Equilibration of a spinless Luttinger liquid

Konstantin Matveev

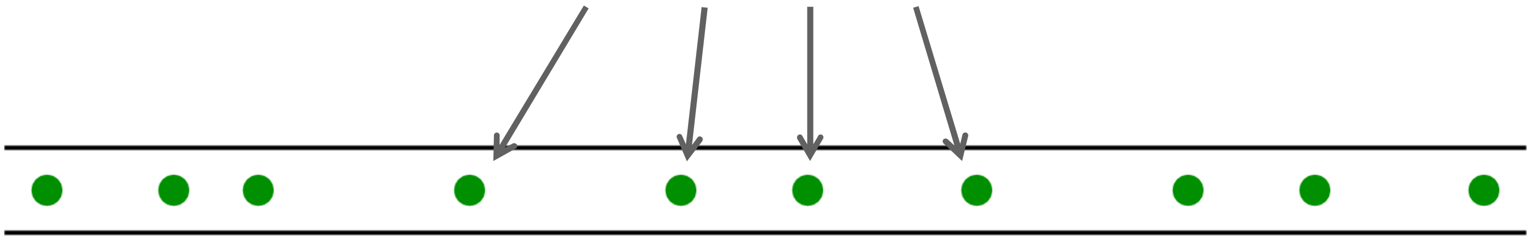
Argonne, November 15, 2012

In collaboration with A. Andreev

Acknowledge: T. Micklitz, M. Pustilnik, J. Rech

The problem

How fast a system of identical 1D particles comes to thermal equilibrium?



Assumptions:

- Low temperature $T \ll (\hbar n)^2/m$
- Arbitrary interaction strength
- No spin
- Fermions (optional)
- Galilean invariance (optional)

Outline

- Luttinger liquid: low temperature theory of 1D systems
- Two relaxation times in one dimension
- Relation to experiments with quantum wires
- Phenomenological (Luttinger-liquid) theory of the equilibration rate
- Comparison with the microscopic calculations

Interacting fermions in one dimension: Luttinger liquid

Particle-hole excitations become acoustic bosons

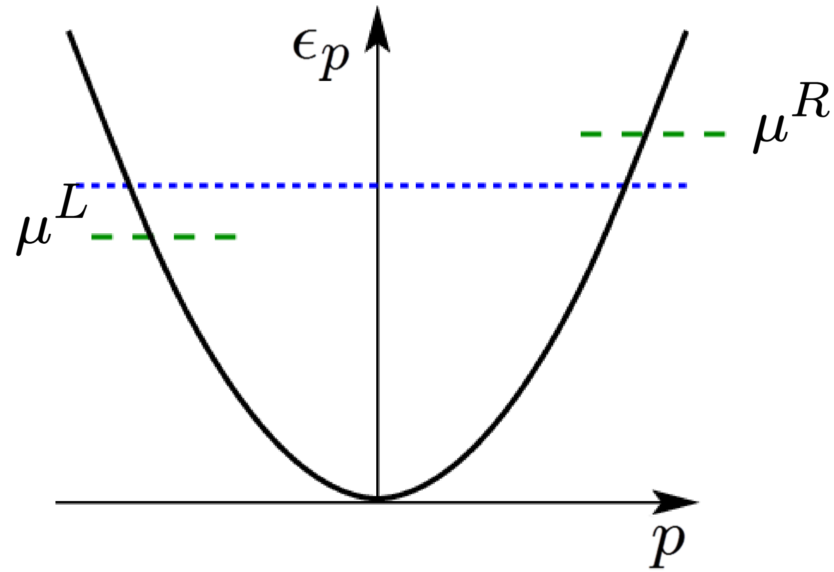
Luttinger liquid theory describes the low-energy properties of one-dimensional electron systems at any interaction strength.

Energy and momentum of a Luttinger liquid

Hamiltonian: $$H = \sum_q \hbar v|q| b_q^\dagger b_q + \frac{\pi\hbar}{2L}[v_N(N-N_0)^2 + v_J J^2]$$

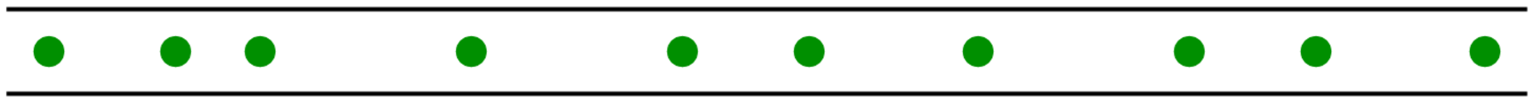
N is the total number of particles, J is the current quantum number:

$$N = N^R + N^L, \quad J = N^R - N^L$$



Momentum: $$P = \sum_q \hbar q\, b_q^\dagger b_q + p_F J$$

Equilibrium state of a uniform 1D system



In the absence of disorder, collisions between particles conserve momentum.
As a result the Gibbs distribution has the form

$$w_i = \exp\left(-\frac{E_i - uP_i}{T}\right)$$

The distribution is fully described by the temperature T and velocity u.

