Topological aspects of Andreev bound state in superconductivity

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A. A. Golubov (Twente University)
Contents of our talk

(1) Surface Andreev bound state up to now
(2) Majorana fermion
(3) Fabrication of Majorana Fermion at Nanowire and Interface
(4) Superconducting doped topological insulator
Bogoliubov-de Gennes equation

\[
\left(-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + U(x) - E_F\right) u(x) + \Delta(x)v(x) = Eu(x), \\
-\left(-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + U(x) - E_F\right) v(x) + \Delta^*(x)u(x) = Ev(x).
\]

\(u(x), v(x)\) : Wave functions
\(\Delta(x)\) : Pair potential
\(U(x)\) : Hartree Fock potential
\(E_F\) : Fermi energy

Andreev reflection
Classical example of Andreev bound state

**ABS with nonzero energy**

ABS has been known since 1963.

P.G. de Gennes and D. Saint-James


High transparent interface

Energy levels of Andreev bound state (nonzero energy)

\[ \varepsilon_n = \pm \frac{\pi u_F x}{2L} (n + 1/2), \quad n = 0, 1, 2, \ldots \]

\[ \Delta_0 \gg |\varepsilon_n| \]

\[ \Delta_0 \] magnitude of pair potential

\[ u_F x \] x component of Fermi velocity
Andreev bound state
(non-topological and topological)

Andreev bound state with non zero energy  (de Gennes, Saint James)

Not edge state
Non topological

Mid gap (zero energy) Andreev bound state
Surface Andreev bound state
Edge state  Topological

L. Buchholtz & G. Zwicknagl (81); J. Hara & K. Nagai : Prog. Theor. Phys. 74 (86)
C.R. Hu : (94)
Surface Andreev bound state (ABS) up to now

(1) $d$-wave (cuprate)
(2) chiral $p$-wave ($\text{Sr}_2\text{RuO}_4$)
(3) helical (NCS superconductor)
(4) 3d superconductor (superfluid $^3\text{He}$)

The presence of ABS is supported by the bulk topological invariant.

Tunneling effect in unconventional superconductors

s-wave

Unconventional superconductor

Important issue of cuprate in the 90s.
Conductance formula and Andreev bound state (ABS)

Tanaka and Kashiwaya PRL 74 3451

\[
\sigma_T(E) = \frac{\int_{-\pi/2}^{\pi/2} d\theta \sigma_N(\theta) \sigma_R(E, \theta) \cos \theta}{\int_{-\pi/2}^{\pi/2} d\theta \sigma_N(\theta) \cos \theta}
\]

\[
\sigma_R(E, \theta) = \frac{1 + \sigma_N(\theta) |\Gamma_+|^2 + [\sigma_N(\theta) - 1] |\Gamma_+\Gamma_-|^2}{1 + [\sigma_N(\theta) - 1]|\Gamma_+\Gamma_- \exp[i(\phi_- - \phi_+)]|^2}
\]

\[
\exp(i\phi_+) = \frac{\Delta(\theta_+)}{|\Delta(\theta_+)|} = \frac{\Delta_+}{|\Delta_+|} \quad \exp(i\phi_-) = \frac{\Delta(\theta_-)}{|\Delta(\theta_-)|} = \frac{\Delta_-}{|\Delta_-|}
\]

\[
\Gamma_\pm = \frac{E - \sqrt{E^2 - \Delta_\pm^2}}{|\Delta_\pm|}, \quad E = eV
\]

Condition for ABS

\[
\sigma_N(\theta) \rightarrow 0 \quad 1 = \Gamma_+\Gamma_- \exp[i(\phi_- - \phi_+)]
\]

\[
1 = \frac{-E + \sqrt{E^2 - \Delta_0^2 \sin^2 2\theta}}{E + \sqrt{E^2 - \Delta_0^2 \sin^2 2\theta}} \quad E = 0
\]

Flat zero energy band

Tunneling conductance in $d$-wave junction


$$\Delta_{\pm} = \Delta_0 \cos[2(\theta \mp \alpha)]$$

$\alpha$ angle between the normal to the interface and the lobe direction

Bulk ldos (blue line)

Normal metal $\Rightarrow$ d-wave superconductor

$\alpha = 0$

$\alpha = \frac{\pi}{8}$

$\alpha = \frac{\pi}{4}$

Zero bias conductance peak

Andreev bound state

Surface zero energy state

Well known example of Andreev bound states in $d$-wave superconductor

Phase change of pair potential is $\pi$

ABS in $d$-wave (110)direction

Flat dispersion!!
Zero energy

$\Delta_+ \Delta_- < 0$

$E = 0$

Surface

Alff, Kashiwaya PRB (1998)

