Phase fluctuations, Percolation, Interactions at the Superconductor-Insulator Transition

ANL 2010

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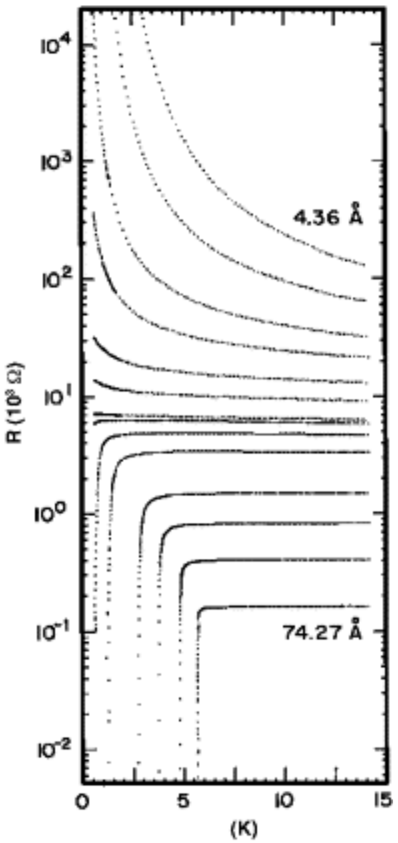
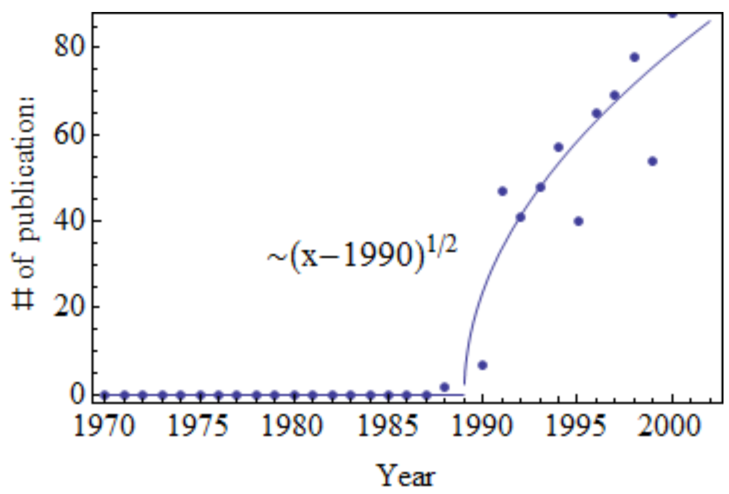
Teaser Trailer

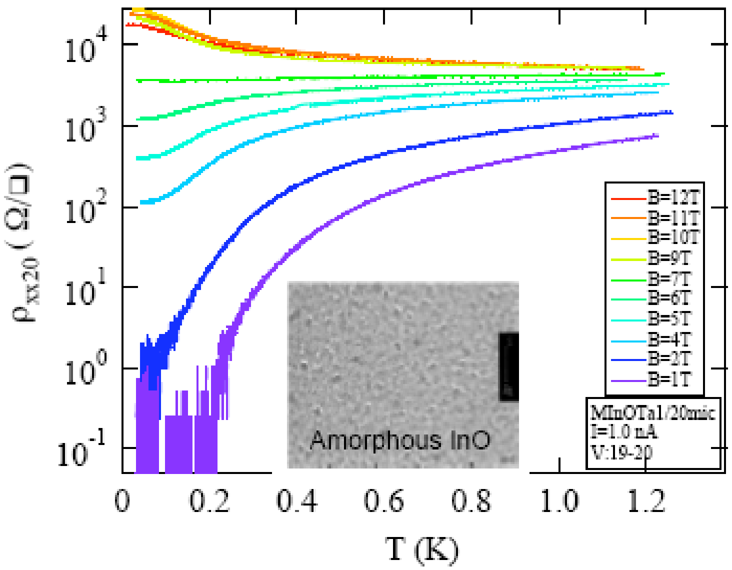


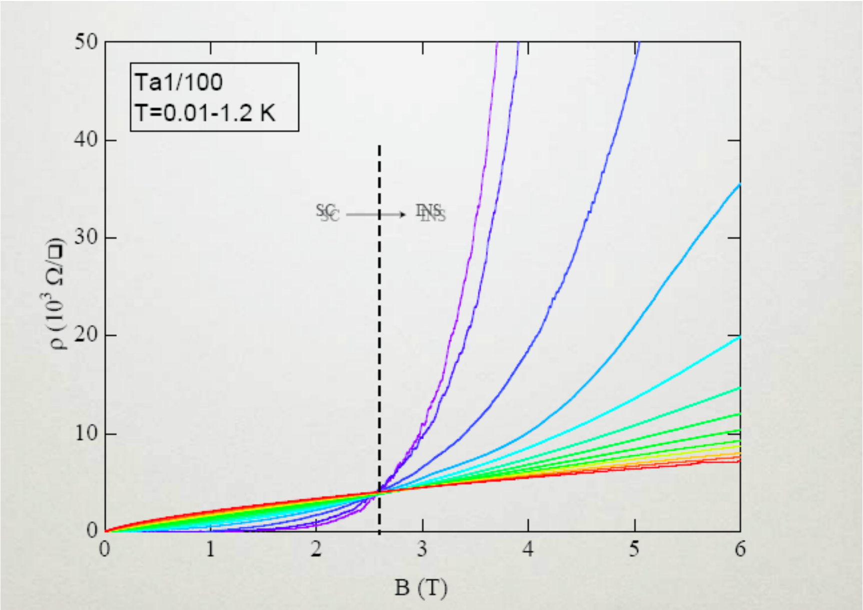


[D.B. Haviland, Y. Liu, A.M. Goldman, Phys. Rev. Lett. **62,** 2180 (1989)]

Teaser Trailer

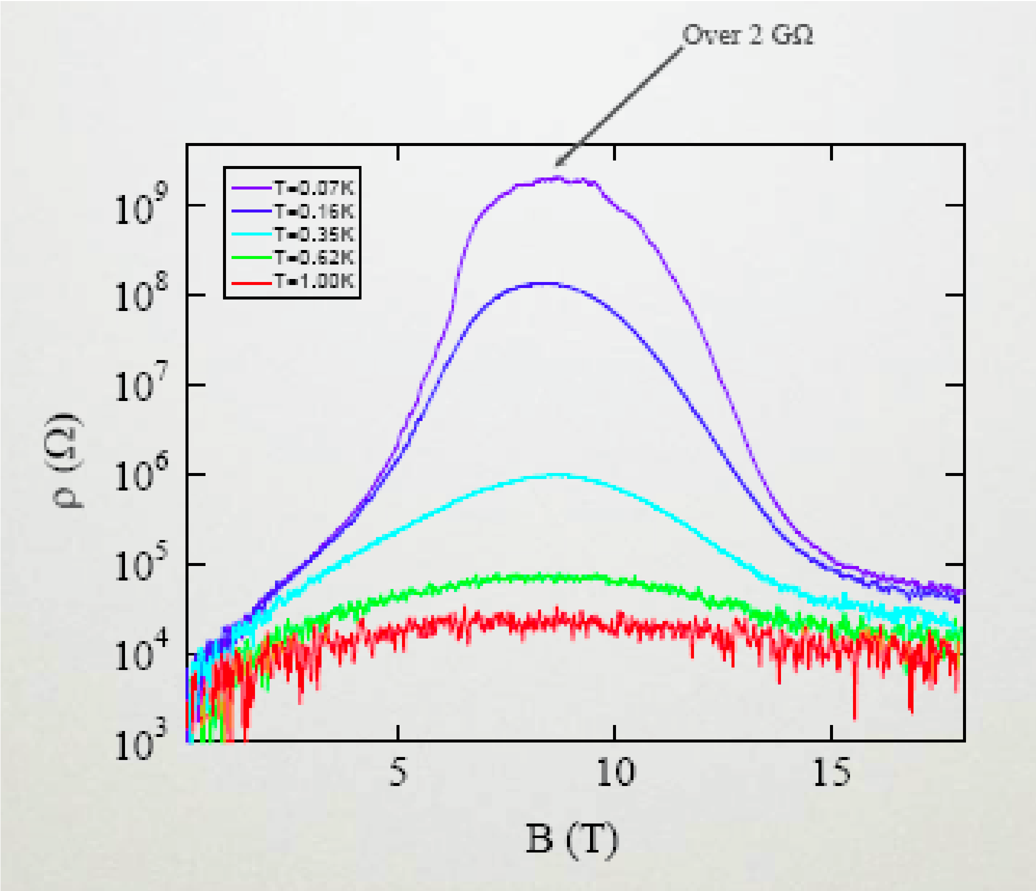
Field-driven transition…

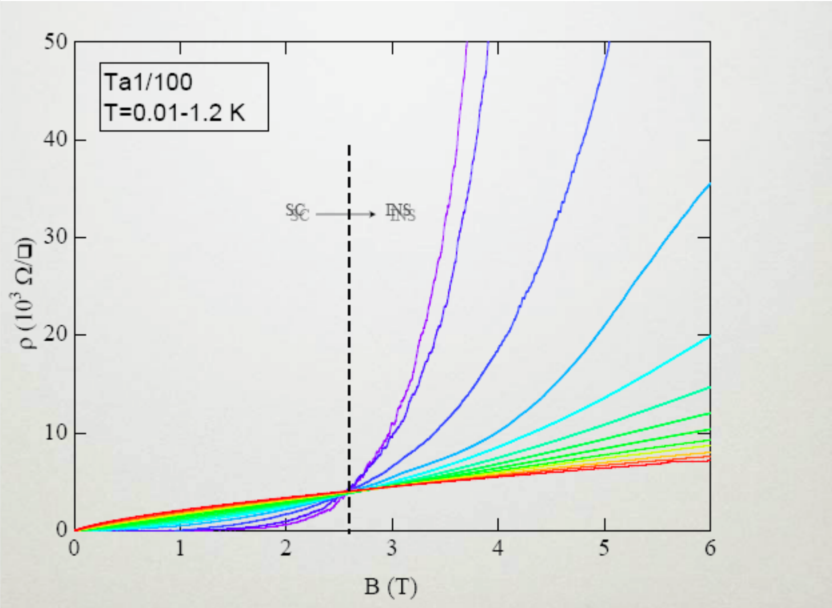




[Sambandamurty, D.Shahar *et.al.* *PRL* **92**, 107005 (2004)]

Teaser Trailer





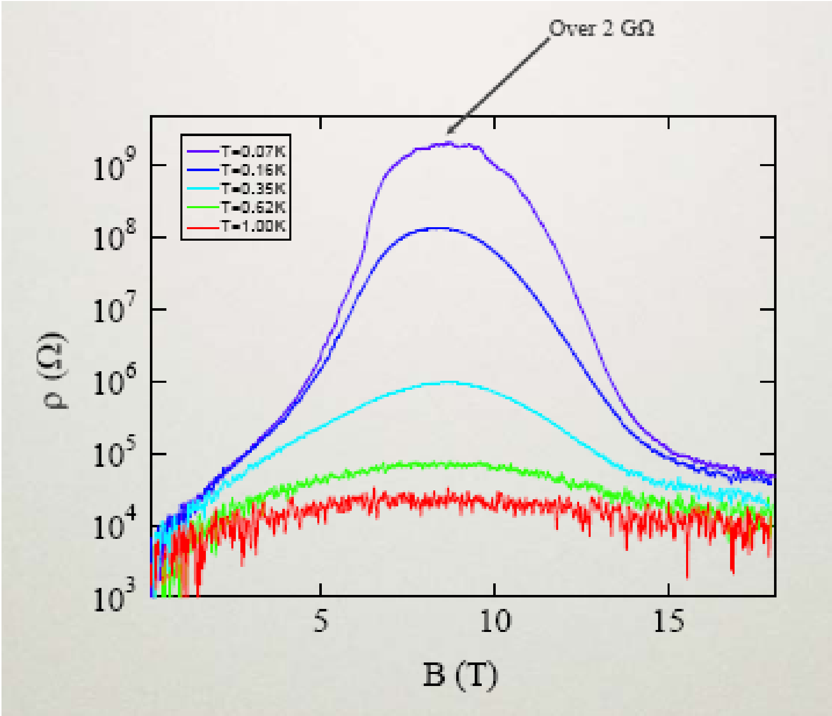
[Sambandamurty, D.Shahar *et.al.* *PRL* **92**, 107005 (2004)]

Teaser Trailer

Aim: a unified description of SIT in thin films –

- ✓ B-SIT
- ✓ huge MR peak
- ✓ critical behavior

- $B_{||}$-dependence
- T-SIT



Outline

- How phase fluctuations drive the B-SIT
- How interactions drive the MR peak
- How thickness drives the T-SIT

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- How phase fluctuations drive the B-SIT
- How interactions drive the MR peak
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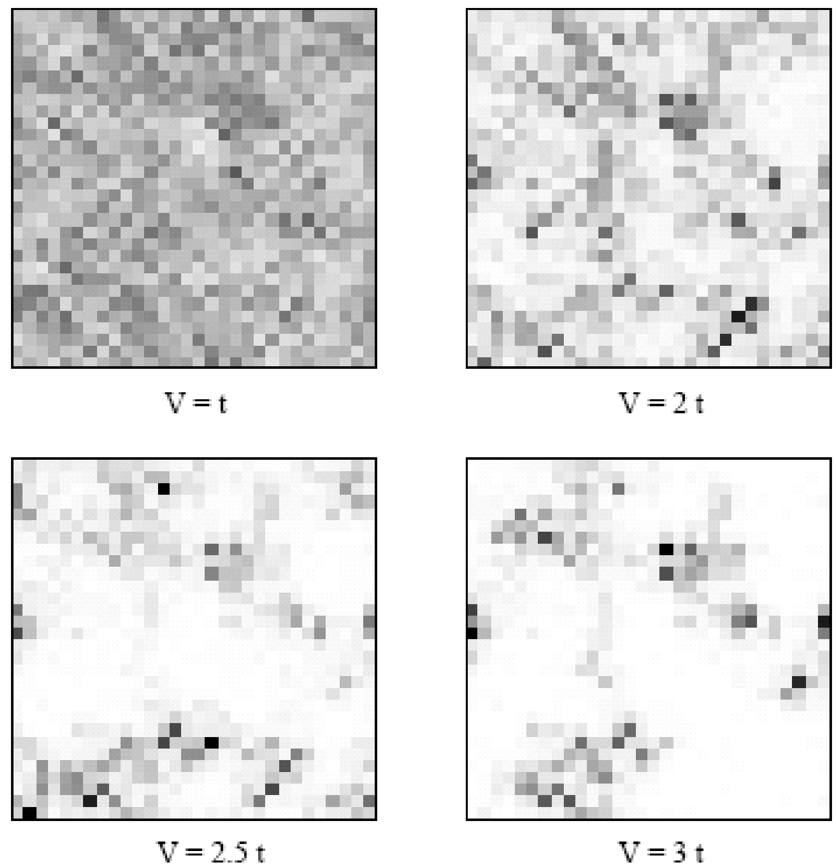
Crash review on the theory of disordered SCs

- Disorder-induced SC gap fluctuations at T,B=0

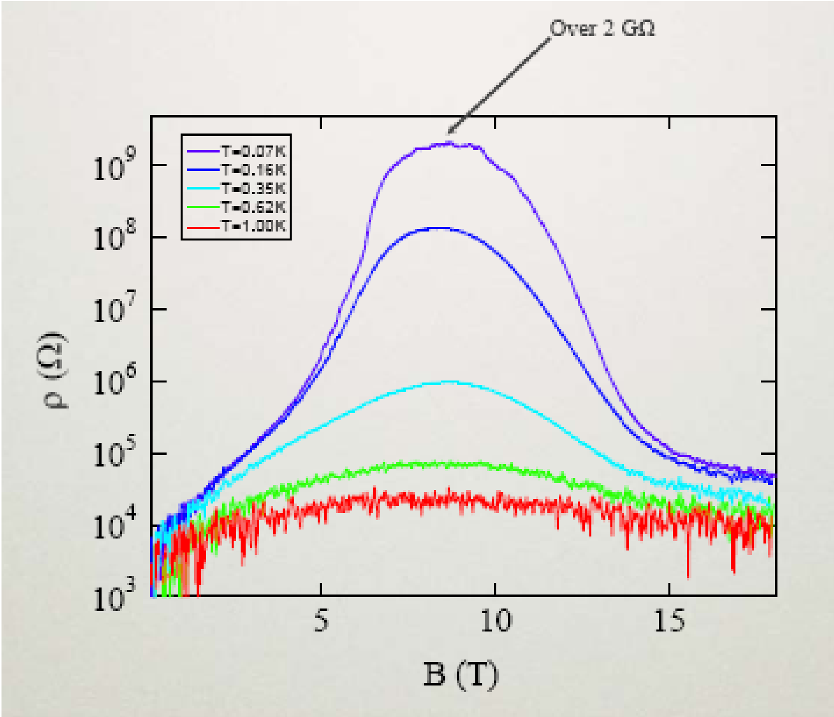
  [Ghosal, Randeria and Trivedi 1998]



and more…

Crash review on the theory of disordered SCs

And still…



Goal - describe both rise and fall of resistance within one framework :

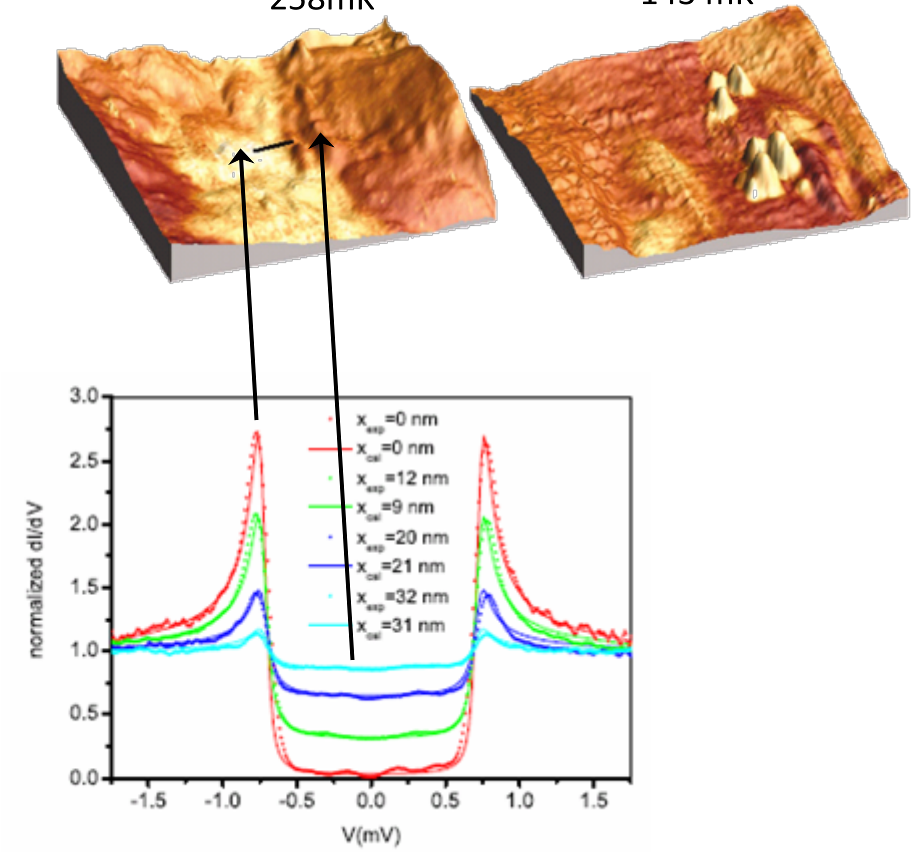
use the idea of SC islands to describe the entire range of magnetic fields

The model

1. The existence of SC islands due to disorder

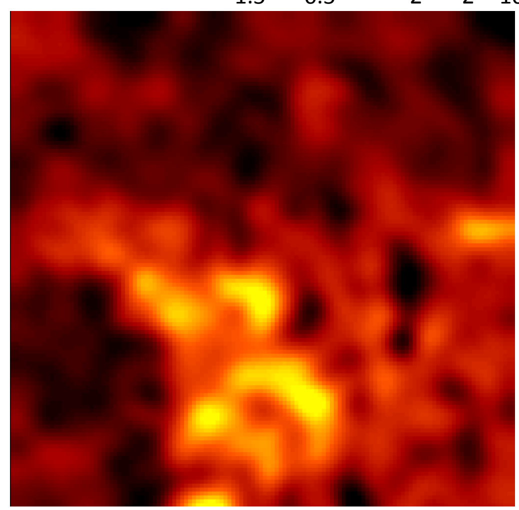
TiN

258mK 143 mK



[Esscofier *et.al.*, Phys. Rev. Lett. **93**, 217005 (2004)
[

near SIT in $Eu_{1.5}Ce_{0.5}NbSr_2Cu_2O_{10-d}$



[Y. Levi, I. Felner, U. Asaf and Oded Millo,

Phys. Rev. B **60**, R15059[

SC islands – a mean-field 1st step

- Mean-Field : Saddle point approximation to the integral, leading to the B-dG equations

Hopping, Diagonal disorder, Magnetic field

Local order parameter

$$\begin{pmatrix} \hat{\xi} & \Delta_i \\ \Delta_i^* & -\hat{\xi} \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}_i) \\ v_n(\mathbf{r}_i) \end{pmatrix} = E_n \begin{pmatrix} u_n(\mathbf{r}_i) \\ v_n(\mathbf{r}_i) \end{pmatrix}$$