Equilibrium state of a Luttinger liquid

1. Distribution of the bosons in equilibrium: $N_q = \frac{1}{e^{\hbar(v|q|-uq)/T} - 1}$

2. Distribution of *J:* $w_J \propto \exp\left\{-\frac{1}{T}\left(\frac{\pi\hbar v_J}{2L} J^2 - u p_F J\right)\right\}$

A sharp peak at $J = \frac{p_F L}{\pi\hbar v_J} u$ $\Rightarrow$ $\boxed{u = v_d}$

In thermal equilibrium, u is the drift (center of mass) velocity: $v_d = \frac{j}{n}$

In the center of mass frame, $u = 0.$

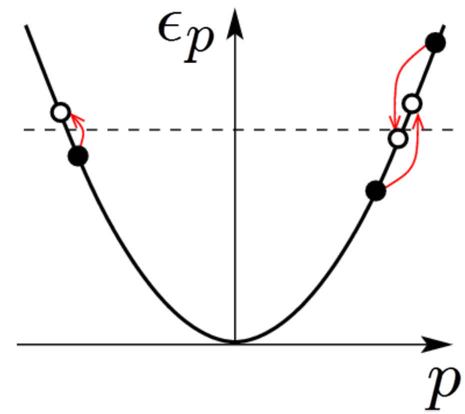
Two relaxation times

Quadratic Hamiltonian: $$H = \sum_q \hbar v |q| b_q^\dagger b_q + \frac{\pi \hbar}{2L}[v_N (N - N_0)^2 + v_J J^2]$$

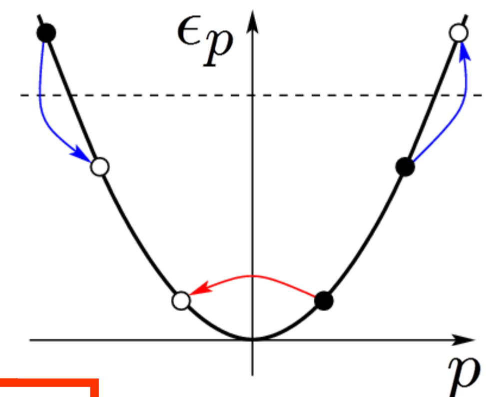
No relaxation

Relaxation of bosons (particle-hole excitations)

- Relaxation rate: $\tau_b^{-1} \propto T^\alpha$



- Relaxation of J (backscattering) $\tau^{-1} \propto e^{-D/T}$



Bosons equilibrate much faster than J

1D system at long time scales

$t > \tau \gg \tau_b$

The bosons are always in equilibrium with each other: $N_q = \dfrac{1}{e^{\hbar(v|q| - uq)/T} - 1}$

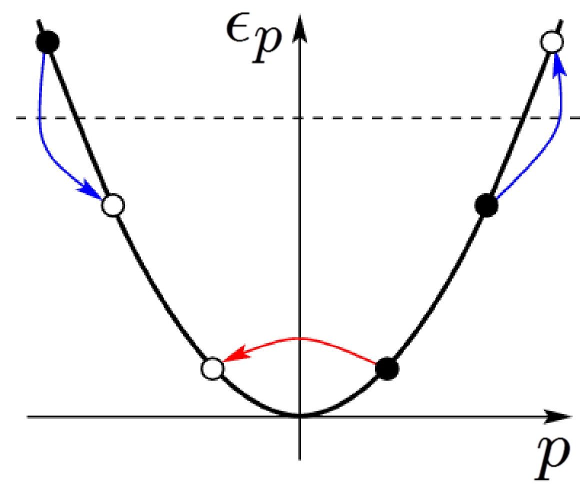
but not yet with the system as a whole: $P = \dfrac{\pi L T^2}{3\hbar v^3} u + p_F J = 0$

$u \neq 0, \;\; J \neq 0$

Slow relaxation to full equilibrium:

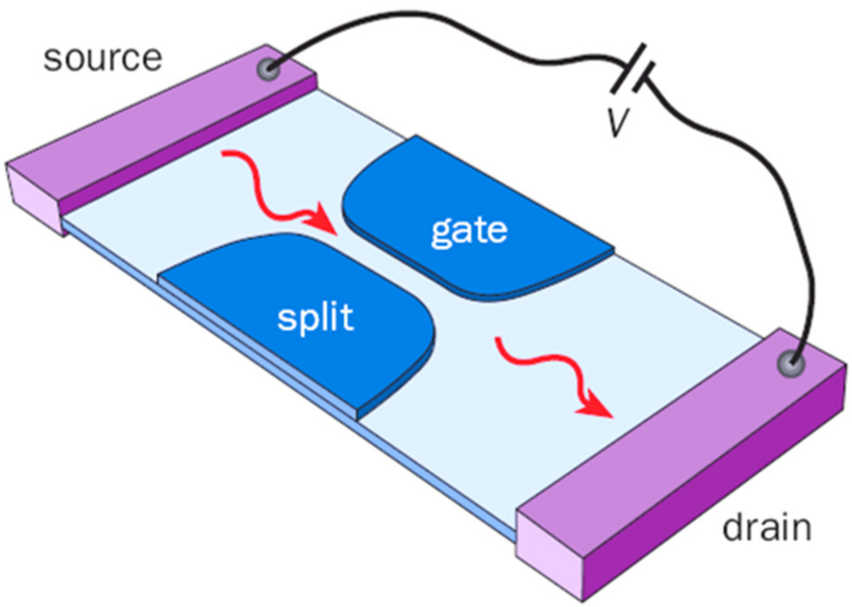
$$\dot{u} = -\frac{u}{\tau}, \;\; \dot{J} = -\frac{J}{\tau}$$

