + \chi_0/2]} \neq 0$

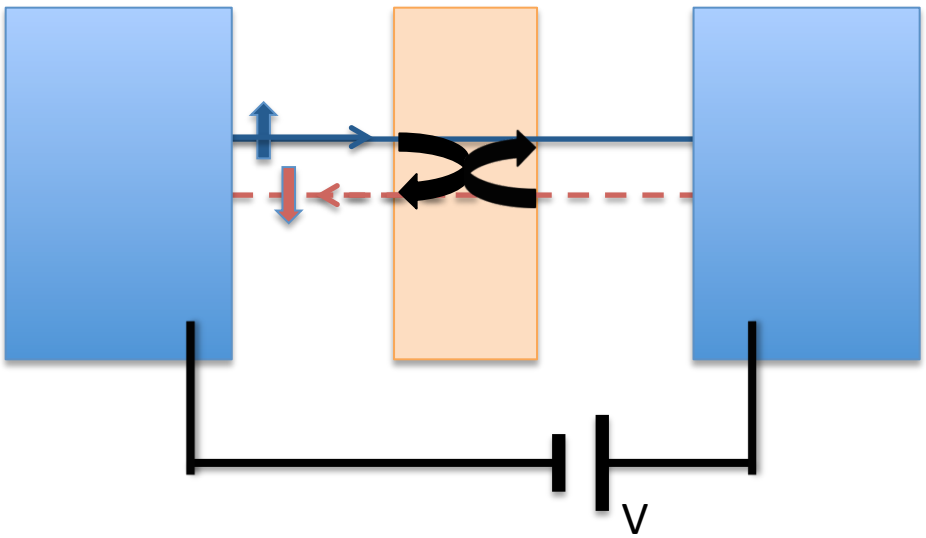
$$eV = n\omega$$

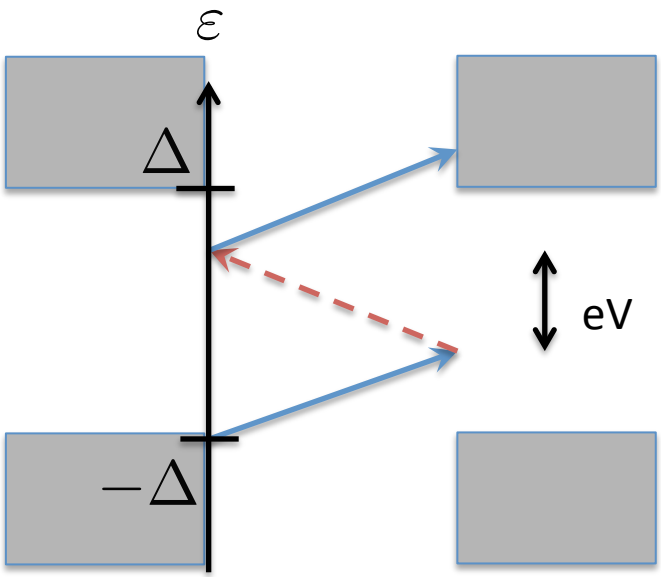
(instead of $2eV = n\omega$ in conventional JJ)

Kitaev, 2001
Fu and Kane, 2009

2. voltage-biased topological junction: scattering theory

Multiple Andreev reflections

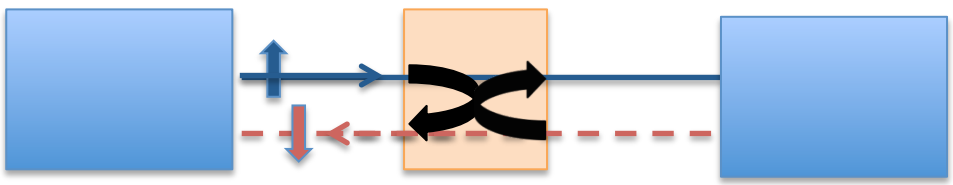




- Electrons and holes gain energy eV at each traversal
- Only quasiparticle states with energy above the gap Δ can escape in the leads

Octavio, Blonder, Tinkham, Klapwijk, 1983

Scattering formalism



$$\Psi = \begin{pmatrix} \psi_{R\uparrow} \\ \psi^\dagger_{L\downarrow} \\ \psi_{L\downarrow} \\ \psi^\dagger_{R\uparrow} \end{pmatrix}$$

e
h
e
h

Andreev reflection

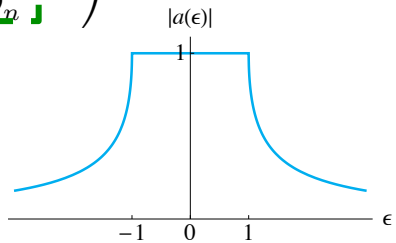
scattering

Andreev reflection

$$\Psi(0) \sim J(\varepsilon) \sum_n \begin{pmatrix} \delta_{n0} + a_{2n} A_n \\ A_n \\ B_n \\ a_{2n} B_n \end{pmatrix} e^{-i(\varepsilon + 2neV)t}$$

$$\Psi(L) \sim J(\varepsilon) \sum_n \begin{pmatrix} C_n \\ a_{2n+1} C_n \\ a_{2n+1} D_n \\ D_n \end{pmatrix} e^{-i(\varepsilon + (2n+1)eV)t}$$

