Argonne, May 2010

# Magneto-transport of 2D Lattice Bosons

Assa Auerbach, Technion, Israel

1. Bose Hubbard Model: Mott - Superfluid transition
2. Vortex dynamics (mass) of Hard Core Bosons
3. Field tuned SIT – Quantum Vortex Liquid
4. Hall conductivity, sign changes.
5. "Bad Metal" Resistivity.

AA, D. P. Arovas and Sankalpa Ghosh, *Phys. Rev. B74, 64511, (2006).*
Netanel H. Lindner and AA, *Phys. Rev. B81, 054512, (2010).*
Netanel H. Lindner AA and D. P. Arovas, *Phys. Rev. Lett. 101, 070403 (2009), PRB 82, (2010).*

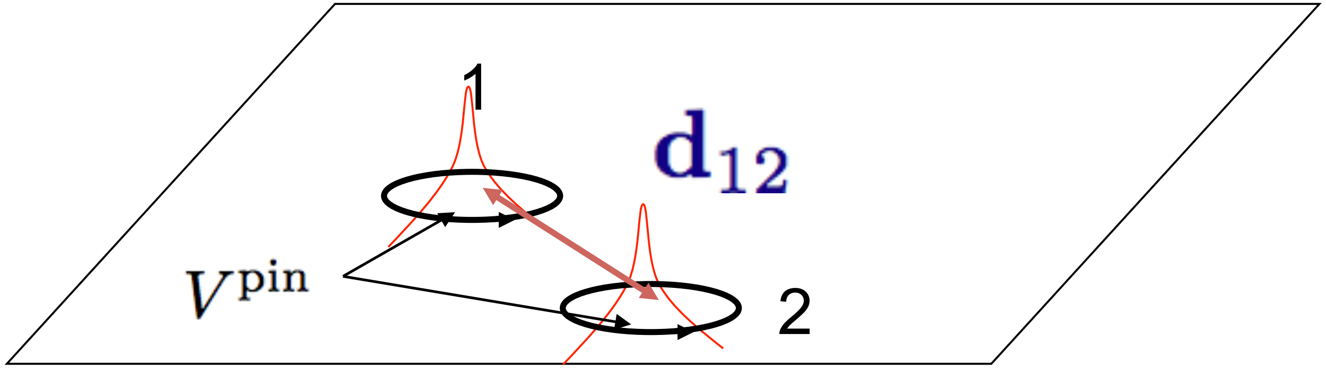
# Continuum bosons vs Lattice bosons

| Gross-Pitaevskii field theory | Quantum XY model |
| --- | --- |
| $^4He$ | JJ arrays, Optical lattices |
| **T=0 Order Parameter** | |
| $\langle b^\dagger \rangle = \sqrt{n}$ | $\langle b^\dagger \rangle \neq \sqrt{n}$ |
| **T=0 Superfluid density** | |
| $\rho_s = \frac{\hbar^2 n}{m_b}$ | $\rho_s \not\propto n$ |
| **Transition temperature** | |
| $T_c = \frac{2\pi\hbar^2}{m}\left(\frac{n}{\zeta_{3/2}(1)}\right)^{2/3}$ | $T_c \propto \rho_s(T=0)$ |

Galilean invariance

# Vortex Tunneling in Continuum BEC

vortex motion = scattering of supercurrent $\Delta\mathbf{P} = hn\hat{\mathbf{z}} \times \Delta\mathbf{d}_{ij}$

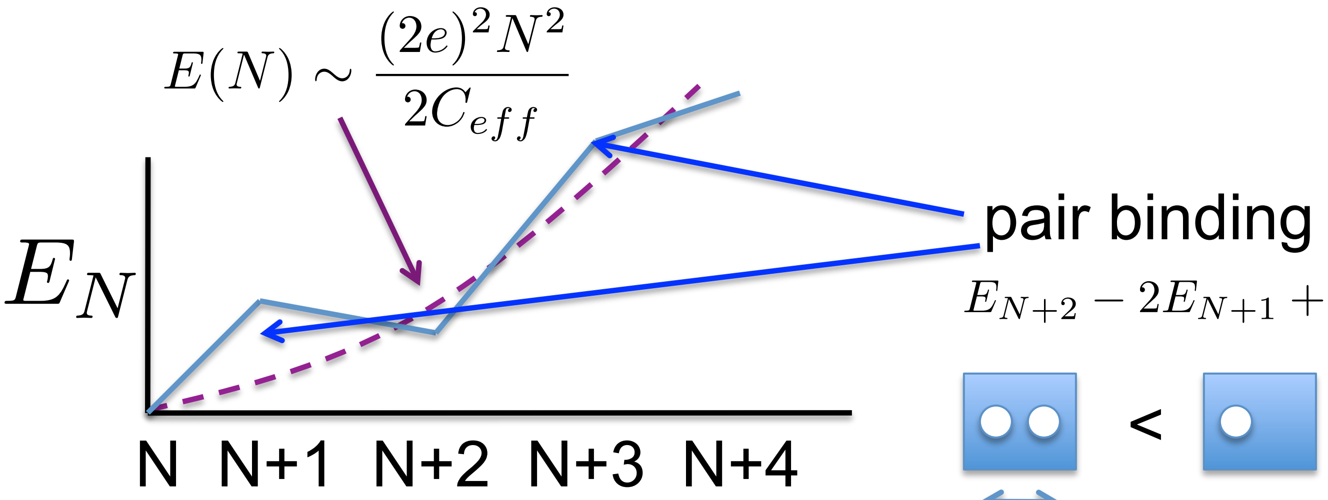



AA, D.P. Arovas, S. Ghosh, Phys Rev B 74, 2006

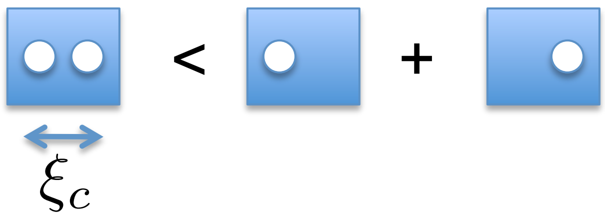
$$t_v = V^{\rm pin} \exp\left(-\frac{\pi}{2} n_0 d_{ij}^2\right)\left(1 + \mathcal{O}(g)\right)$$

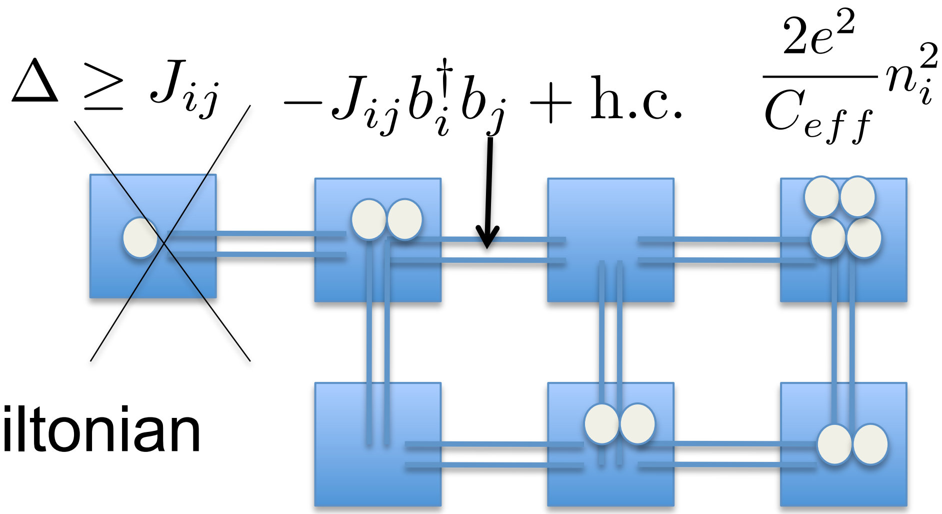
1. Low T resistivity depends on vortex tunneling rate.
2. Lattice potential enhances vortex mobility!

# Low Capacitance Josephson Arrays




$$E_{N+2} - 2E_{N+1} + E_N = \Delta < 0$$



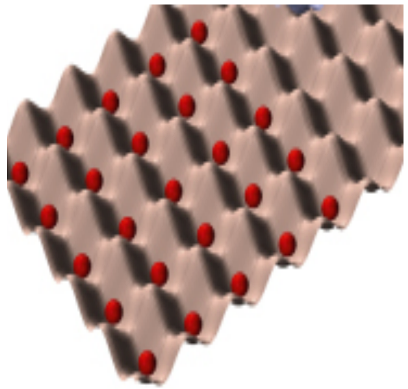




Low energy effective Hamiltonian

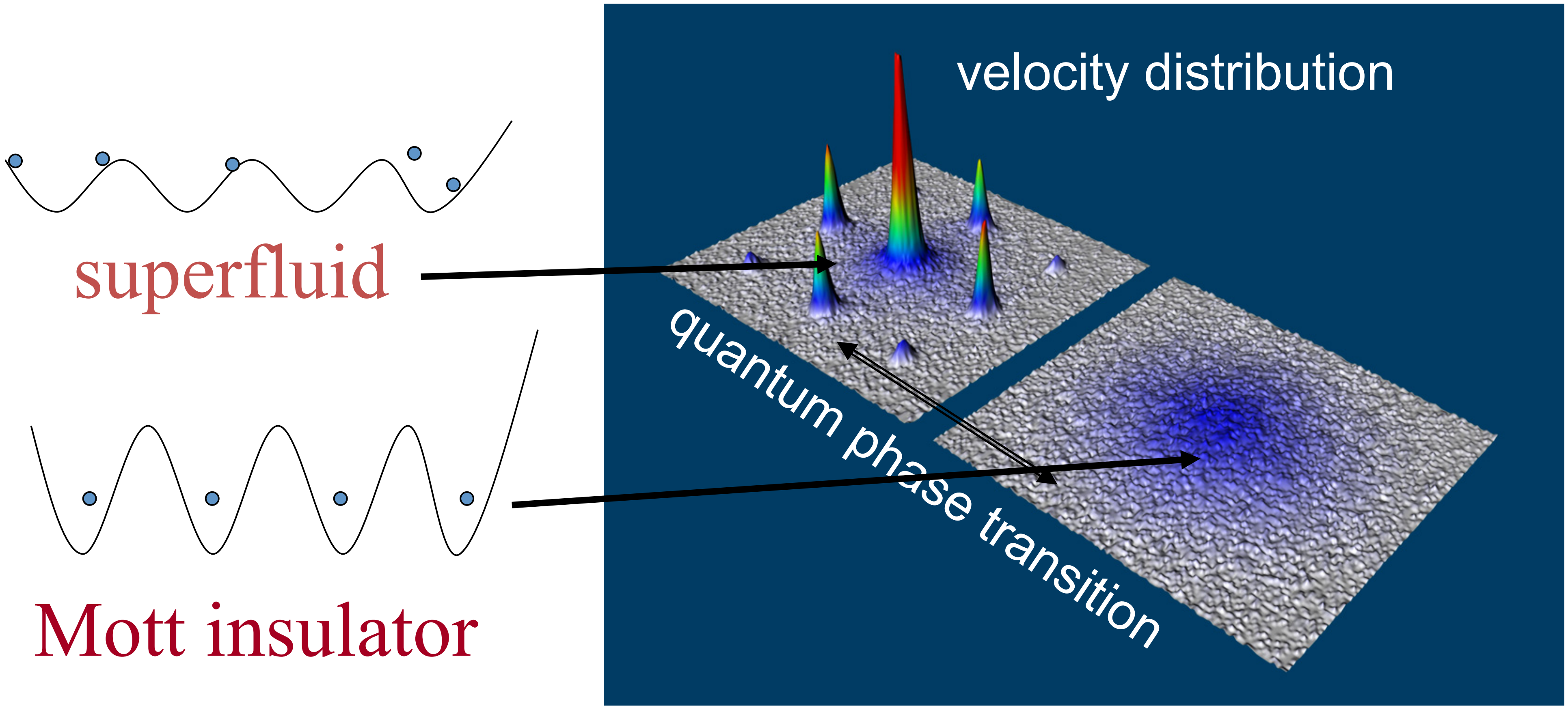
➔ Bose Hubbard model

# Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms

Nature (2002)



Markus Greiner*, Olaf Mandel*, Tilman Esslinger†, Theodor W. Hänsch* & Immanuel Bloch*