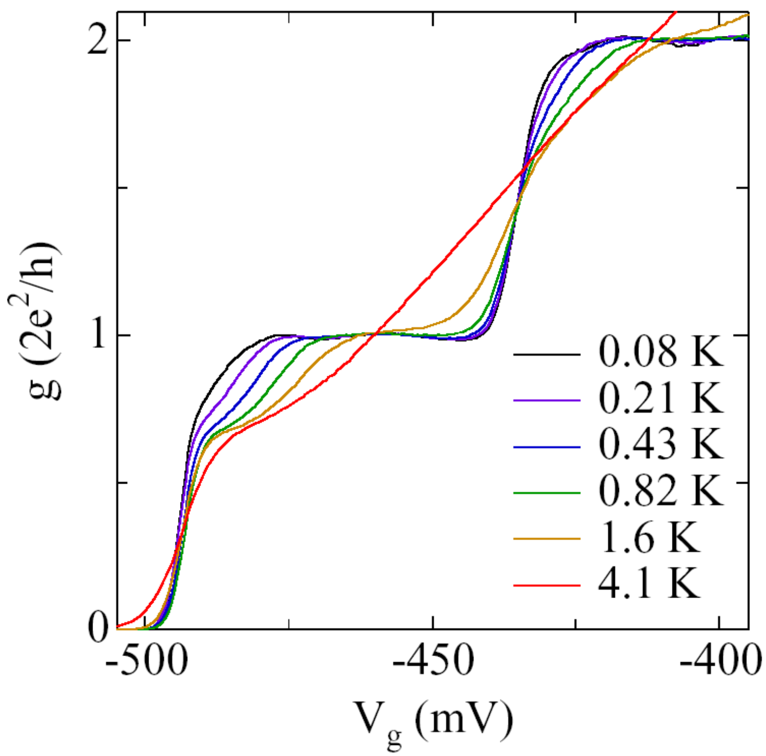
At weak interactions $\tau^{-1} \propto e^{-E_F/T}$



What is the relaxation rate at arbitrary interactions?

Motivation: Conductance of quantum wires



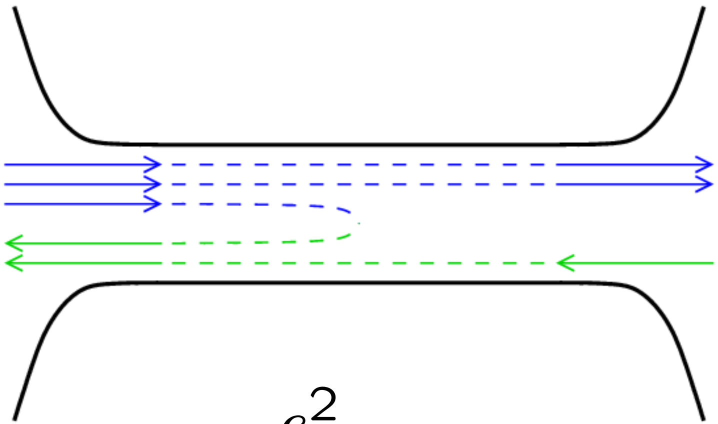


Cronenwett *et al.*, 2001

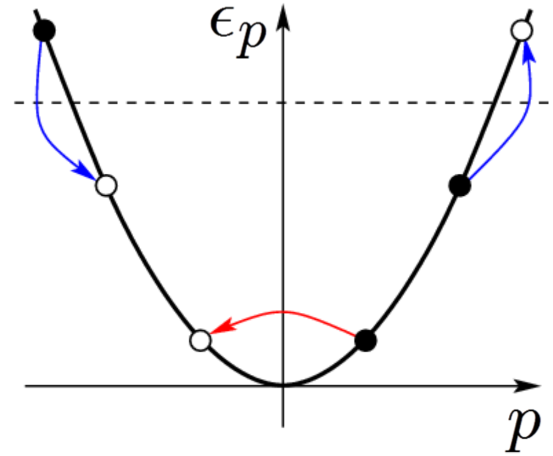
1. Good quantization of conductance $G = \frac{2e^2}{h}$ at $T \to 0$.