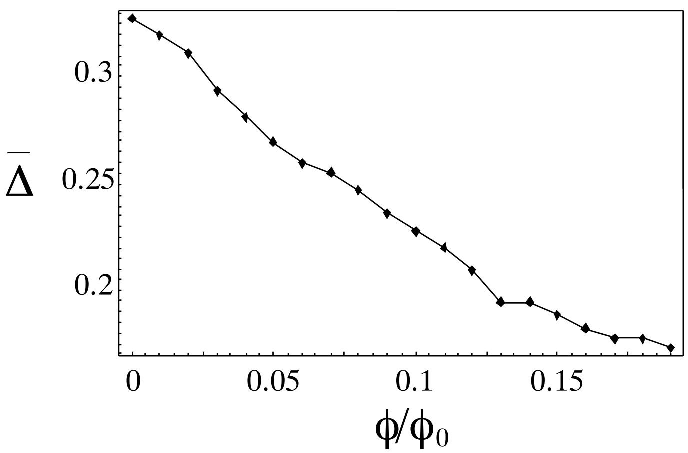
$$\Delta_i = |U| \sum_n u_n(\mathbf{r}_i) v_n^*(\mathbf{r}_i) \ , \ \langle n_i \rangle = 2 \sum_n |v_n(\mathbf{r}_i)|^2$$

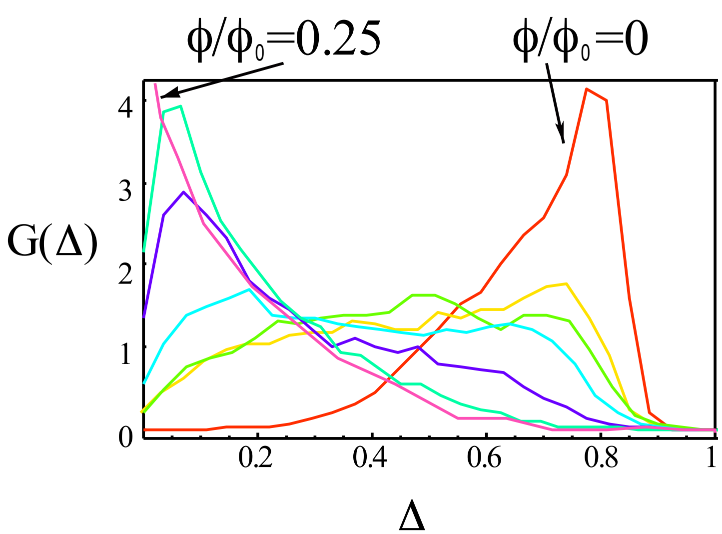
(maintaining $N = \sum_i \langle n_i \rangle$ fixed)

[Ghosal, Randeria and Trivedi 1998]

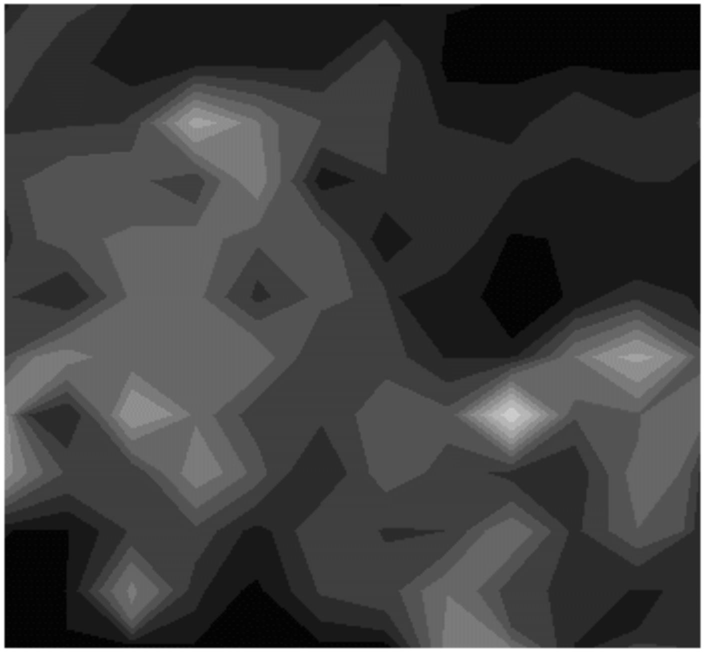
SC islands – a mean-field 1st step

perpendicular magnetic field





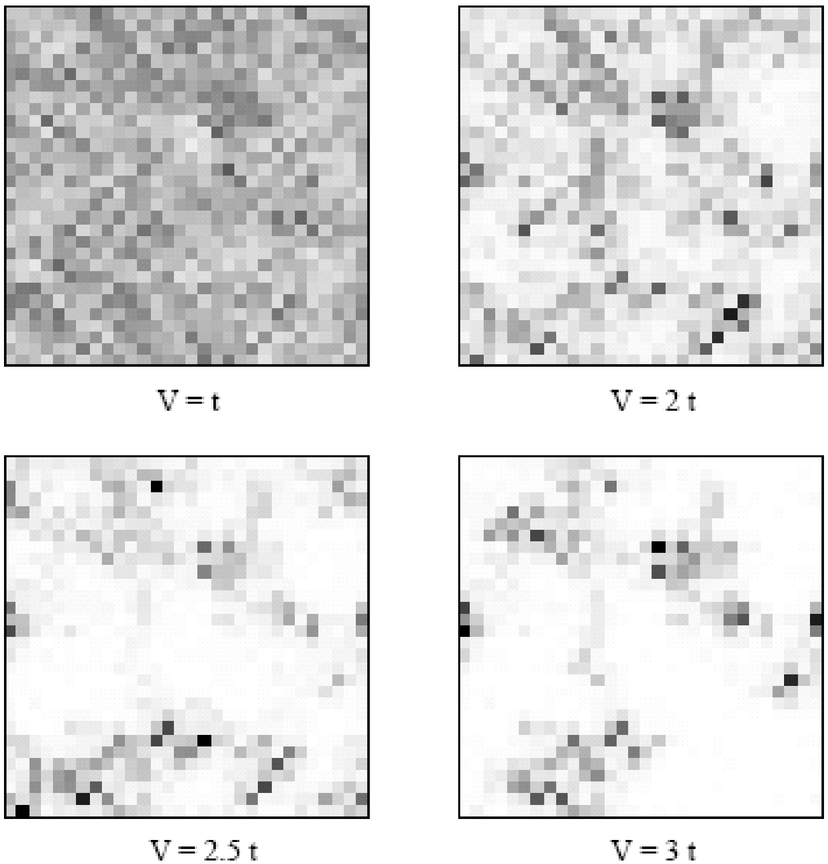
B=0.



Bright=Large Δ

Dark=Small Δ

SC islands – a mean-field 1st step



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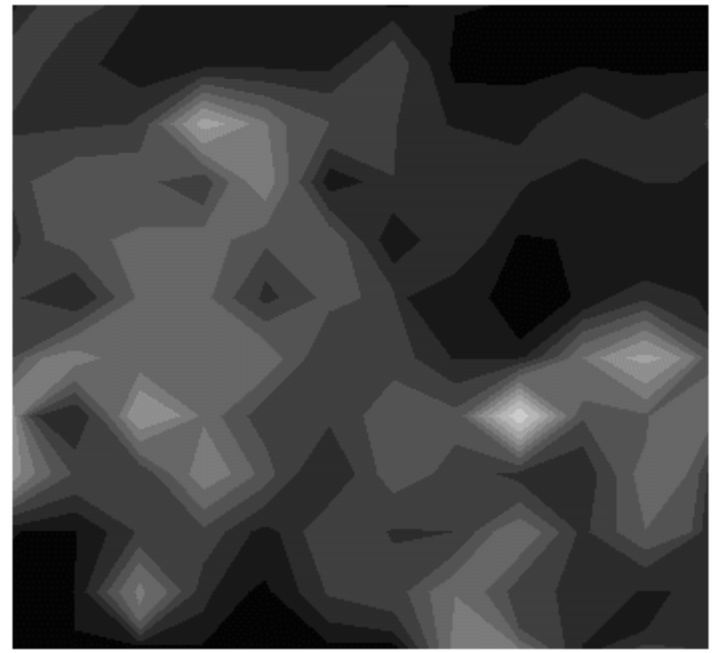


Bright=Large Δ

Dark=Small Δ

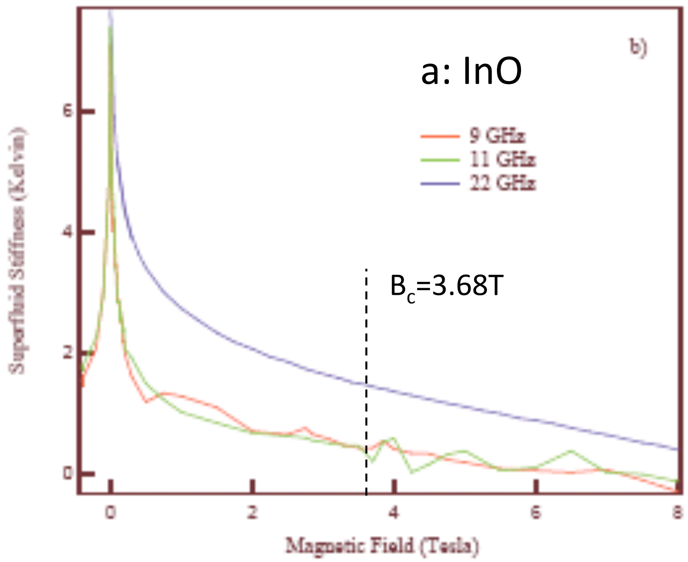
- Disorder-induced SC gap fluctuations at T,B=0

  [Ghosal, Randeria and Trivedi 1998]

Phase fluctuations

- Non-zero pair correlations are not sufficient for superconductivity:

The system is SC if the SC phases on the two sides of the system are correlated.



[Crane *et.al.,*

PRB **75**, 184530 (2007)[

<u>Problem</u> – the BdG formalism does not capture phase fluctuations !

<u>Solution</u> – treat (thermal) phase fluctuations beyond BdG

(using Monte-Carlo simulation) [M. Mayr *et al.*, *PRL* **94**, 217001 (2005)]

SC islands and phase fluctuations – Formalism

- Starting point – Negative-U Hubbard Model

Diagonal disorder

Parallel field

$$Z=\iint D\{C_i,C_i^{+}\}\exp\left(-\int_0^{\beta}\mathrm{d}\tau\left[\sum_{i\sigma}C_{i\sigma}^{+}(\tau)(-\partial_\tau+\varepsilon_i+h_{\|}\sigma)C_{i\sigma}(\tau)-\right.\right.$$

$$\left.\left.-\sum_{<ij>\sigma}(t_{ij}C_{i\sigma}^{+}(\tau)C_{j\sigma}(\tau)+c.c.)-U\sum_i C_{i\uparrow}^{+}(\tau)C_{i\downarrow}^{+}(\tau)C_{i\downarrow}(\tau)C_{i\uparrow}(\tau)\right]\right)$$

Hopping+orbital field

Attractive e-e interaction

- A Hubbard-Stratonovich transformation (exact):

$$Z=\iint D\{\Delta_i,\theta_i\}D\{C_i,C_i^{+}\}\exp\left(-\int_0^{\beta}d\tau\left[\sum_{i\sigma}C_{i\sigma}^{+}(\tau)(-\partial_\tau+\varepsilon_i+h_{\|}\sigma)C_{i\sigma}(\tau)-\right.\right.$$

$$\left.\left.-\sum_{<ij>\sigma}(t_{ij}C_{i\sigma}^{+}(\tau)C_{j\sigma}(\tau)+c.c)-\sum_i(\Delta_i(\tau)e^{-i\theta_i(\tau)}C_{i\uparrow}^{+}(\tau)C_{i\downarrow}^{+}(\tau)+c.c.)+\sum_i\frac{|\Delta_i(\tau)|^2}{U}\right]\right)$$

- Integrate out fermions (still exact)

SC islands and phase fluctuations – Formalism

Ignore quantum fluctuations in Δ (its τ dependence)

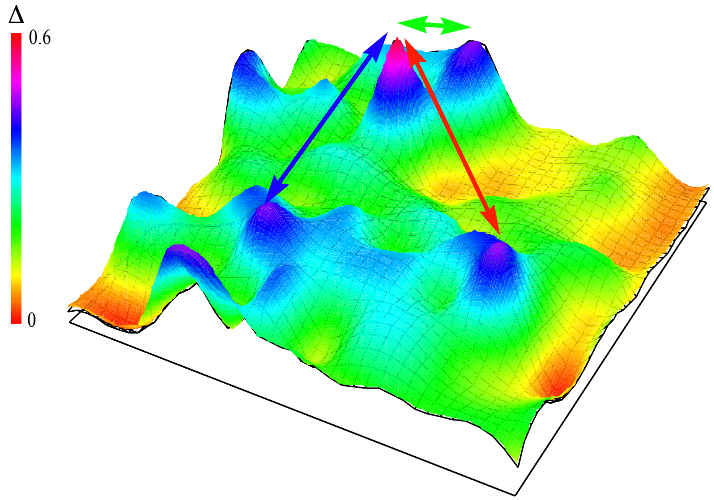
$$Z = \int \prod_i d|\Delta_i| \, d\theta_i \exp\left(-\frac{\beta}{2U} \sum_i |\Delta_i|^2 \right) \mathrm{Tr} \exp(-\beta \mathrm{H}_{\mathrm{BdG}})$$

$$= \int \prod_i d|\Delta_i| \, d\theta_i \exp\left(-\frac{\beta}{2U} \sum_i |\Delta_i|^2 \right) \prod_{i=1}^{2N} (1 + \exp(-\beta E_n))$$

From which phase correlations can be calculated (using Metropolis Monte-Carlo)

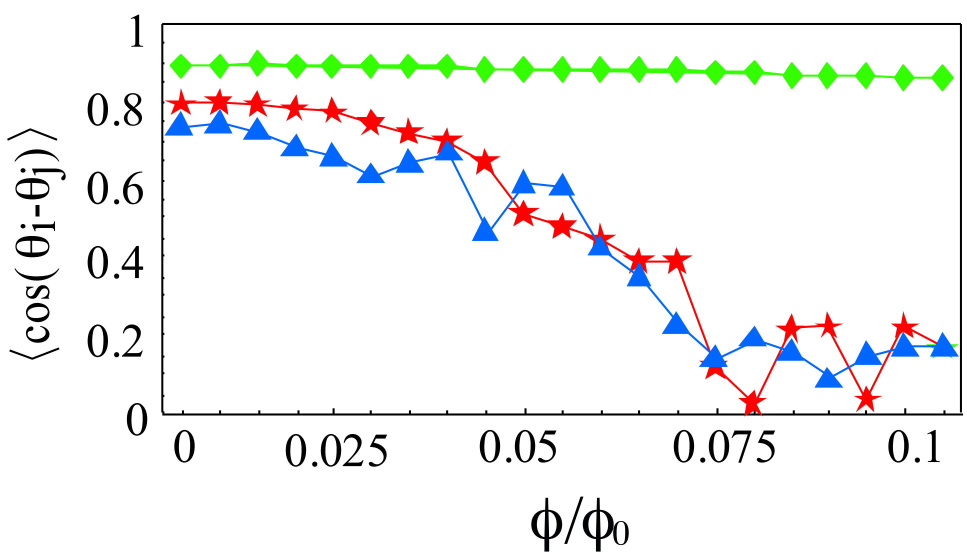
$$\left\langle \cos(\delta\theta_i - \delta\theta_j) \right\rangle = \int \prod_i d|\Delta_i| \, d\theta_i \cos(\delta\theta_i - \delta\theta_j) \exp\left(-\frac{\beta}{2U} \sum_i |\Delta_i|^2 \right) \prod_{i=1}^{2N} (1 + \exp(-\beta E_n))$$

B-induced phase fluctuations

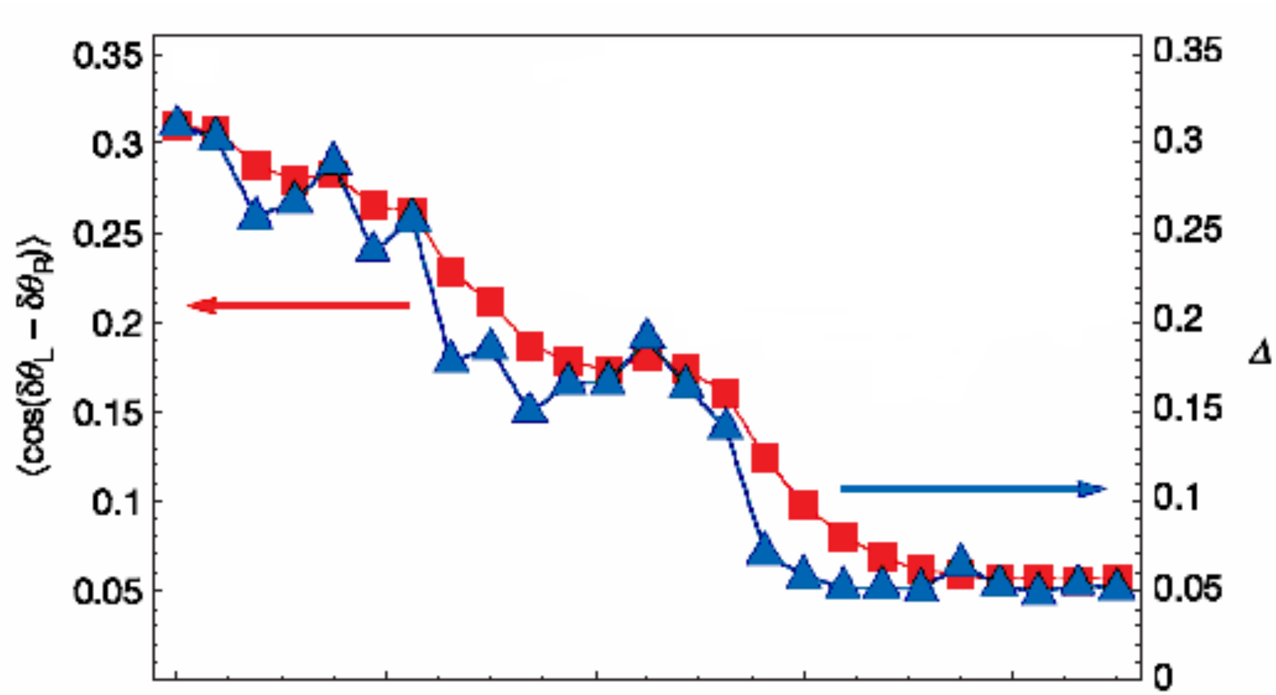


[YD, Y. Meir and Y. Avishai,

Nature **449**, 876-880 (2007)]



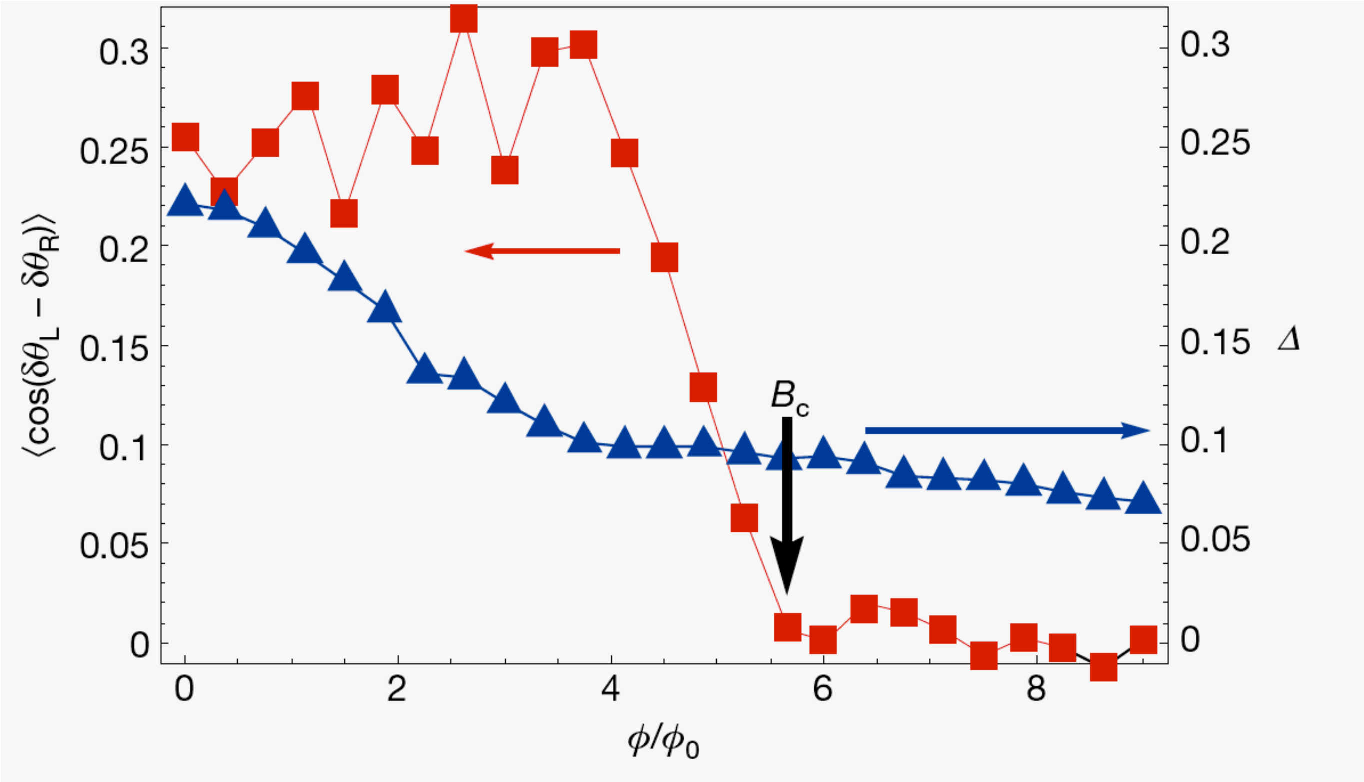
B-induced SIT



<n>=0.92, W/t=0.1

[YYY, Nature **449**, 876-880 (2007)]