Tanaka Kashiwaya PRL 74 3451 (1995),
Condition for ABS (without dispersion)

Spin-singlet $d_{xy}$-wave

$$1 = \frac{-E + \sqrt{E^2 - \Delta_0^2 \sin^2 2\theta}}{E + \sqrt{E^2 - \Delta_0^2 \sin^2 2\theta}} \quad \Rightarrow \quad E = 0$$

Flat zero energy band

Spin-triplet $p_x$-wave

$$1 = \frac{-E + \sqrt{E^2 - \Delta_0^2 \cos^2 \theta}}{E + \sqrt{E^2 - \Delta_0^2 \cos^2 \theta}} \quad \Rightarrow \quad E = 0$$

Flat dispersion

Flat Andreev bound state and topology

Hamiltonian

\[ \mathcal{H} = \sum_k \left( c_{k \uparrow}^\dagger, c_{-k \downarrow} \right) \mathcal{H}(k) \left( \begin{array}{c} c_{k \uparrow} \\ c_{-k \downarrow}^\dagger \end{array} \right) \mathcal{H}(k) = \left( \begin{array}{cc} \varepsilon(k) & \Delta(k) \\ \Delta(k) & -\varepsilon(k) \end{array} \right), \]

\[ \Delta(k) = \begin{cases} \psi(k) = \psi(k) & \text{for spin-singlet} \\ d_z(k) = -d_z(-k) & \text{for spin-triplet} \end{cases}. \]

Winding number for fixed \( k_y \)

\[ \omega_{1d}(k_y) = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk_x \partial_{k_x} \theta(k). \]

\[ \cos \theta(k) = \frac{\varepsilon(k)}{\sqrt{\varepsilon(k)^2 + \Delta(k)^2}}, \quad \sin \theta(k) = \frac{\Delta(k)}{\sqrt{\varepsilon(k)^2 + \Delta(k)^2}}, \]

Sato, Tanaka, et al, PRB 83 224511 (2011)
Midgap Andreev bound state
(Winding number)

$$w_{1d}(k_y) = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk_x \partial_{k_x} \theta(k)$$

$$= -\frac{1}{2} \sum_{k_x; \varepsilon(k)=0} \text{sgn}[\Delta(k)] \cdot \text{sgn}[\partial_{k_x} \varepsilon(k)]$$

Topological invariant defined in bulk

If we consider simple Fermi surface

$$w_{1d} = -\frac{1}{2} \text{sgn}[\partial_{k_x} \varepsilon(-k_x^0, k_y)] \left[ \text{sgn}[\Delta(-k_x^0, k_y)] - \text{sgn}[\Delta(k_x^0, k_y)] \right].$$

$$w_{1d} \neq 0 \iff \Delta(-k_x^0, k_y) \Delta(k_x^0, k_y) < 0$$

Conventional condition

Sato, Tanaka, et al, PRB 83 224511 (2011)
Winding number & Index theorem (1)

From the bulk-edge correspondence, there exists the gapless states on the edge only when integer $w_{1d}$ is nonzero.

BdG Hamiltonian has a symmetry (chiral symmetry)

$$\{ \mathcal{H}(k), \sigma_y \} = 0$$

Zero energy ABS is an eigenstate of $\sigma_y$.

$$n_0^{(+)} \quad \text{Number of ZES where the eigenvalue of } \sigma_y \text{ is } 1$$

$$n_0^{(-)} \quad \text{Number of ZES where the eigenvalue of } \sigma_y \text{ is } -1$$

Index Theorem

$$w_{1d} = (n_0^{(+)} - n_0^{(-)})$$
Winding number & Index theorem (2)

Two-dimensional spin-singlet $d_{xy}$-wave superconductor

$$\varepsilon(k) = \frac{k^2}{2m} - E_F, \quad \Delta(k) = \Delta_0 \frac{k_x k_y}{k^2}.$$  

$$w_{1d}(k_y) = \begin{cases} 
1, & \text{for } 0 < k_y < k_F \\
-1, & \text{for } 0 > k_y > -k_F \\
0, & \text{for } |k_y| > k_F 
\end{cases}, \quad k_F = \sqrt{2mE_F} \quad k_x = \sqrt{k_F^2 - k_y^2}$$

$$|u_0(x)\rangle = C \begin{pmatrix} 1 \\ -i \text{sgn}k_y \end{pmatrix} e^{ik_y y} \sin(k_x x) e^{-x/\xi}$$

superconductor on $x>0$
Winding number & Index theorem (3)