2. Interesting corrections develop at $T > 0$.

Origin of the corrections to conductance: Backscattering



$$I = \frac{e^2}{h}V + e\dot{N}^R$$



Correction to the conductance is determined by the rate of backscattering of right-moving electrons in the wire $\dot{N}^R = \dot{j}/2$

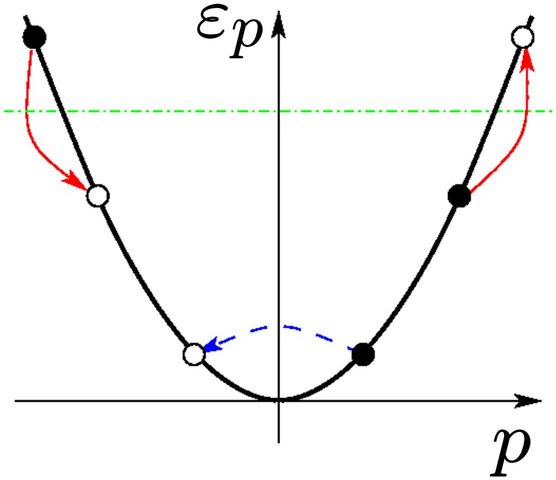
It can be expressed in terms of the relaxation time τ as

$$G = \frac{e^2}{h}\left[1 - \frac{\pi^2}{3}\left(\frac{T}{vp_F}\right)^2 \frac{L}{L + 2v\tau}\right]$$

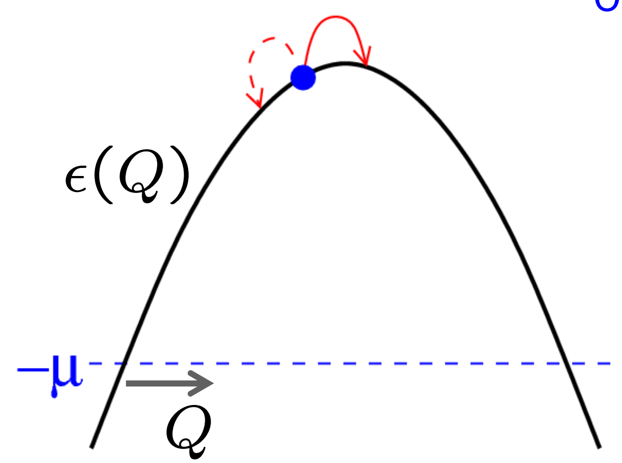
[KM and A. V. Andreev, PRL **107**, 056402 (2011)]

Evaluation of the equilibration rate at weak interactions

Scattering of a hole near the bottom of the band



$$\tau^{-1} = \tau_0^{-1} e^{-E_F/T}$$



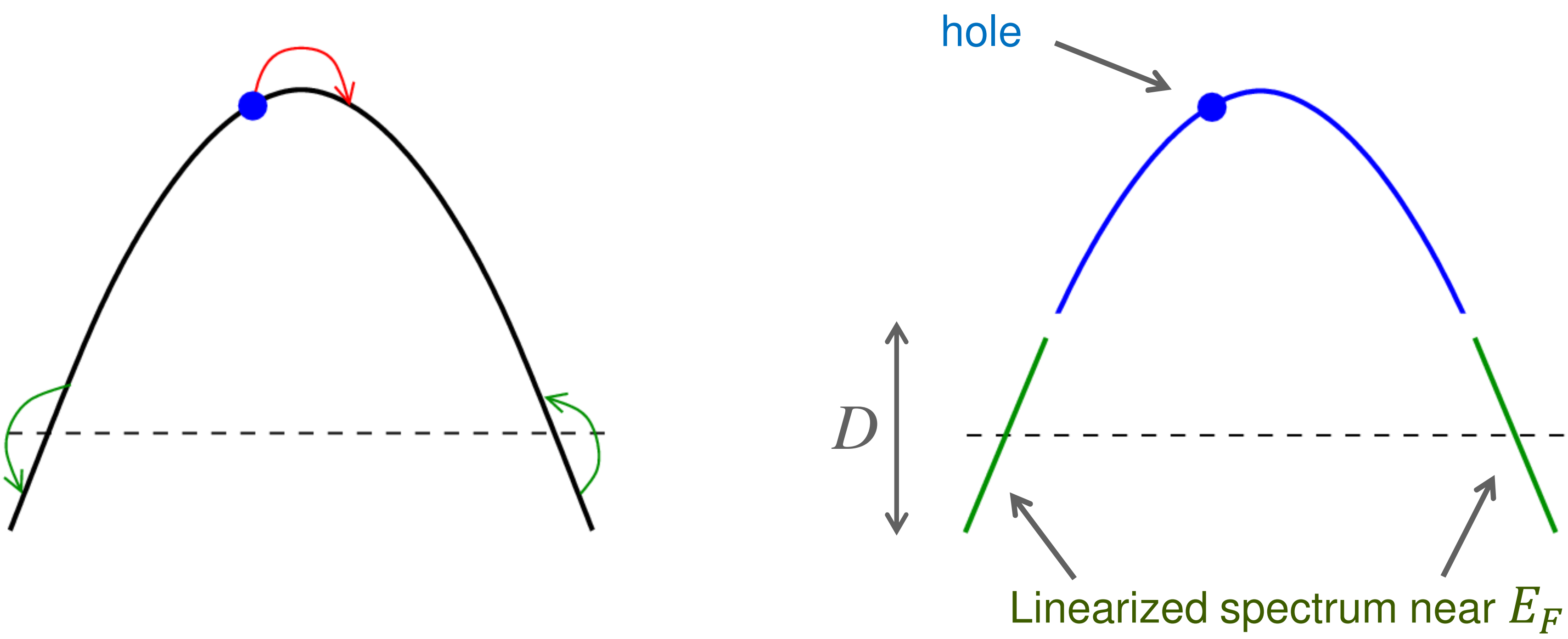
The hole moves in small random steps in momentum space, $\delta Q \sim \dfrac{T}{v_F}$