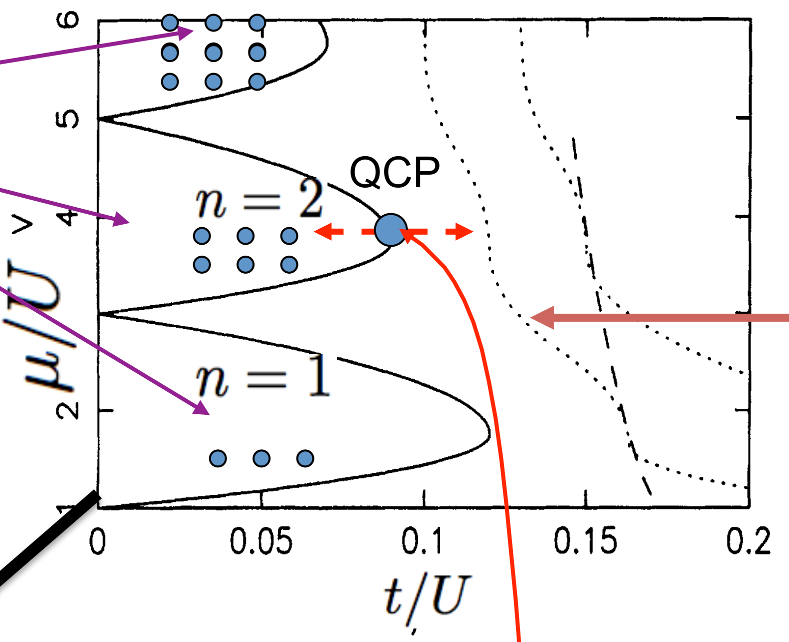


# Bose Hubbard Model

$$\mathcal{H} = -t\sum_{ij} a_i^\dagger a_j + U\sum n_i{}^2 - \mu\sum_i n_i$$

Mott insulators

incompressible



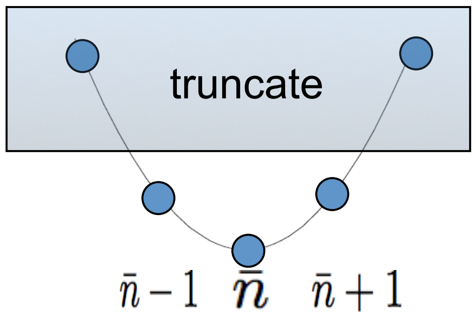

Zimanyi et. al (1994)

superfluid

phase order

B

S=1, O(2) relativistic GP




Introduction of vortices into the superfluid --
vortex lattice melting

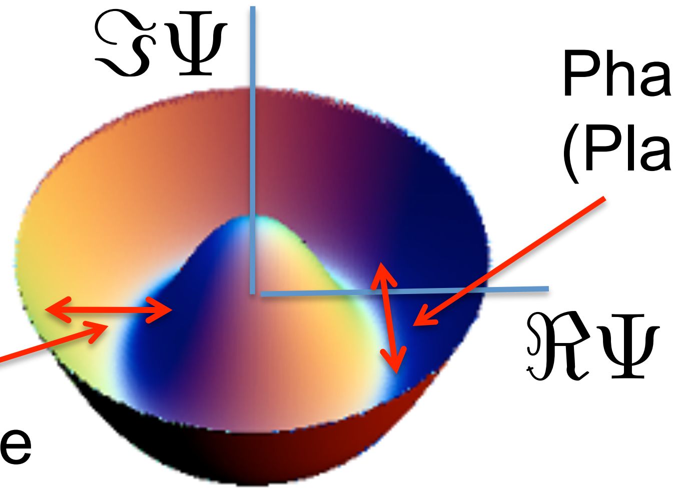
# flu near QCP

order parameter field $\Psi(x) = n_x(x) + i n_y(x)$

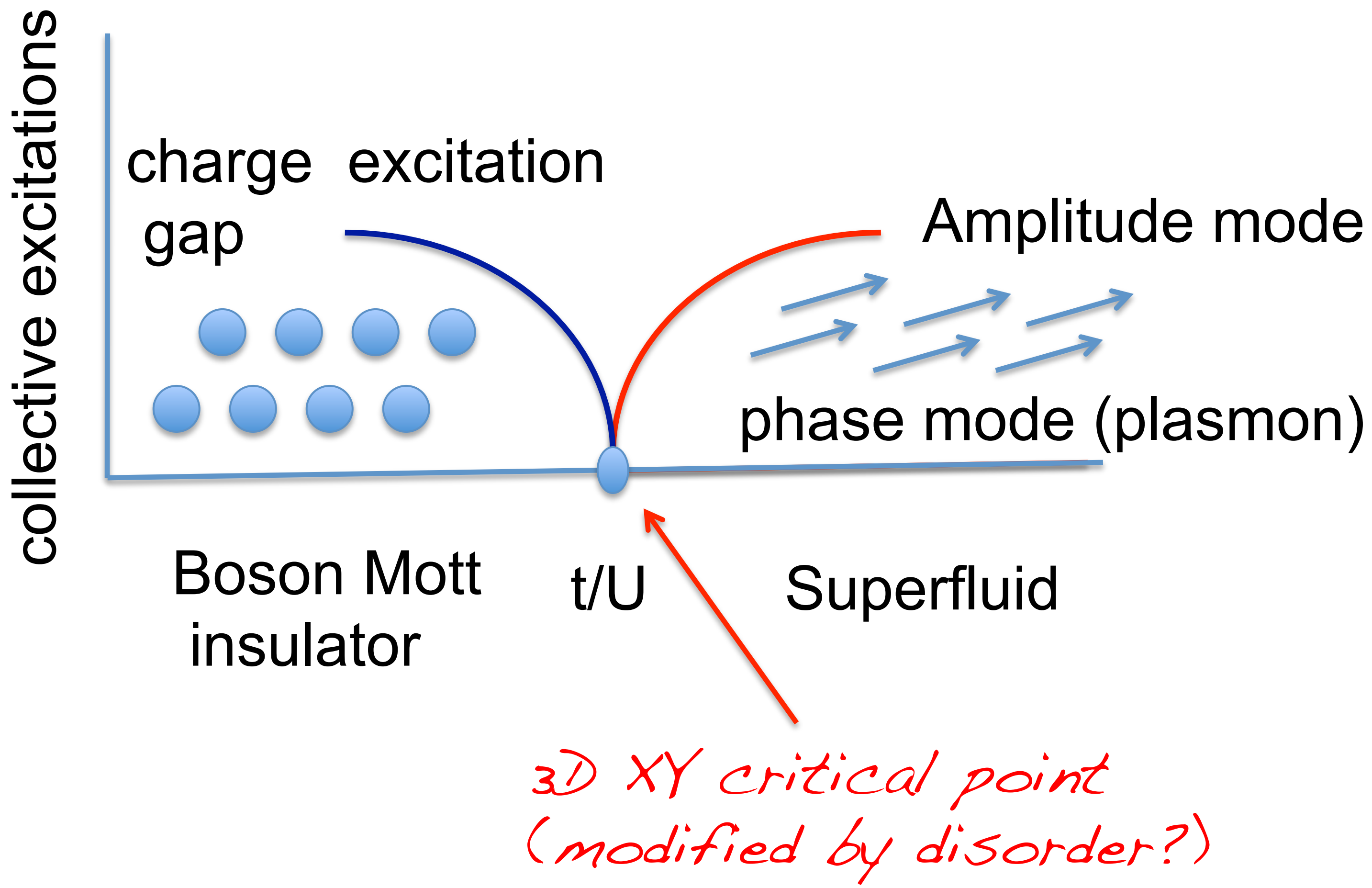
Relativistic Gross-Pitaevskii = O(2) field theory (Higgs)

$$S_{RGP} = \int d^2x \int d\tau \frac{1}{2}|\dot{\Psi}|^2 + \frac{\rho_s}{2\Delta^2}|\nabla\Psi|^2 - \frac{m}{8\Delta^2}\left(|\Psi|^2 - \Delta^2\right)^2$$

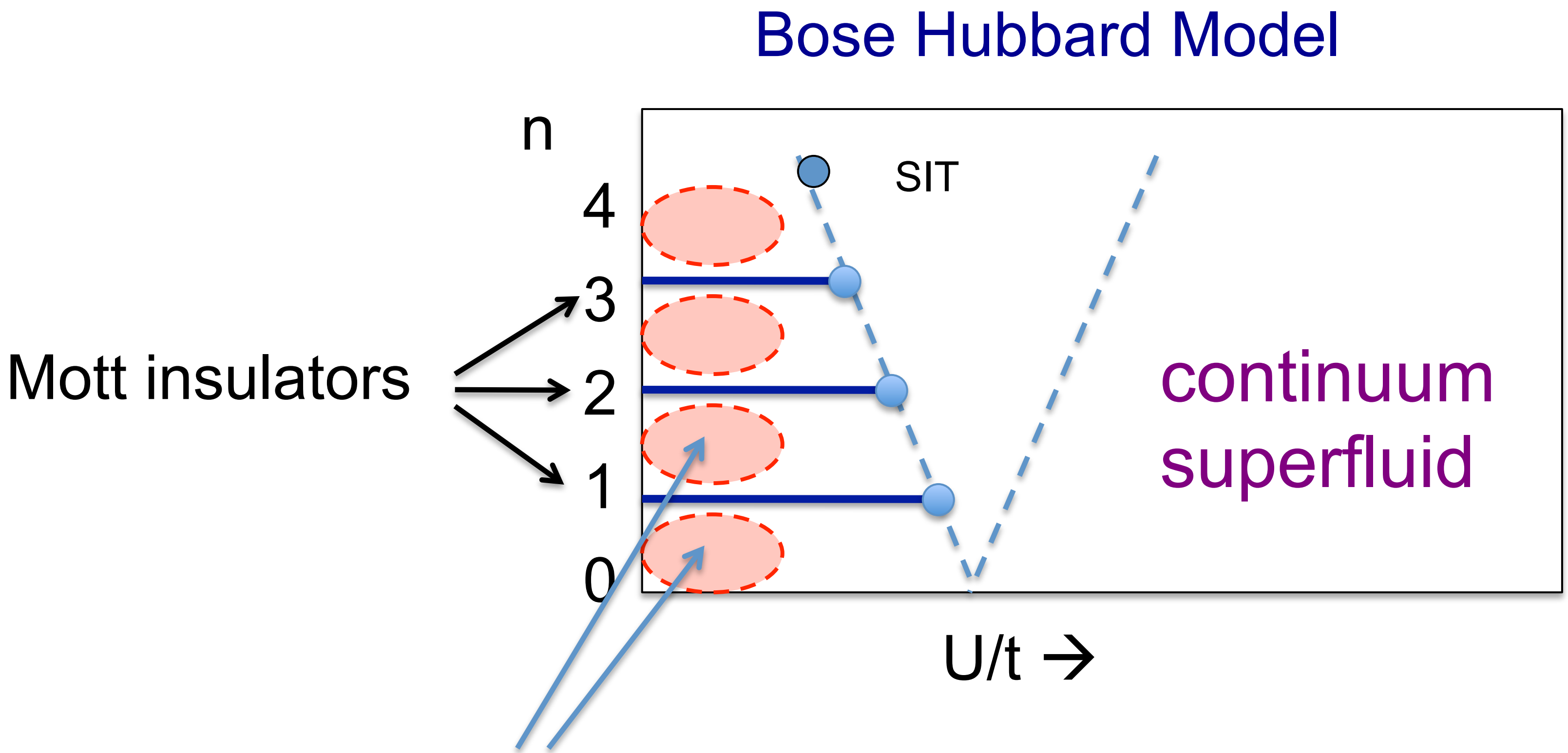
Altman AA, , PRL 89, (2002)



