# Superfluid-Insulator transition in cavity QED systems

*Rosario Fazio*

*NEST, Scuola Normale Superiore, Pisa*
*Istituto di Nanoscienze - CNR, Pisa*

# In collaboration with

| | |
|---|---|
| Dario Gerace | Uni-Pavia |
| Andrea Tomadin | IQOQI -Innsbruck |
| Vittorio Giovannetti<br>Davide Rossini | SNS - Pisa |
| Iacopo Carusotto | BEC - Trento |
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- Phys. Rev. Lett. **99**, 186401 (2007)
- Nature Physics **5**, 281 (2009)
- Phys. Rev. A **81**, 061801(R) (2010)

# CAVITY-QED

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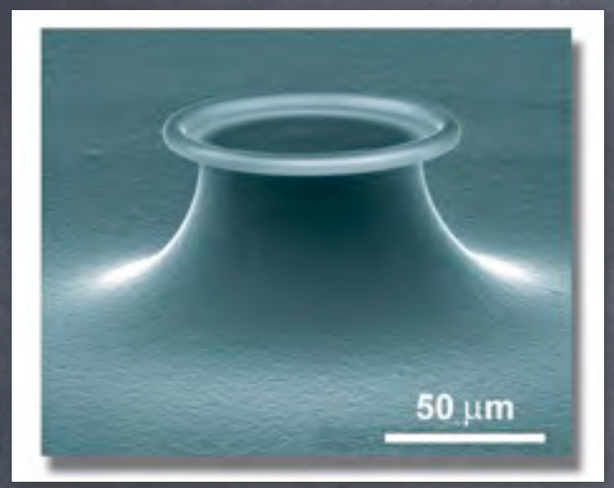
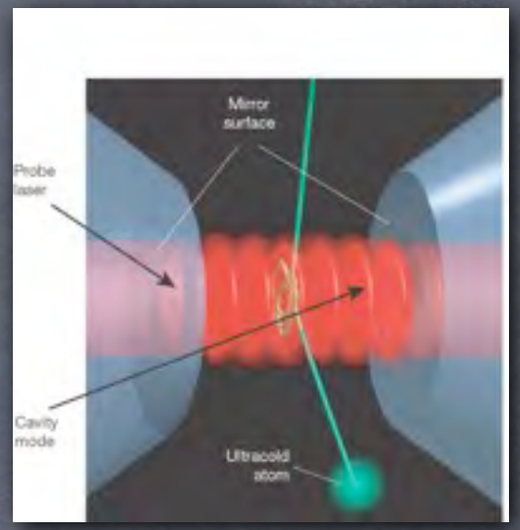
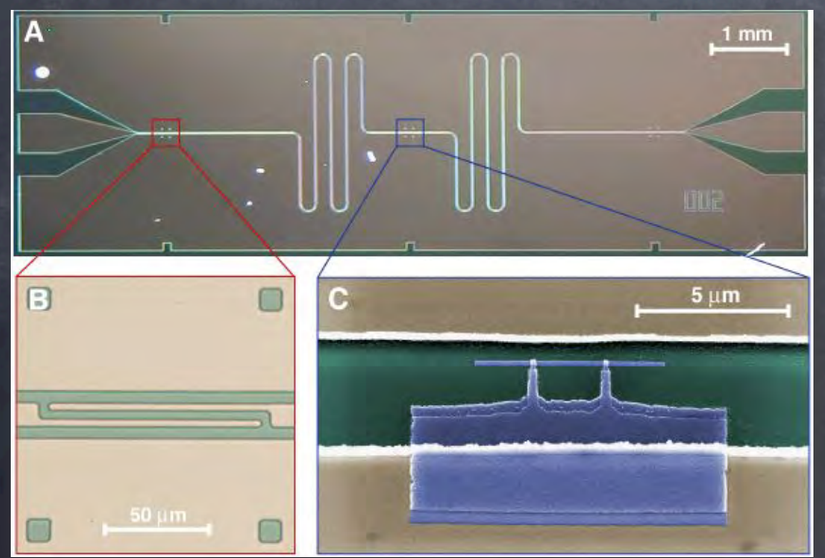


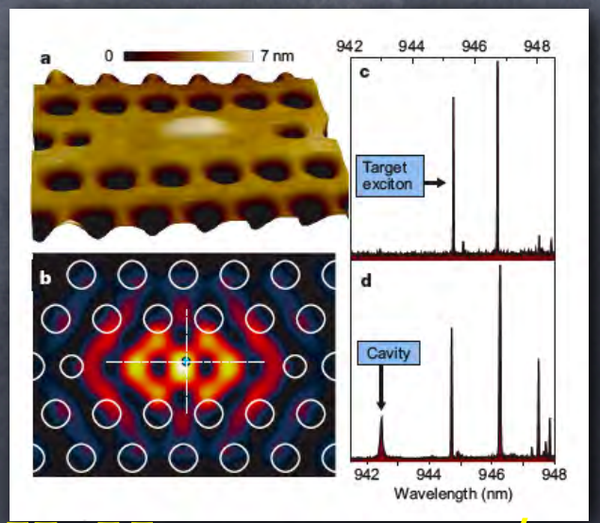

D.K. Armani *et al* Nature **421**, 925




J.M. Raimond *et al*
Rev. Mod. Phys **73**, 565 (2001)

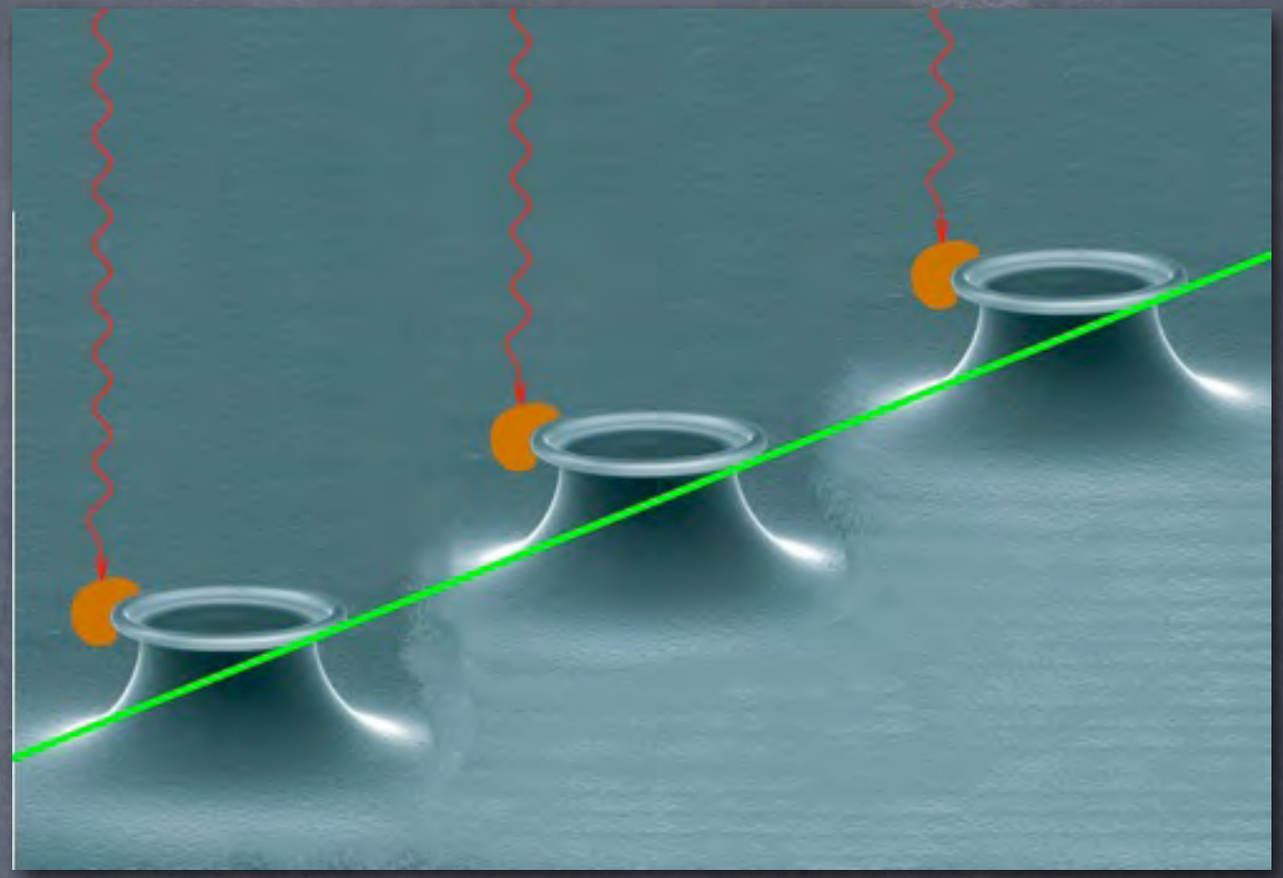


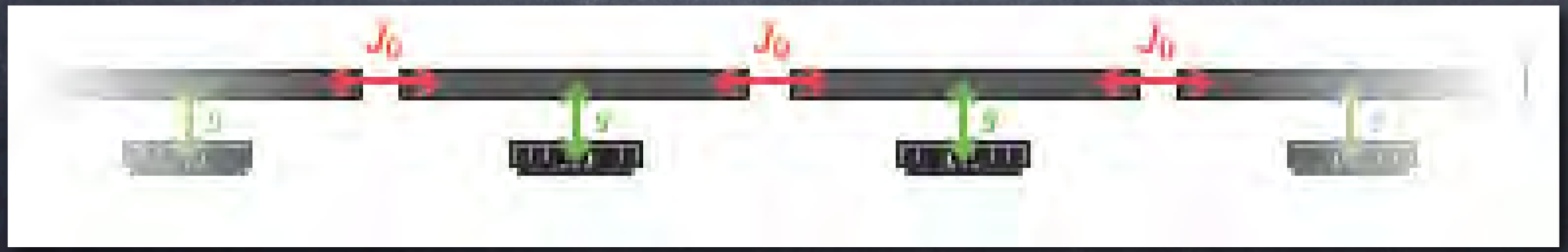

A. Wallraff *et al*, Nature **431**, 162 (2004)




K. Hennessy *et al*, Nature **445**, 895 (2007)

# Arrays of coupled cavities





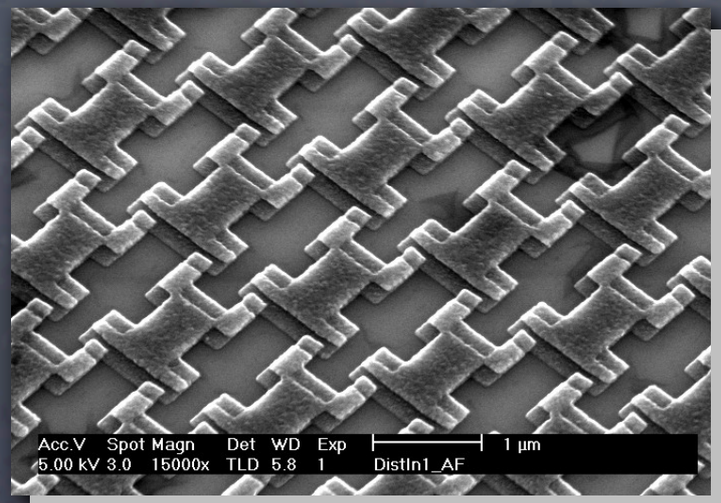

From M. Hartmann

# Arrays of coupled cavities

- M. J. Hartmann, F.G. Brandao, and M. B. Plenio, Nat. Phys. **2**, 849 (2006)
- A.D. Greentree *et al*, Nat. Phys. **2**, 856 (2006)
- D.G. Angelakis, M.F. Santos, and S. Bose, Phys. Rev. A (RC) **76**, 031805 (2007)