Such diffusion is described by the Fokker-Planck equation:

$$\partial_t f = -\frac{B}{2}\partial_Q \left[\frac{\epsilon'(Q)}{T} + \partial_Q\right] f$$

Its solution enables one to find the prefactor $\quad \tau_0^{-1} \propto T^{3/2}$

Arbitrary interactions: Mobile impurity in a Luttinger liquid



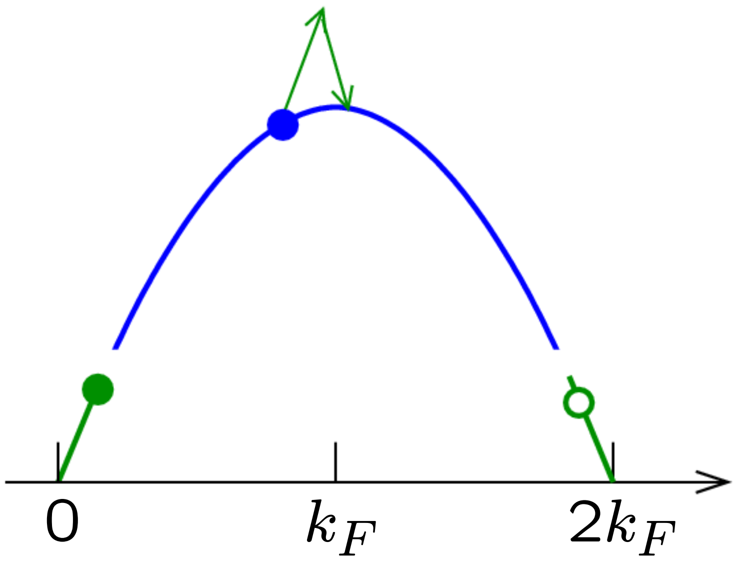
The bandwidth D is in the range $T \ll D \ll E_F$

At non-weak interactions, fermions with linear spectrum form a Luttinger liquid

The hole (dressed by interactions) becomes a mobile impurity

[cf. Ogawa, Furusaki & Nagaosa 1992, Castro Neto & Fisher, 1996]

Mechanism of equilibration



The hole moves in momentum space by absorbing one boson and emitting another

The typical momentum of a boson is small

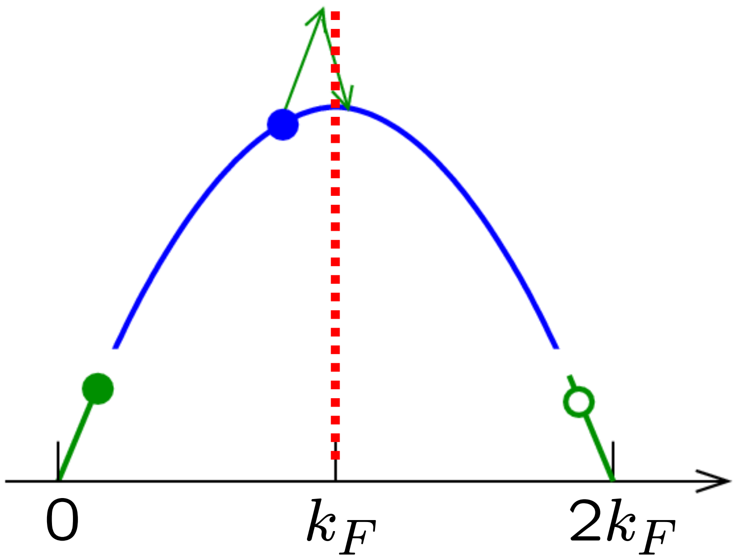
$$Q \sim \frac{T}{v} \ll k_F$$

The hole motion is diffusive → Fokker-Planck equation

$$\partial_t f = -\frac{B}{2}\partial_Q \left[\frac{\epsilon'(Q)}{T} + \partial_Q\right] f$$

$f(Q,t)$ is the occupation probability of the hole; B is the diffusion constant

Equilibration rate from Fokker-Planck equation



At low temperature the hole is in equilibrium with the nearest boson branch

$$f = e^{-\frac{\epsilon(Q)-\hbar u Q}{T}}, \quad Q < k_F$$

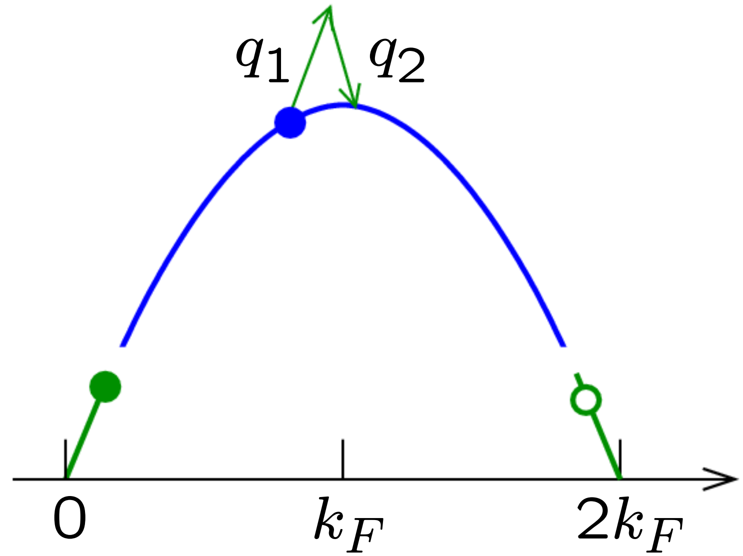
$$f = e^{-\frac{\epsilon(Q)-\hbar u (Q-2k_F)}{T}}, \quad Q > k_F$$