# QCP near Mott phase

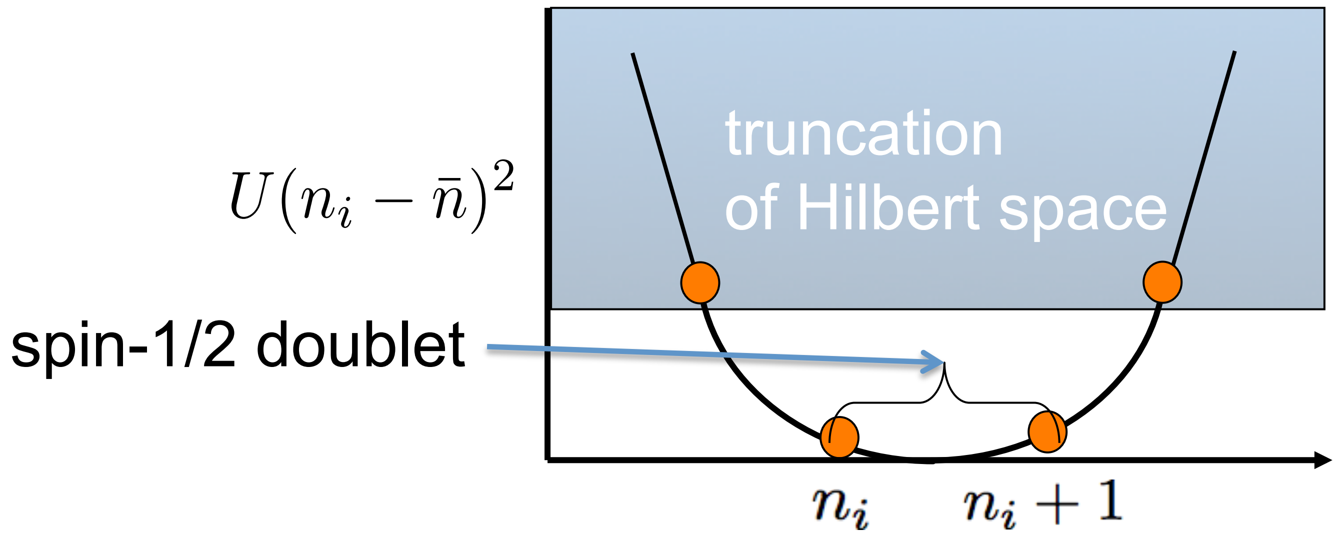


# Hard-Core Regimes




Hard Core Limit: $i \leq n_i \leq i+1$

# Hard Core Bosons = spin half representation




$$c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} = \tilde{a}^\dagger_i = \mathcal{P}\, a^\dagger_i\, \mathcal{P} = S^+_i$$

$$c_{i\downarrow} c_{i\uparrow} = \tilde{a}_i = \mathcal{P}\, a_i\, \mathcal{P} = S^-_i$$

$$n_i = \tilde{a}^\dagger_i \tilde{a}_i = S^z_i + \tfrac{1}{2}.$$

HCB are different than free bosons $\quad [\tilde{a}_i\,,\,\tilde{a}^\dagger_j] = (1 - 2n_i)\,\delta_{ij}$

*especially around half filling (n=1/2)*

# Effective Hamiltonian

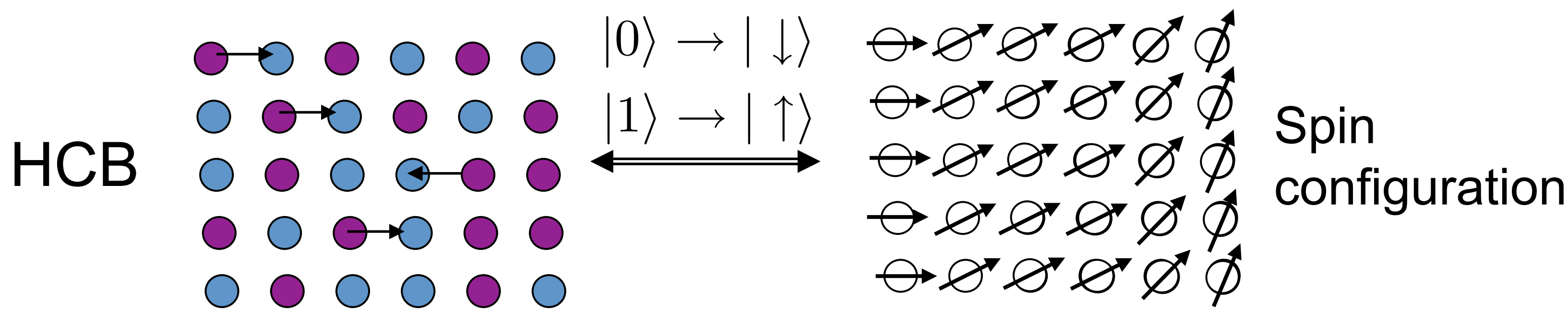
Gauged Spin ½ XXZ model

$$\mathcal{H} = -2J \sum_{\langle ij \rangle} \left( e^{iqA_{ij}} S_i^+ S_j^- + e^{-iqA_{ij}} S_i^- S_j^+ \right)$$

$$+4V \sum_{\langle i,j \rangle} S_i^z S_j^z - \mu \sum_i (S_i^z + \tfrac{1}{2}).$$

$$N_b = \sum_i \langle S_i^z \rangle + \frac{1}{2} \qquad \langle b_i^\dagger \rangle = \langle S_i^x \rangle + i \langle S_i^y \rangle$$

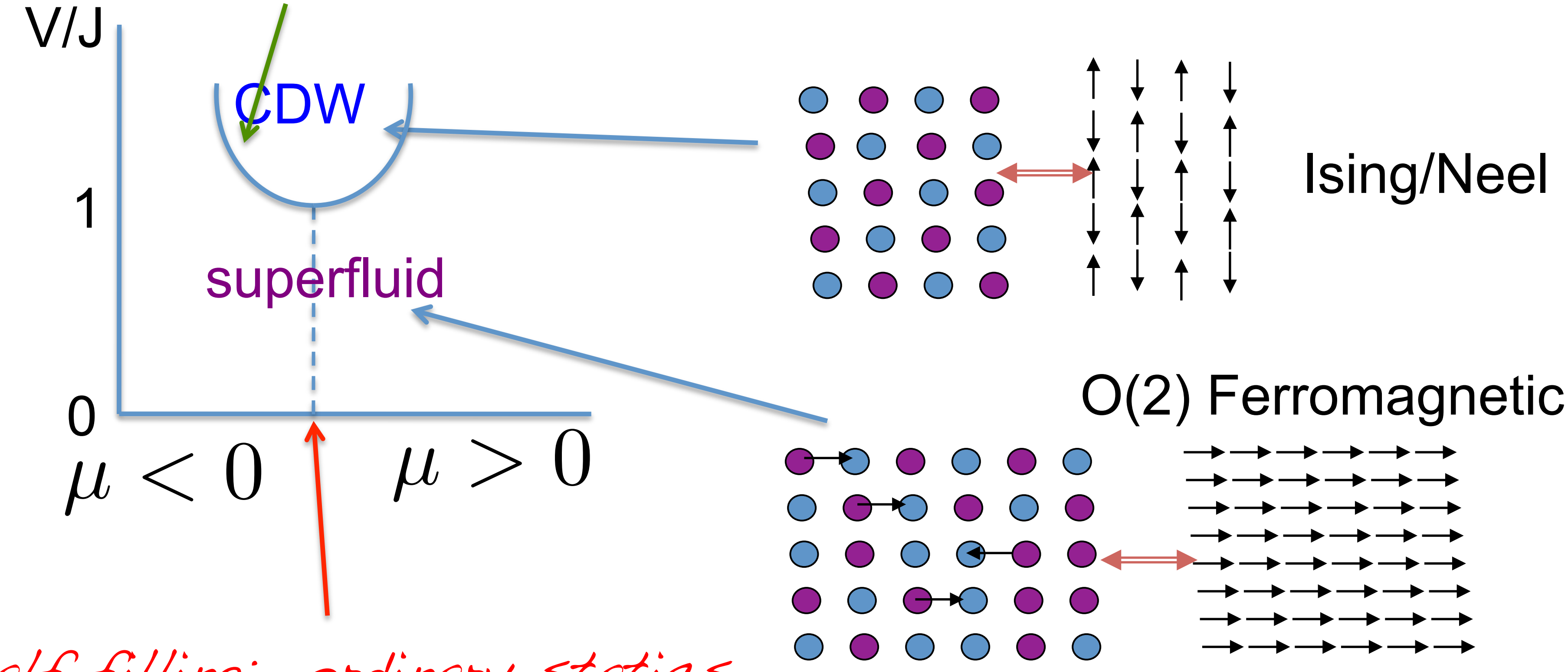
*Solved by techniques of Quantum Magnetism!*



# Ground states of Hard Core Bosons

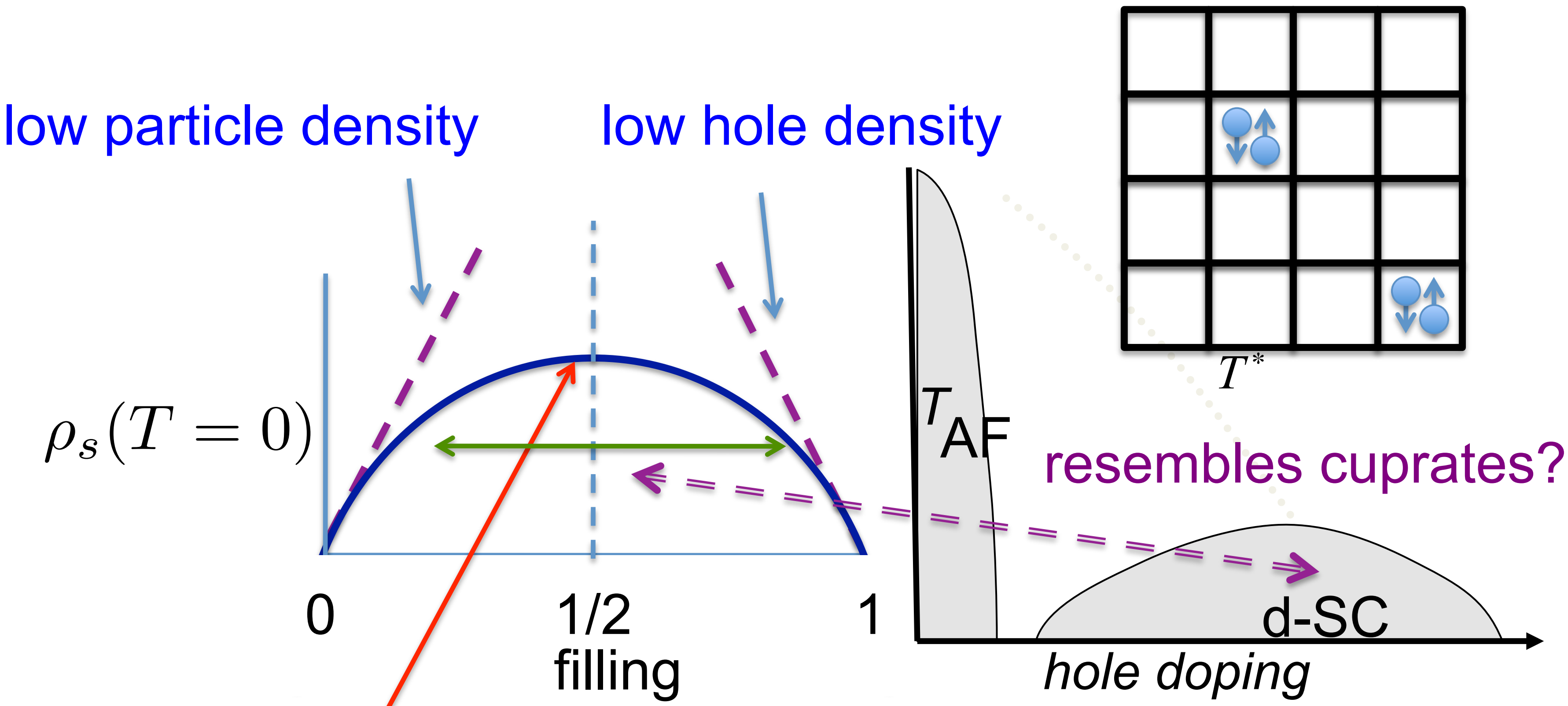
Mean field theory $\quad |\Psi^{var}\rangle = \prod_i |\vec{S}(x_i)\rangle$

"Spin-Flop" transition (Fisher-Liu)