<table>
<thead>
<tr>
<th>$k_y$</th>
<th>$n_0^{(+)}$</th>
<th>$n_0^{(-)}$</th>
<th>$n_0^{(+)} - n_0^{(-)}$</th>
<th>$w_{1d}(k_y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; k_y &lt; k_F$</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>$0 &gt; k_y &gt; -k_F$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$</td>
<td>k_y</td>
<td>&gt; k_F$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$k_y$</th>
<th>$n_0^{(+)}$</th>
<th>$n_0^{(-)}$</th>
<th>$n_0^{(+)} - n_0^{(-)}$</th>
<th>$w_{1d}(k_y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; k_y &lt; k_F$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>$0 &gt; k_y &gt; -k_F$</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>$</td>
<td>k_y</td>
<td>&gt; k_F$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) $w_{1d}(k_y) = (n_0^{(+)} - n_0^{(-)})$ Superconductor $x > 0$

(b) $w_{1d}(k_y) = -(n_0^{(+)} - n_0^{(-)})$ Superconductor $x < 0$
Surface Andreev bound state (ABS) up to now

(1) $d$-wave (cuprate)
(2) chiral $p$-wave ($\text{Sr}_2\text{RuO}_4$)
(3) helical (NCS superconductor)
(4) 3$d$ superconductor (superfluid $^3\text{He}$)

The presence of ABS is supported by the bulk topological invariant.

Extension to spin-triplet superconductors

\[ \sigma_T(\text{eV}) \]

\[ p_x + ip_y \]

\[ eV/\Delta_0 \]

Normal metal

\[ p_x \]

\[ p_y \]

Superconductor

\[ p_x + ip_y \]


Condition for ABS

\[ 1 = \frac{-E + \sqrt{E^2 - \Delta_0^2}}{E + \sqrt{E^2 - \Delta_0^2}} \exp(-2i\theta) \]

\[ E = \Delta_0 \sin \theta \sim \frac{k_y}{k_F} \]

linear dispersion

\[ \Delta = \Delta_0 \exp(i\theta) \]

chiral \( p \)

\[ 1 = \frac{-E + \sqrt{E^2 - \Delta_0^2 \cos^2 \theta}}{E + \sqrt{E^2 - \Delta_0^2 \cos^2 \theta}} \]

flat dispersion

\[ E = 0 \]

\[ \rho_x \]

\[ \Delta = \Delta_0 \cos \theta \]

surface
Chiral superconductor $\text{Sr}_2\text{RuO}_4$

Edge surface current

$p_x + ip_y$

Similar structure to cuprate

Maeno (1994)
Recent experiment of $\text{Sr}_2\text{RuO}_4$

It is possible to fit experimental data taking into account of anisotropy of pair potential.

Tunneling spectrum in two-dimensional topological superconductors

$E/\Delta$

$\Delta$

$-\Delta$

$d_{x^2-y^2}$-wave nodal gap

zero energy flat band of surface ABS

$\theta/\pi$

chiral $p$-wave full gap

chiral edge state (ABS)

broad zero-bias peak due to linear dispersion

Chiral superconductor

Topological invariant defined in bulk

Volovik(88), Furusaki(01)

\[ w_{2d} = -\frac{1}{8\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} dk_x dk_y \epsilon^{ij} \epsilon^{abc} m_a(k) \partial_{k_i} m_b(k) \partial_{k_j} m_c(k), \]

\[ m_1(k) = \frac{\text{Re} \Delta(k)}{\sqrt{\epsilon(k)^2 + |\Delta(k)|^2}}, \]

\[ m_2(k) = \frac{\text{Im} \Delta(k)}{\sqrt{\epsilon(k)^2 + |\Delta(k)|^2}}, \]

\[ m_3(k) = \frac{\epsilon(k)}{\sqrt{\epsilon(k)^2 + |\Delta(k)|^2}}. \]

\[ \Delta(k) \] Energy gap function

\[ \epsilon(k) \] Quasiparticle energy

Measured from Fermi surface

\[ w_{2D} = 1 \]

\( E = 0 \)
\( k_y = 0 \)

Edge state is possible for nonzero \( w_{2D} \)