$$a_p = e^{-i \arccos[(\varepsilon + peV)/\Delta]}$$

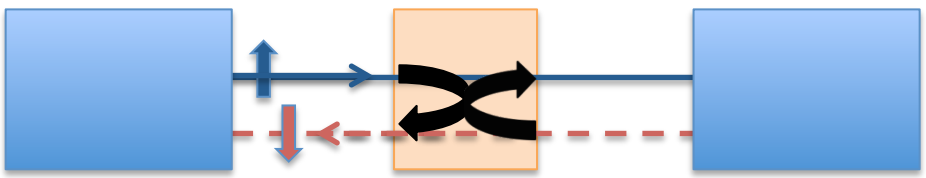


$$s_e = \begin{pmatrix} r & t e^{ieVt} \\ t e^{-ieVt} & r' \end{pmatrix} \qquad s_h = \begin{pmatrix} -r^* & t^* e^{-ieVt} \\ t^* e^{ieVt} & -r'^* \end{pmatrix}$$

$$D = |t|^2$$

Averin, Bardas, 1995

Scattering formalism



Derive recursion relations for A_n and B_n

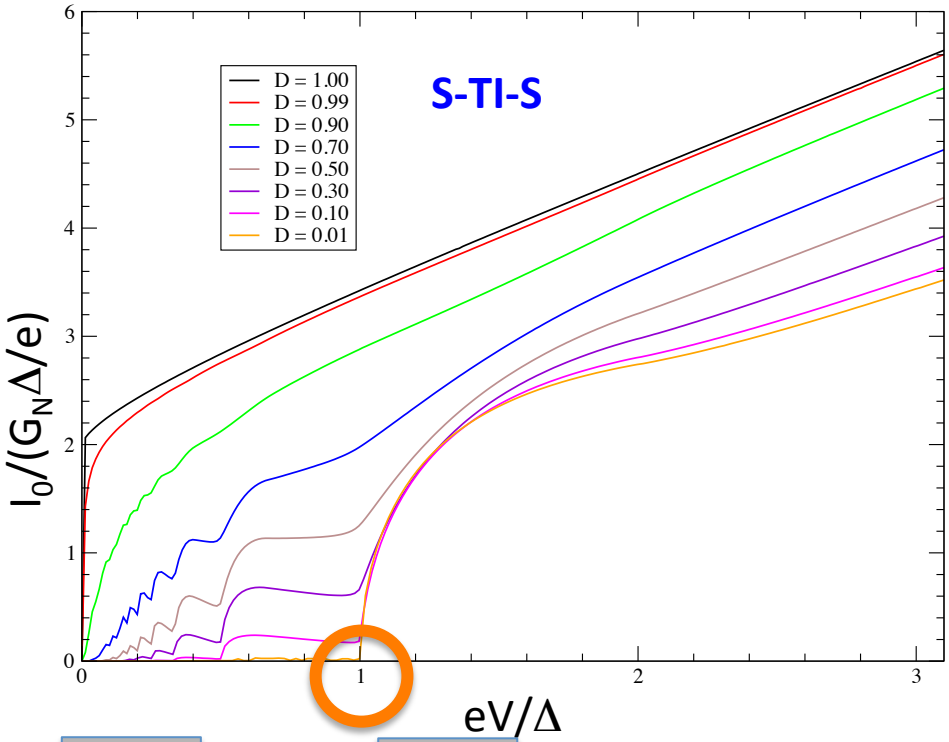
Current: $$\boxed{I(t) = I_0 + \sum_{n\neq 0} I_n e^{i2eVt/\hbar}}$$

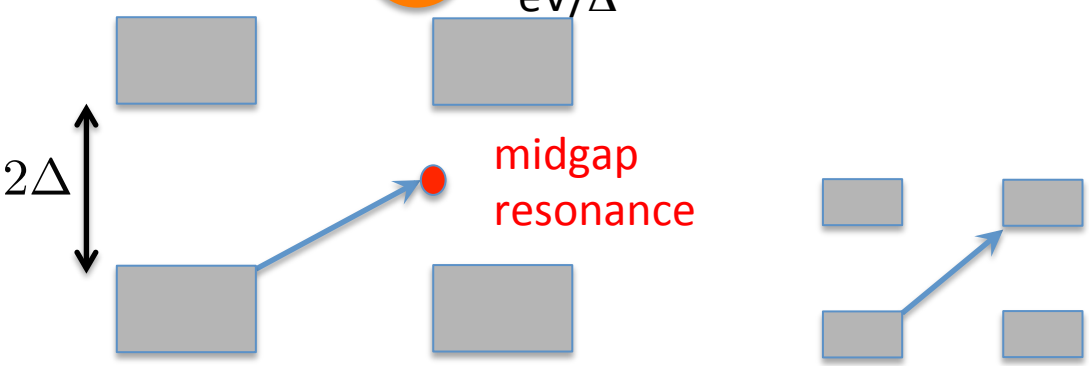
$$I_n = \frac{e}{h}\left\{ DeV\delta_{n0} - \int d\epsilon \, \tanh\frac{\epsilon}{2T} J^2 \left[a^*_{2n}A^*_n + a_{-2n}A_{-n} + \sum_m (1 + a^*_{2(m+n)}a_{2m}) \left(A^*_{m+n}A_m - B^*_{m+n}B_m\right)\right]\right\}$$