**Review:**
M.J. Hartmann, F.G.S.L. Brandao, and M.B. Plenio, Laser & Photon. Rev. **2**, 527 (2008)

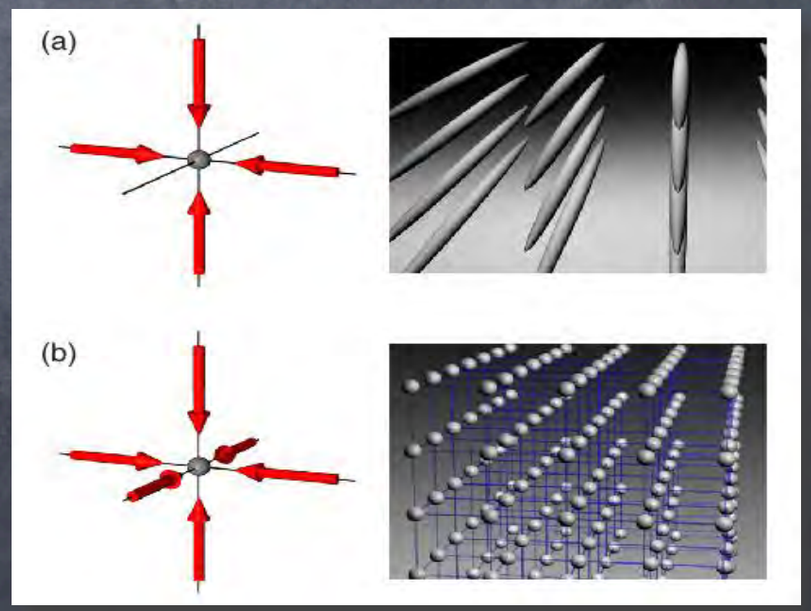
# Competition of local non-linearity with photon hopping




From J.E. Mooij group (TU Delft)

# Josephson arrays

# Optical Lattices




D. Jaksch et al, Phys. Rev. Lett. 81, 3108 (1998)
M. Greiner et al, Nature 415, 39 (2002)

# Coupled QED-cavity arrays

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Differently from Josephson arrays and optical lattices, the will be experimental access to correlation

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# Coupled QED-cavity arrays

- Differently from Josephson arrays and optical lattices, the will be experimental access to correlation
- Coupled cavities can be realized in a number of very different physical systems
- It is (in principle) possible to study a variety of equilibrium and non-equilibrium phase transitions in many-body systems

# Jaynes-Cummings Model

$$\mathcal{H} = \epsilon\sigma_z + \omega a^\dagger a + \beta(a^\dagger\sigma_- + a\sigma_+)$$

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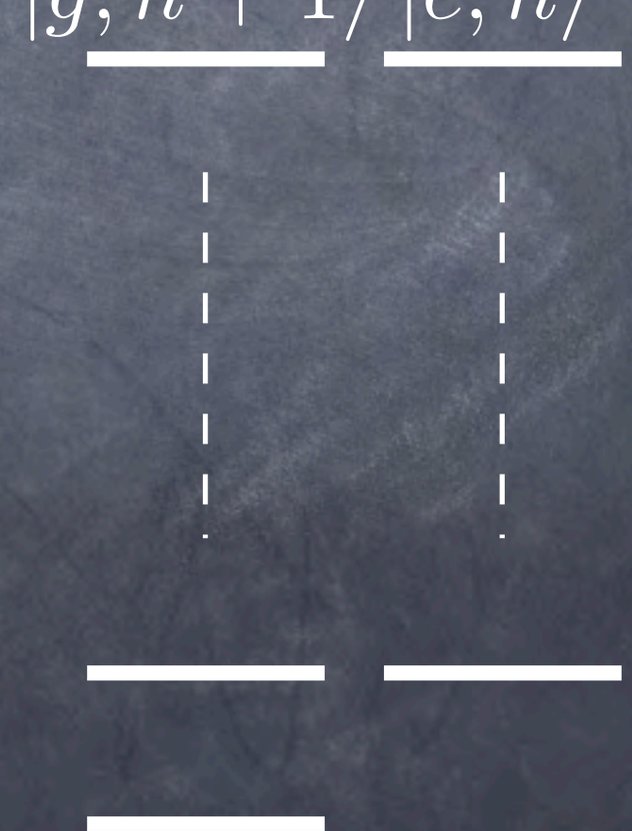

# Jaynes-Cummings Model

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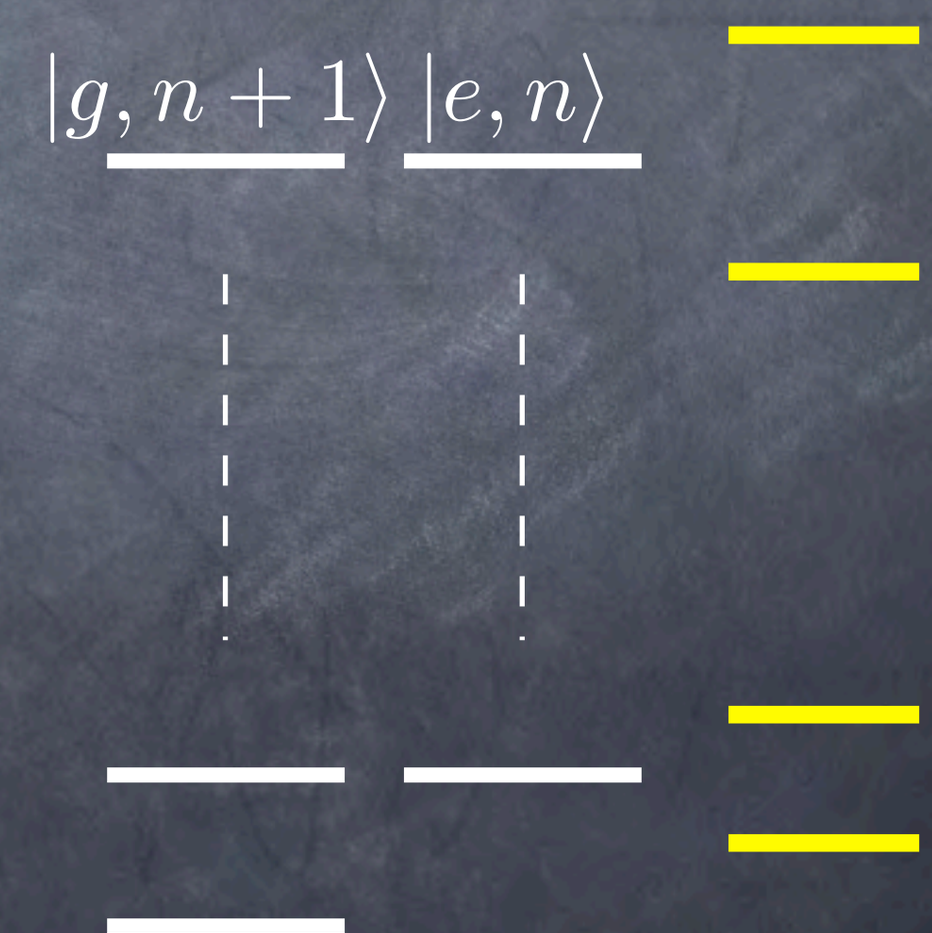
The degeneracy between
$|e, n\rangle$ and $|g, n+1\rangle$
is lifted by the coupling



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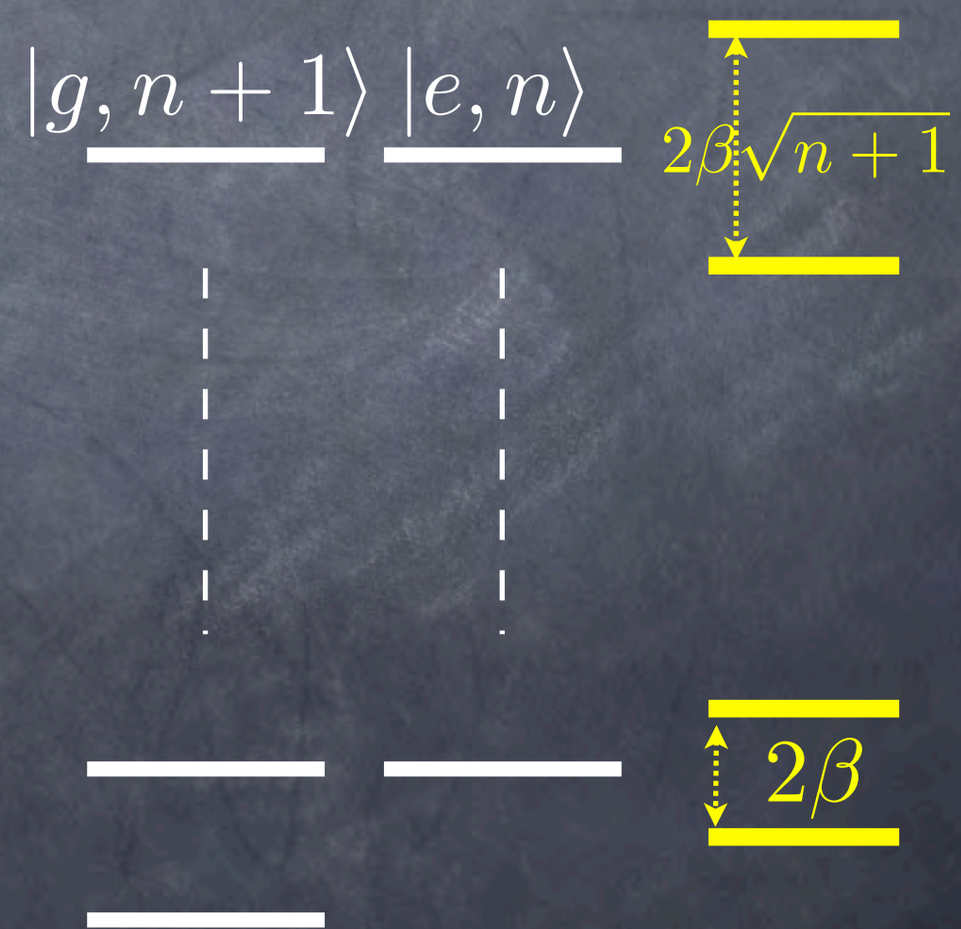
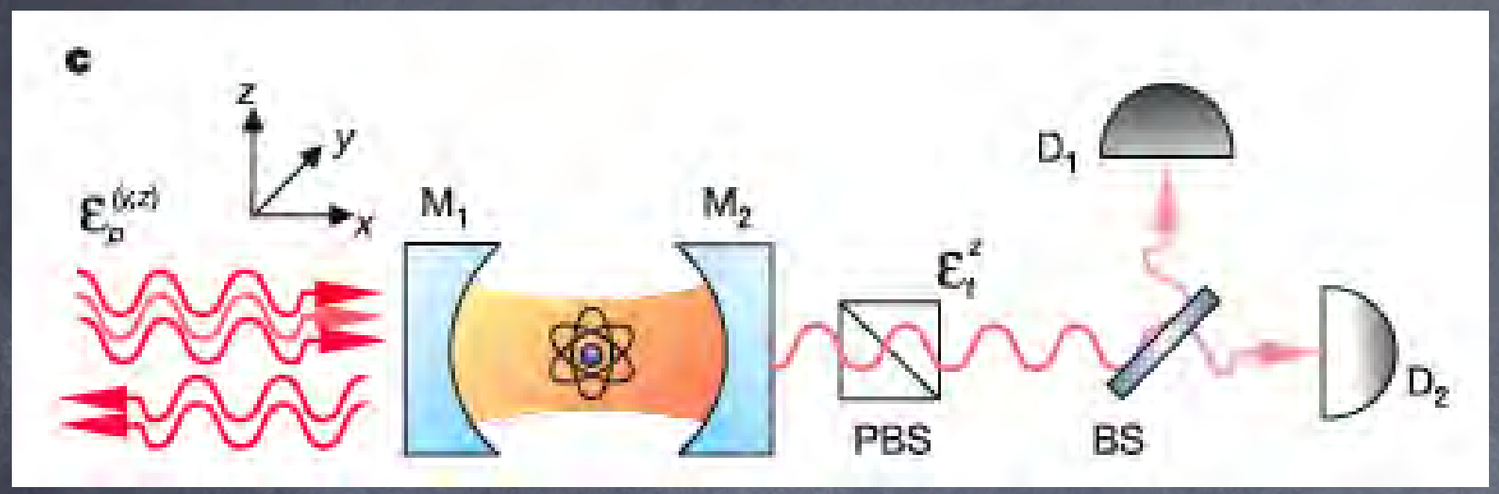
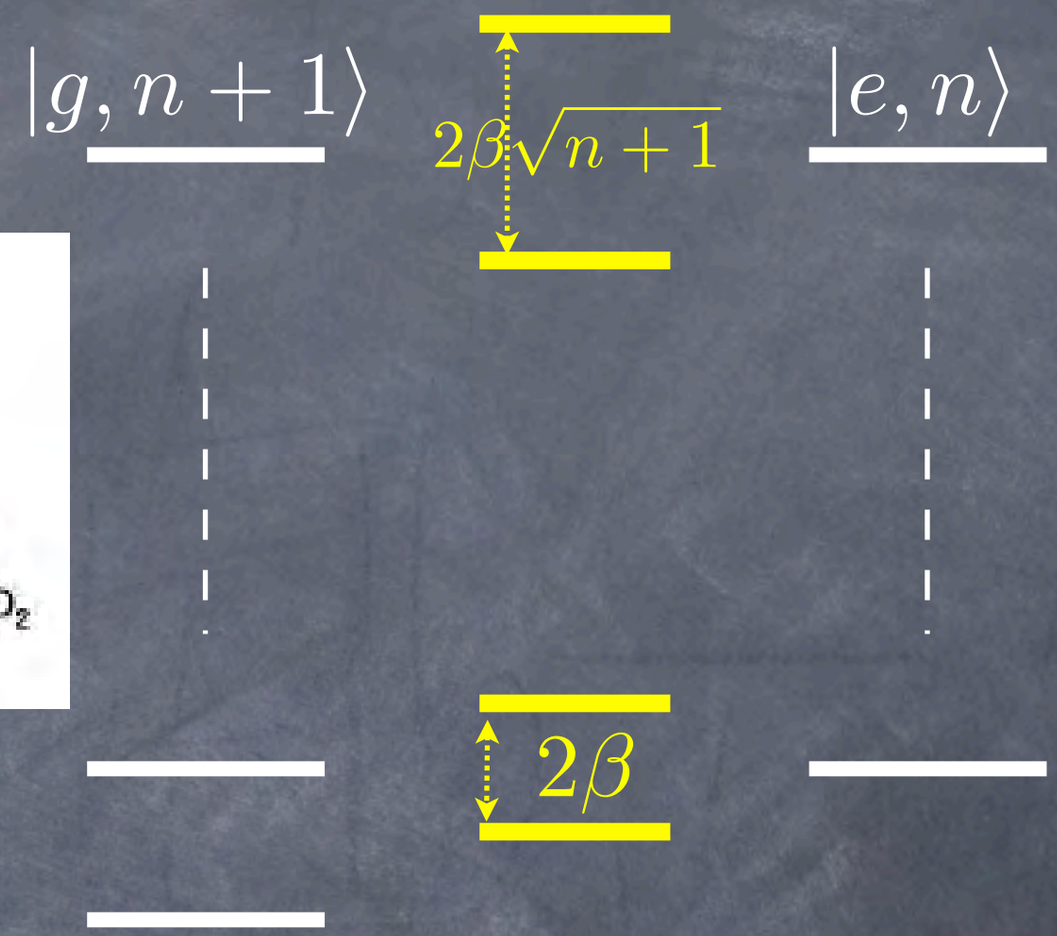