Broken time reversal symmetry
<table>
<thead>
<tr>
<th>Andreev bound state</th>
<th>Topological invariant</th>
<th>Time reversal symmetry</th>
<th>Materials</th>
<th>Theory of tunneling</th>
<th>Insulator (semi-metal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chiral</td>
<td>2d winding number</td>
<td>×</td>
<td>Sr$_2$RuO$_4$, 3He A</td>
<td>PRB (1997)</td>
<td>QHS</td>
</tr>
<tr>
<td>Helical</td>
<td>$z_2$</td>
<td>○</td>
<td>$s \pm p$-wave (NCS)</td>
<td>PRB (2007)</td>
<td>QSHS (2D Topological insulator)</td>
</tr>
<tr>
<td>Cone</td>
<td>3d winding number</td>
<td>○</td>
<td>3He B</td>
<td>PRB (2003)</td>
<td>Topological insulator</td>
</tr>
</tbody>
</table>

M. Sato, et al. Kotai Butsuri in Japan
Surface Andreev bound state (ABS) up to now

(1) $d$-wave (cuprate)
(2) chiral $p$-wave ($\text{Sr}_2\text{RuO}_4$)
(3) helical (NCS superconductor)
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The presence of ABS is supported by the bulk topological invariant.

Andreev bound state in the presence of spin-orbit coupling

Spin-singlet ($s$-wave) $\Delta_s$ spin-triplet ($p$-wave) $\Delta_p$

<table>
<thead>
<tr>
<th>$\Delta_p &gt; \Delta_s$</th>
<th>Andreev bound state</th>
<th>Bulk energy gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_p = \Delta_s$</td>
<td>No Andreev bound state</td>
<td>Gap closes</td>
</tr>
<tr>
<td>$\Delta_s &gt; \Delta_p$</td>
<td>No Andreev bound state</td>
<td>Bulk energy gap</td>
</tr>
</tbody>
</table>

CePt$_3$Si

Calculated conductance

\[ q = \frac{\Delta_s}{\Delta_p} \]

If $q < 1$ Helical superconductor

Zero bias conductance peak by Andreev bound state

(Topological) Andreev bound states

Non-centrosymmetric superconductor (NCS)

- $d_{xy}$-wave
  - Hu (94)
  - Tanaka Kashiwaya (95)
  - Flat

- Chiral $p$-wave
  - $p_x + i p_y$-wave
  - Tanaka Kashiwaya (97)
  - Sigrist Honerkamp (98)
  - Chiral

- NCS (Helical)
  - $p + s$-wave
  - Iniotakis (07)
  - Eschrig (08)
  - Tanaka (09)
  - Helical
Superconducting Materials where zero bias conductance peak by ABS is observed

YBa$_2$CuO$_{7-\delta}$ (Geerk, Kashiwaya, Iguchi, Greene, Yeh, Wei..)
Bi$_2$Sr$_2$CaCu$_2$O$_y$ (Ng, Suzuki, Greene..)
La$_{2-x}$Sr$_x$CuO$_4$ (Iguchi)
La$_{2-x}$Ce$_x$CuO$_4$ (Cheska)
Pr$_{2-x}$Ce$_x$CuO$_4$ (R.L. Greene)
Sr$_2$RuO$_4$ (Mao, Maeno, Laube, Kashiwaya)
κ—(BEDT-TTF)$_2$X, X=Cu[N(CN)$_2$]Br (Ichimura)
UBe$_{13}$ (Ott)
CeCoIn$_5$ (Wei Greene)
PrOs$_4$Sb$_{12}$ (Wei)
PuCoGa$_5$ (Daghero)
Superfluid $^3$He (Okuda, Nomura, Higashitani, Nagai)
Surface Andreev bound state (ABS) up to now

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The presence of ABS is supported by the bulk topological invariant.

ABS in B-phase of superfluid $^3$He

**Cone type ABS**

- Salomaa Volovik (1988)
- Schnyder (2008)
- Roy (2008)
- Nagai (2009)
- Qi (2009)
- Kitaev (2009)
- Volovik (2009)

**BW state (B-phase in $^3$He)**

- full gap superconductor

- no zero-bias peak due to linear dispersion of surface ABS

- perpendicular injection ZES: Buchholtz and Zwicknagle (1981)

- Y. Asano *et al*, PRB '03
Superconductivity in doped topological insulator

topological insulator
……metallic surface states

\[
\text{Cu}_x\text{Bi}_2\text{Se}_3
\]

surface states

L. A. Wray et al., Nature Phys. 10

Y. S. Hor et al., PRL '10
S. Sasaki et al., PRL '11

tunneling conductance
(point contact)

zero-bias peak \( \Rightarrow \) surface states ABS

new type of three-dimensional topological superconductor
(Topological) Andreev bound states (2)

Cone
Superfluid $^3$HeB

Caldera
Doped topological insulator

Okuda Nomura (Review) (12)

Yamakage Yada Sato Tanaka (12)
Summary (1)

(1) Surface Andreev bound state can be interpreted as a topological edge state.