Averin, Bardas, 1995

Dissipative DC current

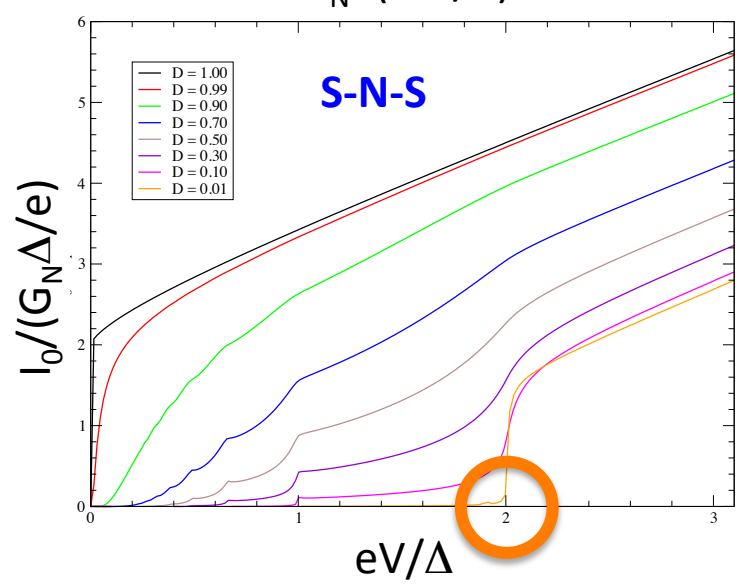
$G_N=(e^2/h)D$



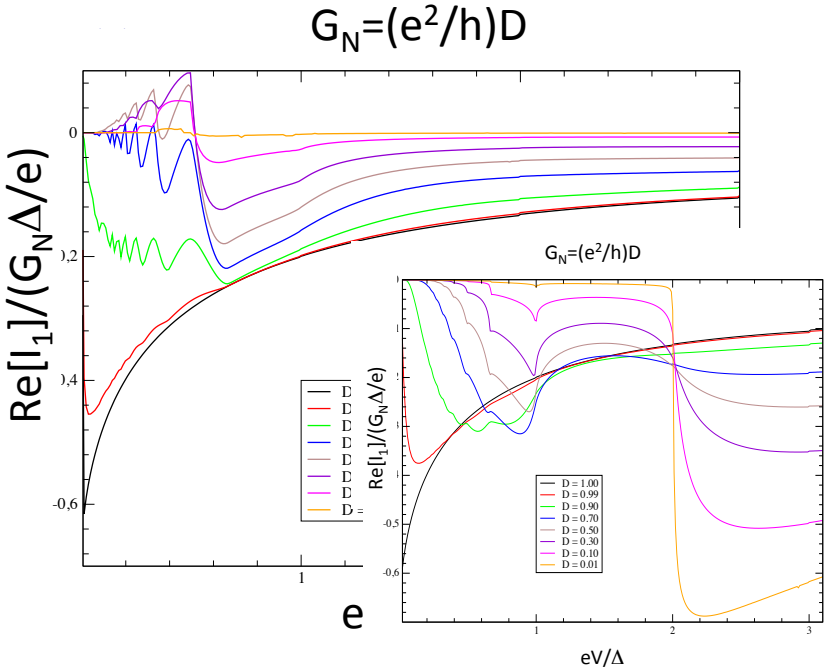


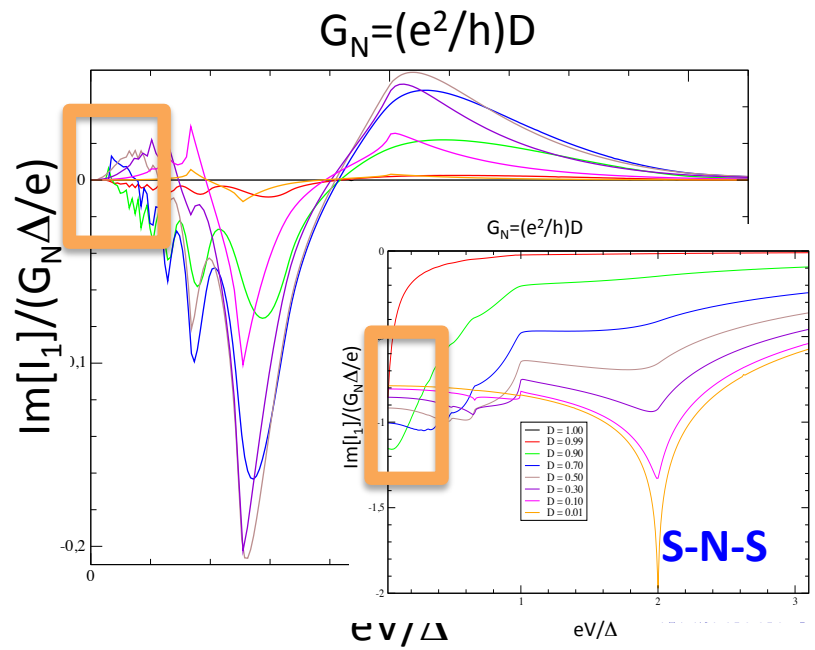
- Onset at $eV=\Delta$ (instead of $eV=2\Delta$) in the tunnel regime
- MAR features at $eV=\Delta/n$ (instead of $eV=2\Delta/n$)
- at D=1: $I_{S\text{-}N\text{-}S} = 2\ I_{S\text{-}N\text{-}S}$