# Photon Blockade




A. Imamoglu *et al* 97, S. Rebic *et al* 99, J. Kim *et al* 99, K. Birnbaum *et al* 05

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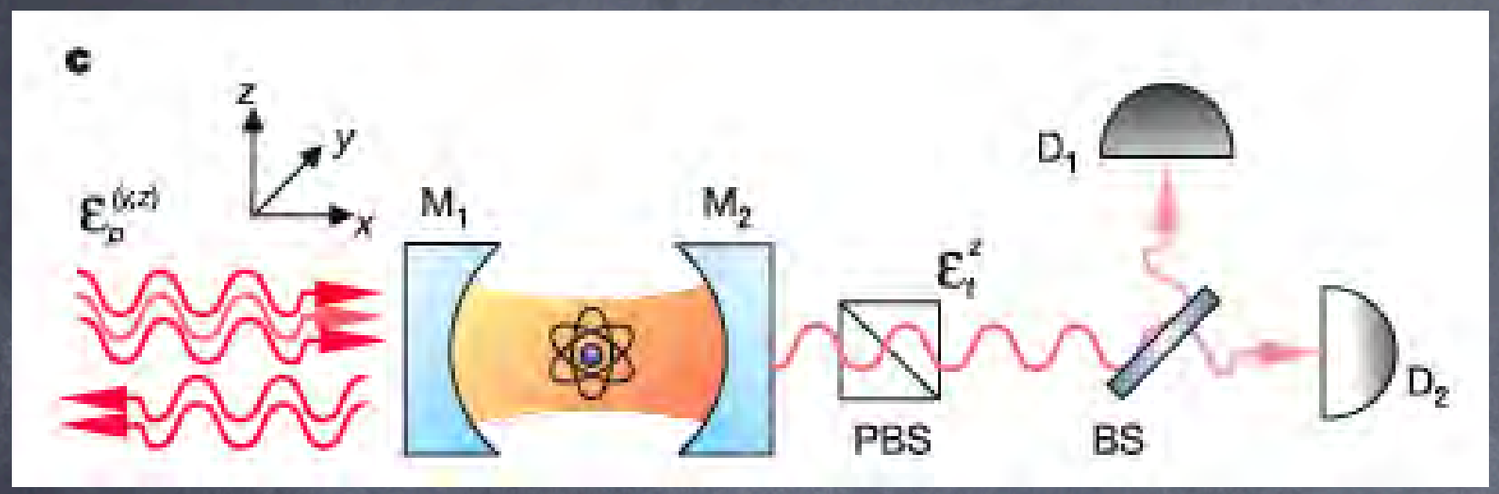