*half filling: ordinary statics,*
*- interesting dynamics!*

# Superfluid density versus charge density



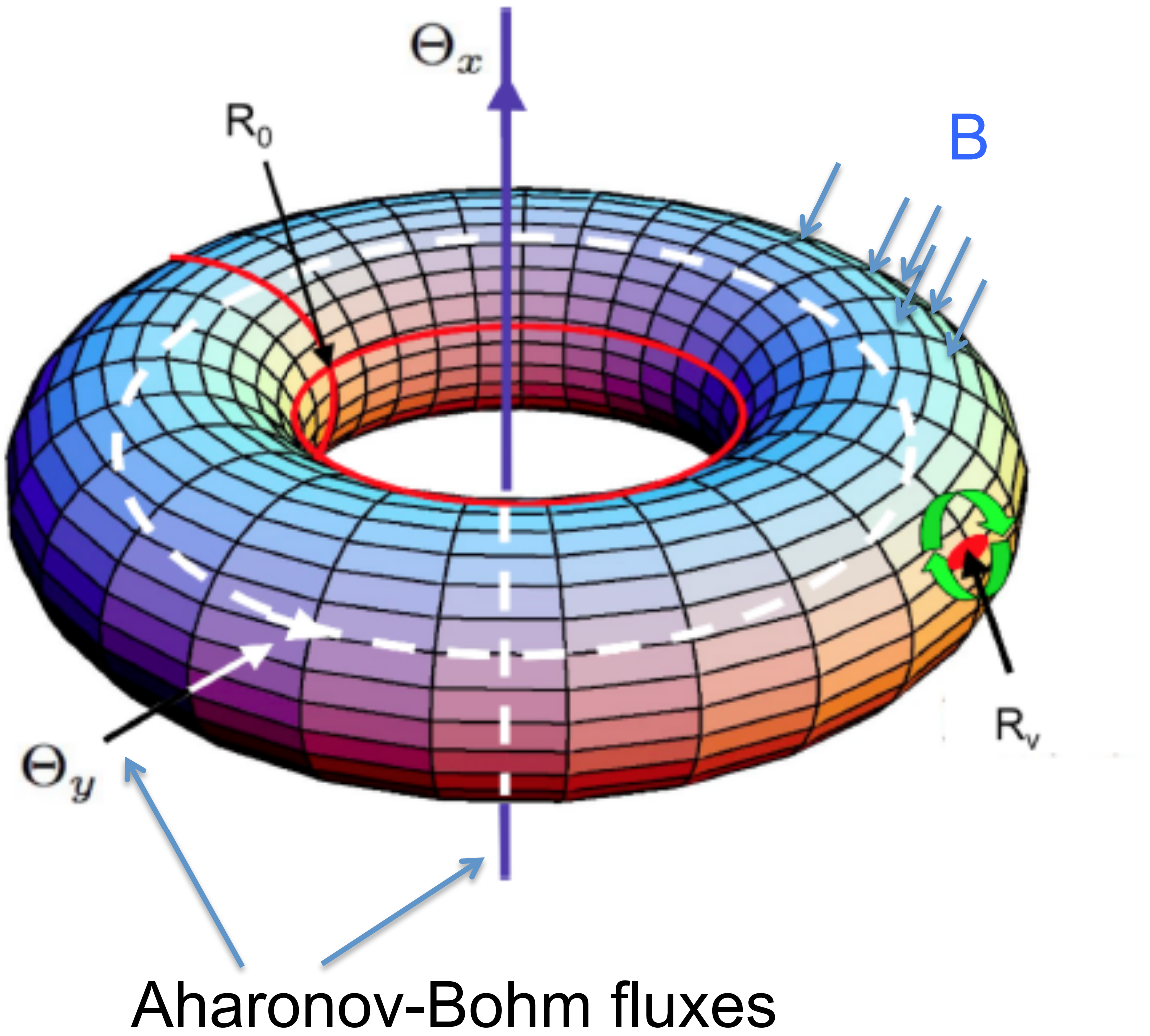

Half filling: "Optimal Doping" (highest Tc)
STRONGEST LATTICE EFFECTS

particle hole symmetry:

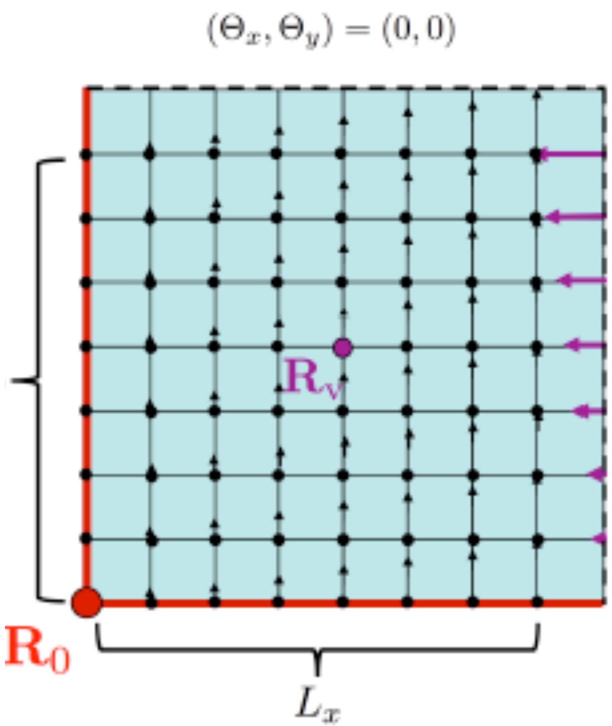
$$\sigma_{xx}(n) = \sigma_{xx}(1-n)$$
$$\sigma_{xy}(n) = -\sigma_{xy}(1-n)$$

# Computing Vortex mass on the Gauged Torus

Lindner. AA Arovas, PRL (2009)
Phys. Rev. B 82 (2010)

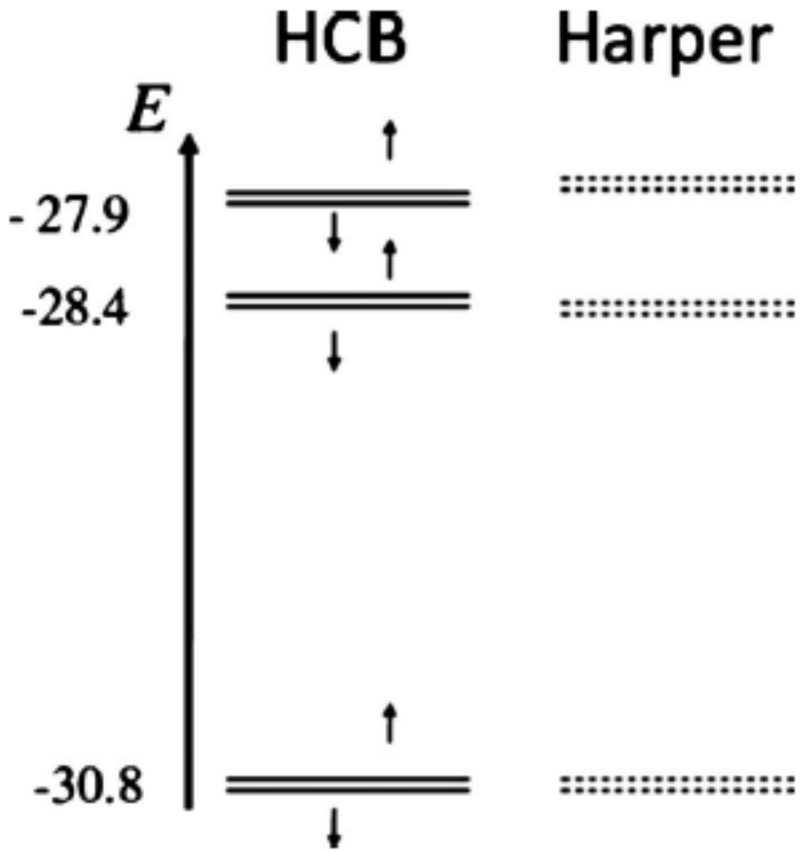


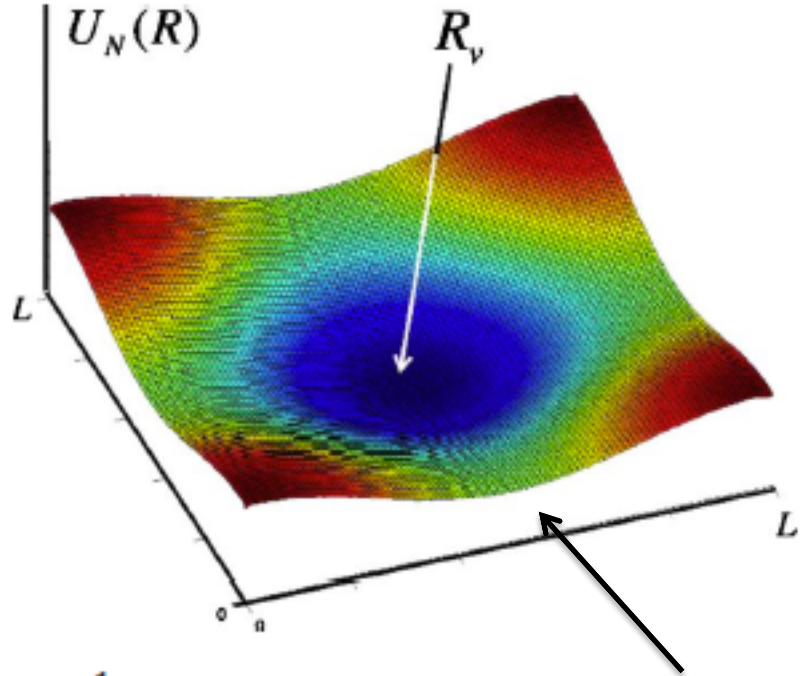

gauge field



# Fitting the vortex hopping rate

Lindner. AA Arovas, PRL (2009)
Phys. Rev. B 82 (2010)




$$H_V^N = -\frac{1}{2} t_V \sum_{R,\eta} \left(e^{iA_{R,R+\eta}} b_R^\dagger b_{R+\eta} + \text{H.c.}\right) + U_N(R) b_R^\dagger b_R$$

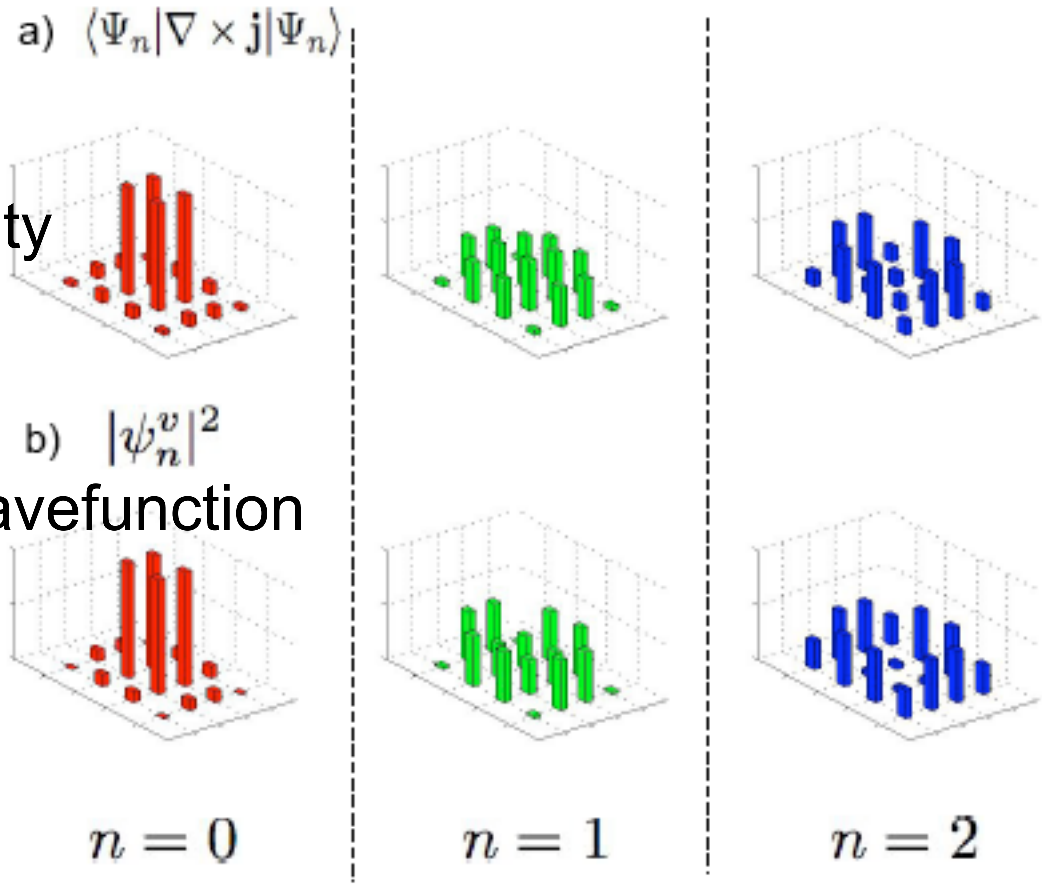
Fit yields: $t_V(n,0) = 4J - 50.4J\left(n - \frac{1}{2}\right)^2 + 5056J\left(n - \frac{1}{2}\right)^4$

Vortex mass ≈ boson mass at half filling

# "Vortex wavefunctions" vs. vorticity density

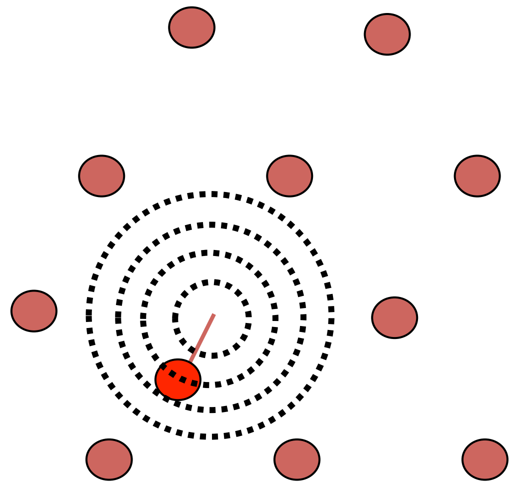
Vorticity density
(HCB, exact)

Single vortex wavefunction
(Harper model)