(2) Topological classification corresponding to bulk topological invariant

### Periodic Table of the Topological Materials

<table>
<thead>
<tr>
<th>Cartan label</th>
<th>T</th>
<th>C</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (unitary)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>AI (orthogonal)</td>
<td>+1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>AII (symplectic)</td>
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<td>0</td>
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<tr>
<td>AIII (ch. unit.)</td>
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</tr>
<tr>
<td>BDI (ch. orth.)</td>
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<td>+1</td>
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</tr>
<tr>
<td>CII (ch. sympl.)</td>
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<td>−1</td>
<td>1</td>
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<tr>
<td>D (BdG)</td>
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<td>0</td>
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<tr>
<td>C (BdG)</td>
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<td>0</td>
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<tr>
<td>DIII (BdG)</td>
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<td>+1</td>
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<tr>
<td>CI (BdG)</td>
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**Dimension**

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<tr>
<td>A</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>AII</td>
<td></td>
<td>0</td>
<td>Z</td>
</tr>
<tr>
<td>AIII</td>
<td>Z</td>
<td>0</td>
<td>Z</td>
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</tbody>
</table>

**Real case**

<table>
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<th>Cartan label</th>
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<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>AI</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>BDI</td>
<td>Z</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>Z</td>
<td>0</td>
<td>Z</td>
</tr>
<tr>
<td>DIII</td>
<td>Z</td>
<td>Z</td>
<td>Z</td>
</tr>
<tr>
<td>AII</td>
<td></td>
<td>Z</td>
<td>Z</td>
</tr>
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<td>Z</td>
<td>Z</td>
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<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>Z</td>
</tr>
</tbody>
</table>

**Ryu et al., NJP 12 (2010) 065005.**

- **IQHE**
- **chiral-p** (Sr$_2$RuO$_4$, $^3$He-A)
- **$^3$He-B**
- **Z$_2$ TI**
- **NCS (helical) Rashba**
- **d(p)-wave fixed $k_y$, flat**
- **chiral-d**
- **QSHE**

Contents of our talk

(1) Surface Andreev bound state up to now
(2) Majorana fermion (mode)
(3) Fabrication of Majorana Fermion at Nanowire and Interface
(4) Superconducting doped topological insulator
Majorana relation

(1) Spinless fermion

Fully polarized, spin degree of freedom is half

\[ \gamma_k = (u_k c_k + v_k c_{-k}^\dagger) \rightarrow \gamma_k = \gamma_{-k}^\dagger \]

\[ u_k = v_{-k}^* \]

(2) Spinfull fermion (Equal spin pairing)

\[ \gamma_{k,\nu} = (u_{k,\nu} c_{k,\nu} + v_{k,\nu} c_{-k,\nu}^\dagger) \rightarrow \gamma_{k,\nu} = \gamma_{-k,\nu}^\dagger \]

\[ u_{k,\nu} = v_{-k,\nu}^* \]

\[ \nu = \uparrow, \downarrow \]

(3) Spinfull fermion (Opposite spin pairing)

\[ \gamma_{k,\nu} = (u_{k,\nu} c_{k,\nu} + v_{k,-\nu} c_{-k,-\nu}^\dagger) \rightarrow \gamma_{k,\nu} = \gamma_{-k,-\nu}^\dagger \]

\[ u_{k,\nu} = v_{-k,\nu}^* \]

\[ \nu = \uparrow, \downarrow \]

(3) It is not called Majorana Fermion for spin-singlet pairing.
## (Topological) Surface Andreev Bound State

<table>
<thead>
<tr>
<th>Type of dispersion</th>
<th>TRS</th>
<th>Examples</th>
<th>Type of Majorana mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat</td>
<td>○</td>
<td>Cuprate</td>
<td>Not Majorana</td>
</tr>
<tr>
<td>Chiral</td>
<td>×</td>
<td>Sr$_2$RuO$_4$ (without orbital effect) $^3$He A phase</td>
<td>double chiral Majorana</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$^3$He B phase</td>
<td>helical Majorana</td>
</tr>
<tr>
<td>Helical</td>
<td>○</td>
<td>$s+ p$–wave (NCS superconductor) $\Delta_p &gt; \Delta_\varepsilon$</td>
<td>helical Majorana</td>
</tr>
<tr>
<td>Cone</td>
<td>○</td>
<td>$^3$He B phase</td>
<td>2D helical Majorana</td>
</tr>
<tr>
<td>Caldera</td>
<td>○</td>
<td>Cu$_x$Bi$_2$Se$_3$</td>
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</tbody>
</table>

Sato, Kashiwaya, Maeno (固体物理)  
Tanaka, Sato, Nagaosa (JPSJ Review)
(1) Majorana fermion (mode) is a special type of Andreev bound state.