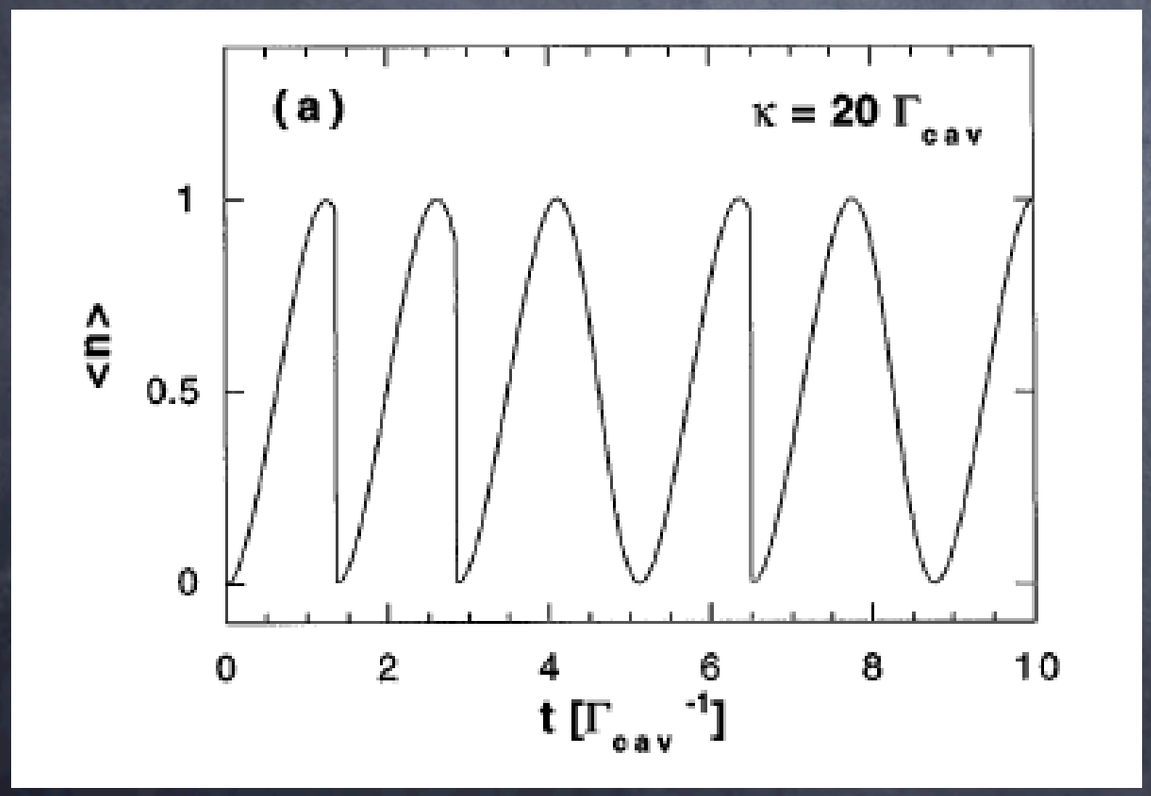


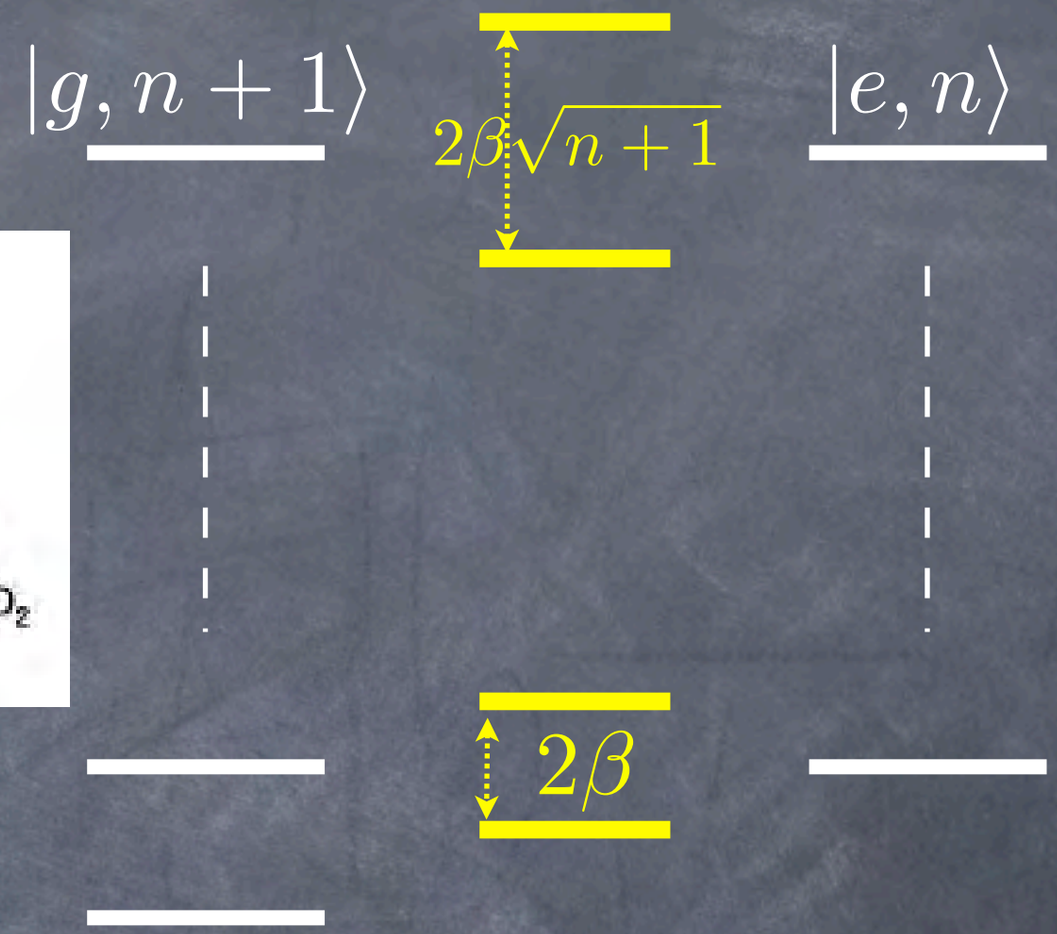


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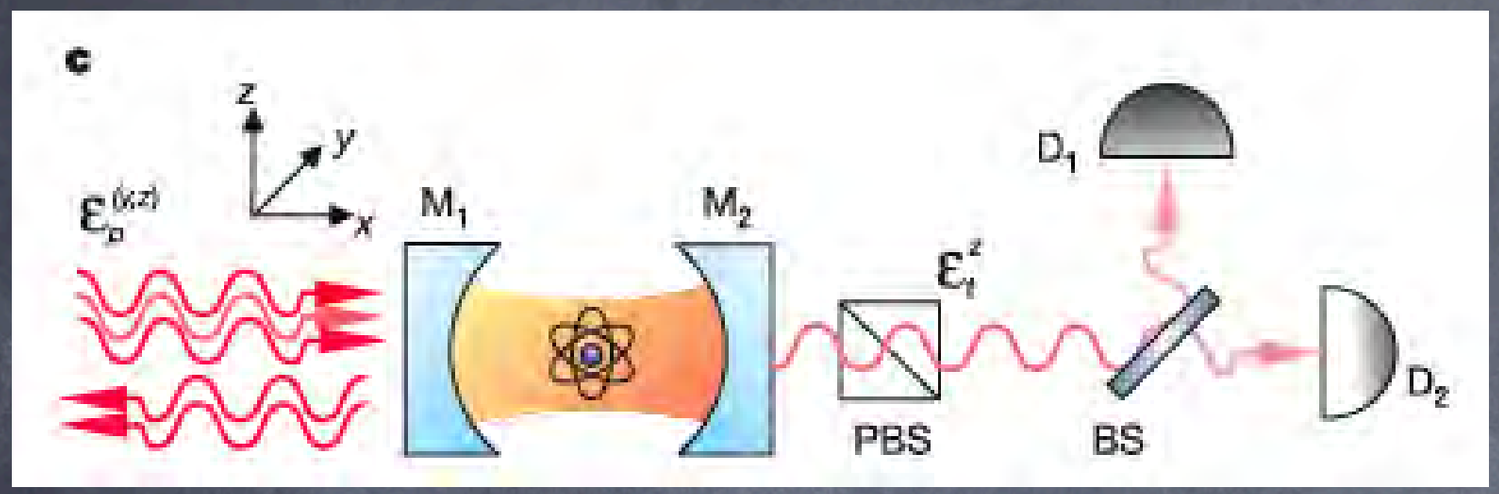
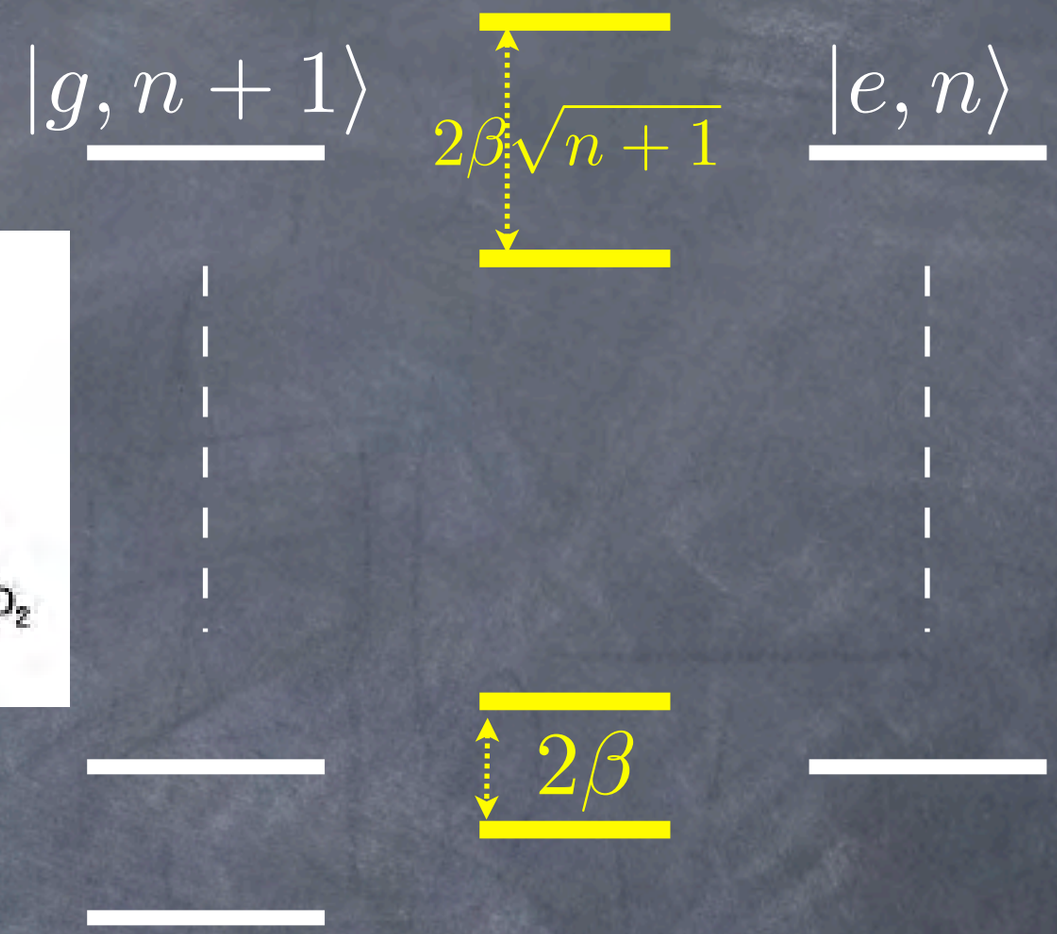
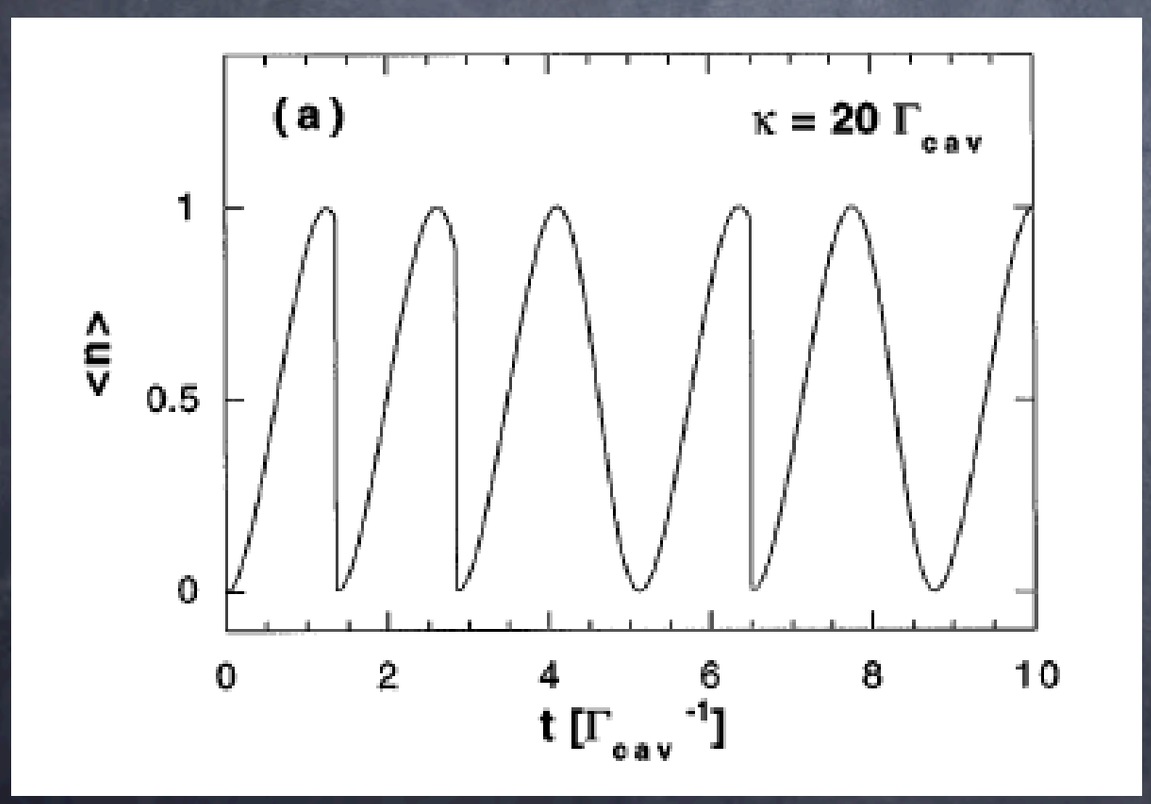


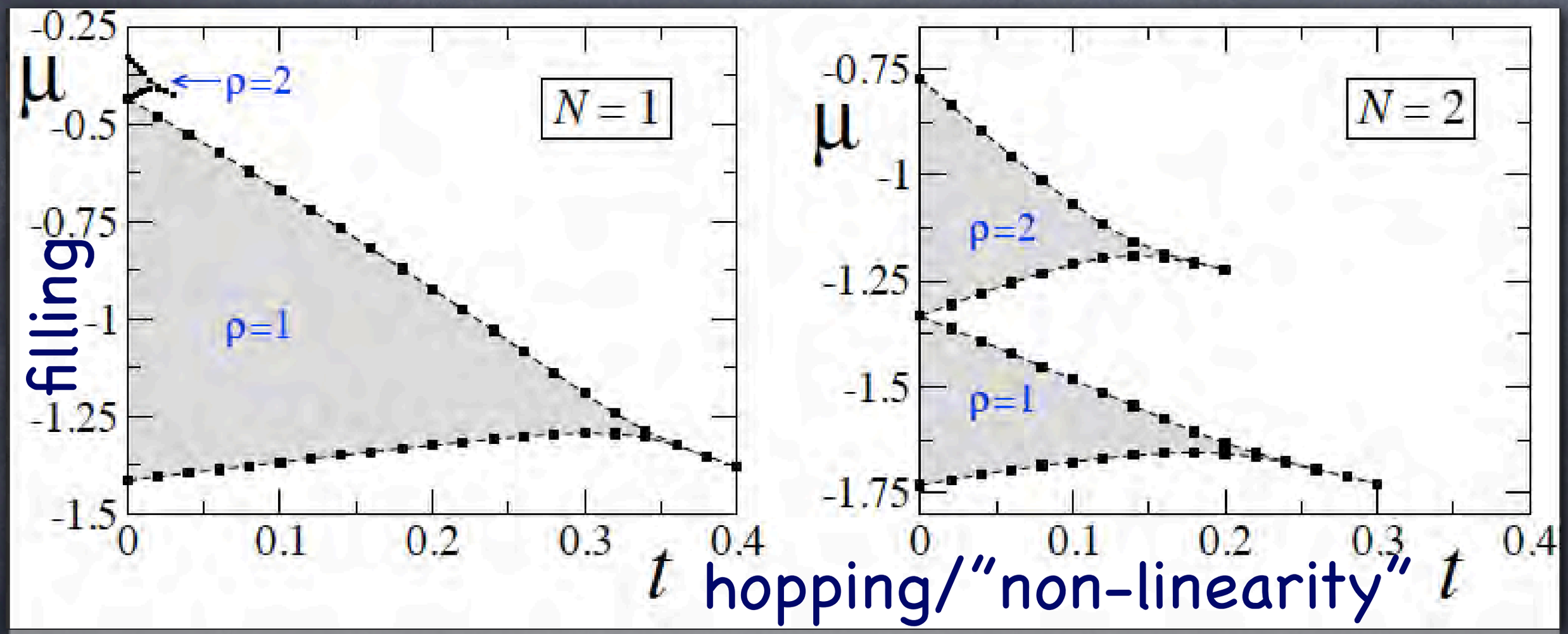

Atom-photon interaction leads to an effec. non-linear medium

A. Imamoglu *et al* 97, S. Rebic *et al* 99, J. Kim *et al* 99, K. Birnbaum *et al* 05

# Coupled Cavities

$$\mathcal{H} = \sum_{i} \mathcal{H}_i^{(JC)} - J \sum_{<ij>} (a_i^\dagger a_j + h.c.)$$