Harper model = particle in a 2D harmonic well + magnetic field

# Quantum Melting



Multivortex hamiltonian = Bose Coloumb liquid

$$\mathcal{H}^{\mathrm{mv}} = \sum_{i,s=\uparrow\downarrow} \frac{\mathbf{p}_i^2}{2M_{\mathrm{v}}} + \frac{\pi t}{4} \sum_{i \neq j} \log(|\mathbf{r}_i - \mathbf{r}_j|) - \frac{n_{\mathrm{v}} \pi^2 t}{4} \sum_i |\mathbf{r}_i|^2 + \mathcal{H}^{\mathrm{ret}}(\omega).$$

Magro and Ceperley: Wigner solid melts at $r_s = 12$

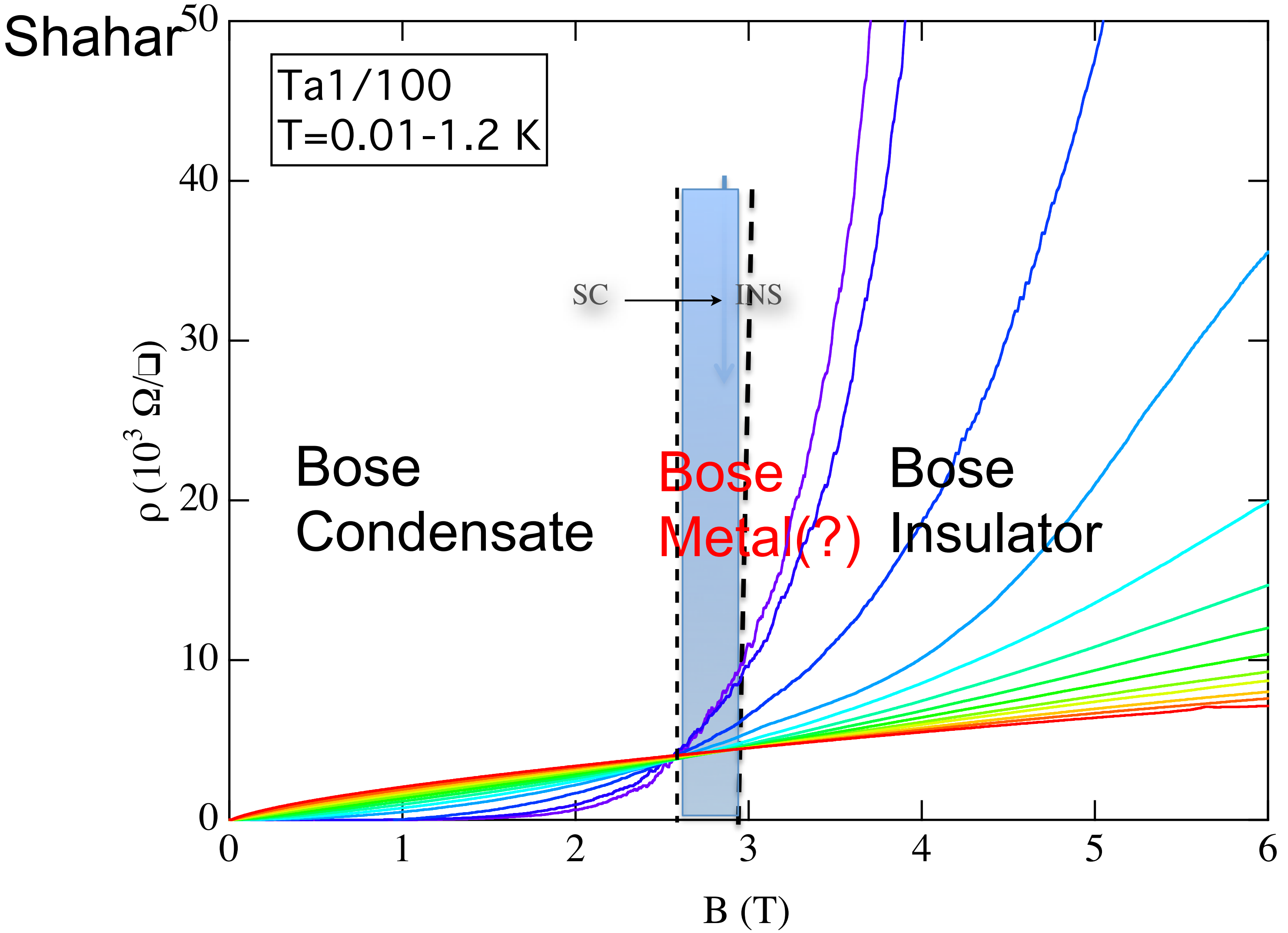
$$r_s^{-2} = \pi n_{\mathrm{v}} a_0^2. \qquad a_0 = \left(\frac{\hbar^2}{\pi t M_{\mathrm{v}}}\right)^{1/2}$$

Therefore, the vortex lattice should quantum melt at

$$n_{\mathrm{v}}^{\mathrm{cr}} \le \left(6.5 - 7.9\frac{V}{t}\right) \times 10^{-3} \text{ vortices per site.} \qquad B_{cr} \simeq 10^{-2} \frac{\Phi_0}{a^2}$$

Quantum Vortex liquid: not Bose condensed!

# Magnetic field induced SIT : vortex lattice quantum melting

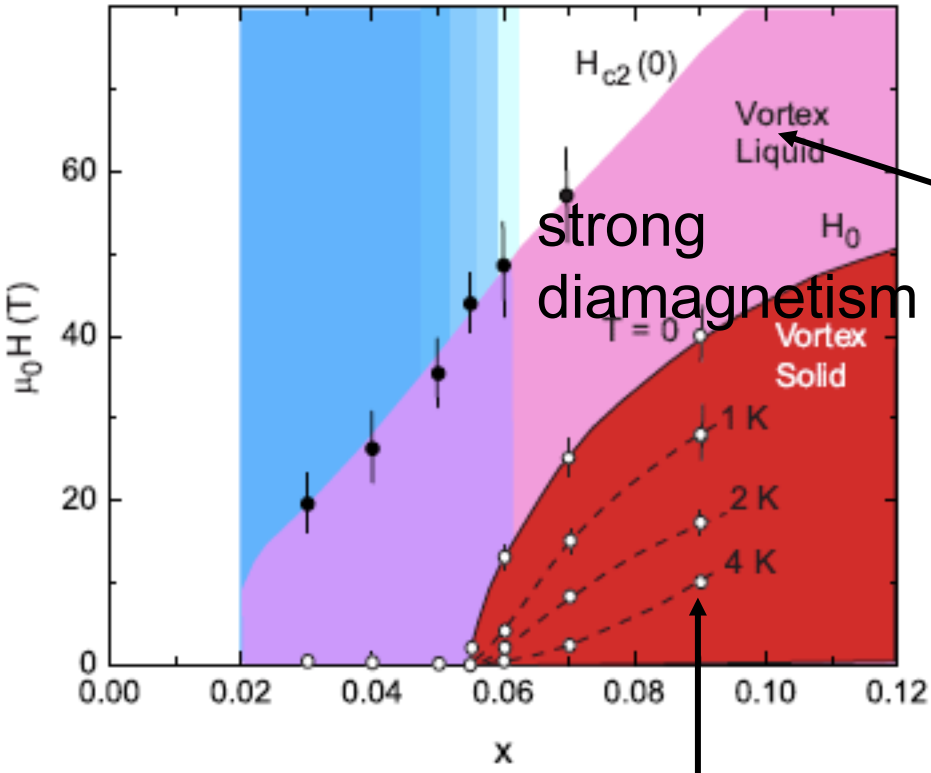


# Quantum melting in cuprates

Low temperature vortex liquid in $La_{2-x}Sr_xCuO_4$

Lu Li[1], J. G. Checkelsky[1], Seiki Komiya[2], Yoichi Ando[2], and N. P. Ong[1*]

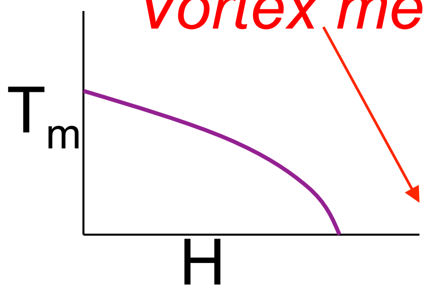
Nature Physics 3, 311-314 (2007)




***A quantum phase:***

*Vortex condensate?*

*Vortex metal?*




$T_m$ decreases with field. (Non classical)

# Hall Conductance of Hard Core Bosons

Thermally averaged Chern numbers

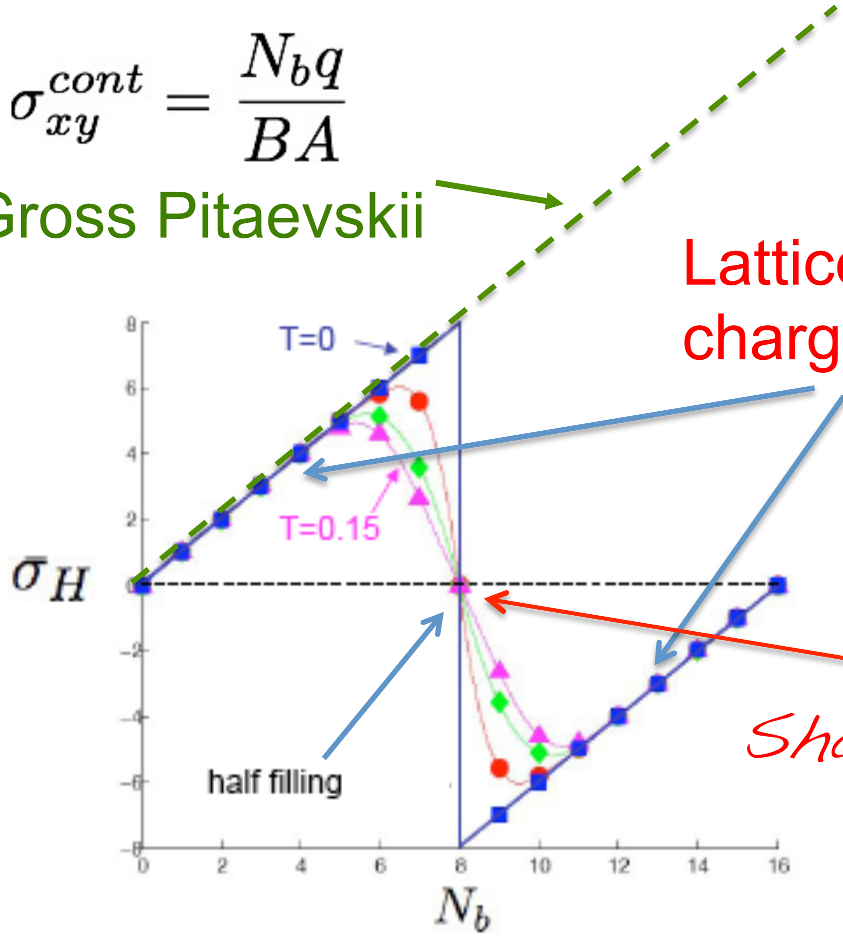
Avron and Seiler, PRL (85)

$$\sigma_{\mathrm{H}}(n_b,T) = \frac{1}{4\pi}\sum_{n=0}^{\infty}\int_0^{2\pi}\int_0^{2\pi} d^2\Theta \; \frac{e^{-E_n/T}}{Z} \; \mathrm{Im}\left\langle \frac{\partial\psi_n}{\partial\Theta_x}\middle|\frac{\partial\psi_n}{\partial\Theta_y}\right\rangle$$

$$\sigma_{xy}^{cont} = \frac{N_b q}{BA}$$

Gross Pitaevskii

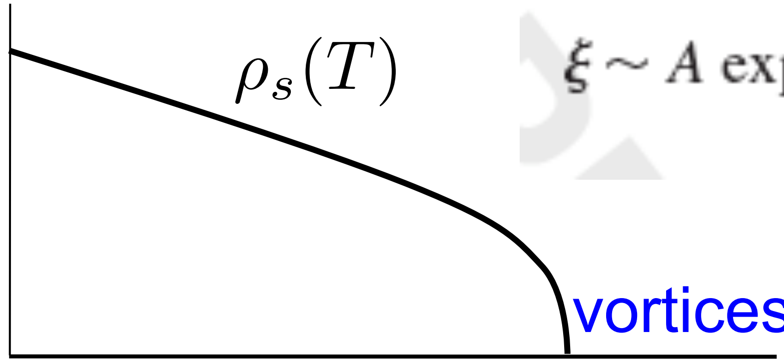
Lattice induced
charge conjugation antisymmetry