Solving the Fokker-Planck equation with these boundary conditions, we indeed find

$$\dot{u} = -\frac{u}{\tau} \qquad \text{with} \qquad \frac{1}{\tau} = \frac{3\hbar k_F^2 B}{\pi^2\sqrt{2\pi m^* T}} \left(\frac{\hbar v}{T}\right)^3 e^{-\Delta/T}$$

Activation energy $\Delta = \epsilon(k_F)$, effective mass $m^* = -\frac{\hbar^2}{\epsilon''(k_F)}$

Evaluation of the diffusion constant



$$B = \sum_{\delta Q} \delta Q^2 W_{\delta Q}$$

$W_{\delta Q}$ is the rate of scattering from state Q to $Q + \delta Q$

Golden rule:

$$W_{\delta Q} = \frac{2\pi}{\hbar} \sum_{q_1 q_2} |t_{q_1 q_2}|^2 \delta_{q_1, q_2 + \delta Q} \, \delta(v q_1 - v|q_2|) N_{q_1}(N_{q_2} + 1) \propto |t_{\delta Q/2, -\delta Q/2}|^2$$

Energy of the hole: $\epsilon(Q, n) = \epsilon(Q, n_0) + (\partial_n \epsilon)\delta n + \frac{1}{2}(\partial_n^2 \epsilon)(\delta n)^2 + \ldots$

Density fluctuation: $\delta n \propto \sum_q \sqrt{|q|}(b_q + b^\dagger_{-q})$

$t_{q_1 q_2} b^\dagger_{q_2} b_{q_1}$ $\qquad$ $t_{q_1 q_2} \propto \sqrt{|q_1 q_2|}$

$\boxed{W_{\delta Q} \propto \delta Q^2}$

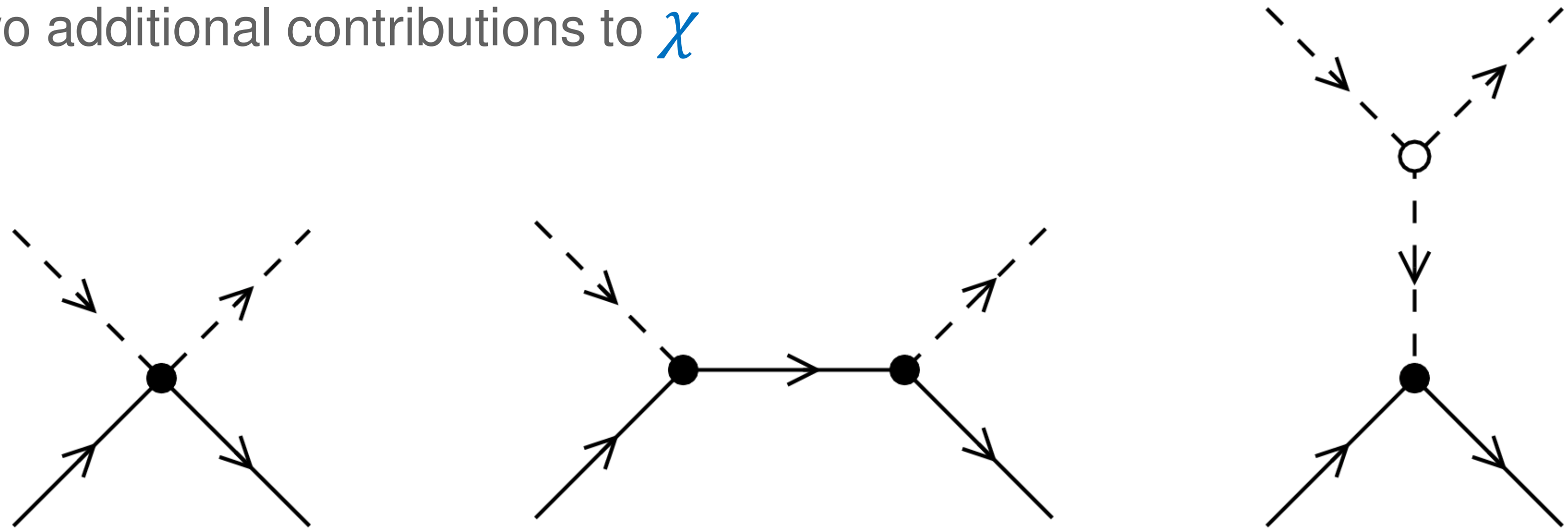
Bosons are thermally excited, $\delta Q \sim T/v$