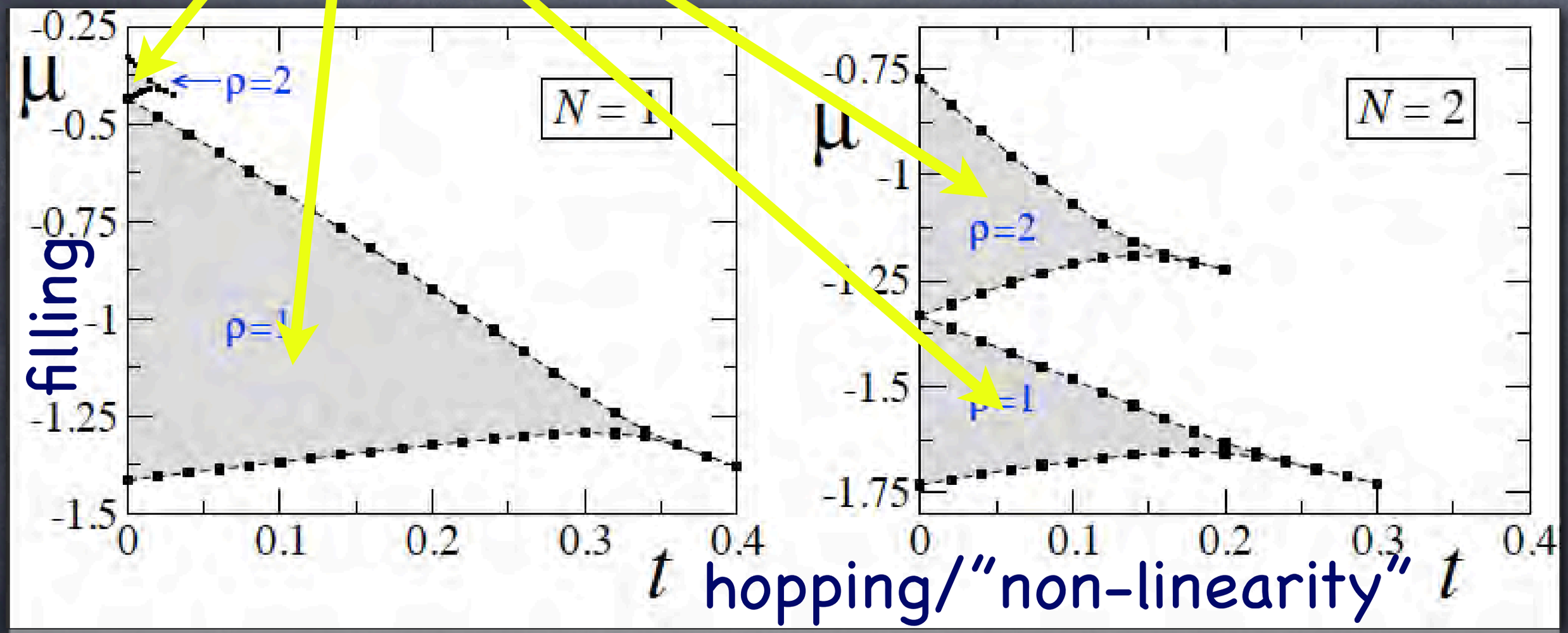
# Competition of local non-linearity with photon hopping




filling

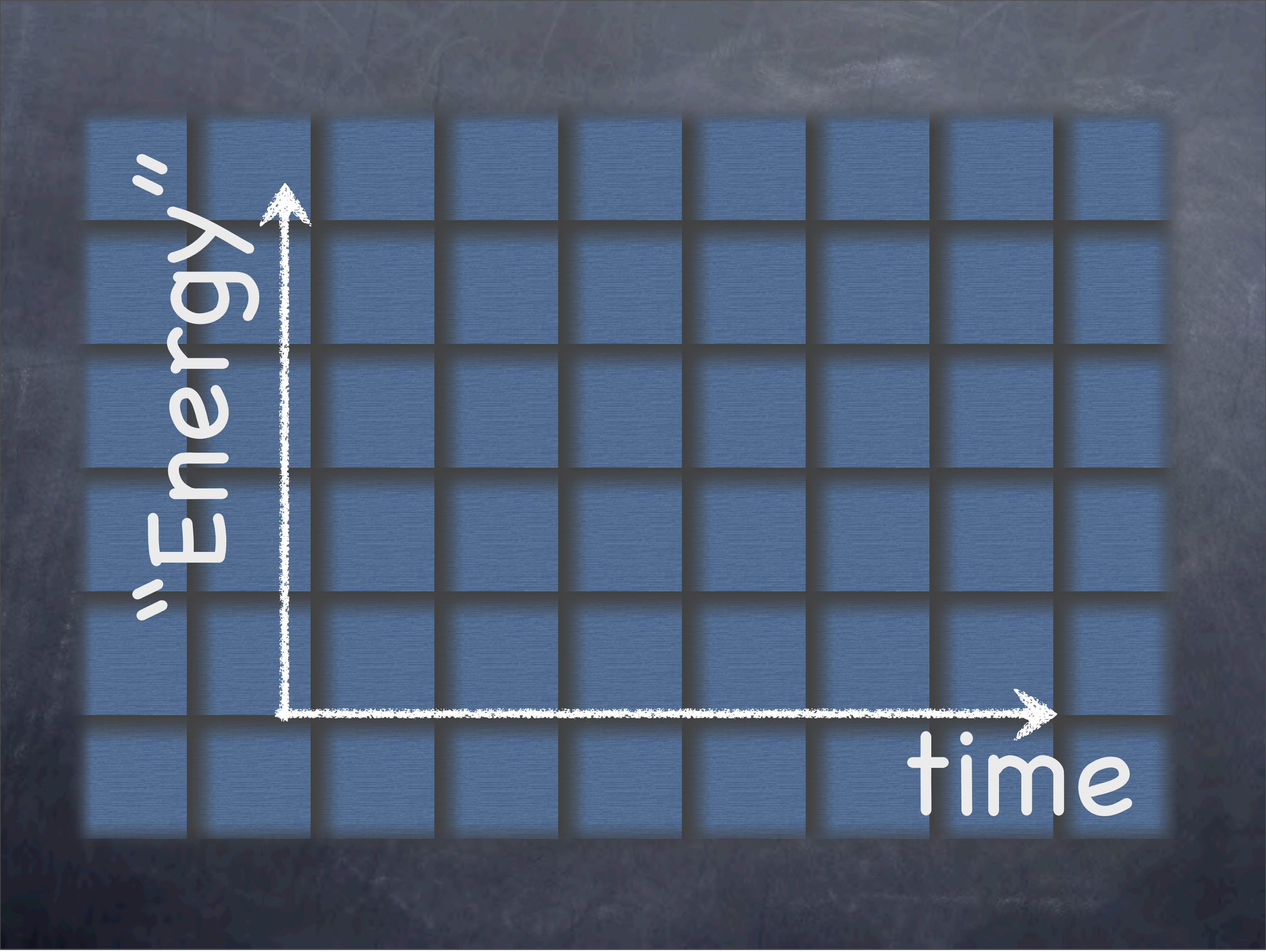
hopping/"non-linearity"