*Sharp transition at half filling*

# Conductivity above BKT transition

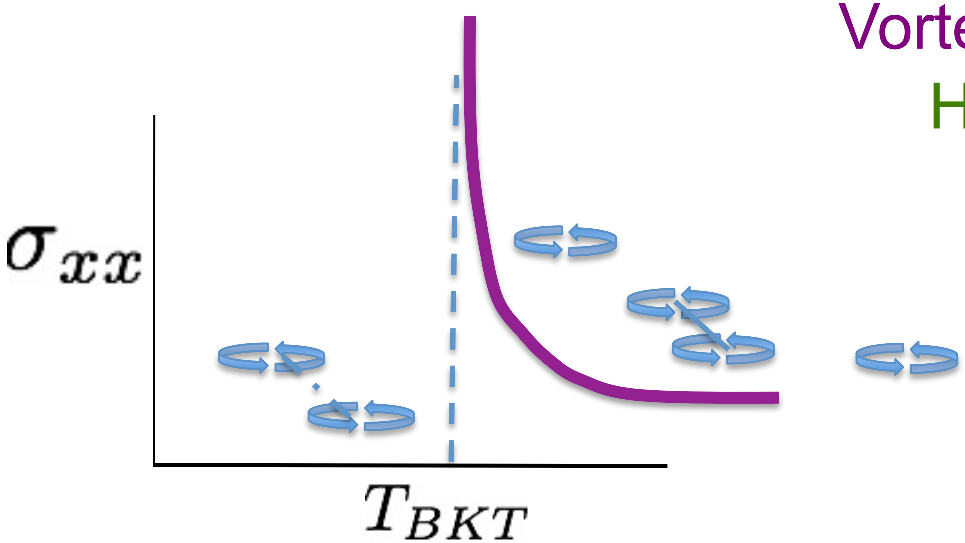
2D




$$\xi \sim A \exp\left(\frac{B}{\sqrt{(T-T_{\mathrm{BKT}})/T_{\mathrm{BKT}}}}\right)$$

vortices proliferation

$$T_{\mathrm{BKT}} \simeq 1.41J.$$




Vortex plasma conductivity

Halperin- Nelson

$$\sigma_{xx}^{HN} = \frac{q^2 T}{h D_v}\xi^2(T) + \sigma_N(T)$$

$$\sigma_N(T), D_v = ?$$

PHYSICAL REVIEW B 81, 054512 (2010)

# Conductivity of hard core bosons: A paradigm of a bad metal

Netanel H. Lindner and Assa Auerbach

HCB Current Operator $J_x = \frac{4qJ}{\sqrt{N}} \sum_{\mathbf{r}} \left( S^x_{\mathbf{r}} S^y_{\mathbf{r}+\hat{\mathbf{x}}} - S^y_{\mathbf{r}} S^x_{\mathbf{r}+\hat{\mathbf{x}}} \right)$

Real Conductivity:

current fluctuations function

superfluid stiffness

$$\sigma(\beta,\omega) = q^2 \pi \rho_s(\beta)\delta(\omega) + \frac{\tanh(\beta\omega/2)}{\omega} G''(\beta,\omega)$$

$$G''(\beta,\omega) = \frac{1}{2}\int_{-\infty}^{\infty} dt e^{-i\omega t} \langle \{ J_x(t), J_x(0) \} \rangle_\beta .$$

# Current fluctuations function

$$G''(\beta,\omega) = -\frac{1}{Z}\mathrm{Im}\mathrm{Tr}\left(e^{-\beta H}\left\{J_x, \frac{1}{\omega - \mathcal{L} + i\epsilon}J_x\right\}\right),$$

Liouvillian hyper-operator $\mathcal{L} = [\mathcal{H}, \cdot]$

Moments expansion:

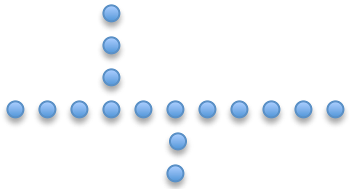
$$\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\omega^k G''(\beta,\omega) = \langle\{J_x, \mathcal{L}^k J_x\}\rangle_\beta \equiv \mu_k(\beta)$$

Static correlators:
amenable to high T expansion

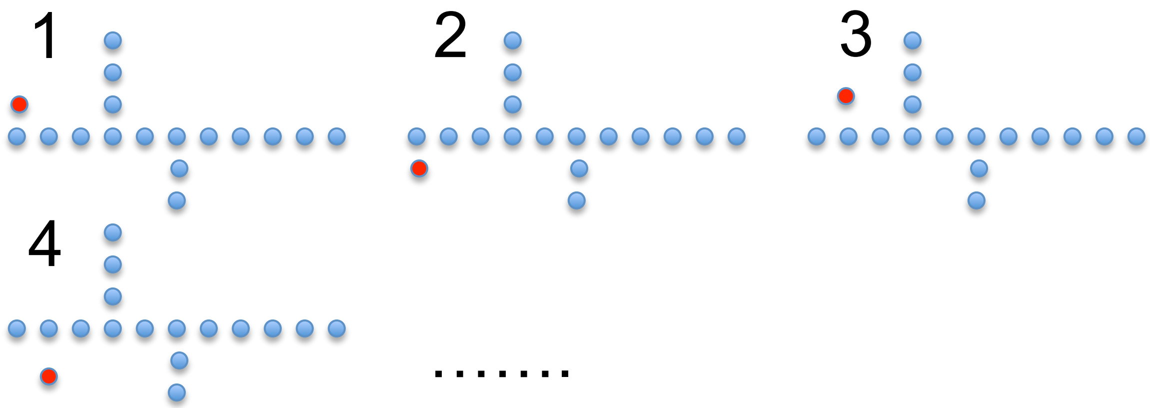
*We can invert a finite set of moments if we know the high order asymptotics!*

# "Gaussian Termination" – high coordination

$$\hat{O}_n = \sum c^{\alpha_1,\alpha_2,\ldots,\alpha_n}_{i_1,i_2,\ldots,i_n} S^{\alpha_1}_{i_1} S^{\alpha_2}_{i_2}, \ldots, S^{\alpha_n}_{i_n}$$



$$\mathcal{L}\hat{O}_n =$$




$z_{eff}\, n$

Linear recurrents → Gaussian dissipation

$\sqrt{n}\Omega$

$$\mathrm{Im}\ \langle 0| \left(\omega + i\epsilon - \mathcal{L}\right)^{-1} |0\rangle =$$

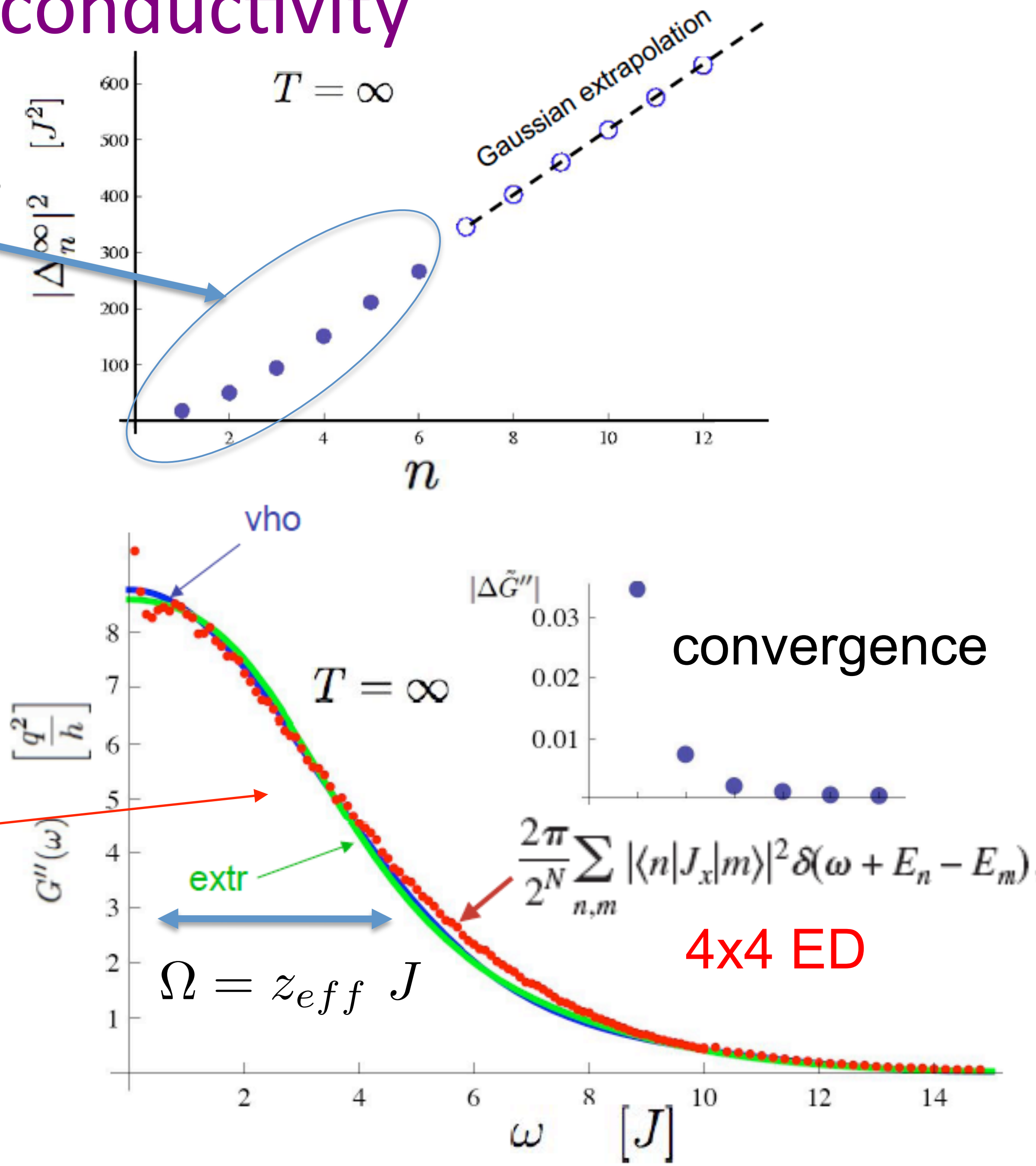
$$\mathrm{Im} \begin{pmatrix} \omega + i\epsilon & -\Omega & 0 & 0 & \ldots \\ \Omega & \omega + i\epsilon & -\sqrt{2}\Omega & 0 & \\ 0 & -\sqrt{2}\Omega & \omega + i\epsilon & -\sqrt{3}\Omega & \\ 0 & 0 & -\sqrt{3}\Omega & \omega + i\epsilon & \end{pmatrix}^{-1}_{11} \propto e^{-\omega^2/\Omega^2}$$

$$G(t) \sim e^{-\Omega^2 t^2}$$

*Different from Boltzmann transport*

# Dynamical conductivity

Computed recurrents




$$\sigma_{\beta\to 0} = \frac{\beta}{2}G''_\infty(\omega)$$

Gaussian decay

# HCB: Field theory at n=1/2

order parameter field $\Psi(x) = n_x(x) + i n_y(x)$

Relativistic Gross-Pitaevskii = O(2) field theory (Higgs)

$$S_{RGP} = \int d^2x \int d\tau \, \tfrac{1}{2}|\dot{\Psi}|^2 + \frac{\rho_s}{2\Delta^2}|\nabla\Psi|^2 - \frac{m}{8\Delta^2}\left(|\Psi|^2 - \Delta^2\right)^2$$

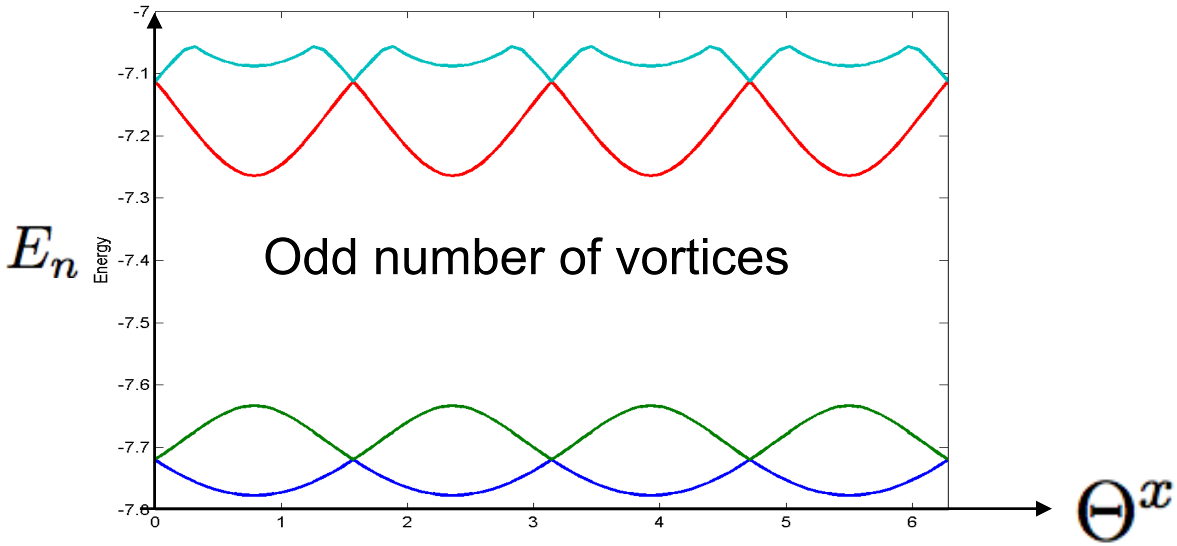
$$+ iS \int d\tau \sum_i (-1)^i \dot{\phi}$$

Berry phases

1. Irrelevant for static corelations in superfluid phase.
2. Relevant for quantum disordered phases, (Haldane, Read, Sachdev)
3. Relevant for vortex dynamics, degeneracies (Lindner AA Arovas).