B-induced SIT



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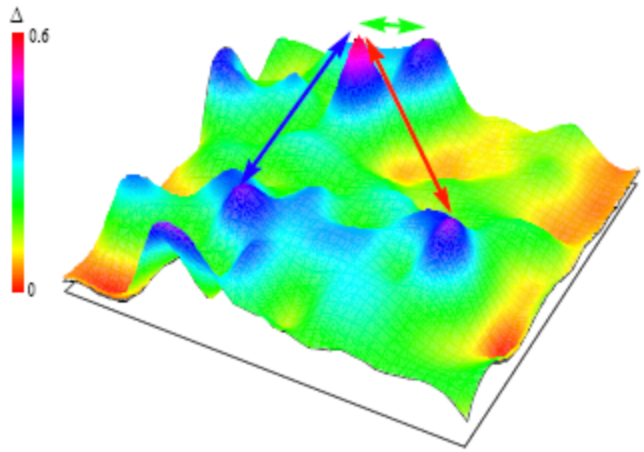
B-induced SIT



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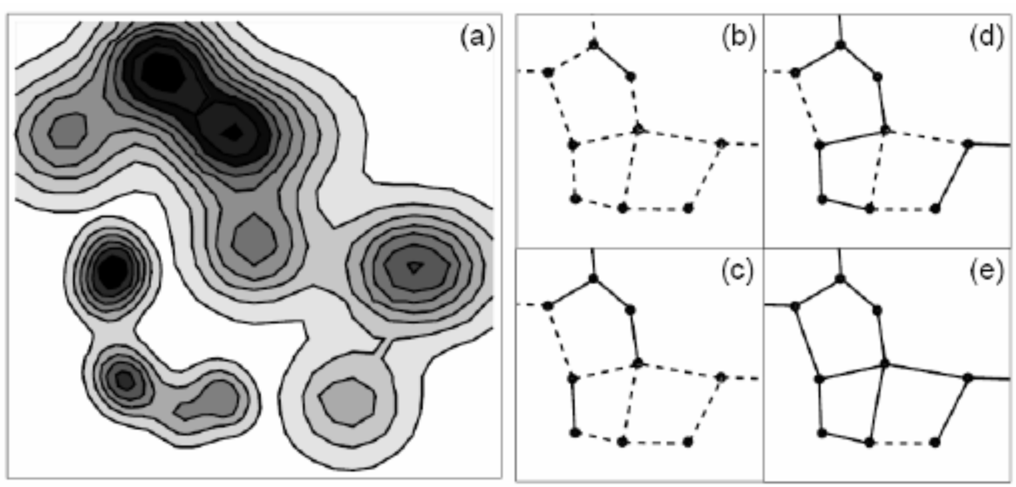
EPL, **91** (2010) 47003 www.e
doi: 10.1209/0295-5075/91/47003

Thermal phase transition in two-dimensional disordered superconductors

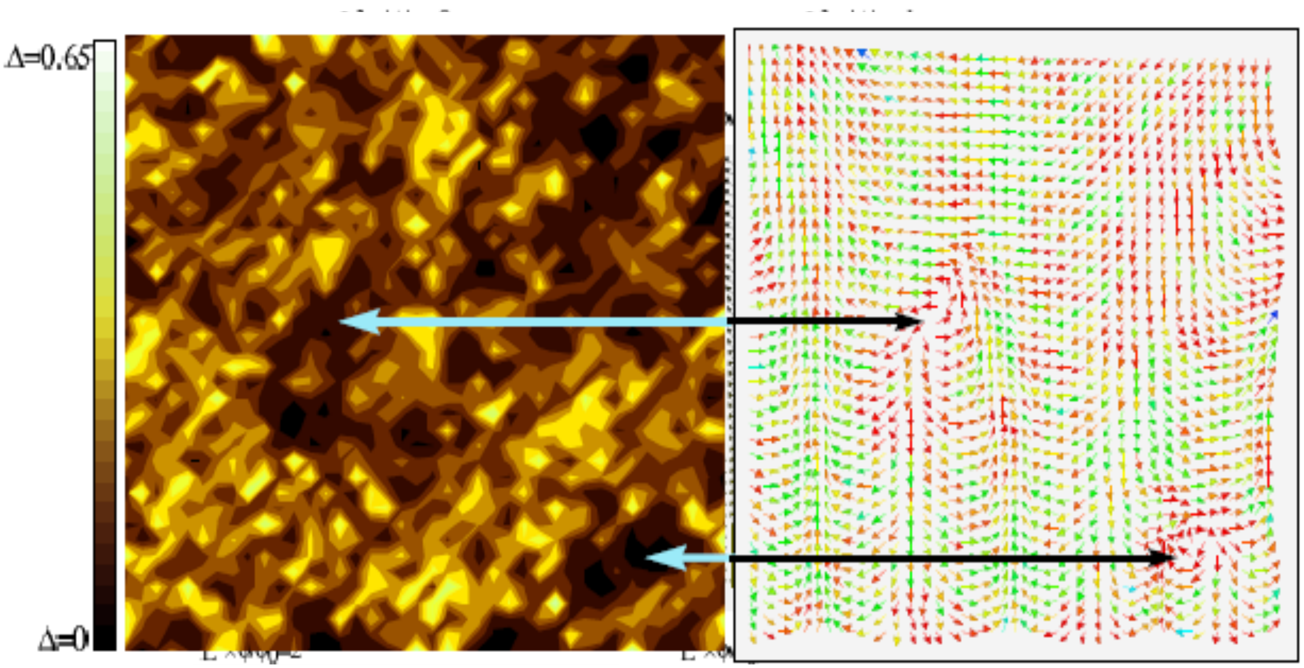
A. Erez[a] and Y. Meir

Physics Department, Ben-Gurion University - Beer Sheva 84105, Israel



[YYY, PRB **71**,125311 (2005);
PRL **94**,156406 (2005)]

B-induced SIT



Vortices weaken the Josephson couplings between the islands. When this coupling becomes of the order of T, phase correlations between the islands disappear.

[YYY, PRB **78**, 024502 (2008)]

SC islands in parallel field

Mean-field equations in the presence of a parallel field

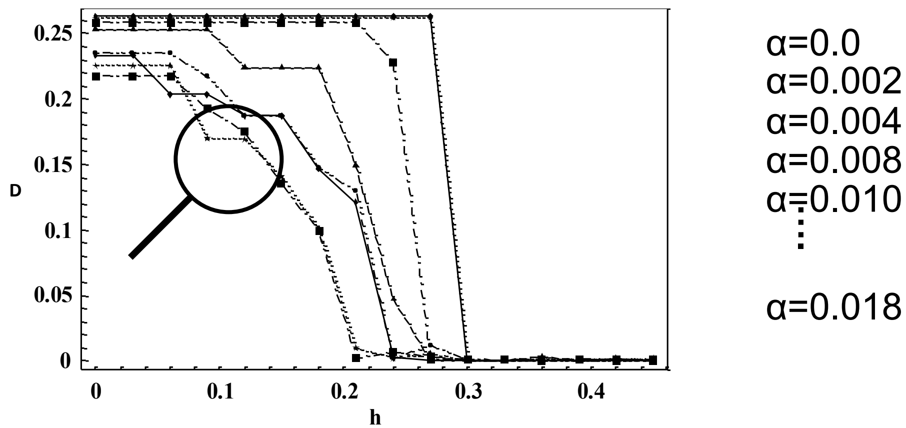
$$\begin{pmatrix} \hat{\xi} + \sigma h & \Delta_i \\ \Delta_i^* & -\hat{\xi} + \sigma h \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}_i) \\ v_n(\mathbf{r}_i) \end{pmatrix} = E_n \begin{pmatrix} u_n(\mathbf{r}_i) \\ v_n(\mathbf{r}_i) \end{pmatrix}$$

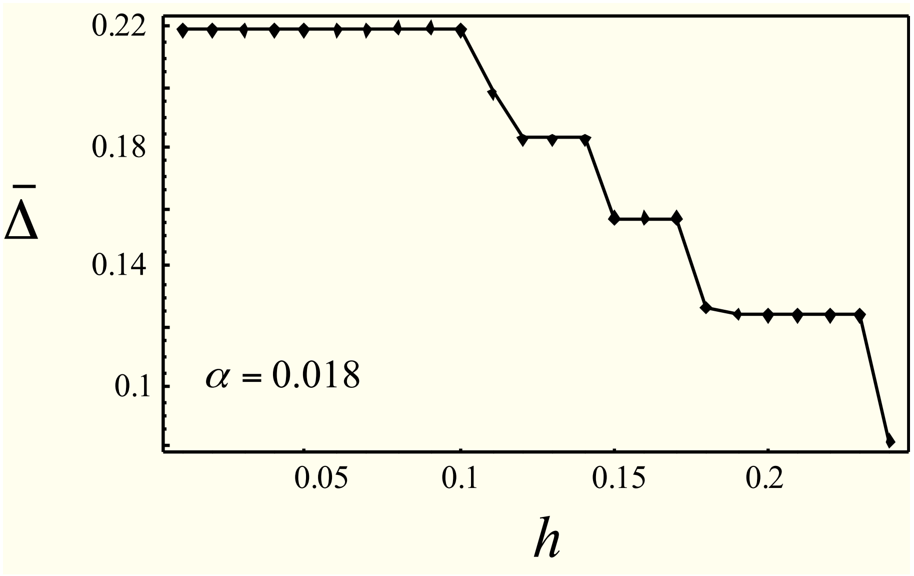
$$\Delta_i = \frac{|U|}{2} \sum_{n\sigma} u_{n\sigma}(\mathbf{r}_i) v_{n\sigma}^*(\mathbf{r}_i) \;,\; \langle n_i \rangle = \sum_{n\sigma} \left| v_{n\sigma}(\mathbf{r}_i) \right|^2$$

reminder : the Clogston-Chandrasekhar critical field (uniform system)

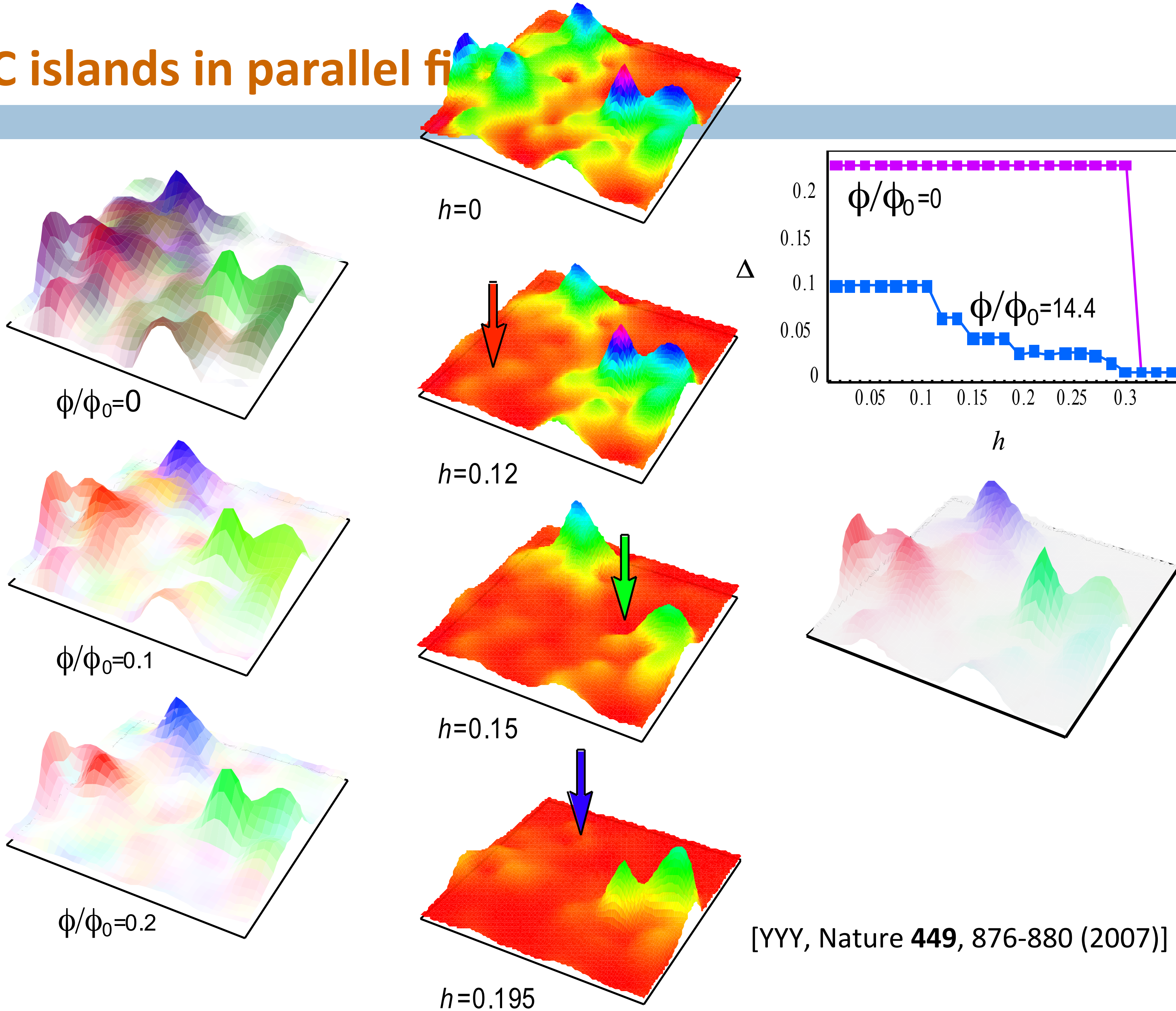
$$h_c = \frac{\Delta_0}{\sqrt{2}} \;,\; \Delta(h, T=0) = \Delta_0 \theta(h - h_c)$$

SC islands in parallel field



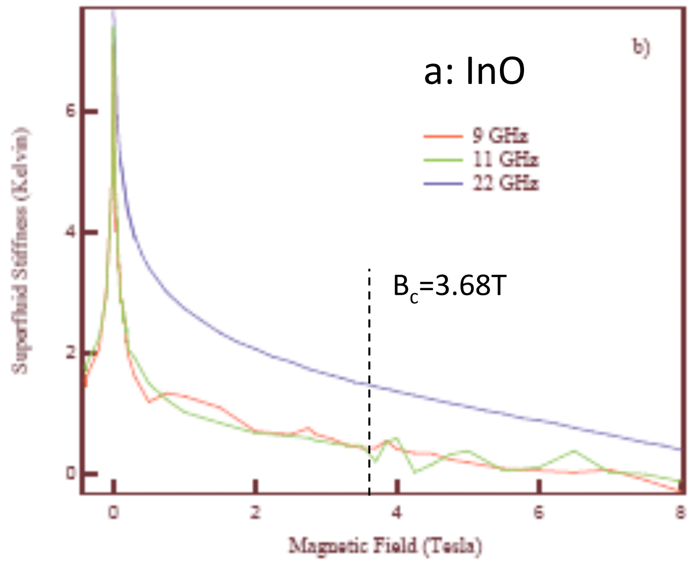


SC islands in parallel fi



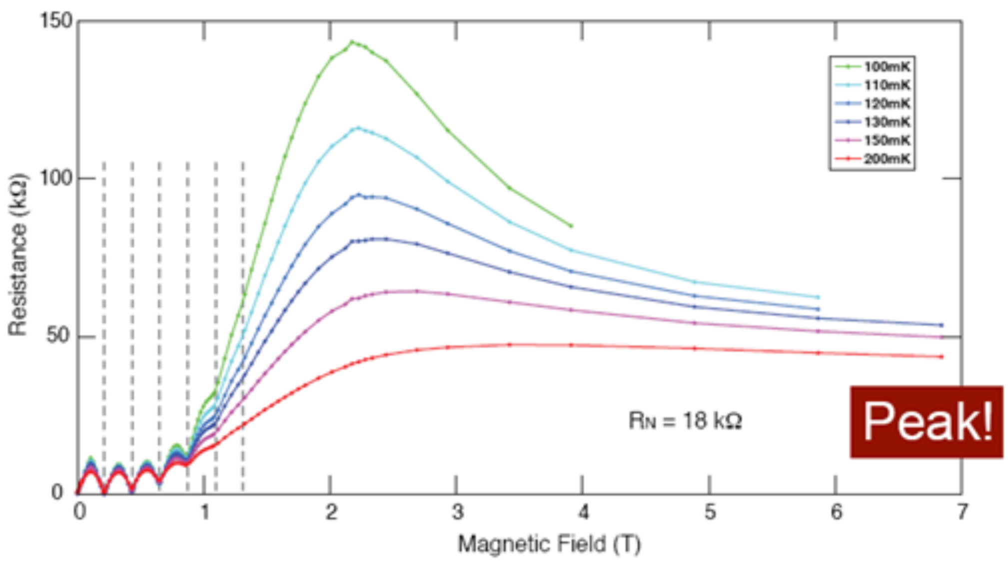
[YYY, Nature **449**, 876-880 (2007)]

B-induced SIT : support from experiment



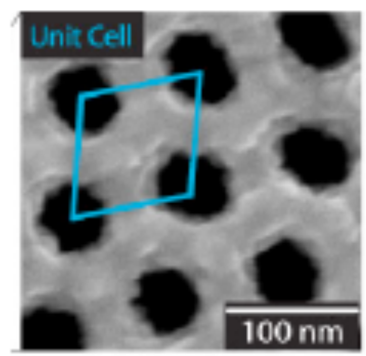
[Crane *et.al.*,

PRB **75**, 184530 (2007)[

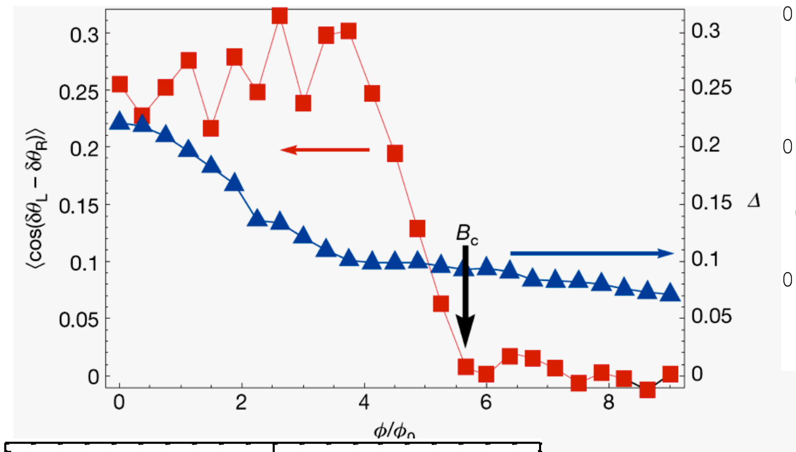


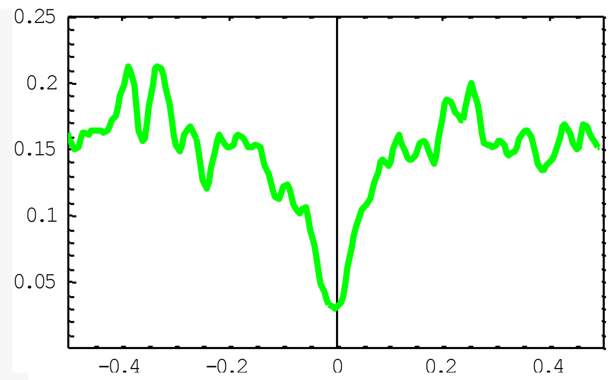
. Q. Nguyen, J. M. Valles, Jr.. *et al.*, Cond-

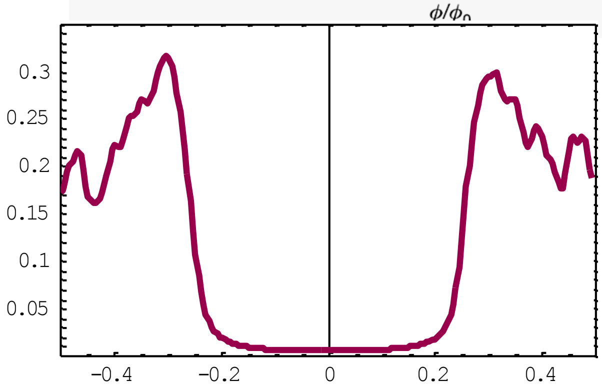
at/0907.4120 (2009)]

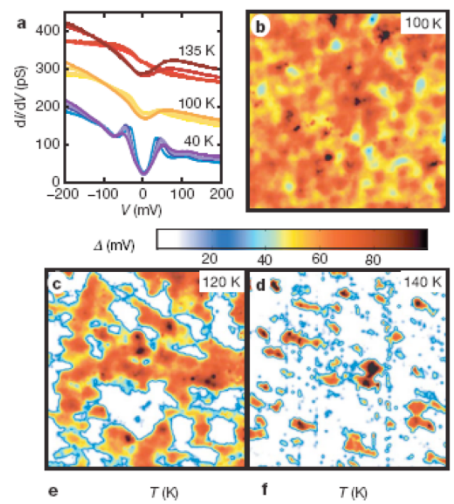


B-induced SIT – pseudogap?