$G_N=(2e^2/h)D$



Vanishing AC current



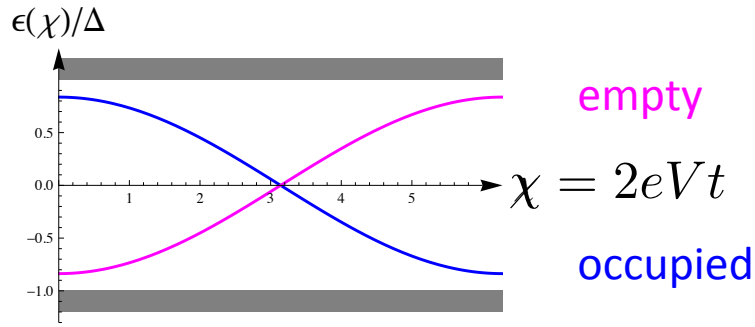


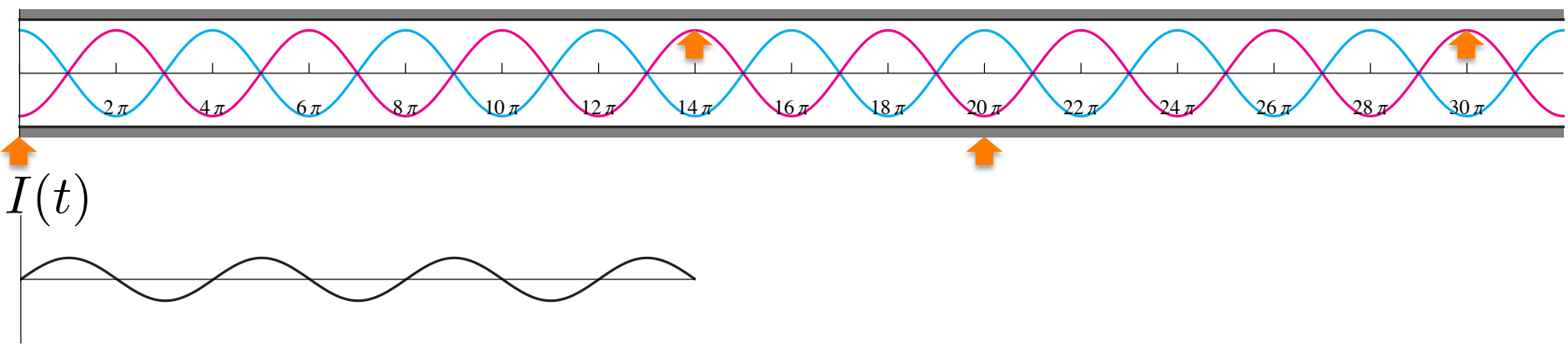
- AC current at conventional frequencies $2neV/\hbar$ (no fractional effect)
- AC current vanishes at $V \to 0$ and/or $D \to 0$ (while $Im[I_n]$ is finite for SNS)

No AC Josephson effect?

DM Badiane, MH, JS Meyer (2011)

Bound state dynamics

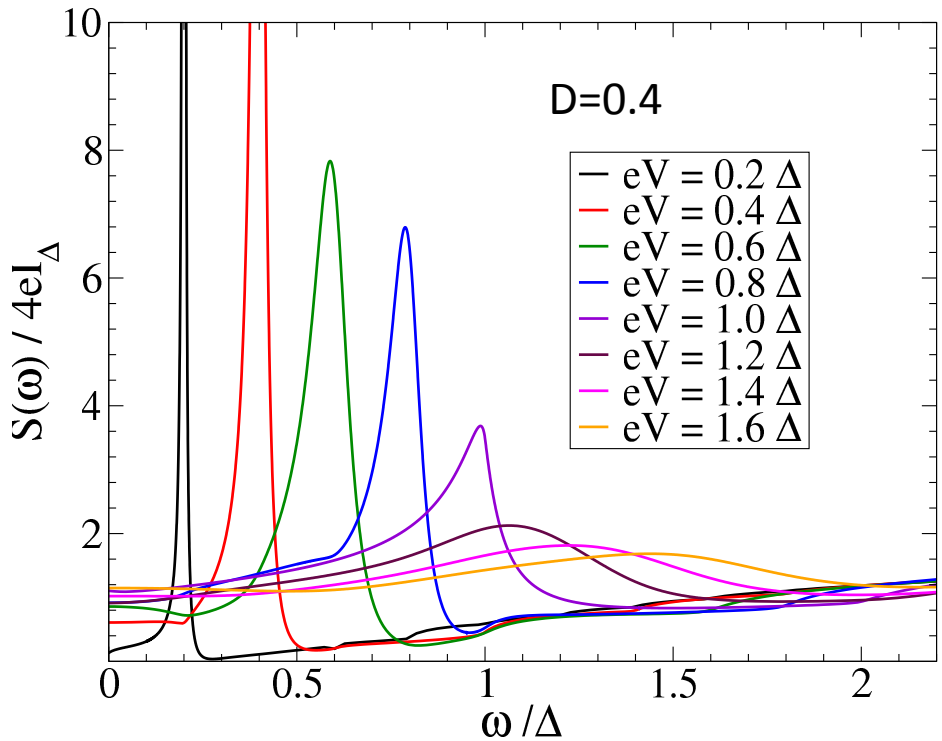




- no average AC current at eV/ħ
  due to dynamical coupling between bound state and continuum
- signature in the noise?