# Quantum Degeneracies in Vortex states

## Exact spectrum of the gauged torus at half filling




<u>Theorem:</u>

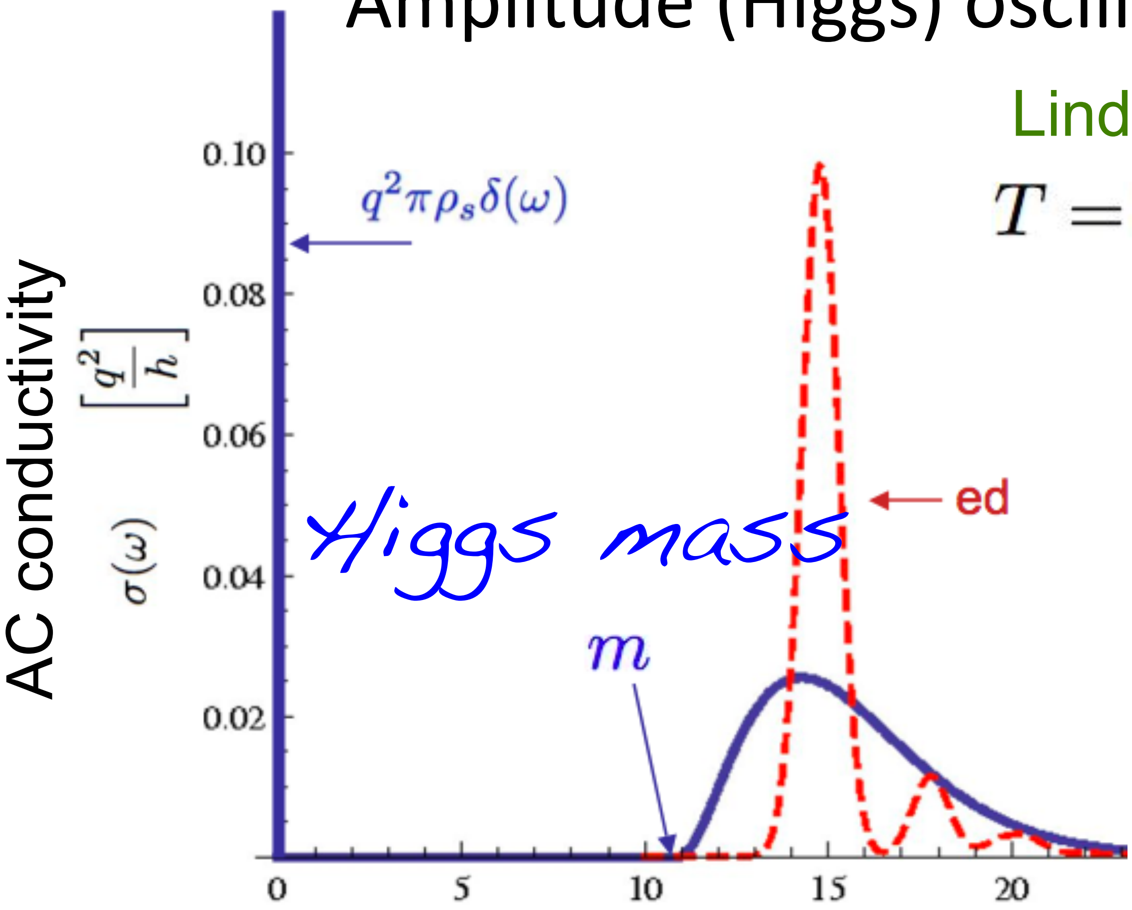
Doublet degeneracies, of all eigenstates, occur when the vorticity center is situated precisely on any lattice site.

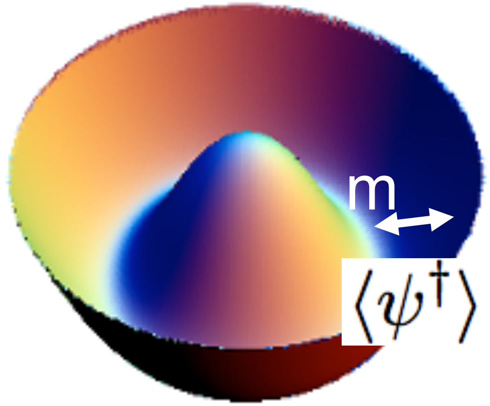
Proof:

We construct a non commuting algebra of symmetries

# Amplitude (Higgs) oscillations

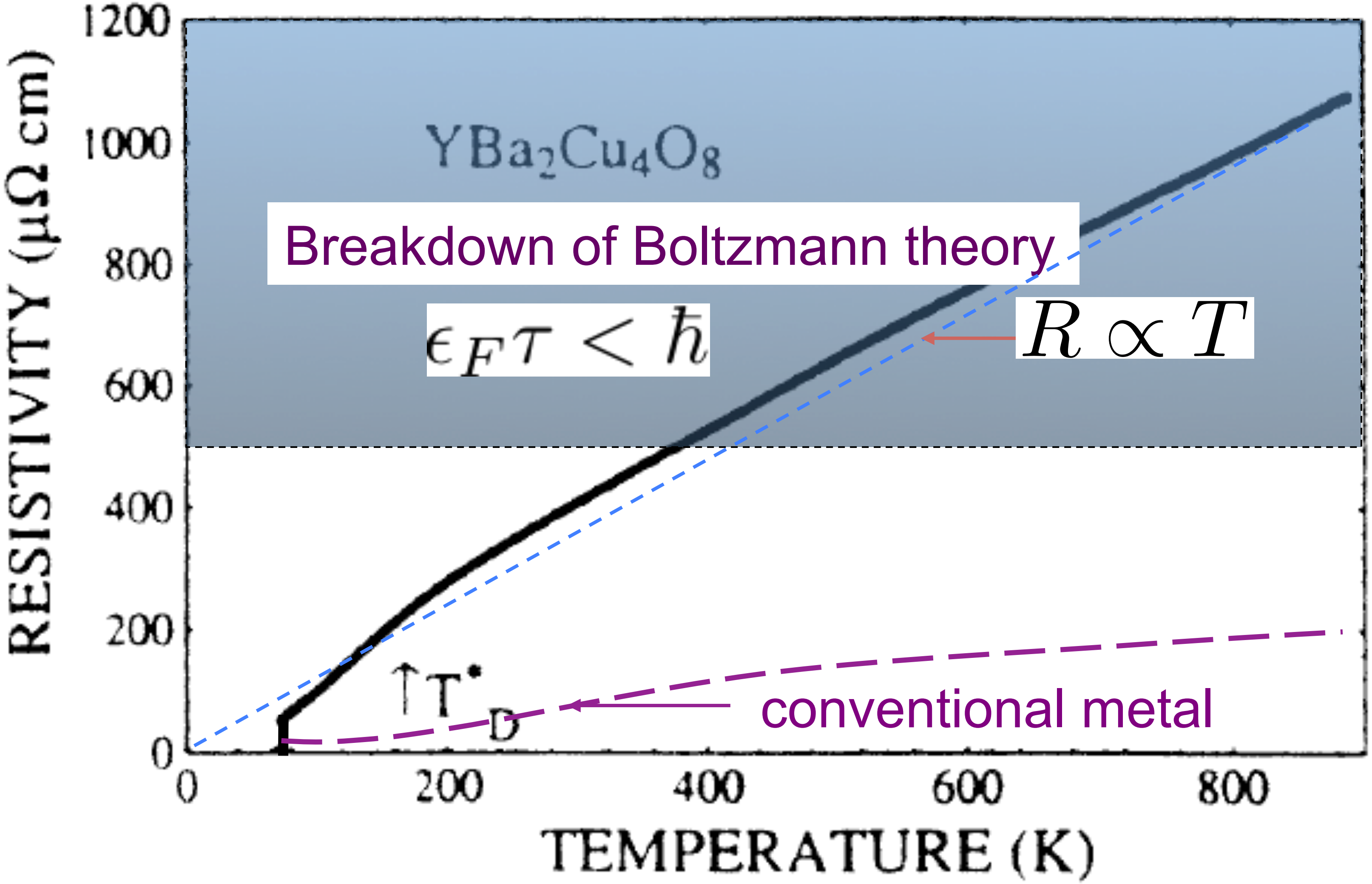
Lindner, AA, PRB (2010)




Analogues: (w Daniel Podolsky)
Oscillating coherence near Mott phase of optical lattices
Magnitude mode in 1-D CDW's
2-magnon Raman peaks in O(3) antiferromagnets

# `Bad Metal'

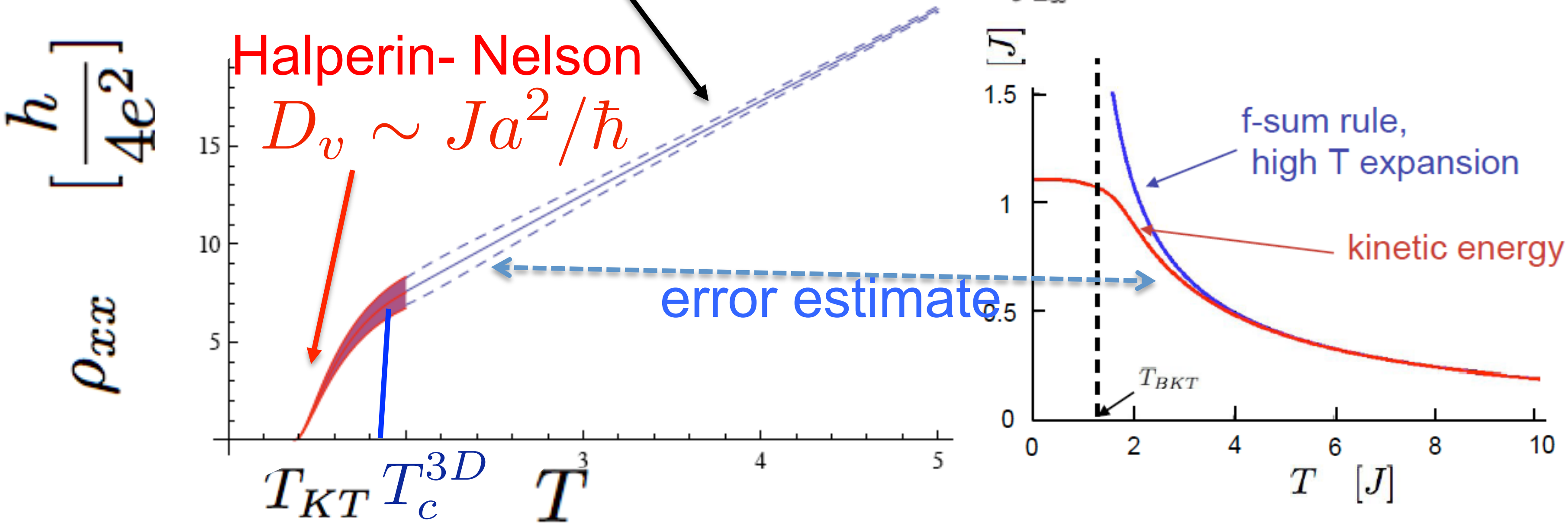



Emery & Kivelson `Bad Metal` behavior

# High temperature resistivity

$$R(T) = 0.23 R_Q \frac{T}{J}\{1 - 2.9(J/T)^2 + \mathcal{O}[(J/T)^4]\}$$

$$\int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{\tanh(\beta\omega/2)}{\omega} G''(\beta,\omega) = \langle -q^2 K_x \rangle_\beta$$