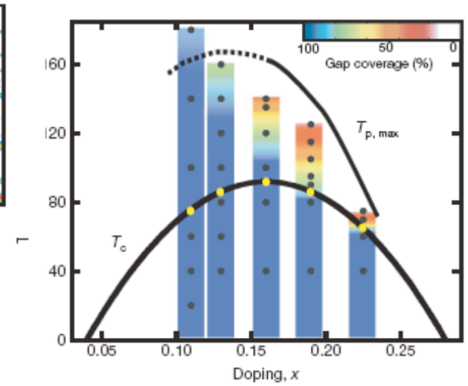






Gomez et al., Nature (2007)

Gaps persist to well above the critical temperature



[YYY, Nature **449**, 876-880 (2007)]

Outline

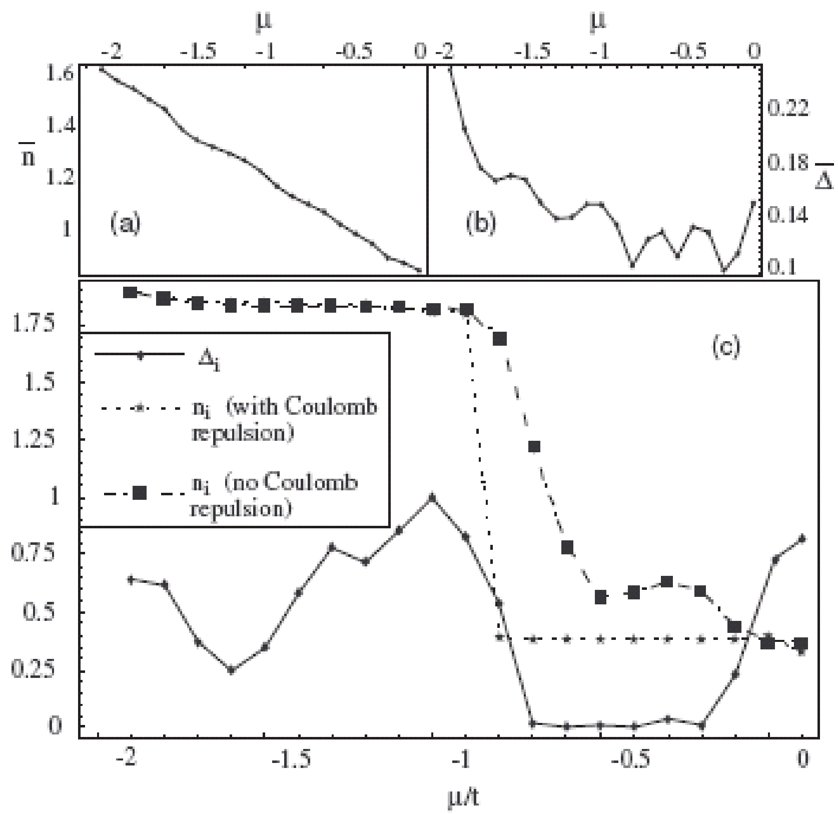
- How phase fluctuations drive the B-SIT
- How interactions drive the MR peak
- How thickness drives the T-SIT

Back to Magneto-Resistance

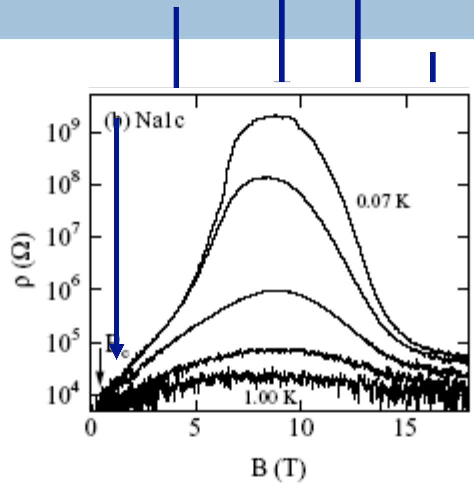
1. The existence of SC islands due to disorder

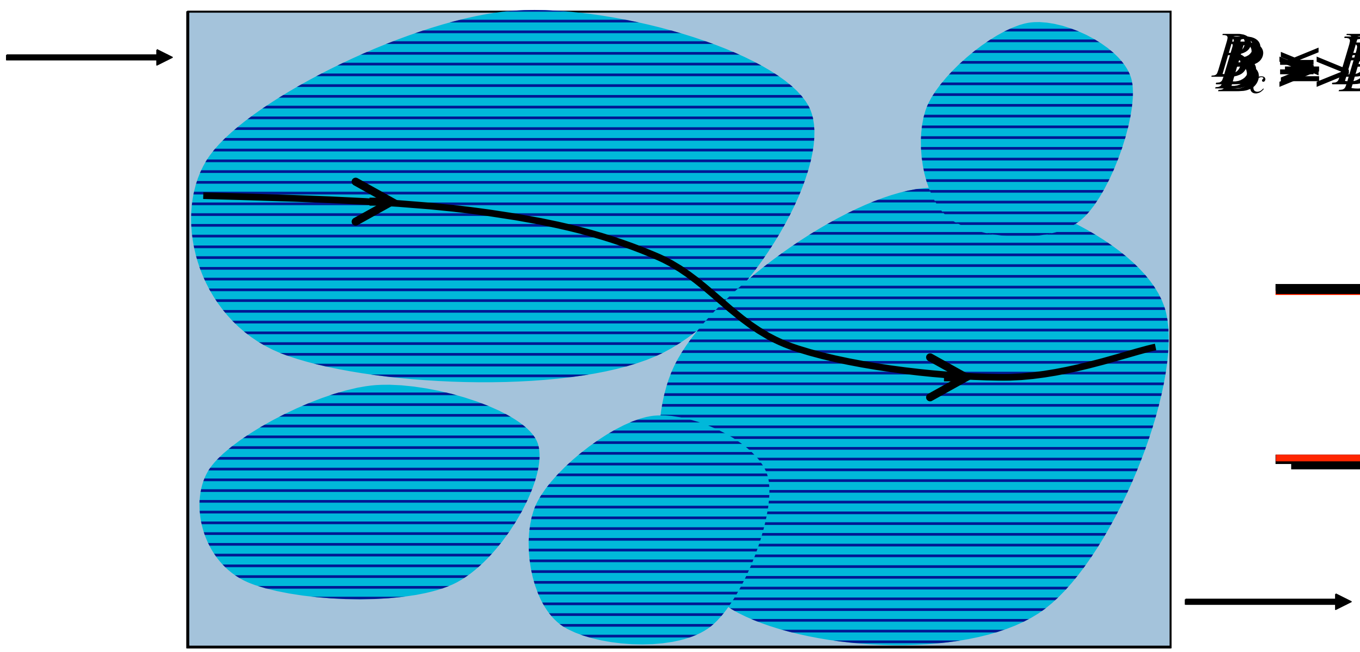
2. A decrease in island size and concentration with magnetic field.

3. These islands have charging energy (and tunnel barrier).



SC Islands and Magneto-Resistance





$B < B_{max}$

$\sim \gamma_A \exp\left(\frac{E_c}{T}\right)$

$\sim \exp\left(\left(\frac{T_0}{T}\right)^{1/(d+1)}\right)$

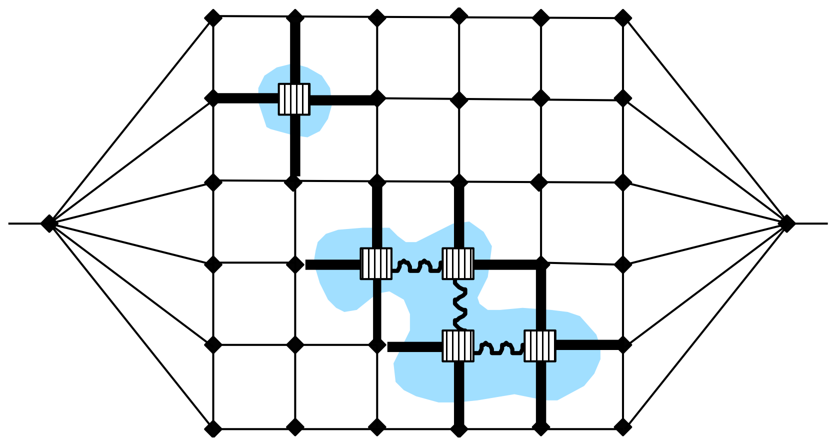
SC Islands and Magneto-Resistance

Numerical Calculations :

disordered system with superconducting sites

- ● Normal site (probability p)
- ||| Superconducting site (probability 1-p)

  p=*p*(*B*)



$$G_{SS} \sim F(T) \xrightarrow{T \to 0} \infty$$

$$G_{NN} \sim \exp\left\{-\left(|\varepsilon_i - \mu| + |\varepsilon_j - \mu| + |\varepsilon_i - \varepsilon_j|\right)/2kT - 2r_{ij}/\xi_{loc}\right\}$$

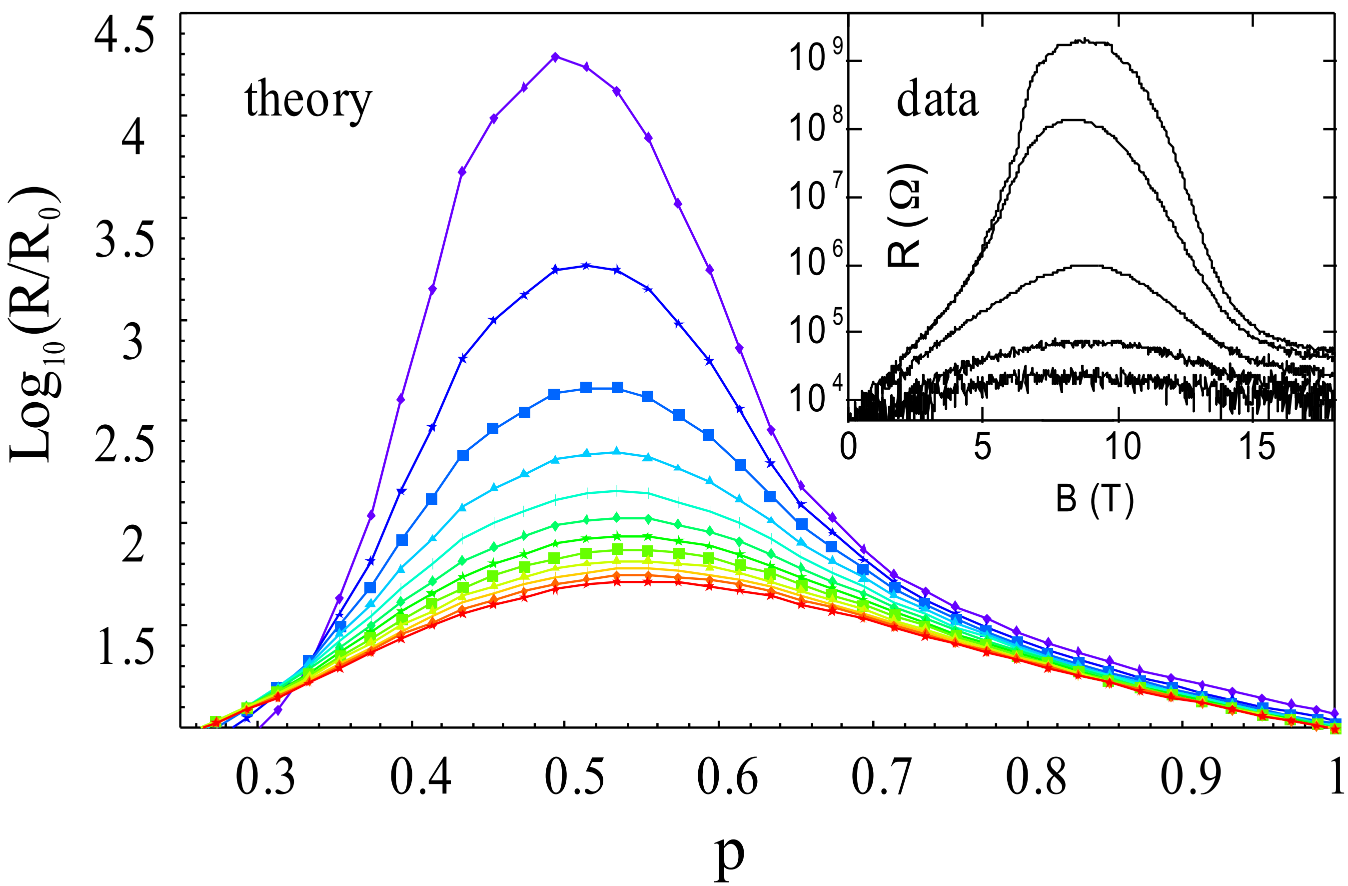
[Miller & Abrahams, Phys. Rev. **120**, (1960)]

$$G_{NS} \sim \exp\left\{-\frac{E_c}{T}\right\}$$

(Nearest neighbors only)

SC Islands and Magneto-Resistance

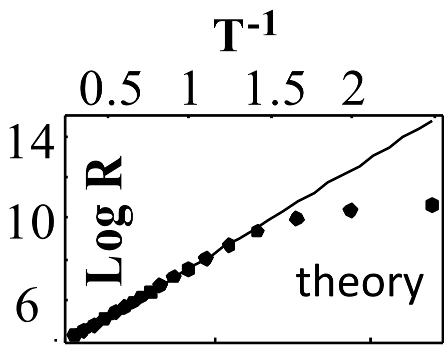
results: magneto-resistance

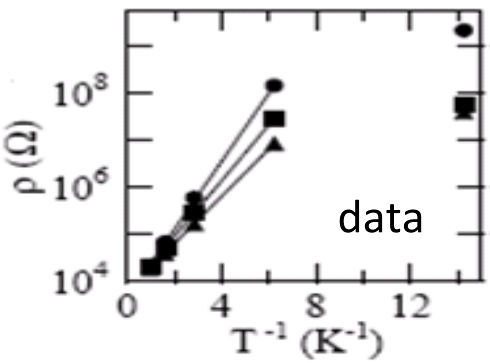


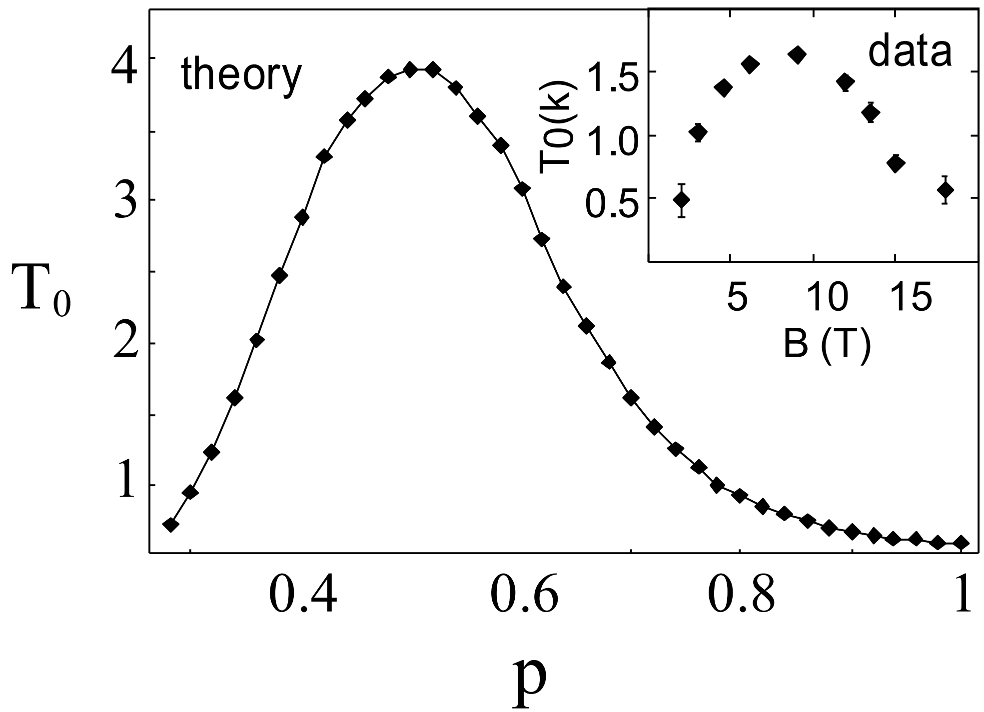
[YD, Y. Meir and Y. Avishai, PRB **73**, 54509 (2005)]

SC Islands and Magneto-Resistance

results: activation energy

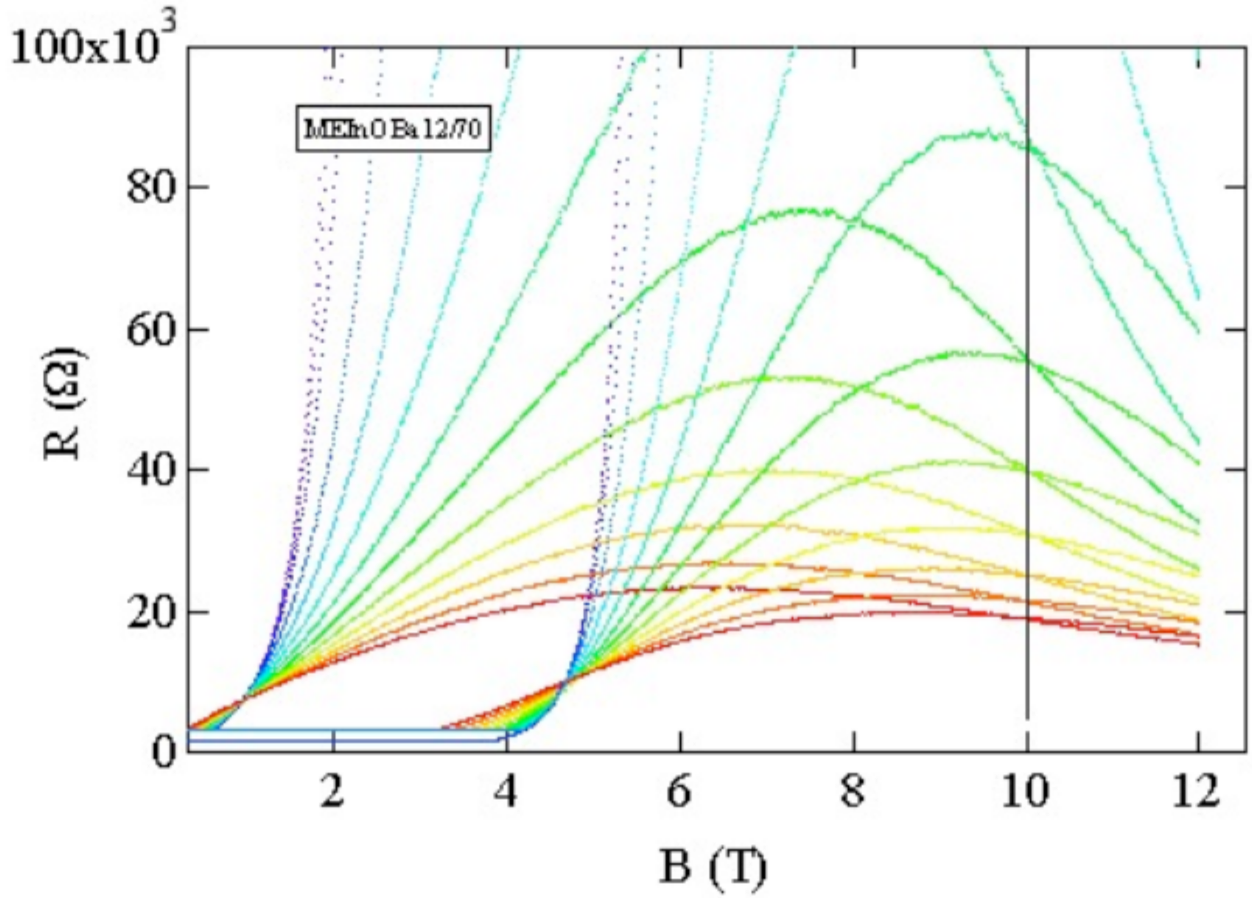






SIT from parallel field

Johansson et al., unpublished



Tilted magnetic field: Phenomenology

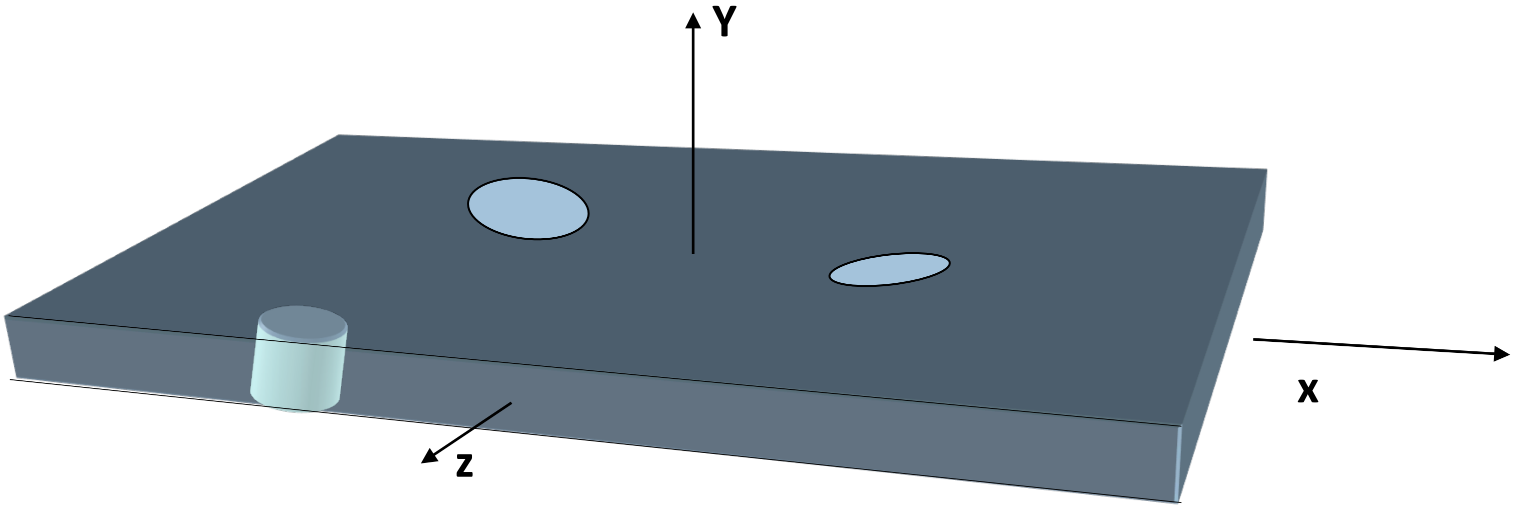
Can this be explained by an orbital effect ?