Current-current correlation (noise)

$$S(\omega) = \int d\tau \langle\langle \{\hat{I}(t+\tau), \hat{I}(t)\} \rangle\rangle e^{i\omega\tau}$$



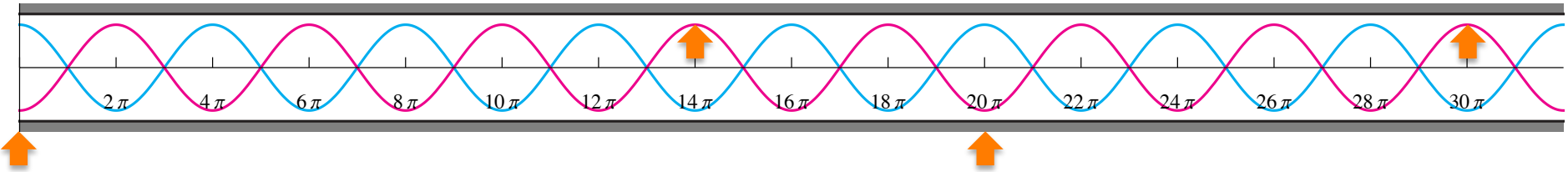
DM Badiane, MH, JS Meyer (2011)

Peak at ω=eV/ħ

Equivalently, the fractional ac effect is seen in transient regime

San-José, Prada, Aguado (2012)

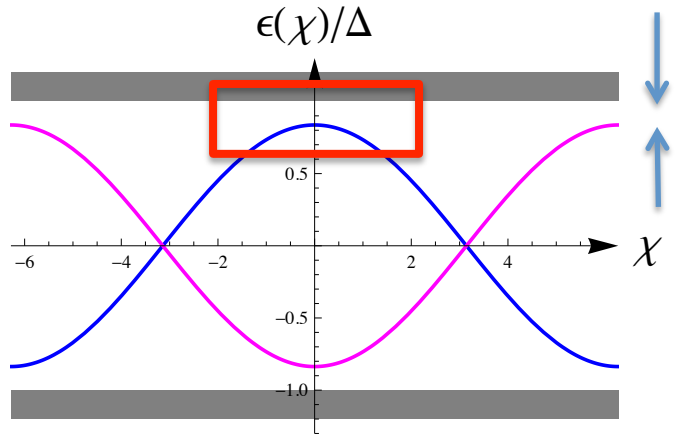
3. Effective model for the bound state dynamics



Weak backscattering

Energy states near Δ

$$R \sim (ML/v)^2 \ll 1$$
$$\chi \ll 1$$
$$vp \ll \Delta$$



$$\delta \sim (1-\sqrt{D})\Delta \propto R\Delta$$

superconducting phase difference:

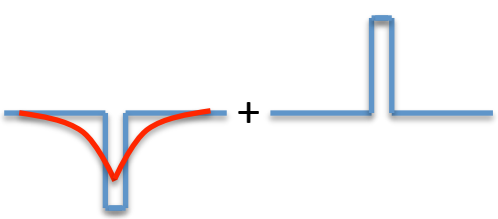
$$\phi(x) = \frac{\chi}{2}\mathrm{sign}(x)$$

$$H = vp\sigma_z\tau_z + \Delta[\cos\phi(x)\tau_x - \sin\phi(x)\tau_y] + M(x)\sigma_x$$

Apply unitary transformation and diagonalize electron-hole space

$$H = \Delta + \frac{v^2p^2}{2\Delta} + \left[\frac{v\chi}{2}\sigma_z + v\sqrt{R}\sigma_x\right]\delta(x)$$

Bound state similar to the Shiba state created by a magnetic impurity in a superconductor



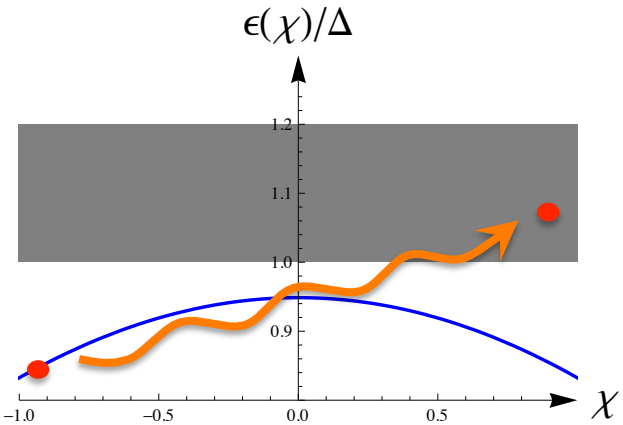
$$\epsilon_A = \Delta\left(1 - \frac{R}{2} - \frac{\chi^2}{8}\right)$$