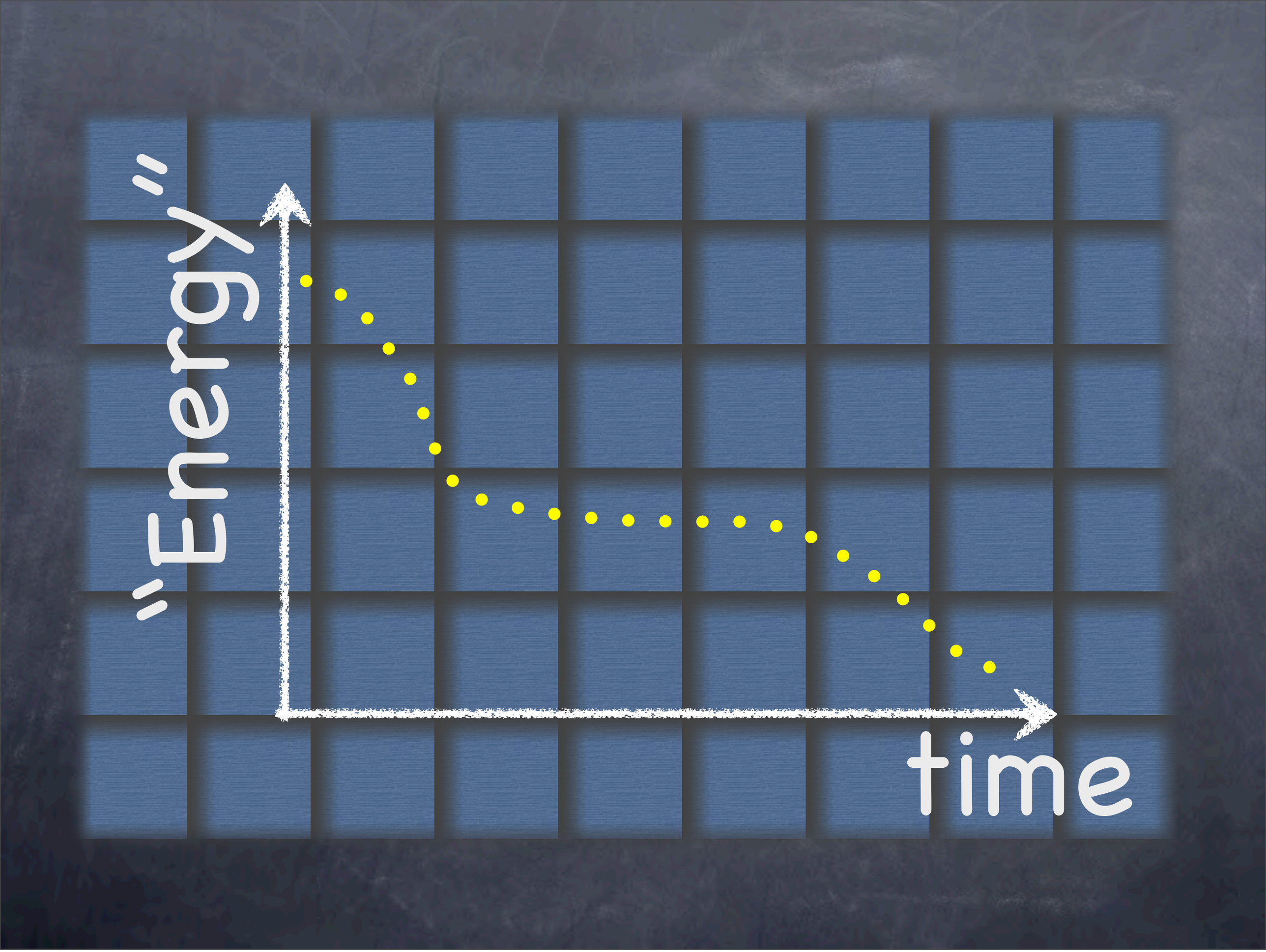
Mott insulator

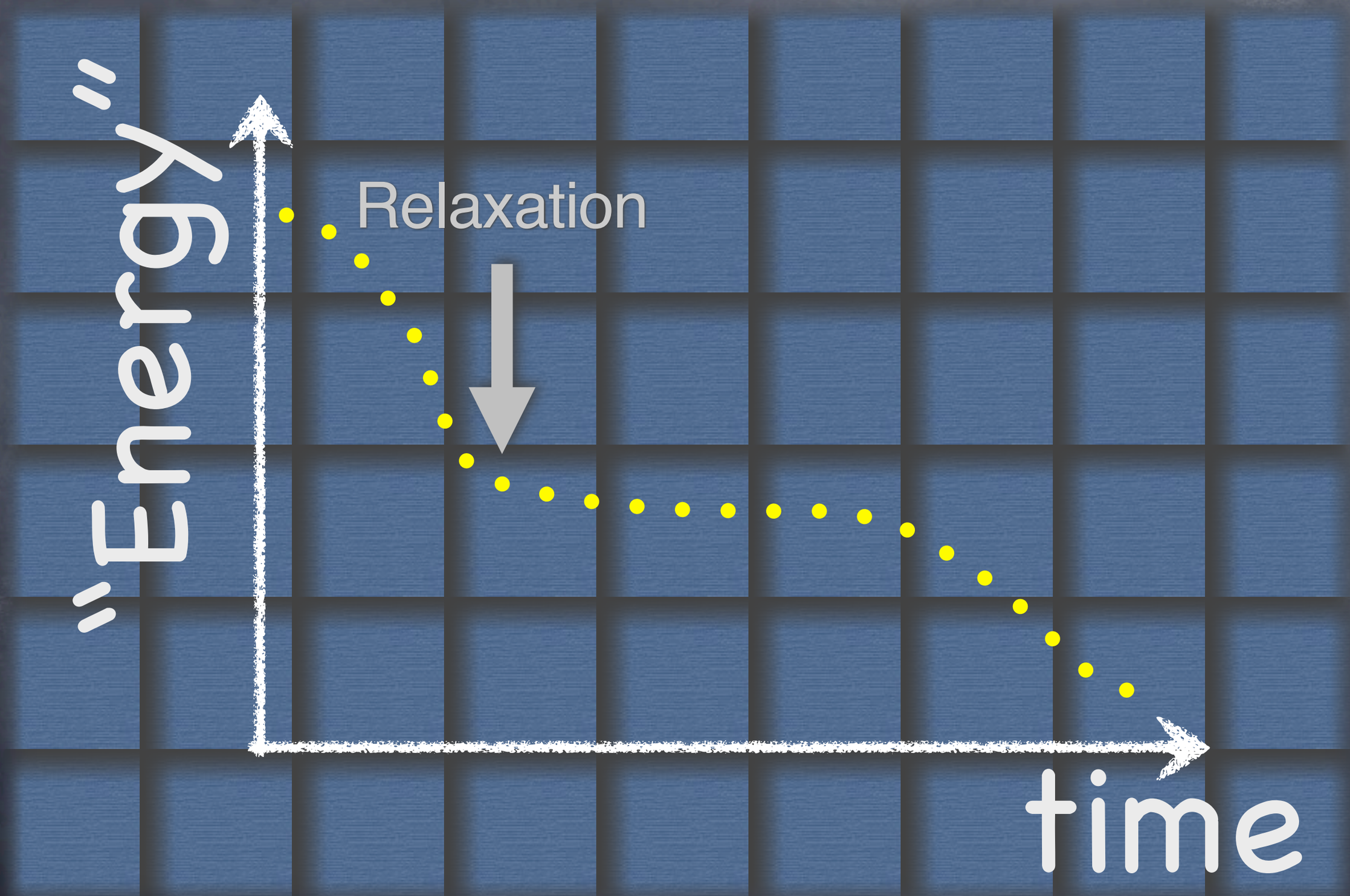


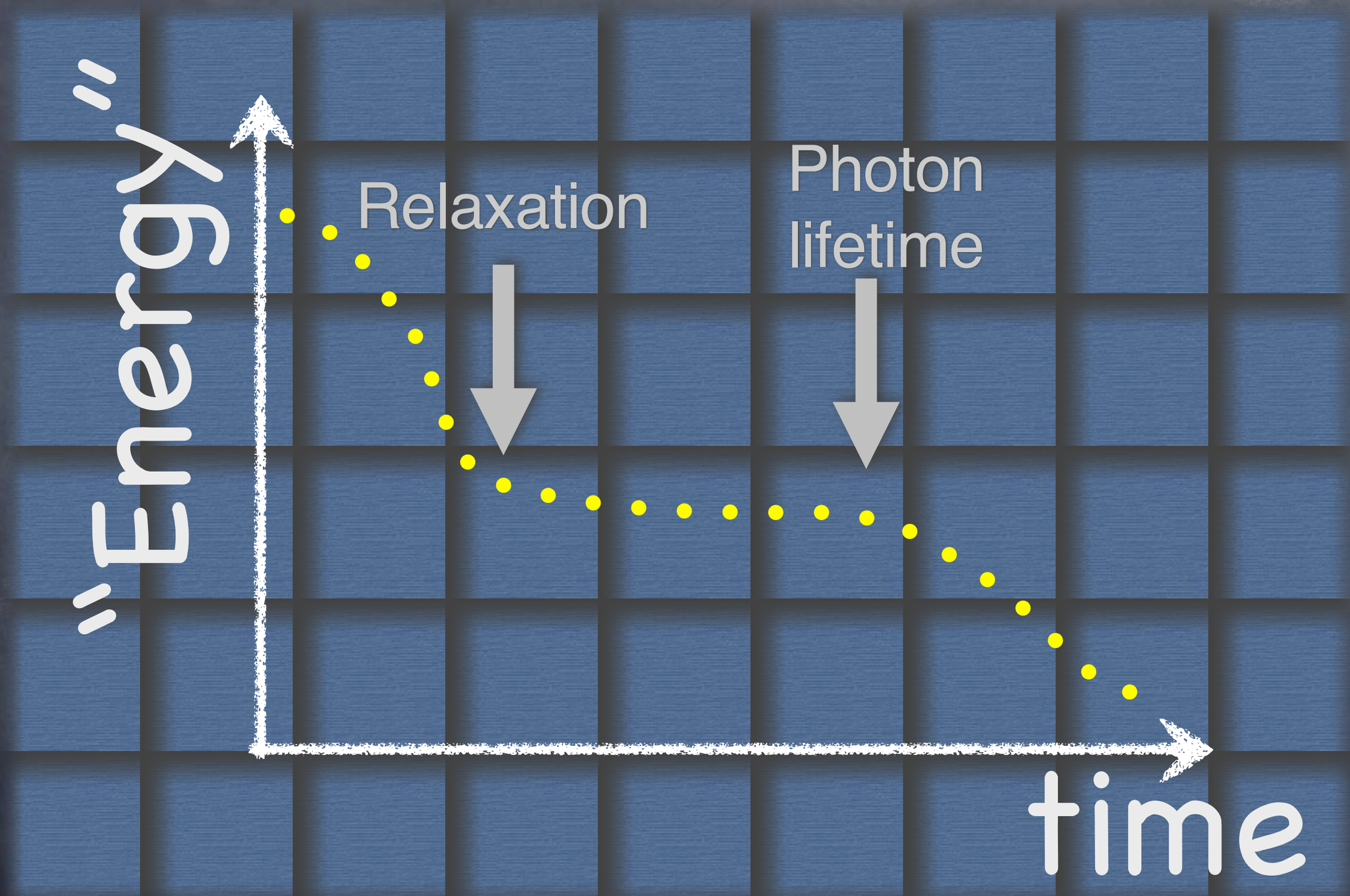

filling

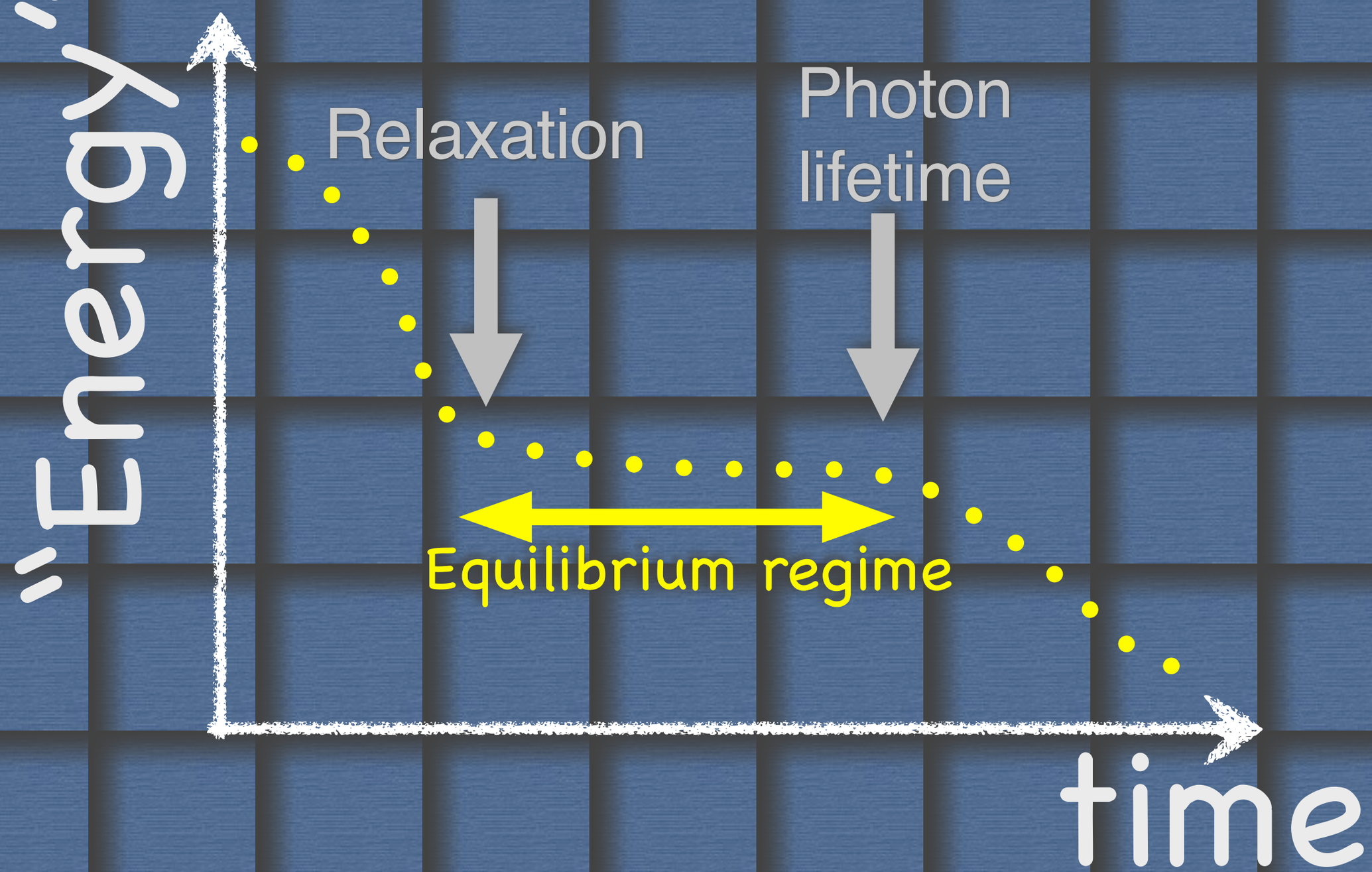
hopping/"non-linearity"