Film thickness typically w~200Å~10ξ

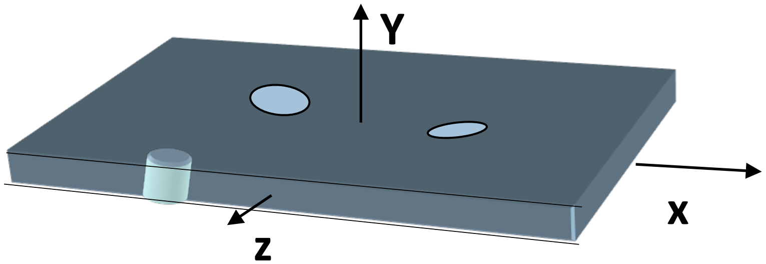
What is the effect of a tilted field ?

$B_\perp$ affects size in the x-z plane

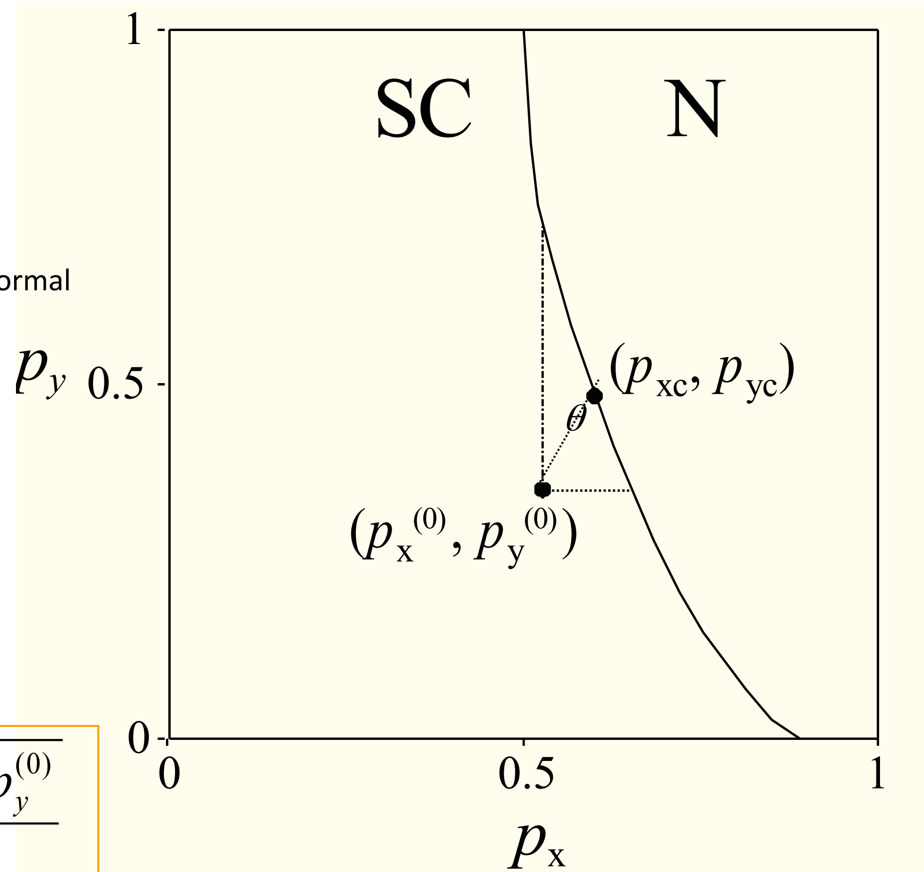
$B_{//}$ affects size in the y-z plane



Tilted magnetic field: Phenomenology



[Y. Meir & A. Aharony]



- construct the p_x-p_y phase diagram

  p_i – probability of a link in the **i**-direction to be normal

- the field-dependence is approximately

$$p_x(B) = p_x^{(0)} + \alpha B_\perp^2$$
$$p_y(B) = p_y^{(0)} + \alpha' B_\parallel^2$$

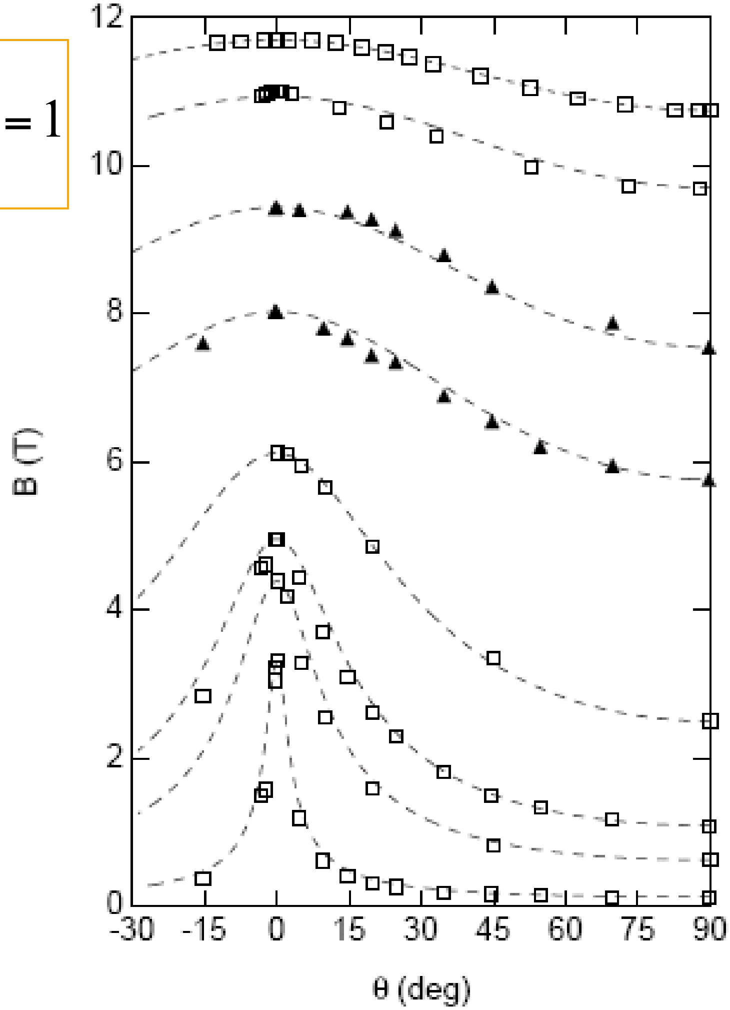
- and thus the critical fields are

$$B_{\perp c} = \sqrt{\frac{p_{xc} - p_x^{(0)}}{\alpha}}\ , \quad B_{\parallel c} = \sqrt{\frac{p_{yc} - p_y^{(0)}}{\alpha'}}$$

- assuming a linear critical line yields

Tilted magnetic field: Phenomenology

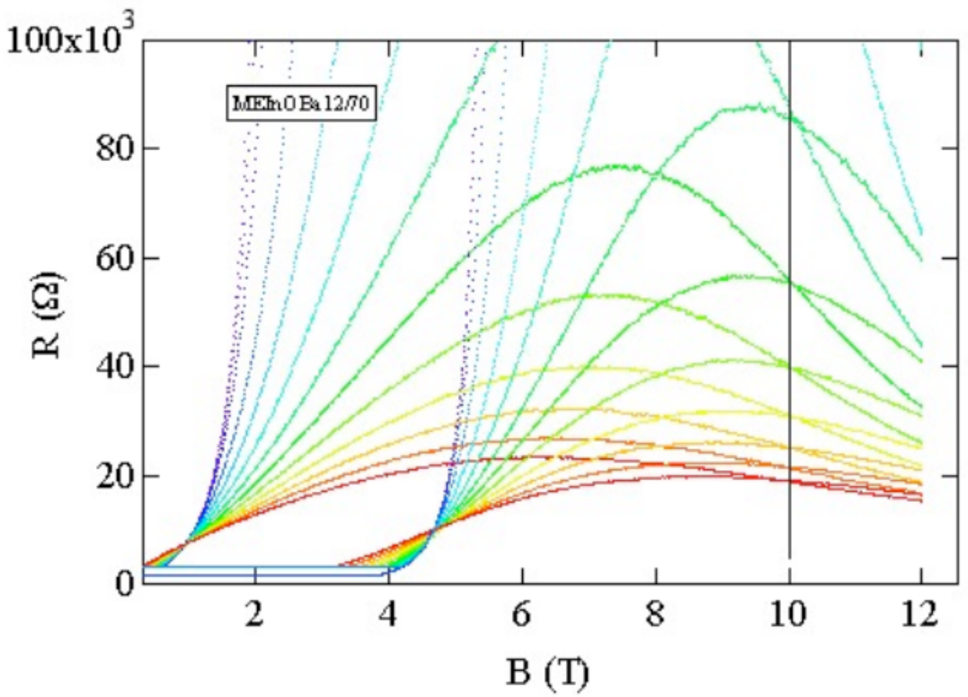
$$\frac{B_c^2 \cos^2 \theta}{B_{\perp c}^2} + \frac{B_c^2 \sin^2 \theta}{B_{\parallel c}^2} = 1$$

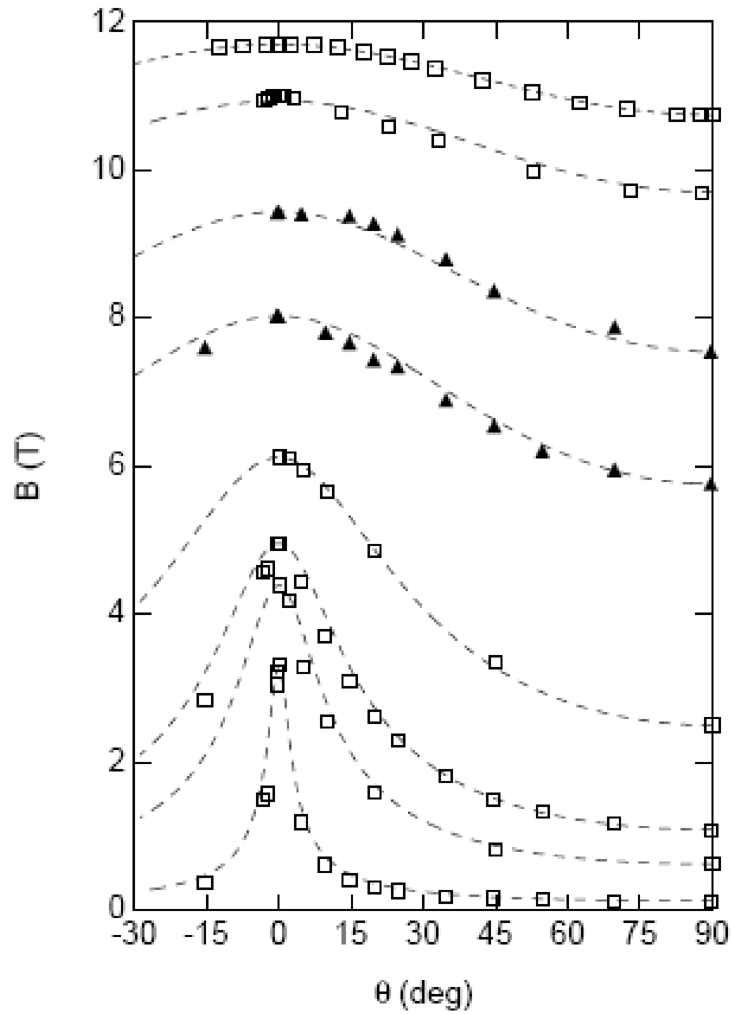


A. Johanssen et.al (cond-mat/0602160)

SC Islands and Magneto-Resistance – More Support

A. Johanssen et.al (cond-mat/0602160)



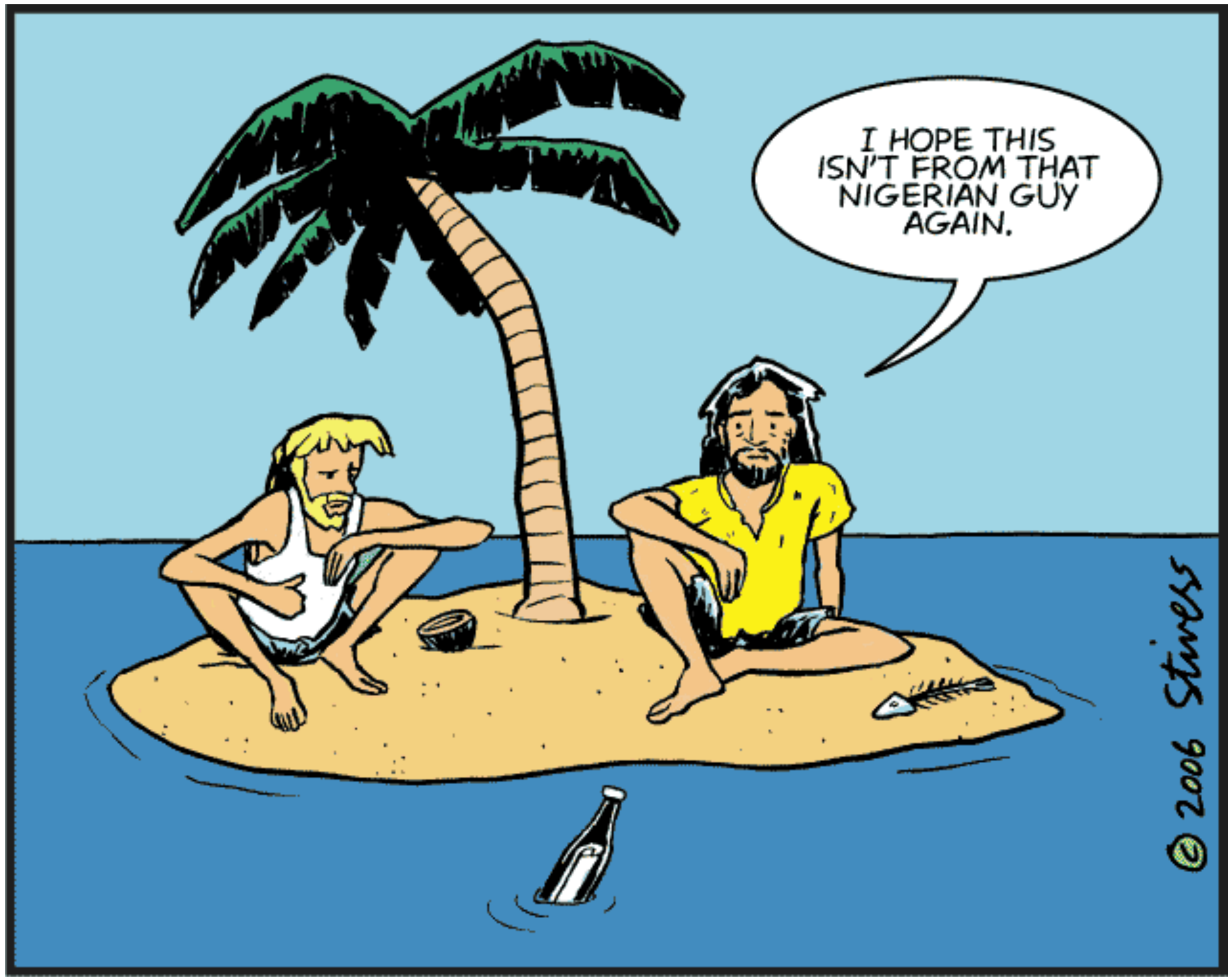


[Y. Meir & A. Aharony]

$$\frac{B_c^2 \cos^2\theta}{B_{\perp c}^2} + \frac{B_c^2 \sin^2\theta}{B_{\parallel c}^2} = 1$$

(almost) Summary

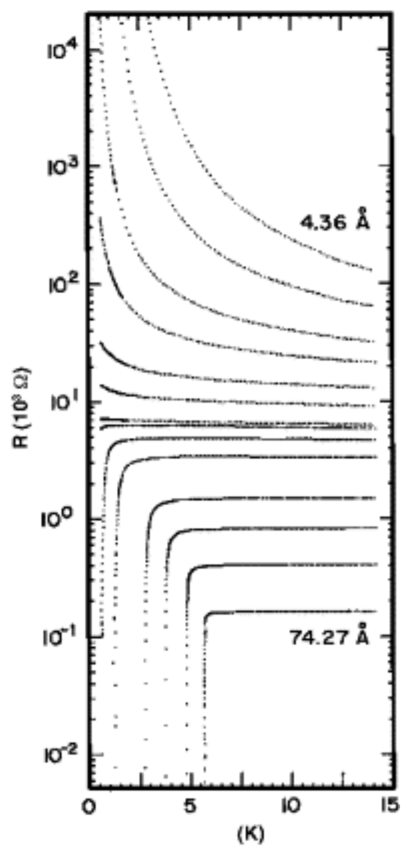
The SIT and the MR peak can be explained by the emergence of SC islands w/ phase fluctuations, interactions

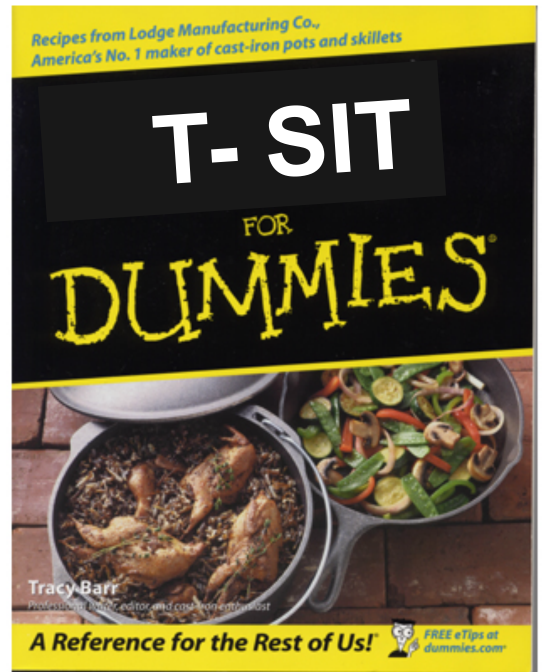


Outline

- How phase fluctuations drive the B-SIT
- How interactions drive the MR peak
- How thickness drives the T-SIT

The SIT in Finite thickness

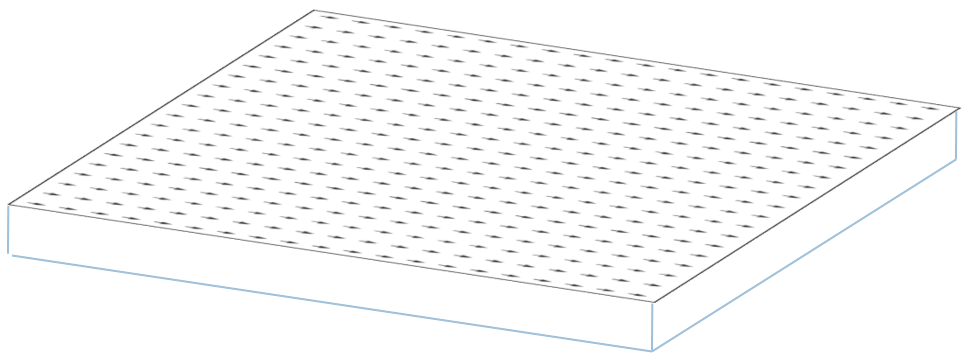




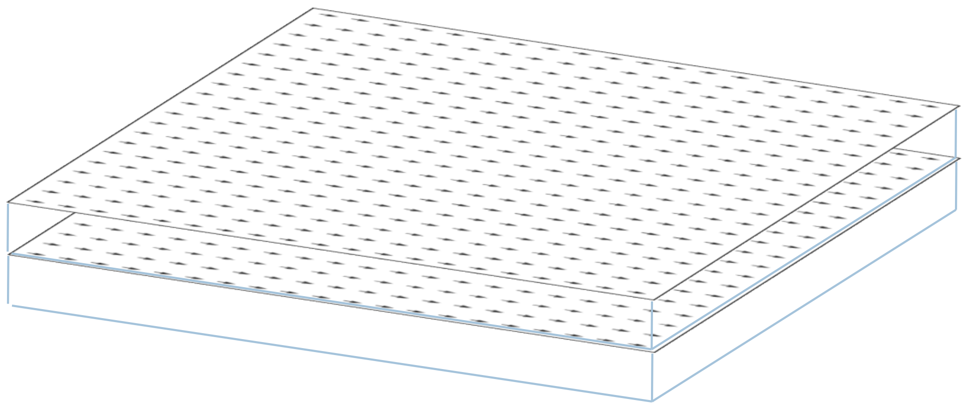
A simple different way to *think* about the SIT