(2) Chiral Majorana (TRS broken)
    Helical Majorana (TRS is not broken)

Future application for quantum computation

Contents of our talk

(1) Surface Andreev bound state up to now
(2) Majorana fermion (mode)
(3) Fabrication of Majorana Fermion in Nanowire and Interface
(4) Majorana fermion in the nanowire
Condition of the formation of Andreev bound state (Majorana fermion)

Sign Change of the Pair potential on the Fermi surface (orbital degree of freedom)
Is it possible to generate Andreev bound state (Majorana fermions) based on the conventional spin-singlet s-wave superconductor?

Yes

Spin-orbit interaction is a key ingredient
Role of spin-orbit (SO) coupling

direction of spin is quenched

Two Fermi surfaces

Without Zeeman

With Zeeman

Only one Fermi surface
Creation of Majorana fermion (spinless) based on the Nano-structures

- Nano-wire, Interface of oxides, surface
- Mainly proximity coupling to conventional spin singlet s-wave pairing

- Spin-orbit coupling
- Reduction of the electron’s spin degree of freedom
  (Exchange field, hybridization of bands)
Chiral Majorana Fermion (mode) (1)

**Spinless, TRS broken**

**zero-dimensional Majorana**

s-wave superconductor
+ InSb Nanowire

Magnetic field

Kitaev(01); Lutchyn ,Sau, Das Sarma(10), Oreg(10)
Beenakker(11)

**one-dimensional Majorana (Chiral )**

s-wave superconductor
+ ferromagnet + topological insulator (TI)

Fu Kane(08), Beenakker (09)
Tanaka & Nagaosa(09), Law Lee (09)

s-wave superconductor
+ ferromagnet + 2D electron gas

Rashba superconductor

Sato 09; Sau 10; Alicea 10, Lutchyn 10 Yamakage 11
Chiral Majorana Fermion
(Special type of Andreev bound state)

Broken Time reversal symmetry!!

\[ \gamma = \gamma^\dagger \]

1d Chiral Majorana fermion
Chiral Majorana mode (CMM) is tunable by material parameters of TI.

1. Normalized conductance has a peak at zero voltage similar to chiral $p$-wave case.

2. $m_z d/v_F = 1$, $m_y/m_z = 0$

3. $\mu/m_z = 1$

4. $\mu/m_z = 0.5$

5. $\mu/m_z = 2$

Tanaka, Yokoyama, Nagaosa, PRL 103, 107002 (2009)
Helical Majorana Fermion (Special type of Andreev bound state)

Time reversal symmetry

$$\gamma_\nu = \gamma_\nu^\dagger$$

1d helical Majorana fermion
Helical Majorana fermion
Using Interface superconductivity

interface of transition metal oxides
- 2d electron gas
- superconductivity
- tunable Rashba SOI

Ohtomo & Hwang Nature 2004
Reyren et al. Science 2007
Caviglia et al. PRL 2010

One-dimensional Majorana (Helical)

\[ \Delta_1 = -\Delta_2 \]

Intra-layer pairing with different sign

Nakosai, Tanaka Nagaosa, PRL(2012)
Model construction

kinetic Hamiltonian

\[ \mathcal{H}_0(\mathbf{k}) = \frac{k^2}{2m} - \varepsilon \sigma_x + \alpha (k_x s_y - k_y s_x) \sigma_z \]

hybridize : transfer \hspace{1cm} \text{SOI} : Rashba SOI

s : spin \hspace{1cm} \sigma : layer

electron density-density interaction

\[ \mathcal{H}_{\text{int}}(\mathbf{x}) = -U (n_1^2(\mathbf{x}) + n_2^2(\mathbf{x})) - 2V n_1(\mathbf{x}) n_2(\mathbf{x}) \]

intra-layer \hspace{1cm} \text{inter-layer}

Bogoliubov de-Gennes Hamiltonian

\[ \mathcal{H}_{\text{BdG}} = \begin{pmatrix} \mathcal{H}_0 - \mu & \Delta \\ \Delta & -\mathcal{H}_0 + \mu \end{pmatrix} \]

cf. Fu and Berg  PRL 2010

S. Nakosai , Y. Tanaka and N. Nagaosa PRL(2012)
Pairing potentials

determination of pairing potentials (= mean-field )