"energy"
time

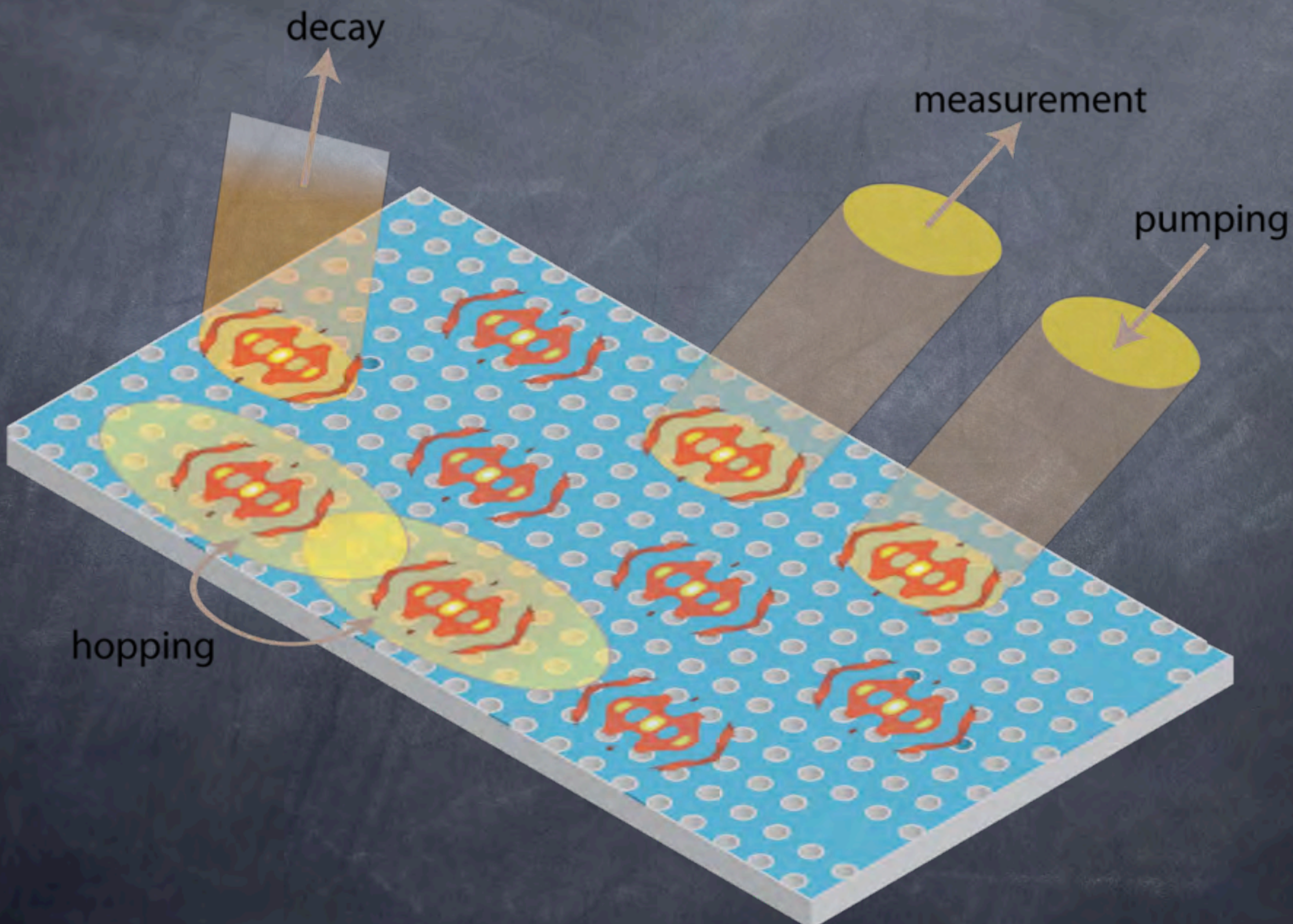

"energy"
time

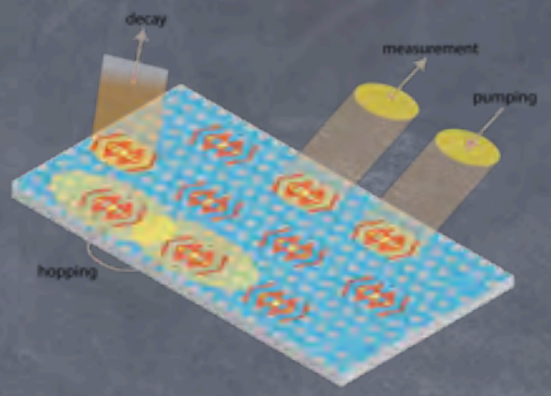

"energy"
Relaxation
time

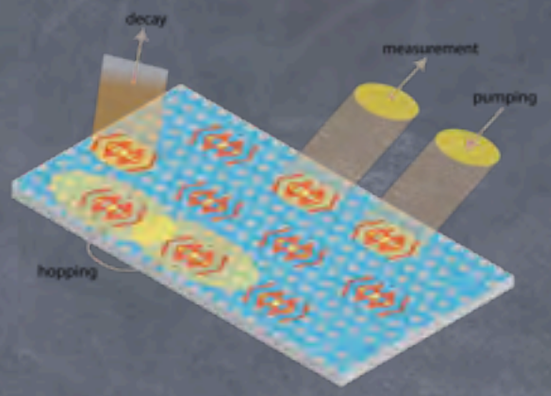

"energy"
Relaxation
Photon
lifetime
time


"energy"
Relaxation
Photon
lifetime
Equilibrium regime
time

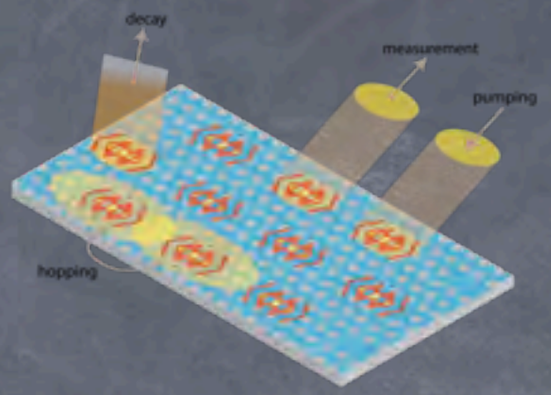
# Coupled cavities out of equilibrium




decay
measurement
pumping
hopping

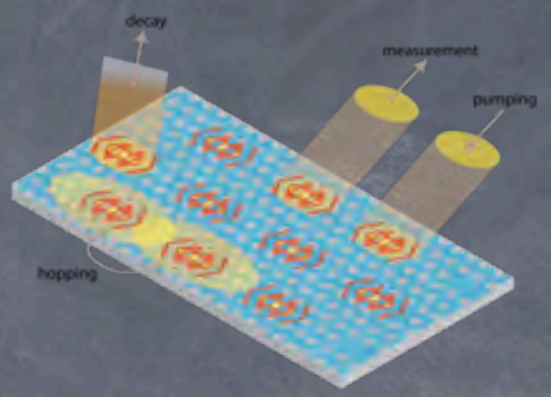



QED arrays to explore non-equilibrium quantum many-body systems




- QED arrays to explore non-equilibrium quantum many-body systems

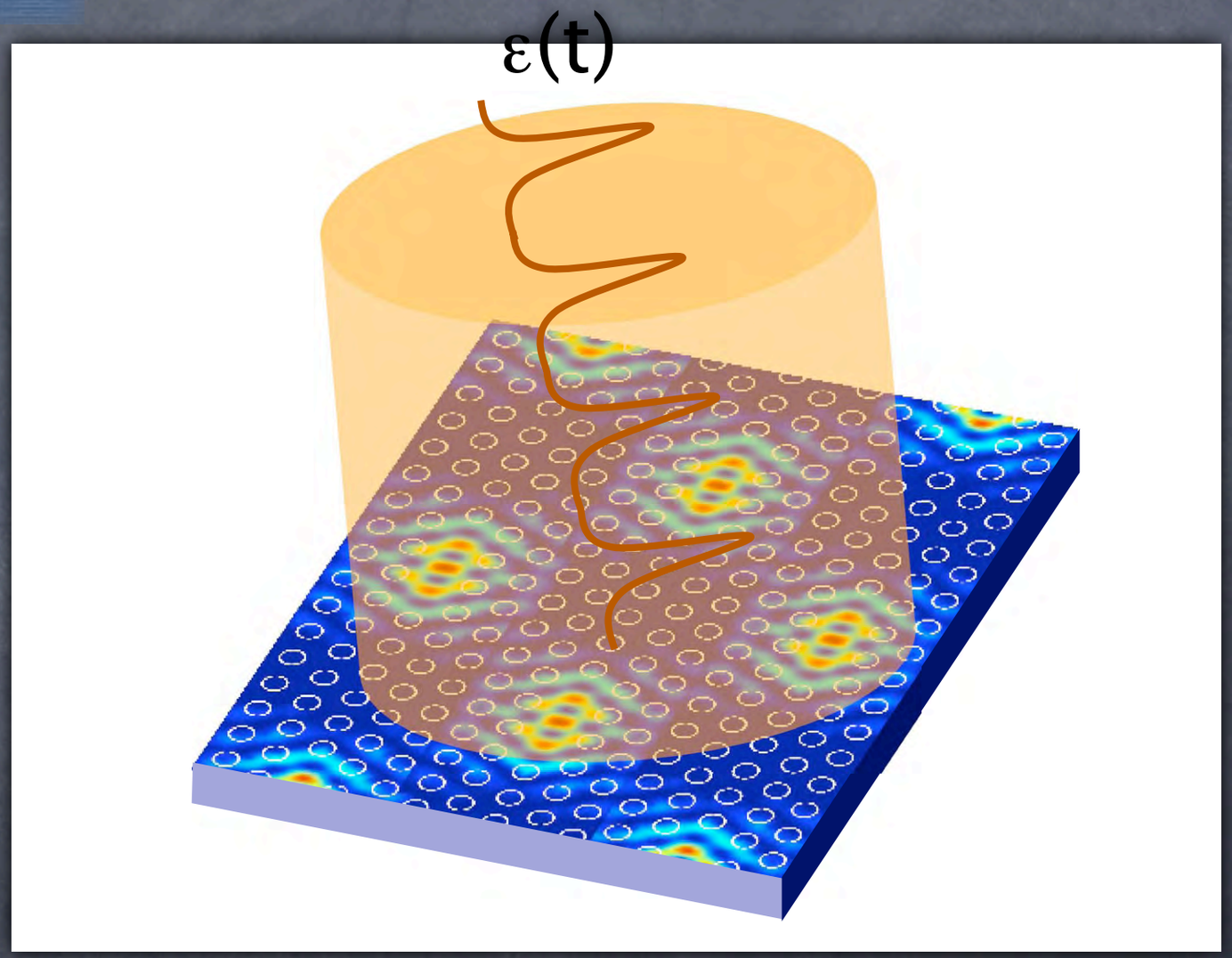
- How to realize and detect, under realistic non-equilibrium conditions, the very rich phase diagram predicted in QED arrays?