[D.B. Haviland, Y. Liu, A.M. Goldman, Phys. Rev. Lett. 62 2180 (1989)]

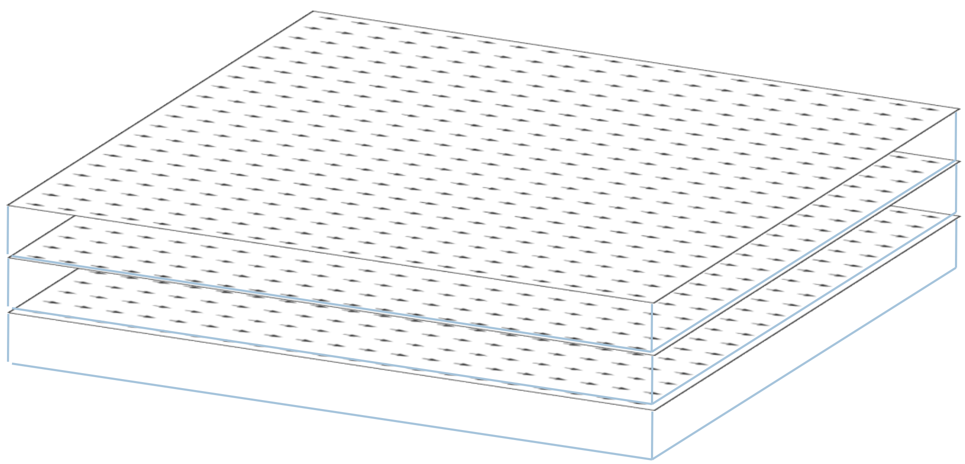
The SIT in Finite thickness – assume KTB transition



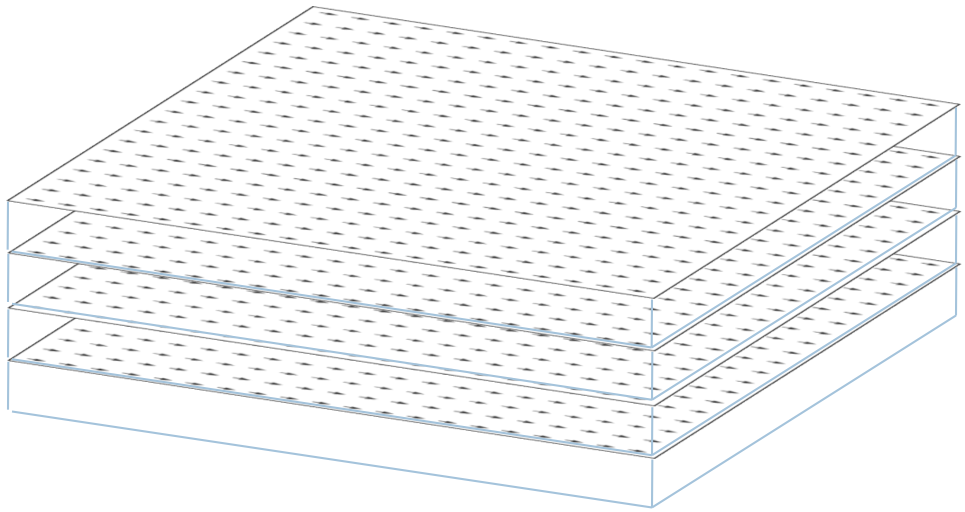
The SIT in Finite thickness – assume KTB transition



The SIT in Finite thickness – assume KTB transition



The SIT in Finite thickness – assume KTB transition



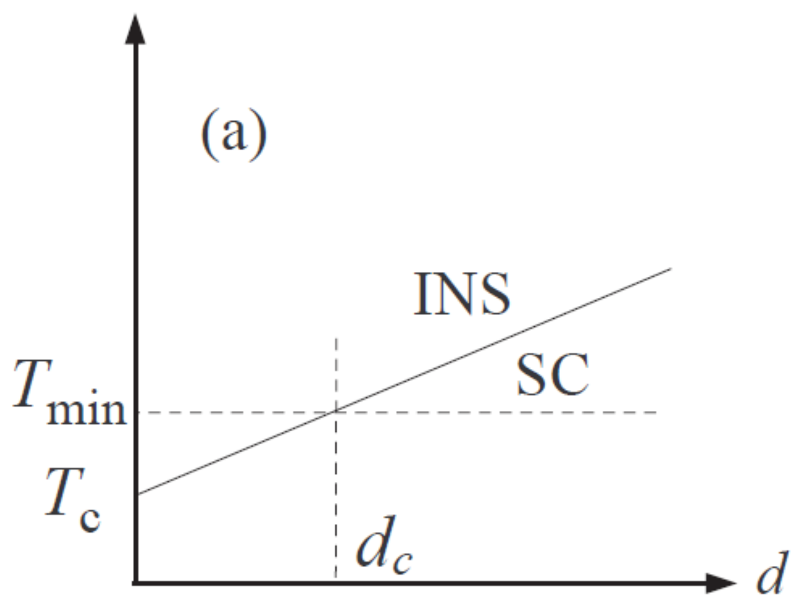
$$K(d) \simeq \frac{1}{3} K_0 \delta$$

The SIT in Finite thickness

$$K(d) \simeq \frac{1}{3} K_0 \delta \longrightarrow T_c \sim T_{c,0}\, d$$

Quantum Fluctuations ⟷

T_Q



The SIT in Finite thickness

$$K(d) \simeq \frac{1}{3} K_0 \delta$$

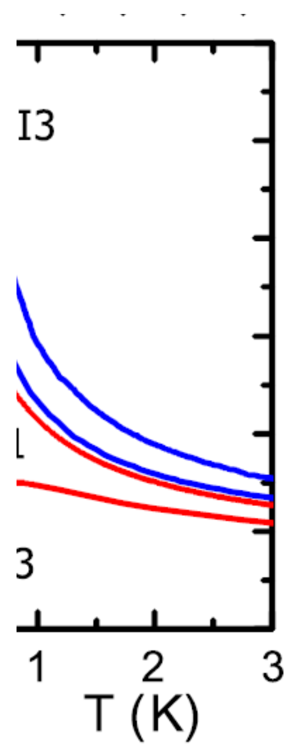
Implications:

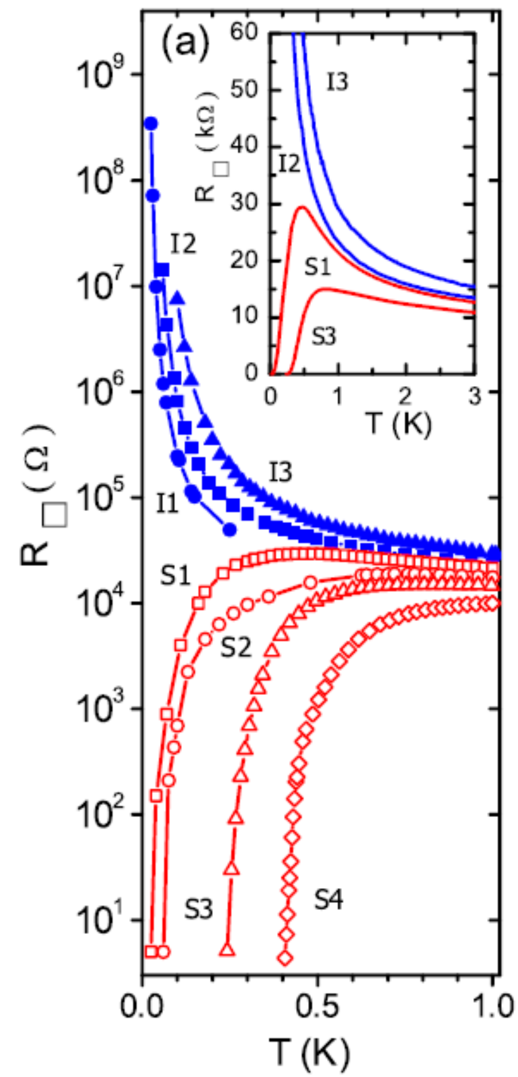
1. The T-SIT @

$$d_c = \frac{6}{\pi} \frac{a}{K_0} T_{\min}$$

$$= \frac{6}{\pi} \frac{a}{K_0} T_{\text{Quantum fluctuations}}$$

[Baturina *et al*, PRL **99,** 257003 (2007)]





The SIT in Finite thickness

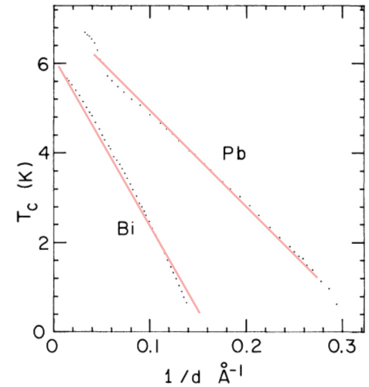
$$K(d) \simeq \frac{1}{3} K_0 \delta$$

2. Dependence of *Tc* on *d*: (thicker films)

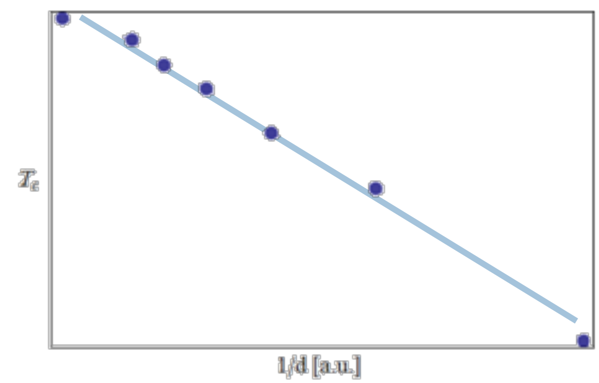
$$K_0 \sim \Delta^2 \sim (T_{c,0} - T)$$

$$K(T_c) = \frac{2}{\pi} T_c \Rightarrow \gamma (T_{c,0} - T_c)\delta = T_c$$

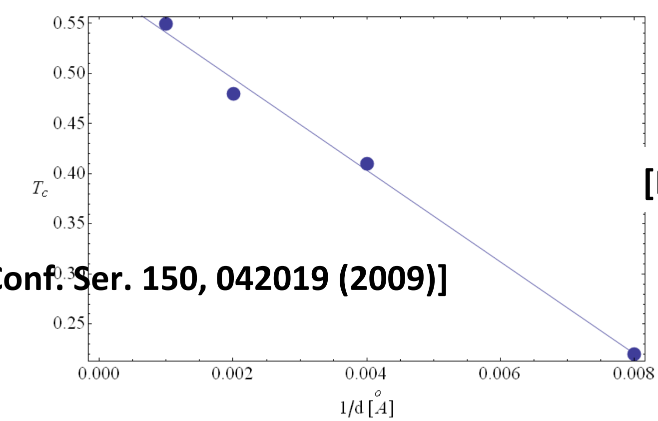
$$T_c \approx T_{c,0}(1 - \frac{1}{\gamma} d^{-1})$$



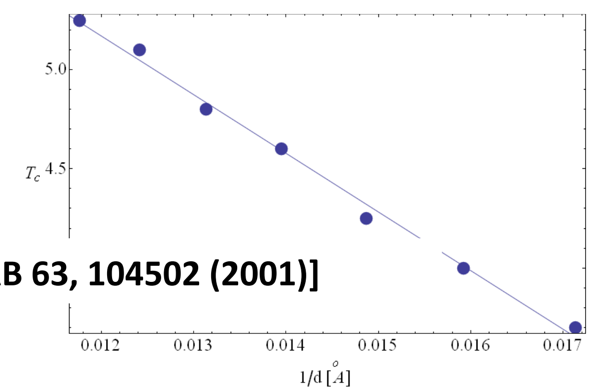
[D. B. Haviland *et al.*,
Phys. Rev. Lett. 62, 2180 (1989)]



[PRB 48, 10498 (1993)]



[J. phys Conf. Ser. 150, 042019 (2009)]



[PRB 63, 104502 (2001)]

Summary

- B-SIT can be by the emergence of SC islands+ *Phase Fluctuations*
- for MR peak – just add interactions
- Phase-Fluctuations may account for the T-SIT : scaling of phase stiffness with thickness

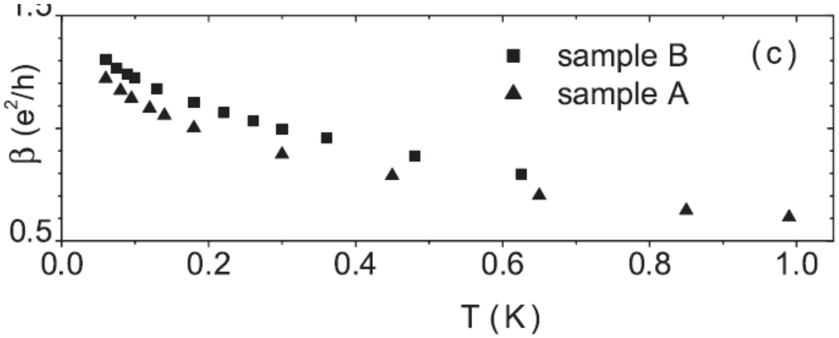
Thank you for listening

תודה על ההקשבה

Many mysteries left

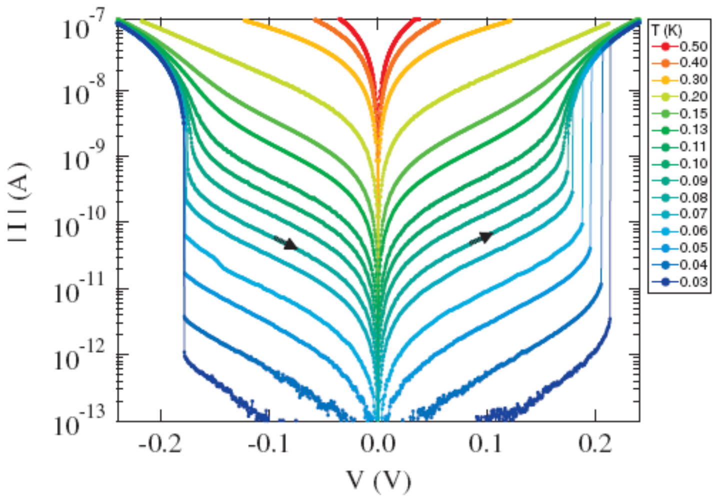
- Is the true ground-state insulating ?

- Is there a high-field metallic state?



[Baturina *et al*, PRL **98**, 127003 (2007)]

- What is the nature of the e-ph coupling in amorphous systems?



[Ovadia *et al*, PRL **102**, 176802(2007)]

Thank you for listening

תודה על ההקשבה

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Application to the T-SIT

$$K(d) \simeq \frac{1}{3} K_0 \delta$$

2. Dependence of *Tc* on *d*:

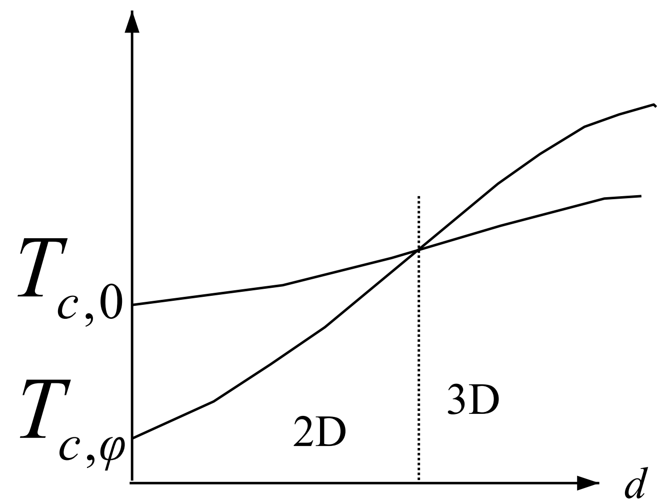
$$K_0 \sim \Delta^2 \sim (T_{c,0} - T)$$

$$K(T_c) = \frac{2}{\pi} T_c \Rightarrow \gamma (T_{c,0} - T_c)\delta = T_c$$

↓

$$T_c \approx T_{c,0}(1 - \frac{1}{\gamma}\delta^{-1})$$

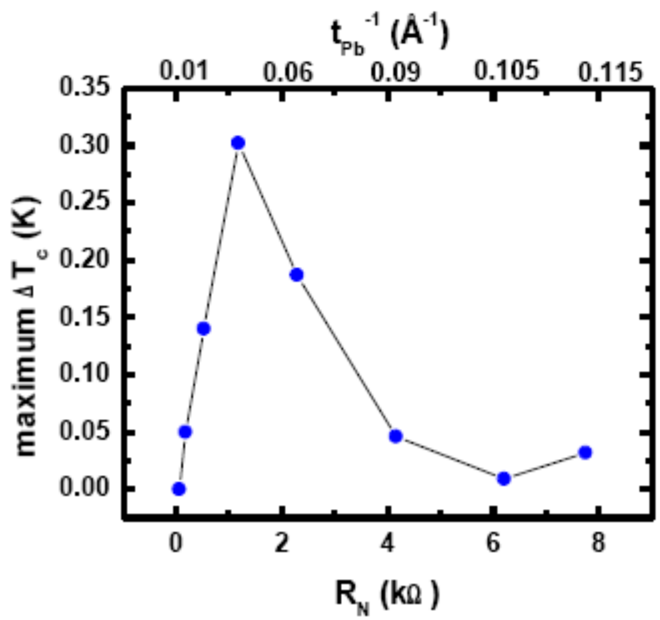
3. Dimensional crossover :

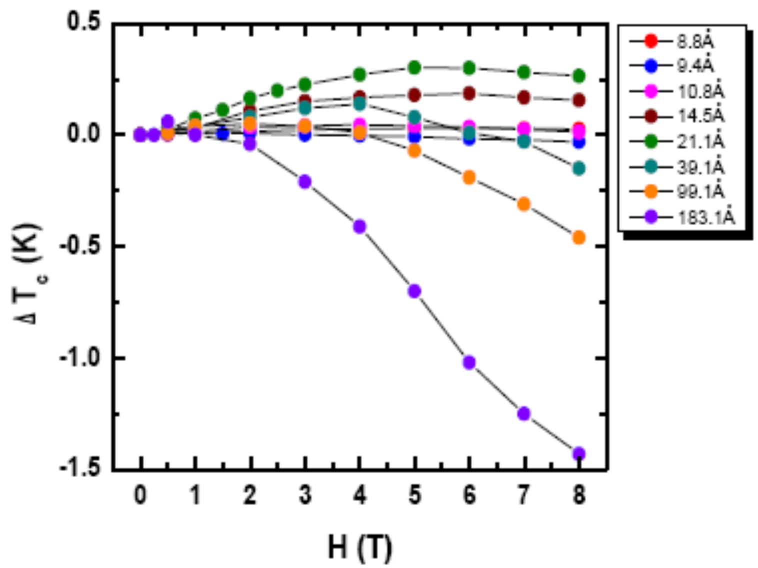


[D.B. Haviland *et al*,
Phys. Rev. Lett. **62,** 2180 (1989)]

Finite thickness: Two experiments

B-induced Tc enhancement

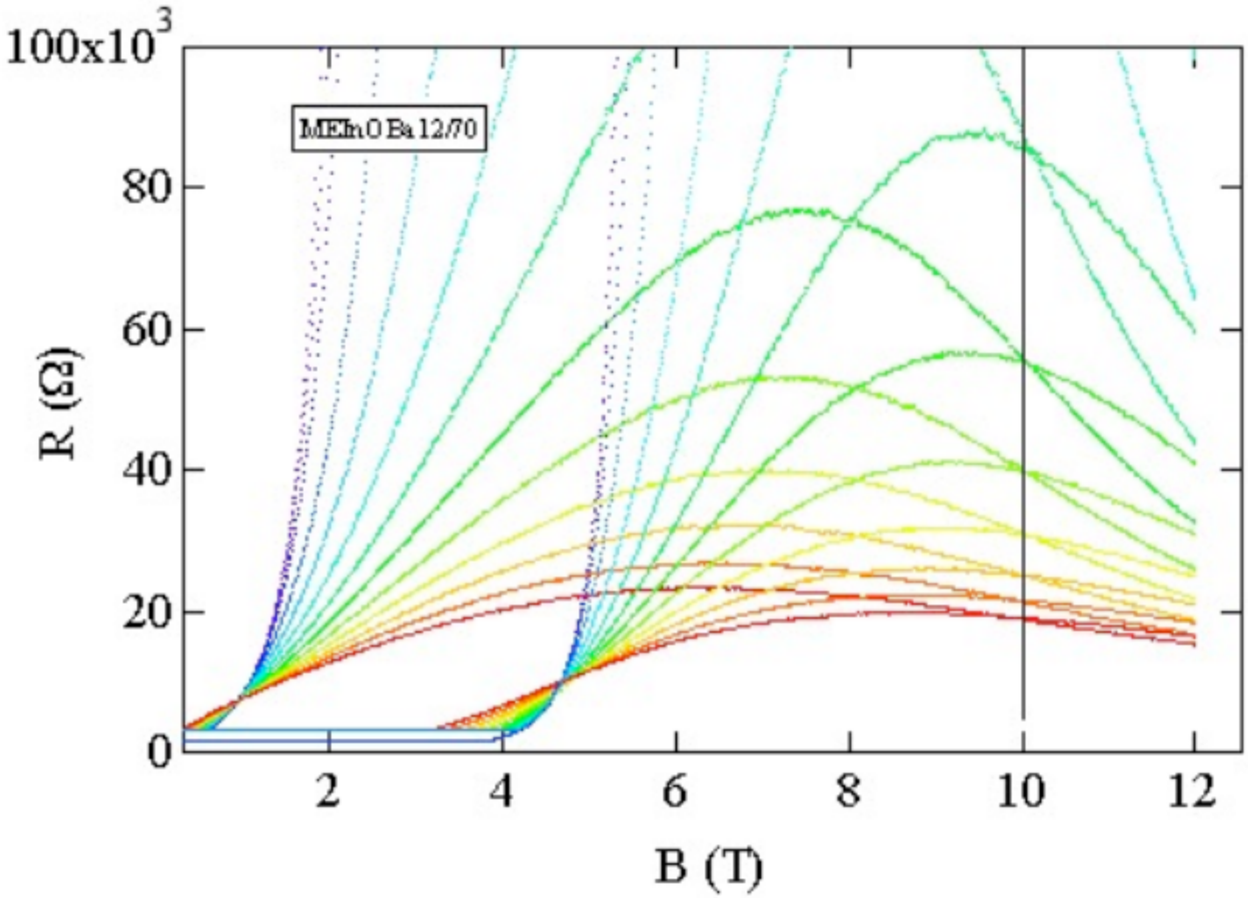




[H. J. Gardner, L. Yu, A. Kumar and Peng Xiong,
http://meetings.aps.org/link/BAPS.2009.MAR.W33.11
http://meetings.aps.org/link/BAPS.2009.MAR.W33.05]