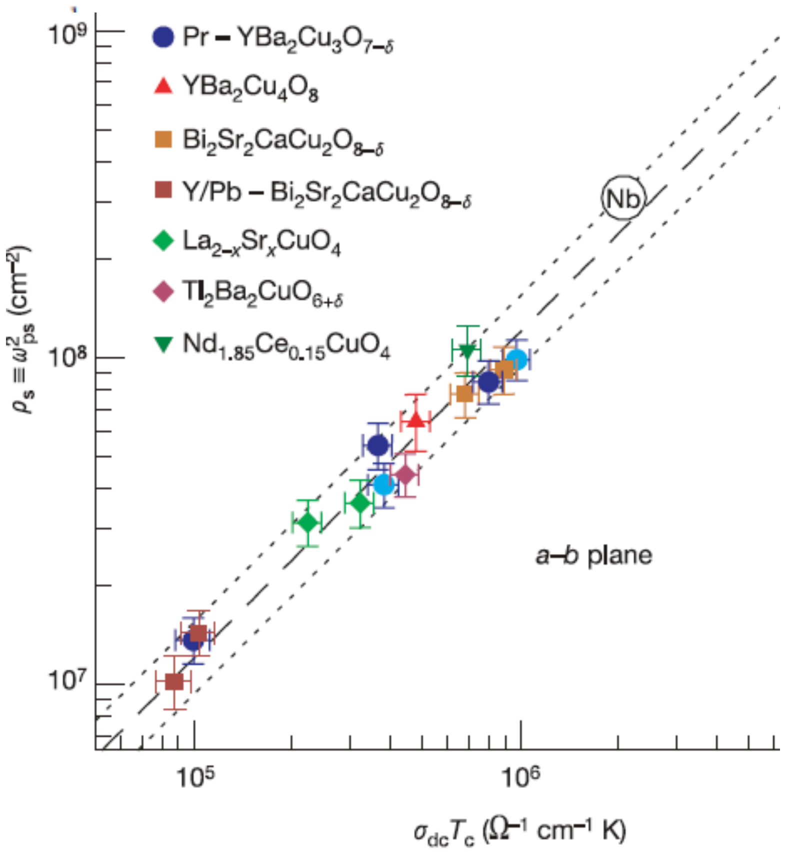

"Bad Metal":
linear increase, no resitivity saturation

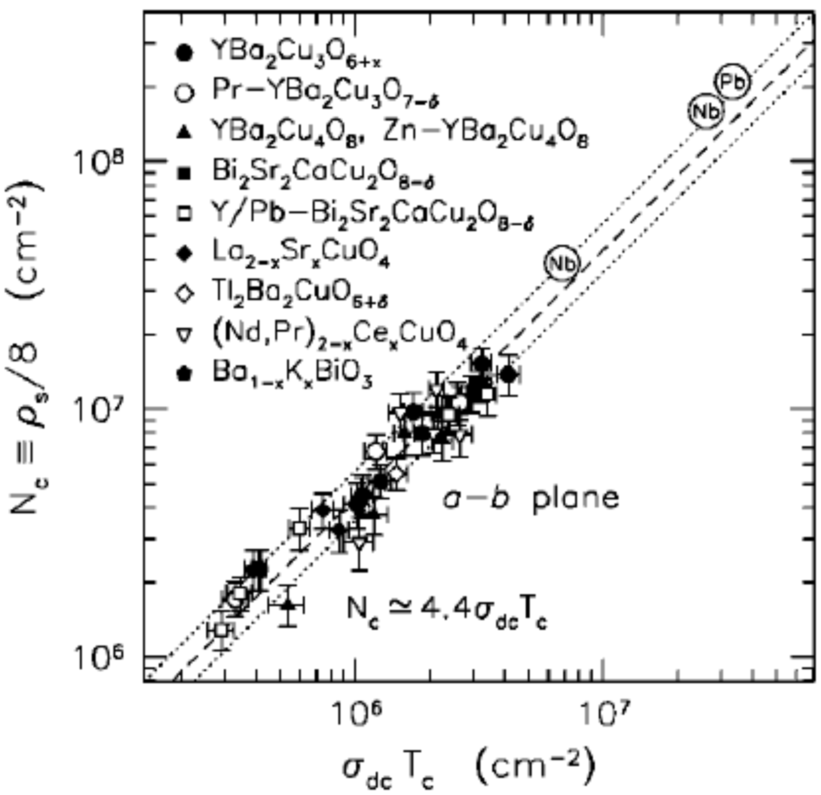
# "Homes Law"

## A universal scaling relation in high-temperature superconductors

$$\frac{dR(T_c)}{dT} \propto \frac{R_Q}{\rho_s(0)}$$

C. C. Homes[1], S. V. Dordevic[1], M. Strongin[1], D. A. Bonn[2], Ruixing Liang[2], W. N. Hardy[2], Seiki Komiya[3], Yoichi Ando[3], G. Yu[4], N. Kaneko[5*], X. Zhao[5], M. Greven[5,6], D. N. Basov[7] & T. Timusk[8]



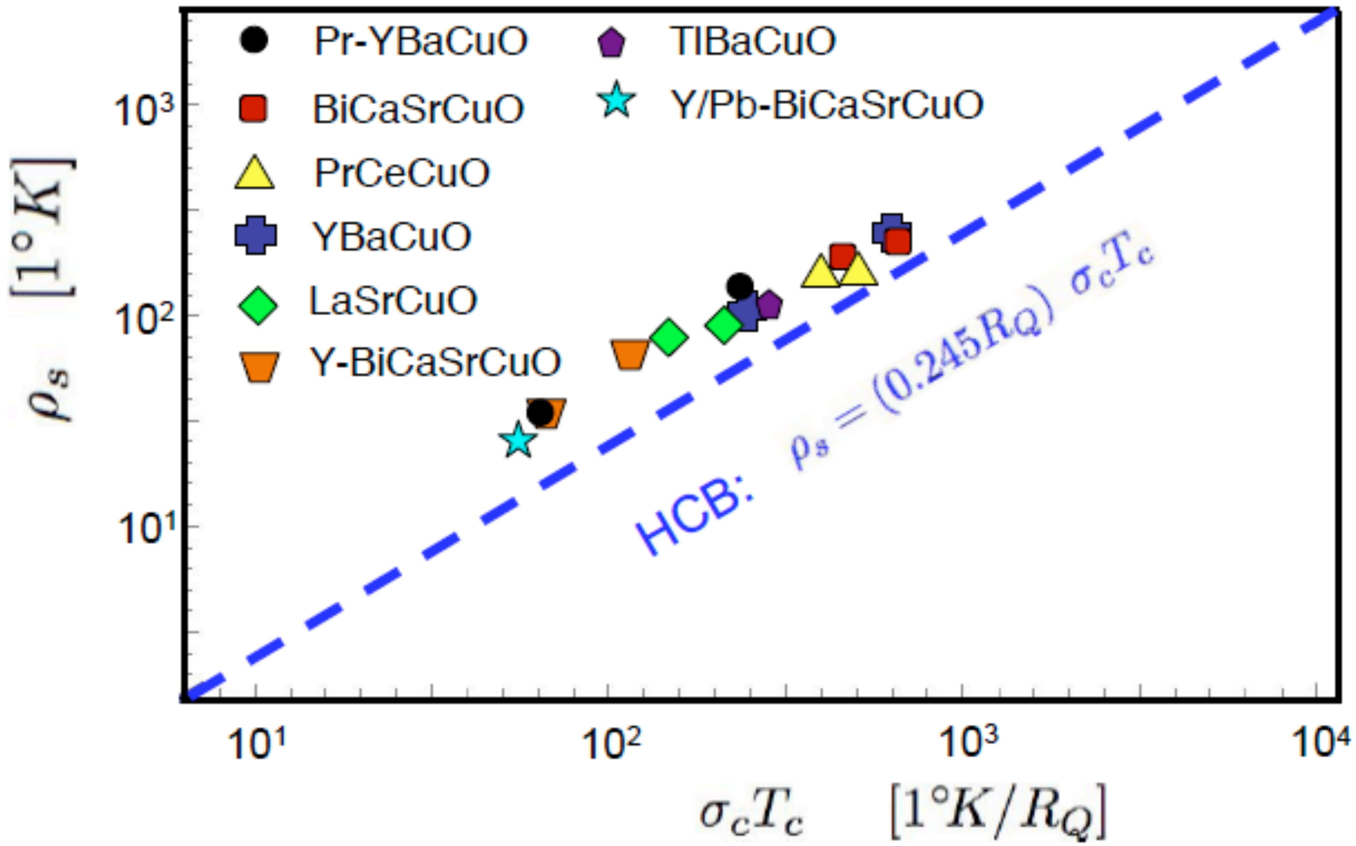



# "Homes law" of HCB

$$\rho_s(0) = 0.245 \frac{R_Q}{R_c} T_c.$$

$R_Q = h/q^2$ is the boson quantum of resistance $= 6453\ \Omega$

Data: Homes et. al.



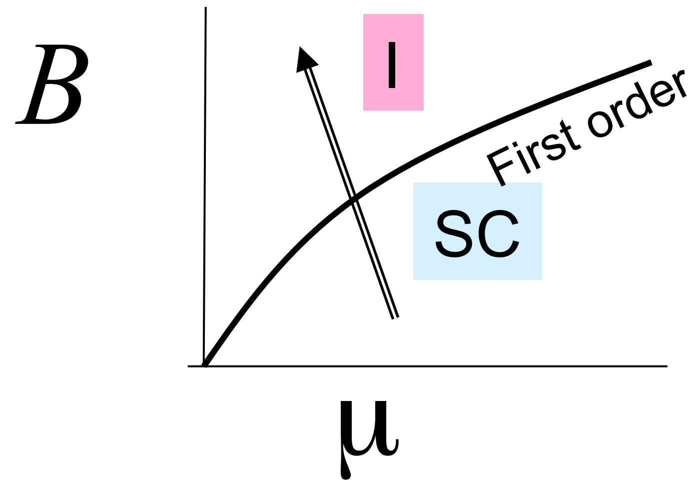
# Summary

1. SIT in granular films can be described by Lattice bosons models = frustrted quantum antiferromagnets
2. Lattice (granularity) mobilizes vortices and give rise to insulating phases.
3. Hall conductivity oscillates with boson filling. Vortices acquire spin-half ("V-spin") degeneracies at half filling.
4. The Higgs amplitude mode should be observable near the SIT Transition.
5. In clean systems: HCB exhibit non-Boltzmann "bad metal" resistivity.
6. (Weak) magnetic field → quantum vortex liquid (perhaps an intermediate metallic phase)

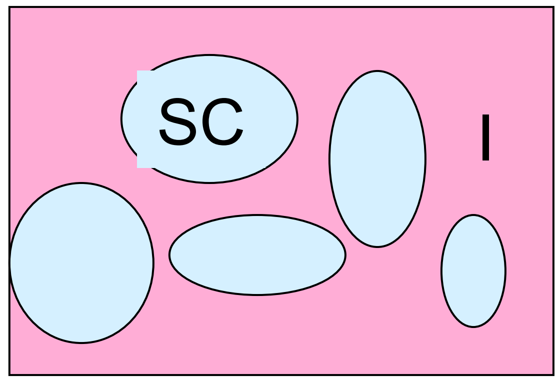
# Why puddles are ubiquitous

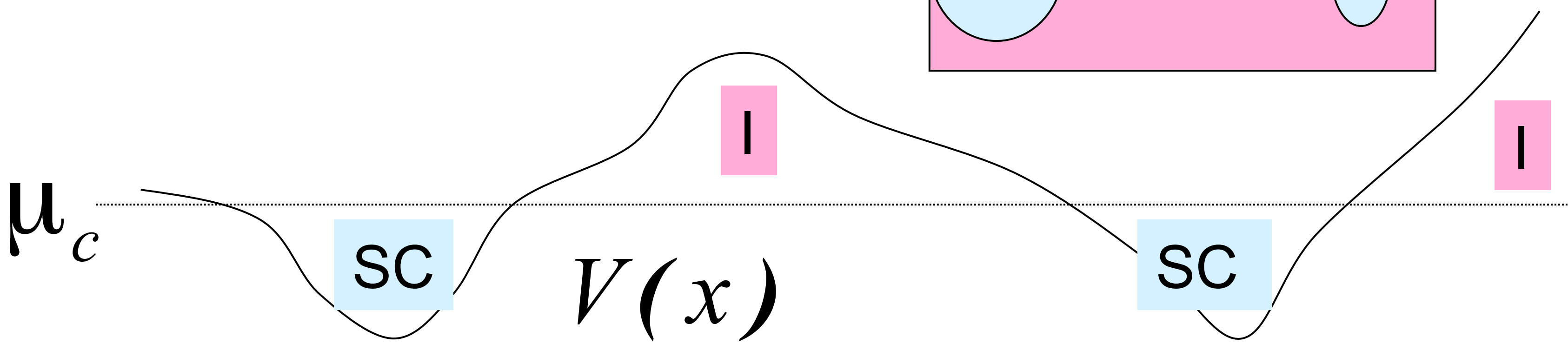
Shimshoni, AA, Kapitulnik, PRL 80 (1998).

Clean system (no disorder)




Imry-Ma: In 2D,
Arbitrary weak disorder eliminates the 1st order transition and breaks the system into domains.




Henceforth: consider a 2D Josephson-capacitors array