Resulting diffusion constant

$$B = \chi T^5$$

[cf. Castro Neto & Fisher, 1996]

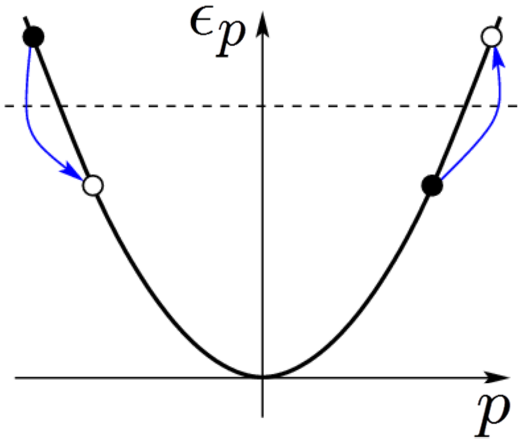
Two additional contributions to χ



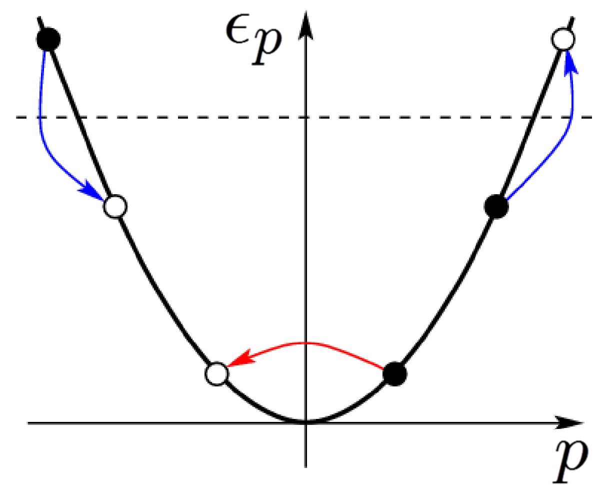
$$\chi = \frac{4\pi^3 n_0^2}{15\hbar^5 m^2 v^8}\left(\Delta'' + \frac{\Delta'^2}{m^* v^2} - \frac{2v'}{v}\Delta'\right)^2$$

$$\Delta' = \frac{d\Delta}{dn},\ \ \Delta'' = \frac{d^2\Delta}{dn^2},\ \ v' = \frac{dv}{dn}$$

Microscopic theory: Weak interactions



Two-particle collisions do not change the momenta of the electrons because of the conservation laws



Three-particle collisions are obtained in second order in interaction strength

The expression for the equilibration rate is recovered if the energy of the hole is calculated in second order in interactions:

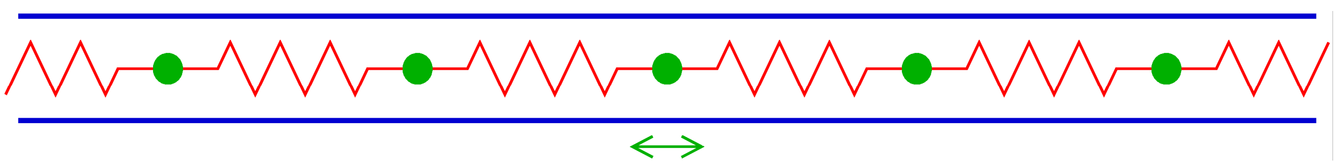
$$\Delta \approx E_F + \delta_1 \Delta + \delta_2 \Delta$$

Microscopic theory: Strong interactions

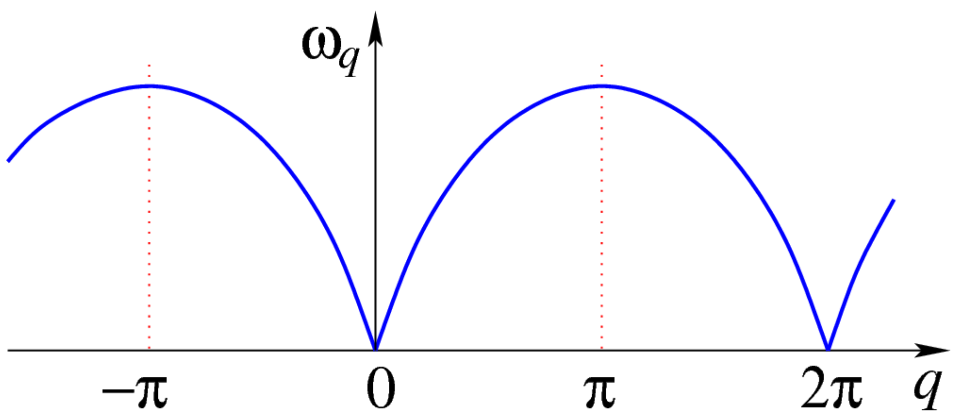
Kinetic energy vs. Coulomb repulsion:

$$E_{\text{kin}} = \frac{\hbar^2 k_F^2}{2m}, \qquad E_{\text{Coul}} = \frac{e^2}{r} \sim e^2 k_F.$$