Shiba 1968,
Rusinov 1969

Switching probability

$$H = \Delta + \frac{v^2 p^2}{2\Delta} + \left[\frac{v\chi}{2}\sigma_z + v\sqrt{R}\sigma_x\right]\delta(x)$$

Time-dependent phase: $\chi = 2eVt$



Adiabatic basis

$$|\Psi(t)\rangle = c_A(t)|\Psi_A(t)\rangle + \sum_{p,\sigma} c_{p,\sigma}(t)|\Psi_{p,\sigma}(t)\rangle$$

$$c_A(-\infty) = 1$$

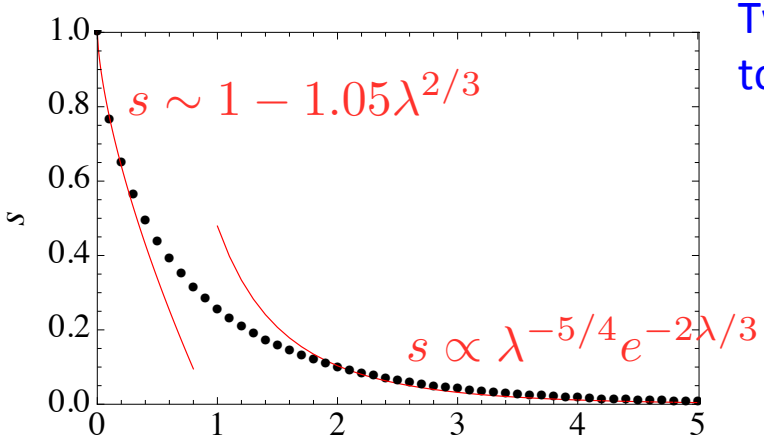
$$c_{p,\sigma}(-\infty) = 0$$

switching probability

$$s = \sum_{p,\sigma} |c_{p,\sigma}(+\infty)|^2$$

Two-band version for transition from discrete state to continuum

Demkov-Osherov (1967)



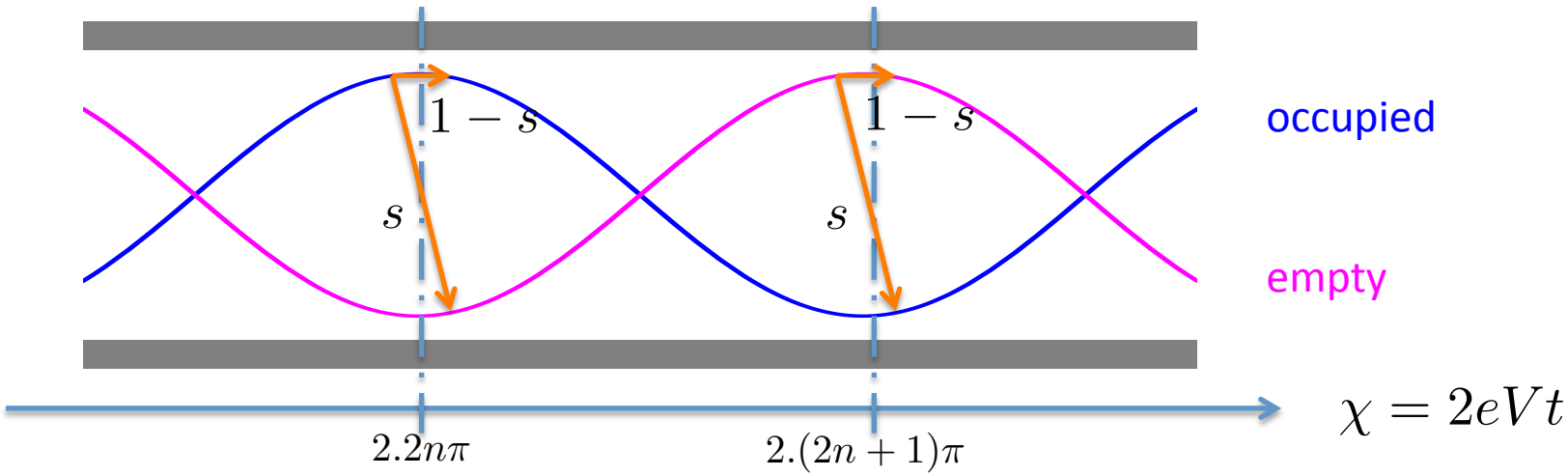
Adiabaticity parameter $\lambda = R^{3/2}\Delta/eV$

Time scale for switching:

$$\tau \sim \begin{cases} \sqrt{R}/eV \\ 1/[\Delta(eV)^2]^{1/3} \end{cases} \ll \pi/eV$$