SIT from parallel field

Johansson et al., unpublished



Tilted magnetic field: Phenomenology

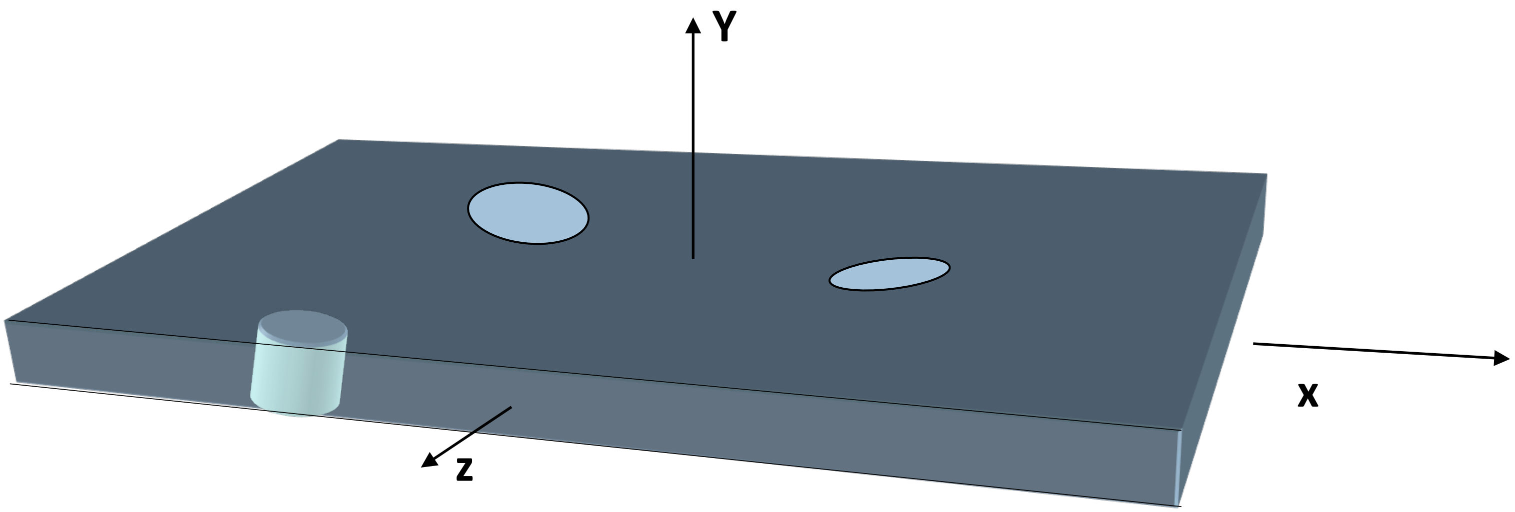
Can this be explained by an orbital effect ?

Film thickness typically w~200Å~10ξ

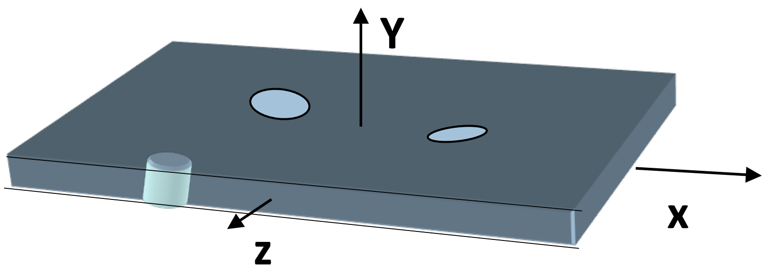
What is the effect of a tilted field ?

$B_{\perp}$ affects size in the x-z plane

$B_{//}$ affects size in the y-z plane



Tilted magnetic field: Phenomenology



[Y. Meir & A. Aharony]

- construct the p_x-p_y phase diagram

  p_i – probability of a link in the **i**-direction to be normal

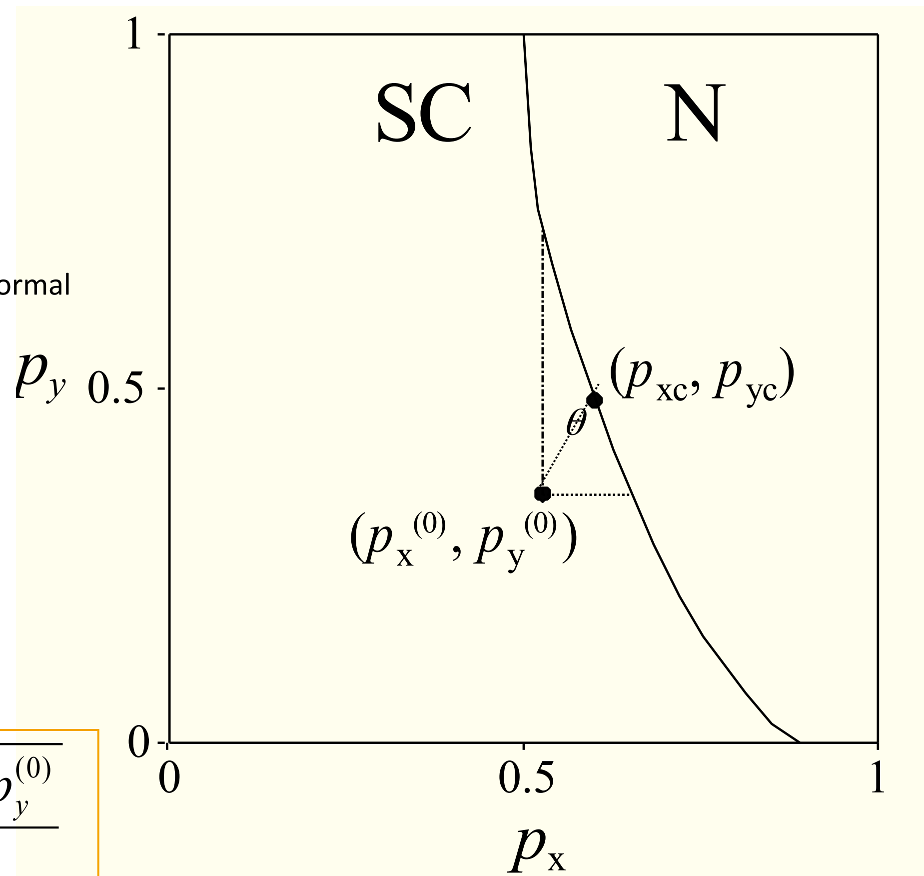
- the field-dependence is approximately

$$p_x(B) = p_x^{(0)} + \alpha B_\perp^2$$
$$p_y(B) = p_y^{(0)} + \alpha' B_\parallel^2$$

- and thus the critical fields are

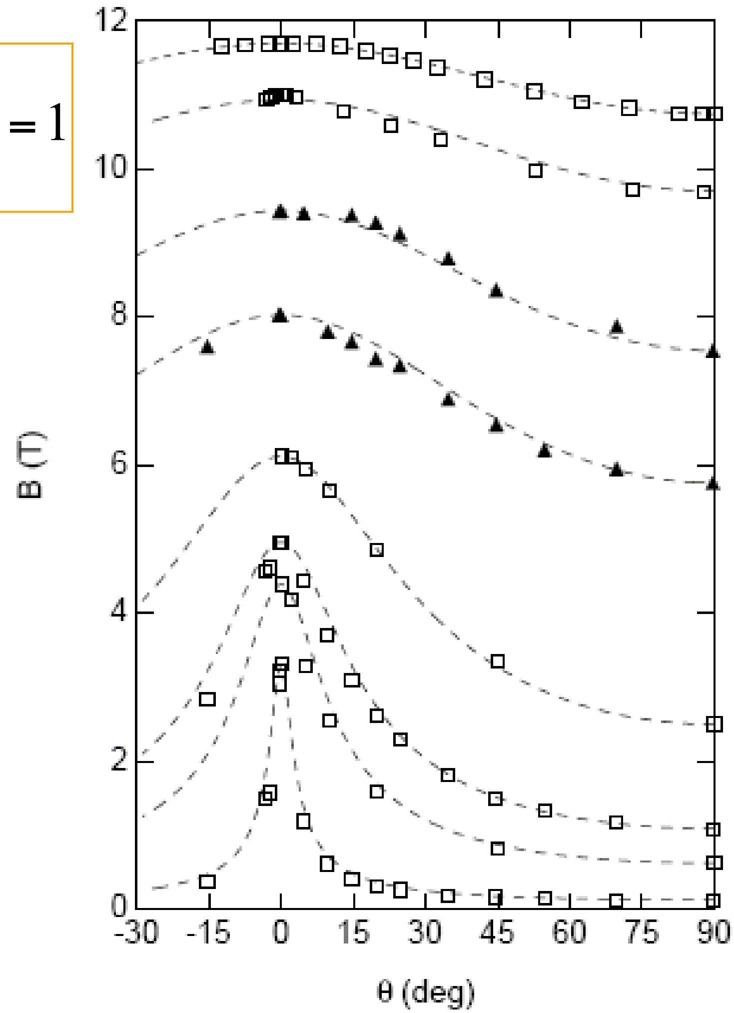
$$B_{\perp c} = \sqrt{\frac{p_{xc} - p_x^{(0)}}{\alpha}} \;,\quad B_{\parallel c} = \sqrt{\frac{p_{yc} - p_y^{(0)}}{\alpha'}}$$

- assuming a linear critical line yields



Tilted magnetic field: Phenomenology

$$\frac{B_c^2 \cos^2\theta}{B_{\perp c}^2} + \frac{B_c^2 \sin^2\theta}{B_{\parallel c}^2} = 1$$



A. Johanssen et.al (cond-mat/0602160)

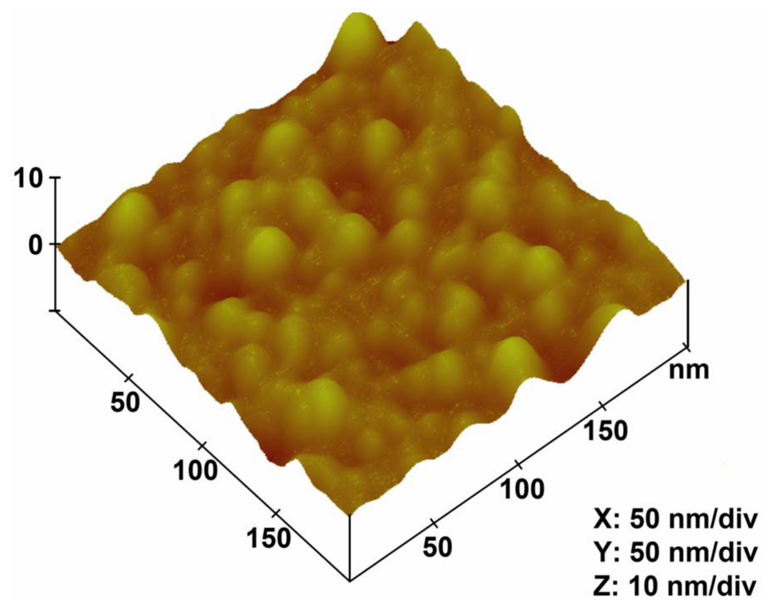
Model for ultra-thin layers

We assume strong surface roughness:

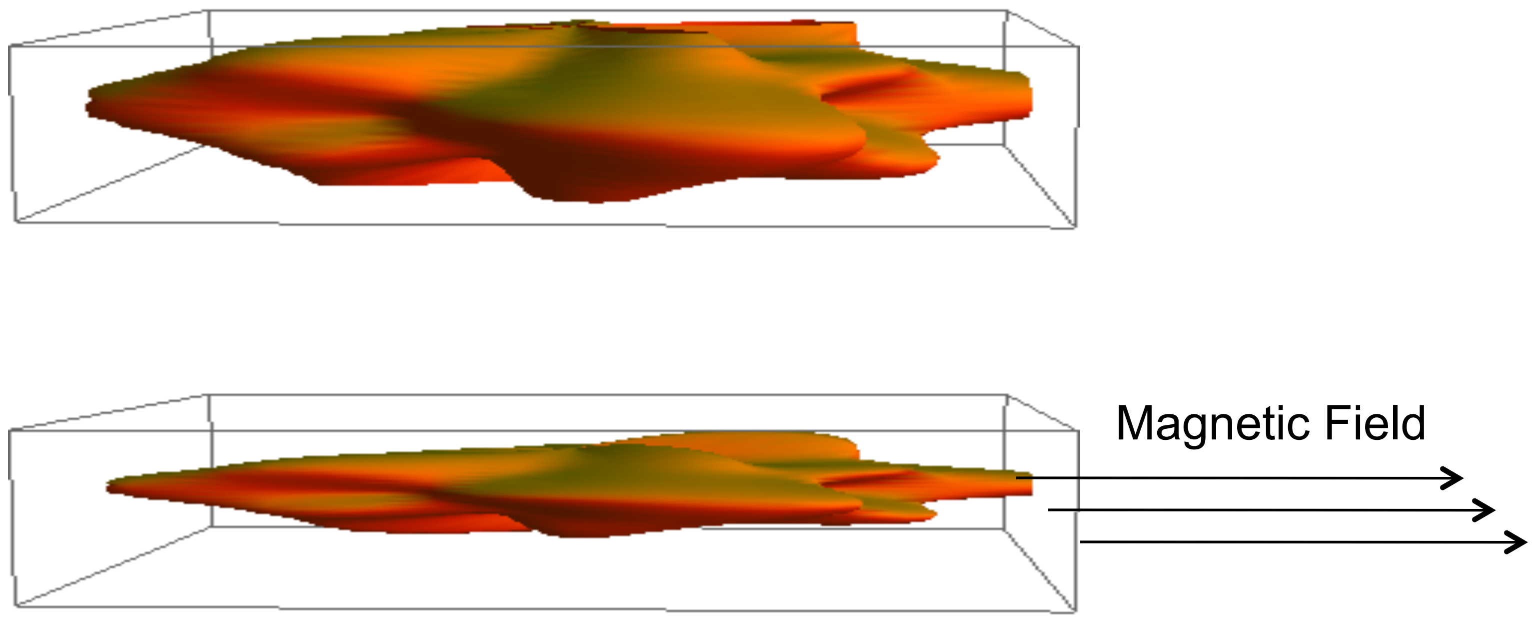
PHYSICAL REVIEW B **78**, 014506 (2008)

Evidence of spatially inhomogeneous pairing on the insulating side of a disorder-tuned superconductor-insulator transition

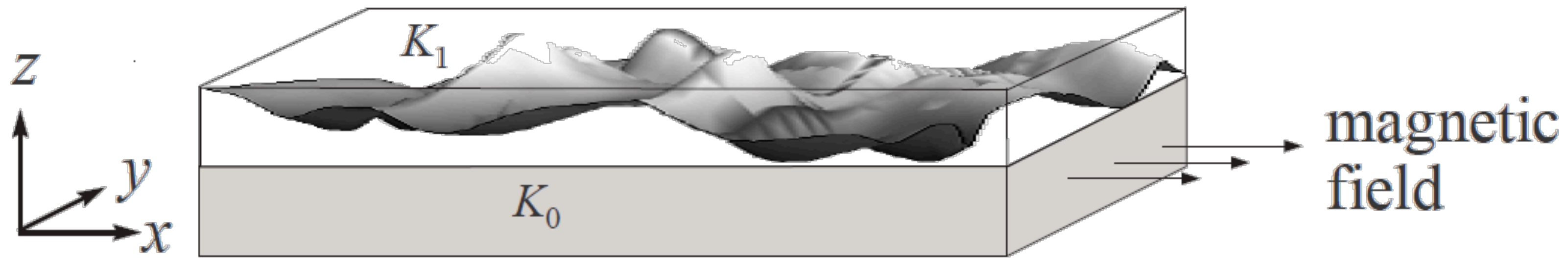
K. H. Sarwa B. Tan, Kevin A. Parendo, and A. M. Goldman
School of Physics and Astronomy, University of Minnesota, 116 Church Street SE, Minneapolis, Minnesota 55455, USA
(Received 15 June 2008; published 10 July 2008)