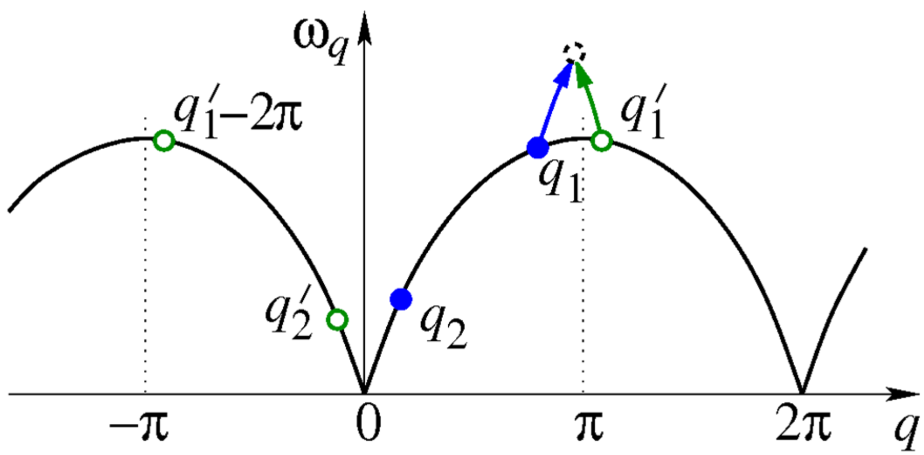
Strong repulsion forces (Wigner) crystallization of the system:



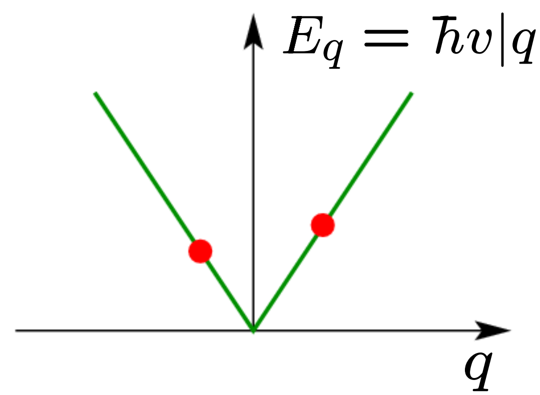
Excitations are phonons with acoustic spectrum



Equilibration of 1D Wigner crystal



At low energies the phonons coincide with the bosons in the Luttinger liquid theory



Coupling of a high-energy phonon with the acoustic ones is accounted for microscopically by expansion in inverse interaction strength

$$\chi = \frac{4\pi^3[(\partial_n\omega_\pi)\partial_n v^2 - v^2\partial_n^2\omega_\pi + \omega_\pi''(\partial_n\omega_\pi)^2 n^{-2}]^2}{15\hbar^3 m^2 v^{12}}$$

(same result as in Luttinger liquid theory)

Integrable models

Calogero-Sutherland model: inverse-square repulsion $$V(x) = \frac{A}{x^2}$$

Lieb-Liniger model: bosons with δ-function repulsion $$V(x) = A\delta(x)$$

No three-particle collisions in integrable models.
There should be no equilibration

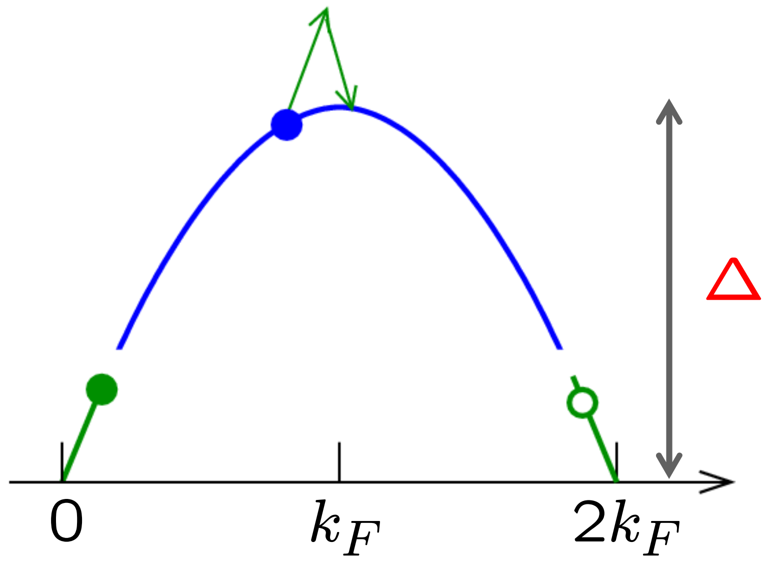
$$\tau^{-1} = 0$$

Using the known expressions for the excitation energies, we indeed find

$$\chi = 0$$

Summary

Equilibration of an interacting 1D system involves excitations passing between the right- and left-moving branches



At low temperatures, the equilibration rate is exponentially small

$$\frac{1}{\tau} = \frac{3\hbar k_F^2 T^5 \chi}{\pi^2 \sqrt{2\pi m^* T}} \left(\frac{\hbar v}{T}\right)^3 e^{-\Delta/T}$$

The prefactor can be expressed in terms of the excitation spectrum

$$\chi = \frac{4\pi^3 n_0^2}{15\hbar^5 m^2 v^8} \left(\Delta'' + \frac{\Delta'^2}{m^* v^2} - \frac{2v'}{v}\Delta'\right)^2$$