L. Glazman, MH, JS Meyer, unpublished

Markov process



$$P^e_{2n-1} + P^o_{2n-1} = 1 \qquad P^e_{2n} + P^o_{2n} = 1 \qquad P^e_{2n+1} + P^o_{2n+1} = 1$$

$$\begin{pmatrix} P^e_{2n} \\ P^o_{2n} \end{pmatrix} = \begin{pmatrix} s & 0 \\ 1-s & 1 \end{pmatrix} \begin{pmatrix} P^e_{2n-1} \\ P^o_{2n-1} \end{pmatrix} \qquad \begin{pmatrix} P^e_{2n+1} \\ P^o_{2n+1} \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1-s \end{pmatrix} \begin{pmatrix} P^e_{2n} \\ P^o_{2n} \end{pmatrix}$$

$$I(t) = -I_0 \sin(eVt) \left(P^o_{\mathrm{Int}[eVt/\pi]} - P^e_{\mathrm{Int}[eVt/\pi]} \right)$$

Similar models have been used for Landau-Zener transitions near avoided crossings in conventional junctions:
Averin et al. 1995, Pikulin et al. 2012, Sau et al 2012…

$I_0 = e\sqrt{T}\Delta$

Average current and noise

The long time probability is independent of initial state and 4π-periodic :

$$\bar{P}_n^e = \frac{1}{2} - (-1)^n \frac{s}{1-s}$$

2π-periodic average current:

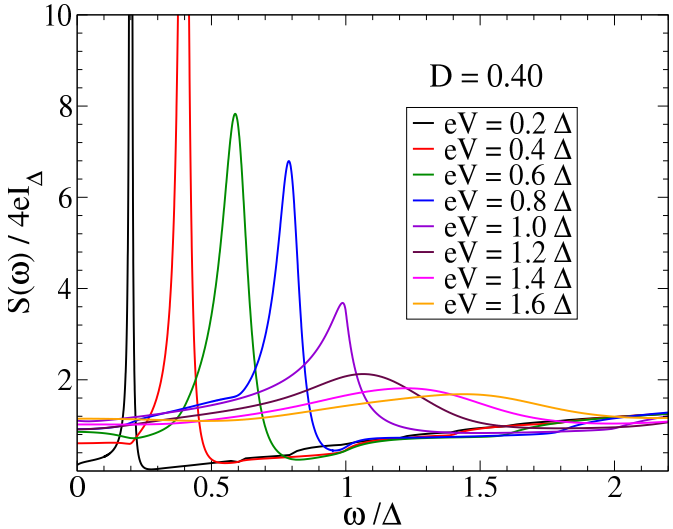
$$\bar{I}(t) = \frac{sI_0}{2-s}|\sin(eVt)| = \frac{2}{\pi}\frac{sI_0}{2-s}\sum_n \frac{1}{1-4n^2}\cos(2neVt)$$

- no average AC current at fractional frequency (*eV/ħ*)
- dc and ac components (at *2neV/ħ*) vanish at $s \to 0$

Peak at $\omega \approx eV$ in the finite-frequency noise

$$S(\omega) = \frac{I_0^2}{2}\frac{seV/\pi}{(\omega - eV)^2 + (seV/\pi)^2}$$

$$s \ll 1 \qquad \omega \approx eV$$



Shapiro steps

$$\chi(t) = 2eVt + \frac{2eV_{ac}}{\Omega}\cos(\Omega t) + \chi_0$$

$V_{ac} \ll V$ The phase increases monotonously with time $\alpha = eV_{ac}/\Omega$

Average dc current

$$\bar{I}_{dc} = \frac{2}{\pi}\frac{sI_0}{2-s}\sum_{n,k}\frac{J_k(2n\alpha)}{1-4n^2}\cos(n\chi_0)\delta_{2neV,k\Omega}$$

even and odd "steps" (in k)
+ fractional "steps" (in n)

Zero-frequency noise

$$S(\omega) = I_0^2\sum_k J_k(\alpha)^2\frac{seV/\pi}{(k\Omega - eV)^2 + (seV/\pi)^2}[1 - \cos\chi_0\delta_{eV,k\Omega}]$$

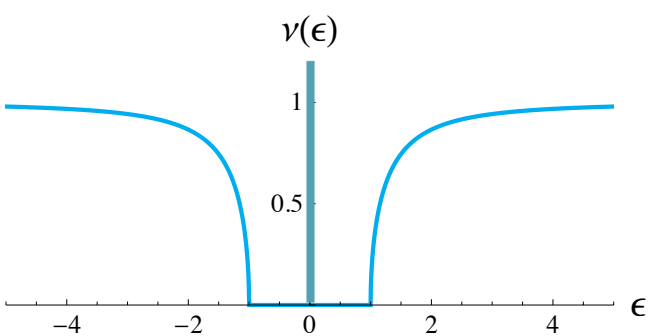
Broadened peaks at
eV=kΩ

Tunneling regime $D \to 0$

use tunneling Hamiltonian

$\to$ current & noise expressed in terms of (normalized) *edge* density of states

$$\nu(\epsilon) = \pi\Delta\delta(\epsilon) + \sqrt{1 - \left(\frac{\Delta}{\epsilon}\right)^2}\,\theta(|\epsilon| - \Delta)$$