Model for ultra-thin layers

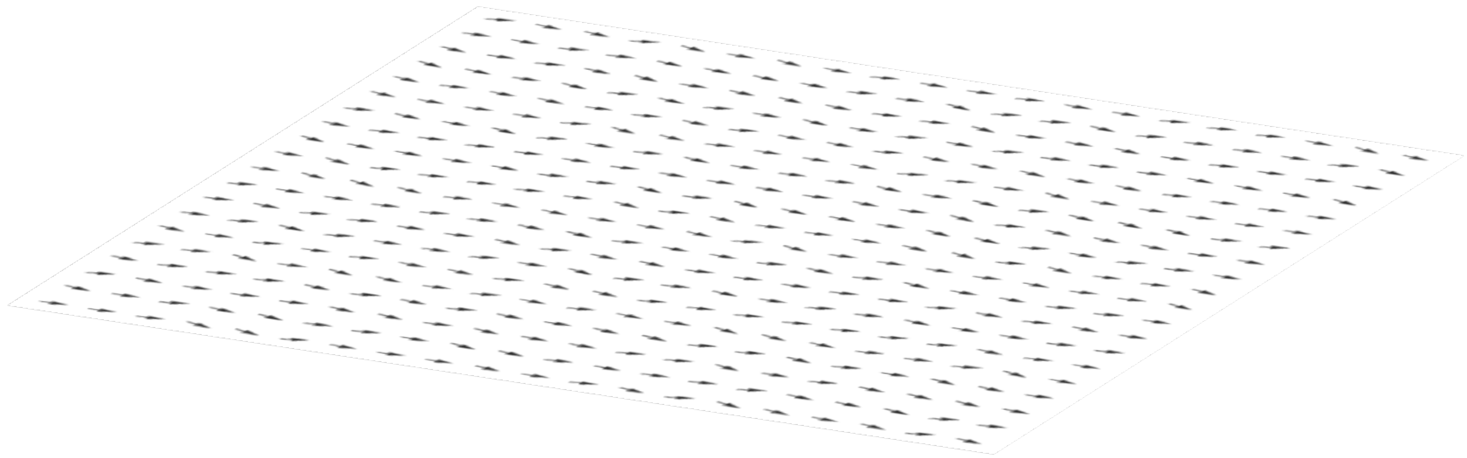


Model for ultra-thin layers

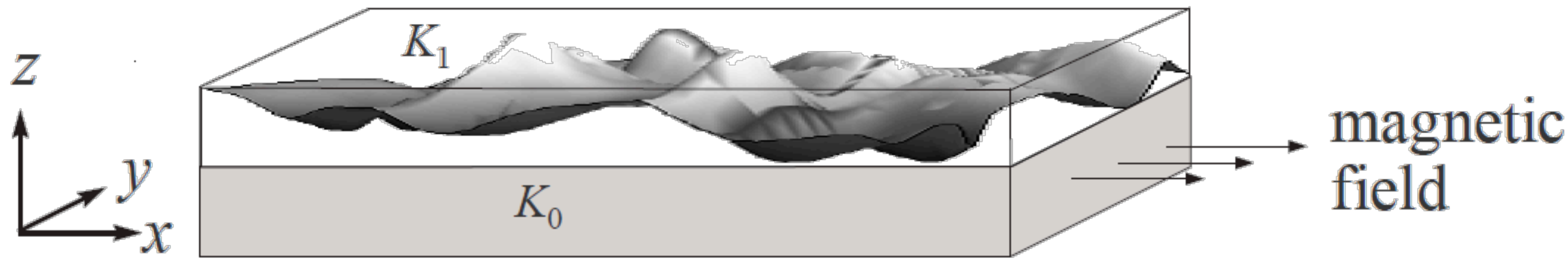


Assumption : this is a KT transition

T=0.2

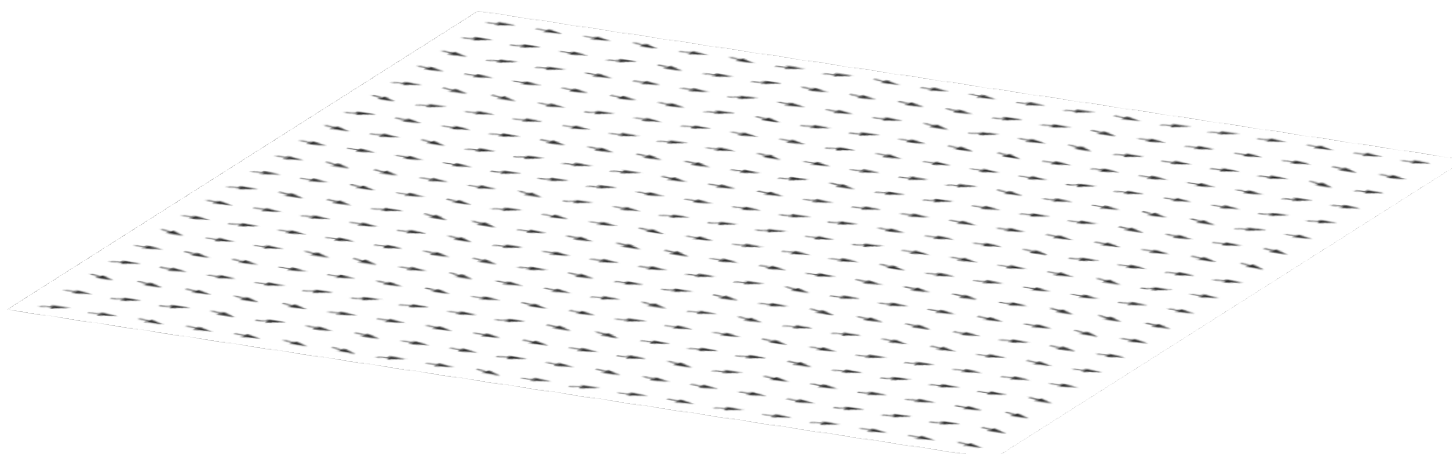


Model for ultra-thin layers

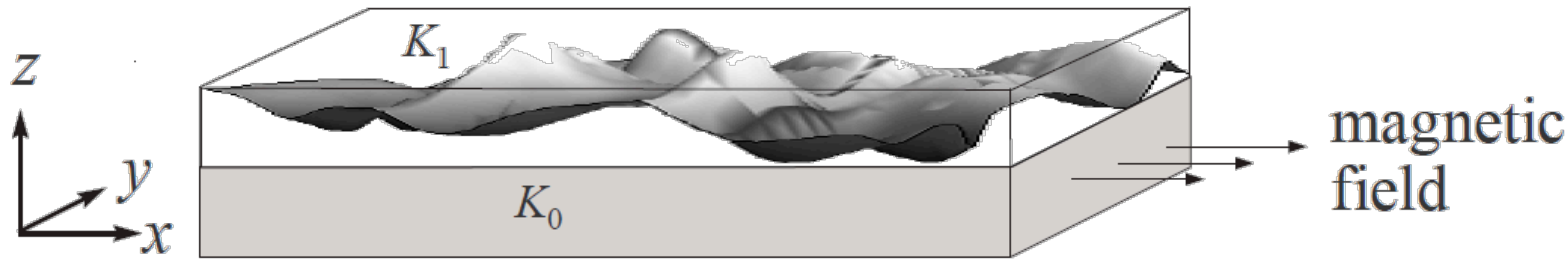


Assumption : this is a KT transition

T=0.4

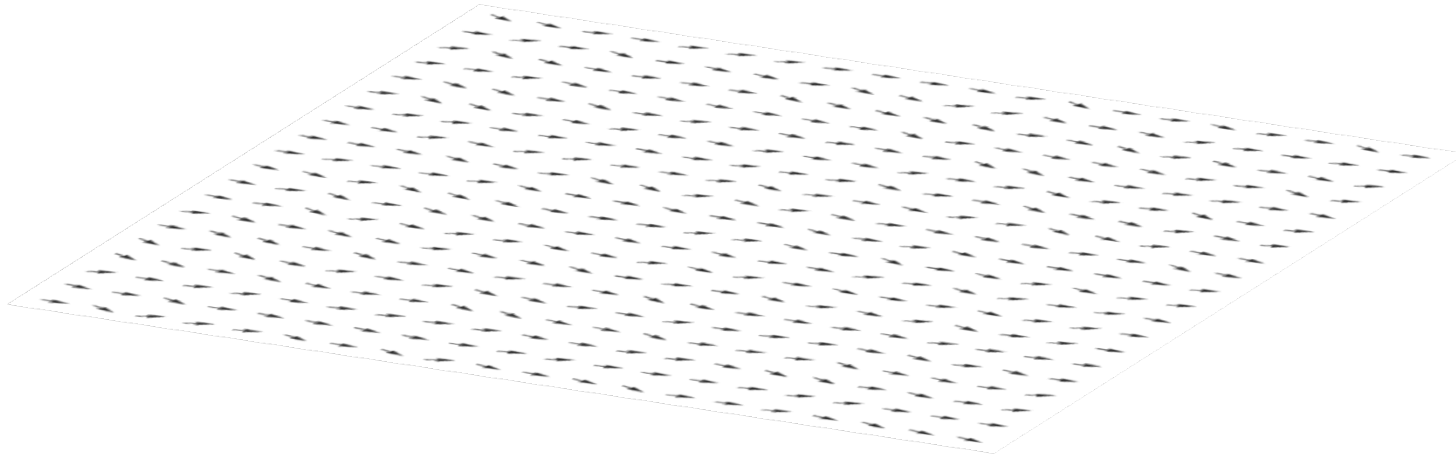


Model for ultra-thin layers

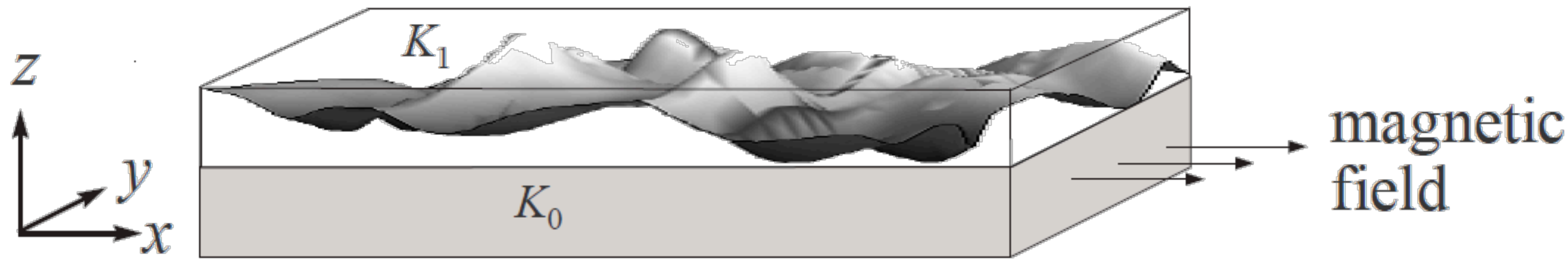


Assumption : this is a KT transition

T=0.6

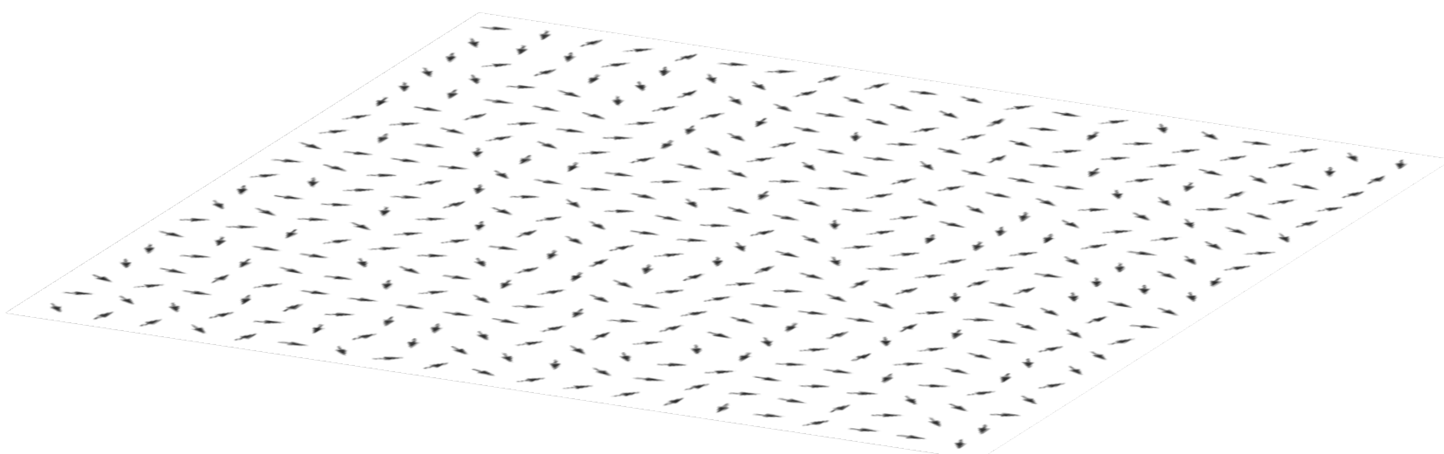


Model for ultra-thin layers

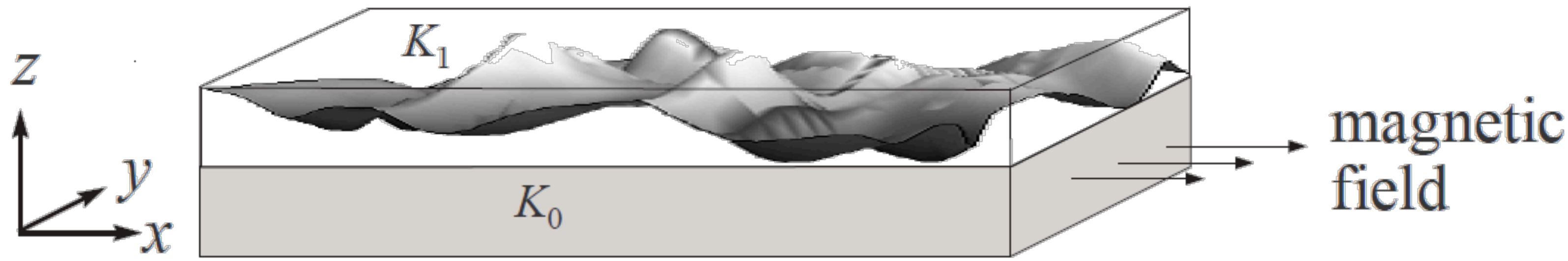


Assumption : this is a KT transition

T=0.8

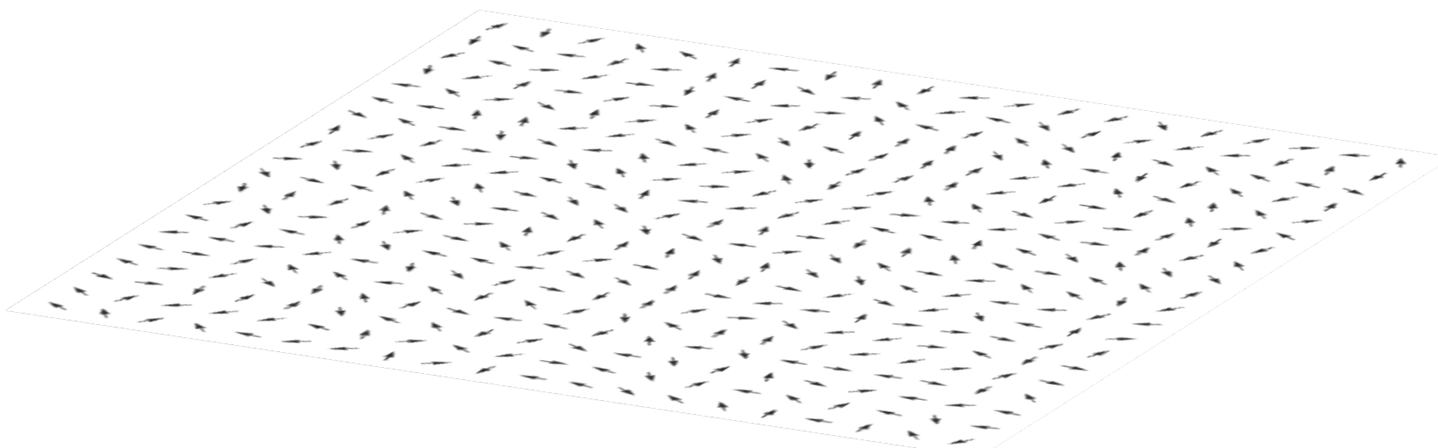


Model for ultra-thin layers

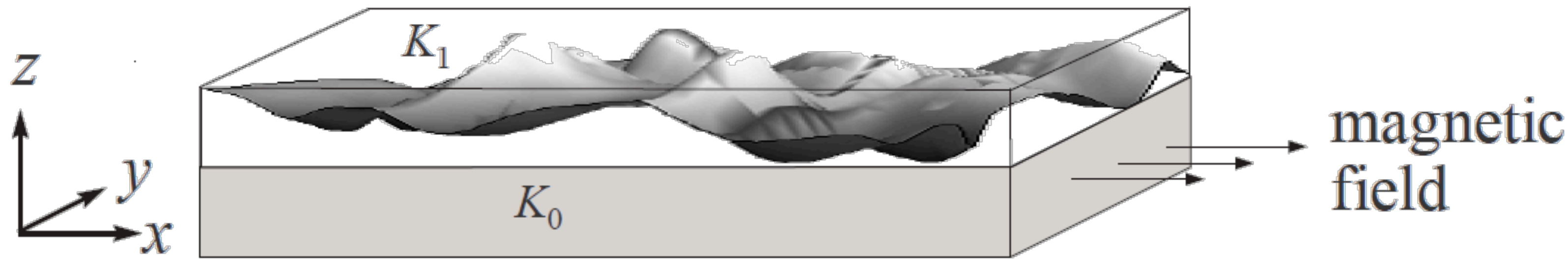


Assumption : this is a KT transition

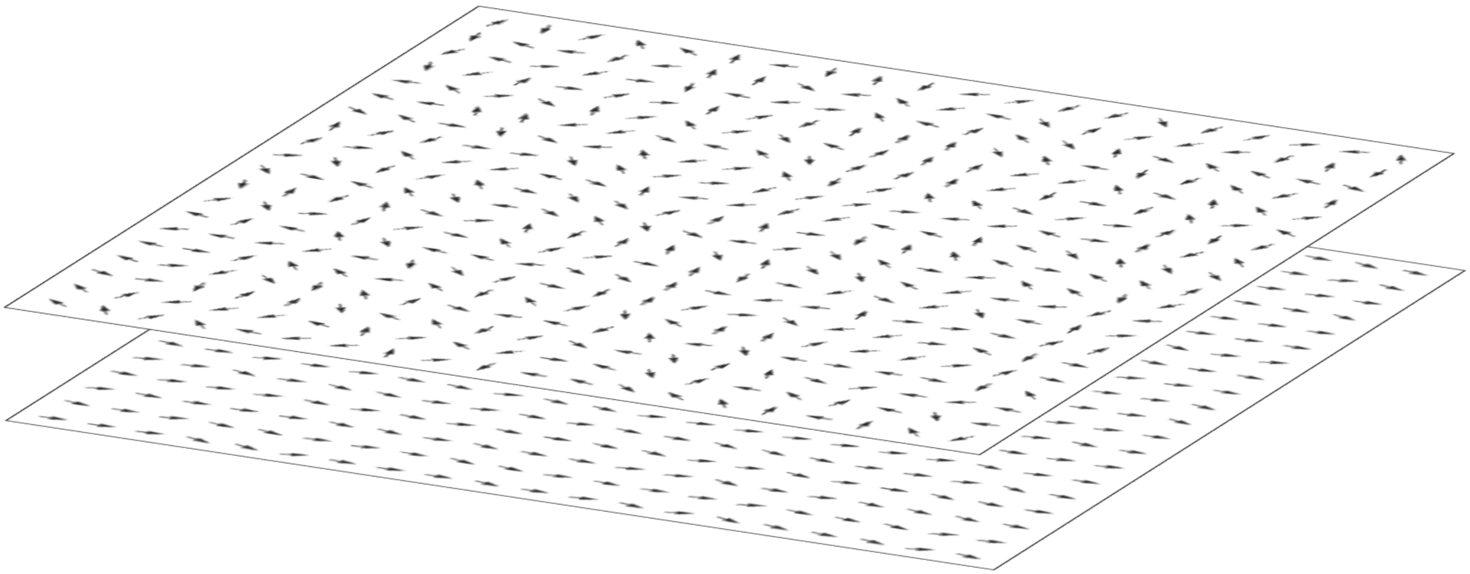
T=1



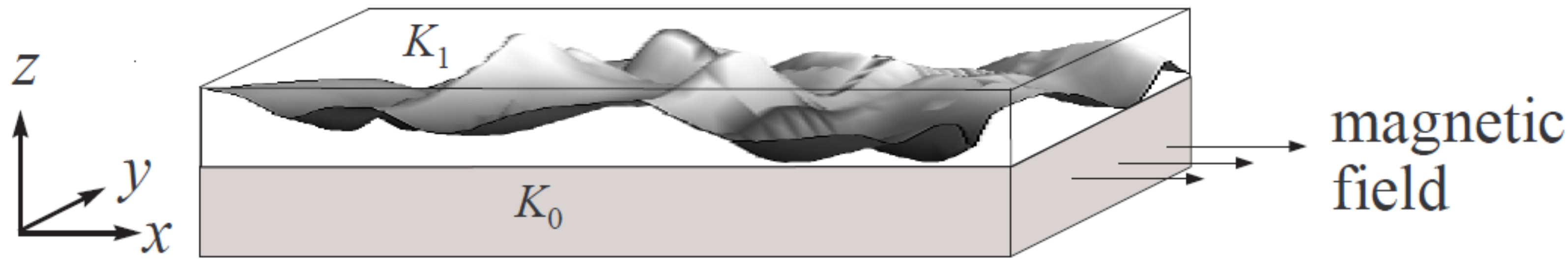
Model for ultra-thin layers



Assumption : this is a KT transition



Model for ultra-thin layers

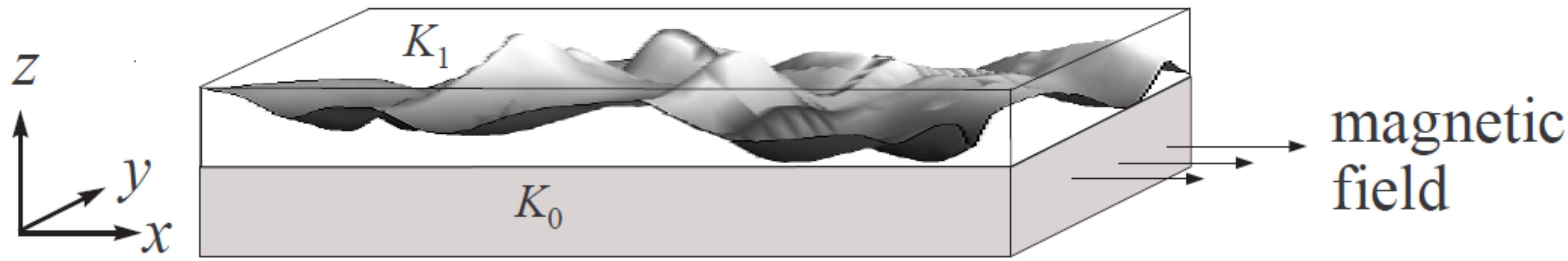


$$S = S_0 + S_1 + S_c$$

$$S_i = \frac{K_i}{2} \int \mathrm{d}^2 r (\nabla \phi_i)^2 \longrightarrow T_{c,i} \sim \frac{\pi}{2} K_i$$

$$S_c = J \int \mathrm{d}^2 r (\phi_0 - \phi_1)^2$$

Model for ultra-thin layers



$$S = S_0 + S_1 + S_c$$

$$S_i = \frac{K_i}{2} \int \mathrm{d}^2 r (\nabla \phi_i)^2 \longrightarrow T_{c,i} \sim \frac{\pi}{2} K_i$$

$$S_c = J \int \mathrm{d}^2 r (\phi_0 - \phi_1)^2$$

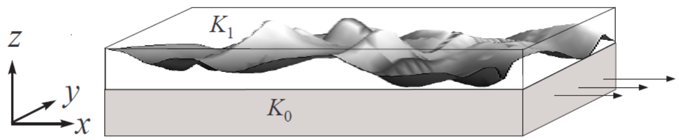
(not interesting)

$$S_c = J_1 \int \mathrm{d}^2 r (\nabla \phi_0) \cdot (\nabla \phi_1)$$

Model for ultra-thin layers

$$S = S_0 + S_1 + S_c$$

$$S_i = \frac{K_i}{2} \int \mathrm{d}^2 r (\nabla \phi_i)^2$$



$$S_c = J \int d^2 r (\varphi_1 - \varphi_0)^2 - J_1 \int d^2 r \nabla \varphi_1 \cdot \nabla \varphi_0$$

Integrate out ϕ_1 :

$$K = K_0 + K_1 - J1$$