- spin and layer degrees of freedom
- short range int. $\rightarrow$ k-indep. pairing amplitudes
- lattice symmetry $D_{4h}$

parity under an inversion operation

<table>
<thead>
<tr>
<th>irreps</th>
<th>matrix</th>
<th>symmetry</th>
<th>I node</th>
<th>spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\Delta}_1$</td>
<td>$A_{1g}$</td>
<td>$I \sigma_x$</td>
<td>$\langle c_{1\uparrow} c_{1\downarrow} \rangle = \langle c_{2\uparrow} c_{2\downarrow} \rangle = \Delta_1/2$</td>
<td>$\uparrow$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\langle c_{1\uparrow} c_{2\downarrow} \rangle = - \langle c_{1\downarrow} c_{2\uparrow} \rangle = \Delta_1'/2$</td>
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</tr>
<tr>
<td>$\hat{\Delta}_2$</td>
<td>$A_{1u}$</td>
<td>$s_z \sigma_y$</td>
<td>$\langle c_{1\uparrow} c_{2\downarrow} \rangle = \langle c_{1\downarrow} c_{2\uparrow} \rangle = \Delta_2/2$</td>
<td>$\downarrow$</td>
</tr>
<tr>
<td>$\hat{\Delta}_3$</td>
<td>$A_{2u}$</td>
<td>$\sigma_z$</td>
<td>$\langle c_{1\uparrow} c_{1\downarrow} \rangle = - \langle c_{2\uparrow} c_{2\downarrow} \rangle = \Delta_3/2$</td>
<td>$\downarrow$</td>
</tr>
<tr>
<td>$\hat{\Delta}_4$</td>
<td>$E_u$</td>
<td>$\begin{pmatrix} s_x \sigma_y \ s_y \sigma_y \end{pmatrix}$</td>
<td>$\langle c_{1\uparrow} c_{2\uparrow} \rangle = \langle c_{1\downarrow} c_{2\downarrow} \rangle = \Delta_4/2$</td>
<td>$\downarrow$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\langle c_{1\uparrow} c_{2\downarrow} \rangle = - \langle c_{1\downarrow} c_{2\uparrow} \rangle = \Delta_4'/2$</td>
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</tr>
</tbody>
</table>

S. Nakosai, Y. Tanaka and N. Nagaosa PRL(2012)
Topological SC?

We set the Fermi energy within the hybridization gap.

1. [Fermi level]  **OK**

2. [odd parity pairing potential]  **OK**

NOTE:
- Pairing amplitudes for $\Delta_2$ and $\Delta_3$ are proportional to $\alpha$.

SOI-induced SC phases

Unconventional SC phase appears in a feasible parameter region.

- intra-layer: attractive (phonon mechanism)
- inter-layer: repulsive (Coulomb interaction)
Spectrum and Edge states

For each Fermi level

**case: A**  
Bulk state  
trivial

**case: B**  
non-trivial

**case: C**  
trivial

\[ \Delta_3 \]

Fermi energy

helical Majorana edge states

S. Nakosai, Y. Tanaka and N. Nagaosa PRL(2012)
Summary (3)

(1) There are many interesting systems where spinless Majorana fermion can be generated from conventional s-wave pairing.

(2) Fabrication of Majorana Fermion in nanowire, surface and interface is a new direction of condensed matter physics.


Majorana Fermion with flat dispersion

Unconventional (anisotropic) pairing at the interface

One-dimensional Majorana (Flat)

Time reversal symmetry

Interface superconductivity
Possible d+p-wave pairing

Reyren et al (2007)
Yada Onari (2009)

K. Yada, M. Sato
PRB 83 064505 (2011)
Contents of our talk

(1) Surface Andreev bound state up to now
(2) Majorana fermion
(3) Fabrication of Majorana Fermion at Nanowire and Interface
(4) Superconducting doped topological insulator
Doped topological insulator

Topological insulator \( \text{Cu}_x\text{Bi}_2\text{Se}_3 \) \( \rightarrow \) Superconducting topological insulator \( \text{Cu}_x\text{Bi}_2\text{Se}_3 \)

\[ \text{Y. S. Hor et al, PRL '10} \]

\[ \text{S. Sasaki et al, PRL '11} \]

\[ \Rightarrow \text{Andreev bound state} \]

\[ \text{3d topological superconductor} \]