Tunneling regime $D \to 0$

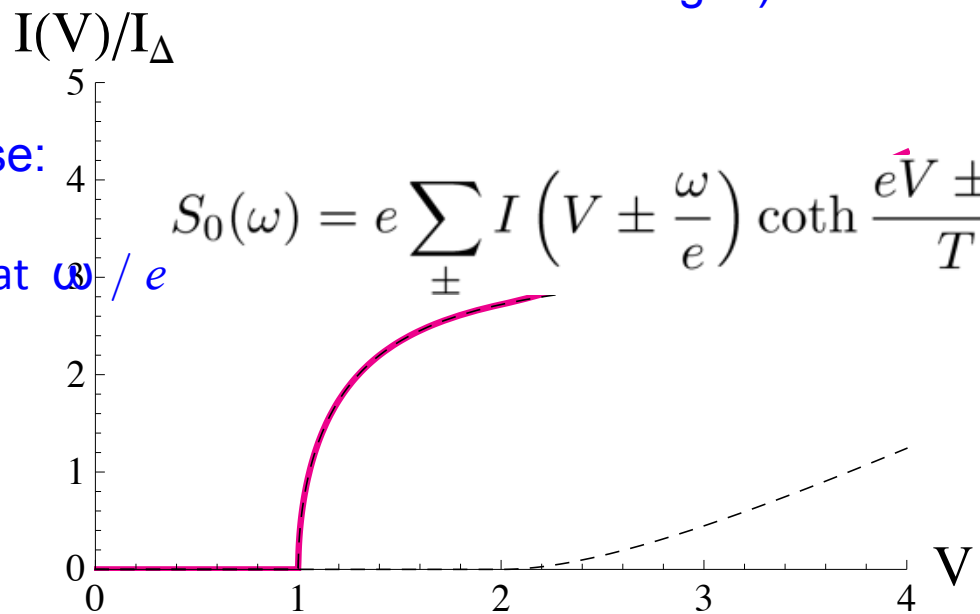
DC bias:

- average current: $$I(V) = D\frac{e}{2\pi}\int d\epsilon\, \nu(\epsilon)\nu(\epsilon - eV)\,[f(\epsilon) - f(\epsilon - eV)]$$

no AC current!
(zero anomalous Green function at the edges)

- finite-frequency noise: $$S_0(\omega) = e\sum_{\pm} I\left(V \pm \frac{\omega}{e}\right)\coth\frac{eV \pm \omega}{T}$$

peak at ω / e



Tunneling regime $D \to 0$

DC + AC bias:

- average DC current:

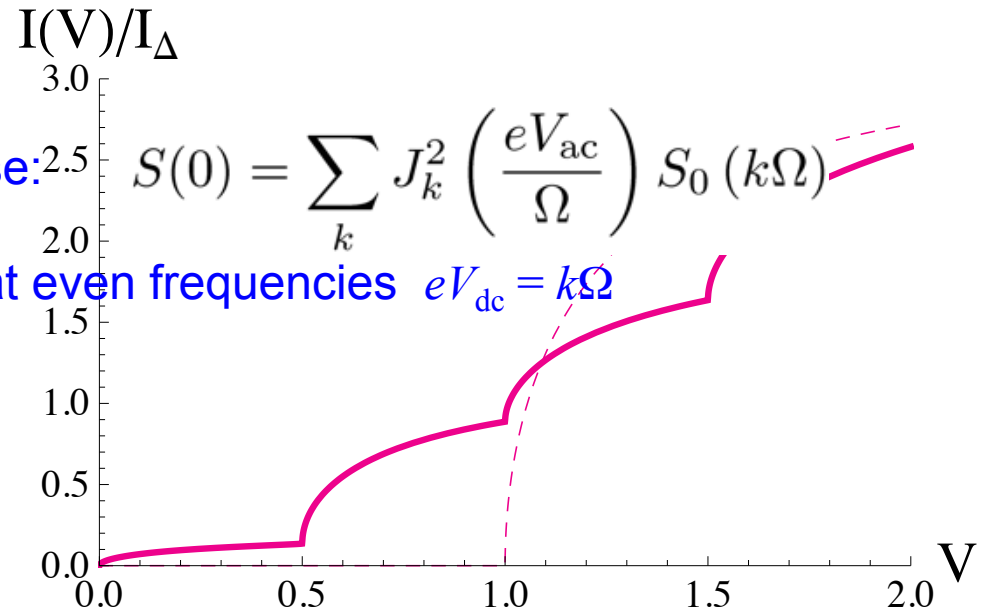
$$I_{\rm dc} = \sum_k J_k^2\left(\frac{eV_{\rm ac}}{\Omega}\right) I\left(V + k\frac{\Omega}{e}\right)$$

no Shapiro steps!
photo-assisted current

- zero-frequency noise:

$$S(0) = \sum_k J_k^2\left(\frac{eV_{\rm ac}}{\Omega}\right) S_0\left(k\Omega\right)$$

peaks at even frequencies $eV_{\rm dc} = k\Omega$



Conclusions

Majorana states are actively searched… but their signatures are subtle

- DC current:

  - MAR at $eV=\Delta/n$
  - tunnel limit: onset at $eV=\Delta$

- No signatures of fractional AC Josephson effect in average current
  non-adiabatic coupling between bound state and continuum

- Clear signature of fractional AC Josephson effect in finite-frequency noise (without microwave) and zero-frequency noise (with microwave)
  rate of non-adiabaticity

Refs: DM Badiane, MH, JS Meyer, PRL **107**, 177002 (2011)
L. Glazman, MH, JS Meyer, unpublished