L. A. Wray et al, Nature Phys. 10
Crystal structure of topological insulator

Zhang, et al., 09
Effective Hamiltonian of topological insulator

\[ H_{\text{TI}}(k) = m(k) \sigma_x + v_z k_z \sigma_y + v \sigma_z (k_x s_y - k_y s_x) \]

Zhang, et al., 09
Model of superconducting topological insulator

\[ H_{\text{TI}}(k) = m(k)\sigma_x + v_z k_z \sigma_y + v \sigma_z (k_x s_y - k_y s_x) \]

\[ H_{\text{STI}}(k) = \begin{pmatrix} H_{\text{TI}}(k) - \mu & \Delta_{\text{STI}} \sigma_y s_z \\ \Delta_{\text{STI}} \sigma_y s_z & -H_{\text{TI}}(k) + \mu \end{pmatrix} \]

\[ k_i \to \sin k_i, \]
\[ k_i^2 \to 2(1 - \cos k_i). \]

mapping to lattice model

Fu and Berg, 10
Emergence of new type of Andreev Bound State

$\Delta_2$

(b) caldera

transition point:
group velocity = 0

$\mu = \frac{v_z^2}{m_1}$

L. Hao and T. K. Lee, PRB ’11  
T. H. Hsieh and L. Fu, PRL ’12  
Tunneling conductance in superconductor realized in doped topological insulator

structural transition
- group velocity $\sim$ zero
- large surface DOS

$eV/\Delta$

zero-bias peak even in the full gap case

Current phase relation in Josephson junctions

Spin structure

singlet/singlet :  \[ J(\varphi) \sim \sin \varphi \]
singlet/triplet :  \[ J(\varphi) \sim \sin 2\varphi \]

J. A. Pals, 76

Free energy

\[ F(\varphi) = \frac{\hbar}{2e} \int^{\varphi} d\varphi' J(\varphi') \]

sin2\(\varphi\) \rightarrow\) quantum two level system
Anomalous current phase relation
\[ \sin 2\varphi \]

spin-singlet / spin-triplet

d-wave / s-wave

The effect of spin-orbit coupling

\[ J(\varphi) \sim \sin 2\varphi \rightarrow \cos \varphi \]

Asano, et al., PRB, 03

Robust in superconducting topological insulator

\[ J(\varphi) \sim \sin 2\varphi \]

Yamakage, et al., arXiv1208.5306
Mirror symmetry and current phase relation

Mirrorsymmetry and current phase relation

Even parity

Mirror inversion: \((x, y, z) \rightarrow (-x, y, z)\)

If one of the pair potential has an odd parity as mirror inversion operation \([\phi \rightarrow \phi + \pi]\)

\[J(\phi) = J(\phi + \pi) \quad \Rightarrow \quad J(\phi) \sim \sin 2\phi\]

If spin-orbit coupling has an odd-parity by mirror inversion

\[J(\phi) \sim \sin 2\phi \quad \text{Yip, et al., PRB, 90}\]

Pair potential in superconducting Topological insulator has an odd-parity by mirror inversion

\[J(\phi) \sim \sin 2\phi \quad \text{robust feature}\]

Fu and Berg, 10
Hsieh and Fu, 12
Yamakage, et al., 12
Current phase relation in superconducting topolgical insulator

Yamakage, et al, arXiv1208.5306

\[ J(\varphi) \sim \sin 2\varphi \]
Josephson current depending on spin-helicity

Hao and Lee, 11
Hsieh and Fu, 12
Yamakage, et al, 12

Surface Andreev bound state of superconducting topological insulator

Cone-type ABS

H\_surf(k\_||) = h\_s v\_surf(k \times s)\_z

Caldera-type ABS
different helicity
Spin-helicity dependent Josephson current

Hao and Lee, 11
Hsieh and Fu, 12
Yamakage, et al, 12

Yamakage, et al, arXiv1208.5306
Summary (4)

(1) New type of Andreev bound state (surface state) is expected in superconducting doped topological insulator.

(2) Josephson current has an anomalous current phase relation.

(3) Anomalous temperature dependence due to spin helicity
Summary

Josephson current in superconducting topological insulator

- symmetry protected $J(\varphi) \sim \sin 2\varphi$
- Josephson current depending on the spin-helicity of Andreev bound state (Majorana fermion)