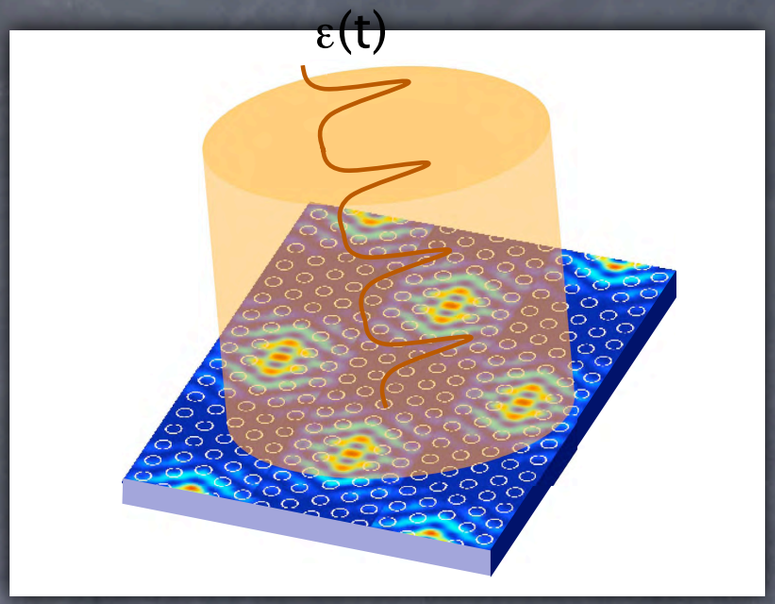

- QED arrays to explore non-equilibrium quantum many-body systems
- How to realize and detect, under realistic non-equilibrium conditions, the very rich phase diagram predicted in QED arrays?
- Explore the competition between correlations and hopping in small networks

# Coupled cavities & Quantum quenches

array is driven by a pulsed external laser



# Coupled cavities
# &
# Quantum quenches




*time*

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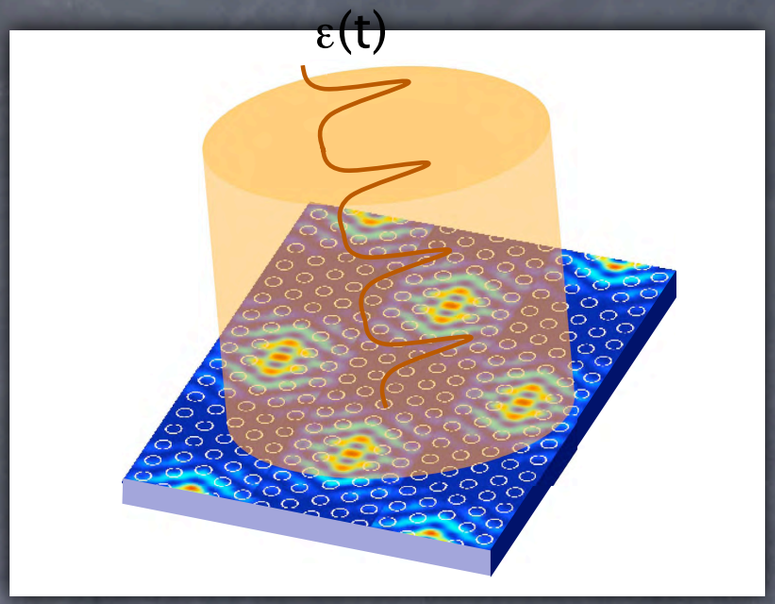


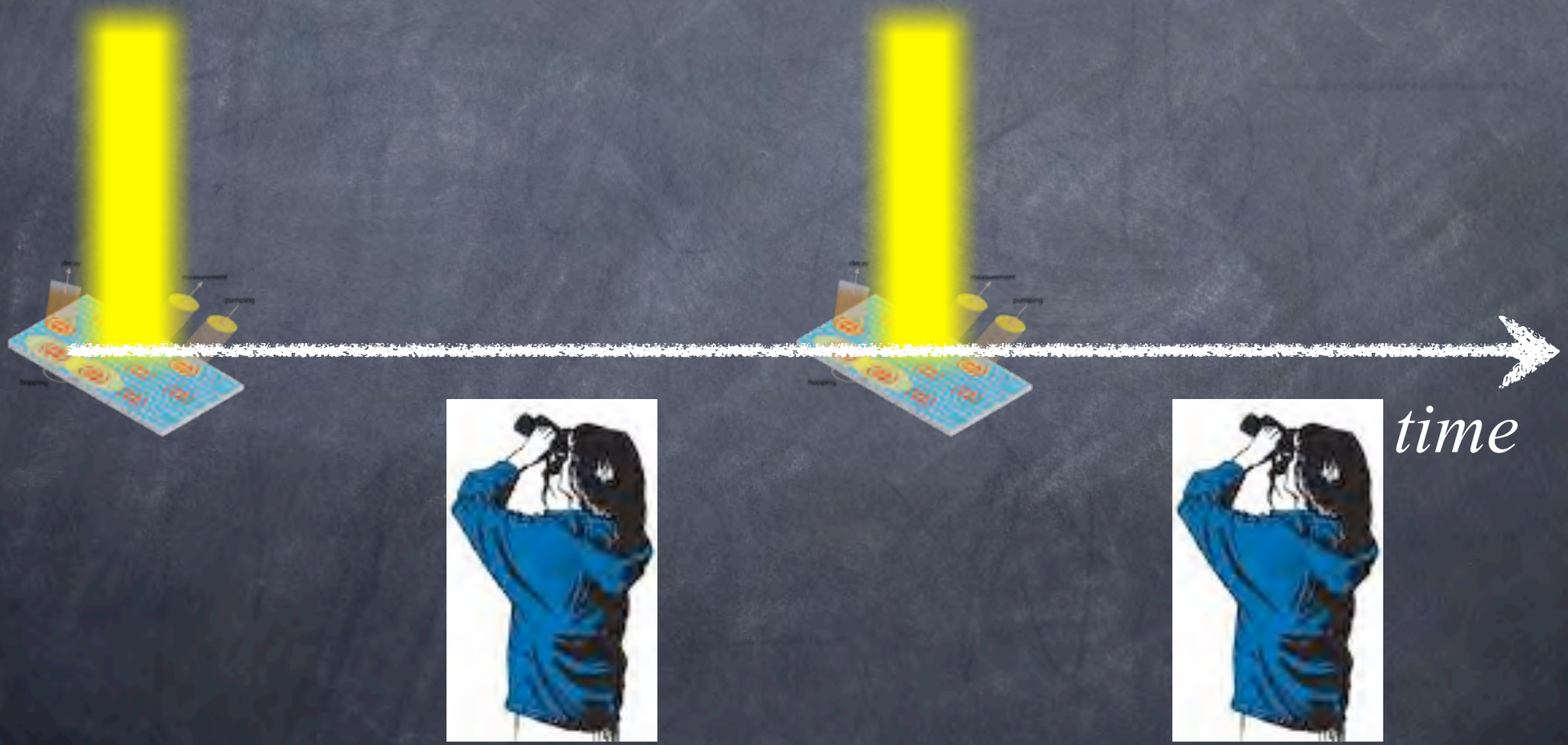

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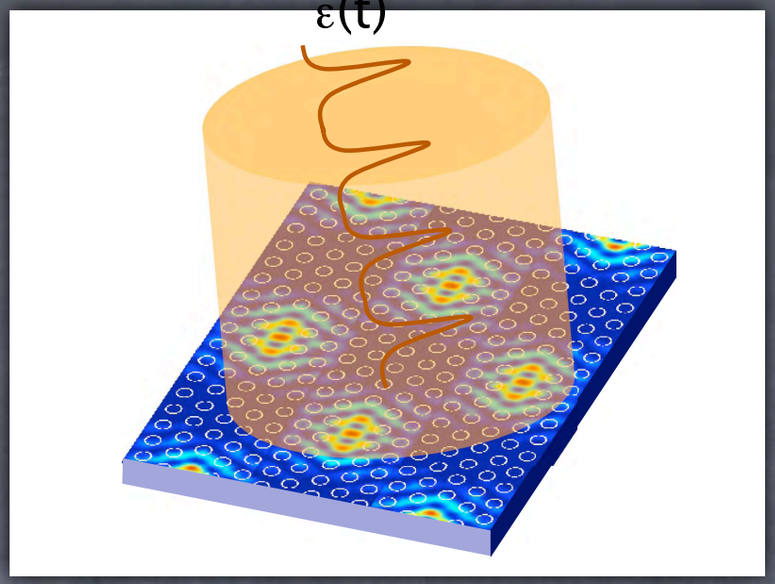
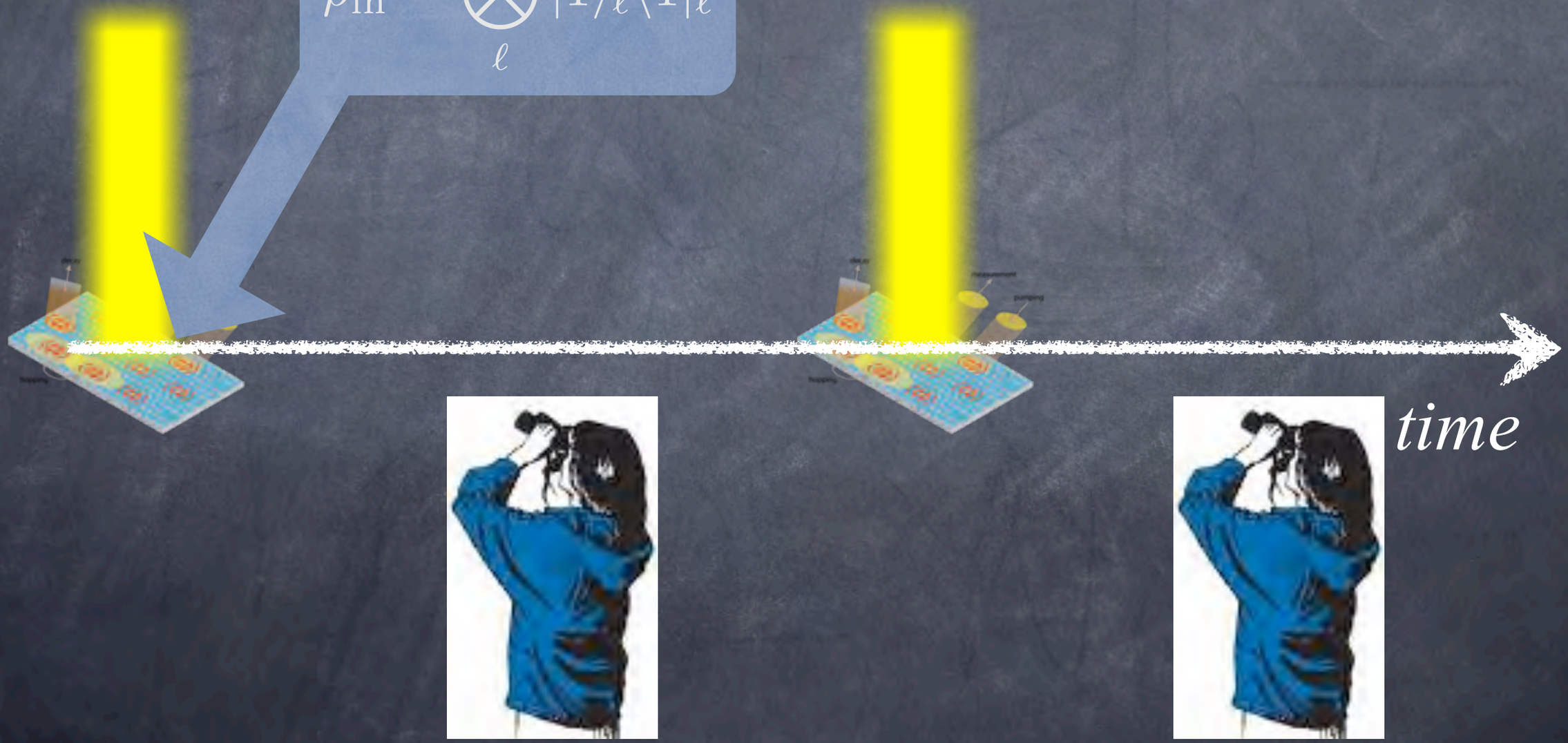



$$\rho_{\rm in} = \bigotimes_{\ell} |1\rangle_\ell \langle 1|_\ell$$



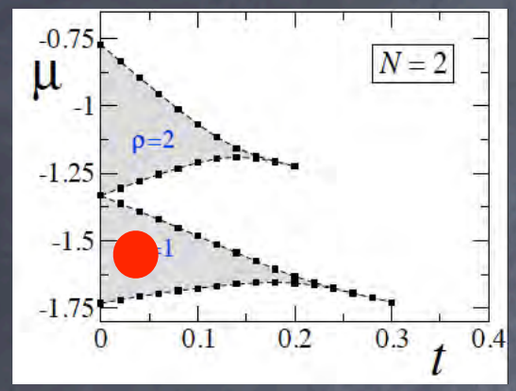
# The order parameter

$$\psi \equiv \langle a \rangle$$

$\psi$
time

$\psi$
time

$J < J_{cr}$



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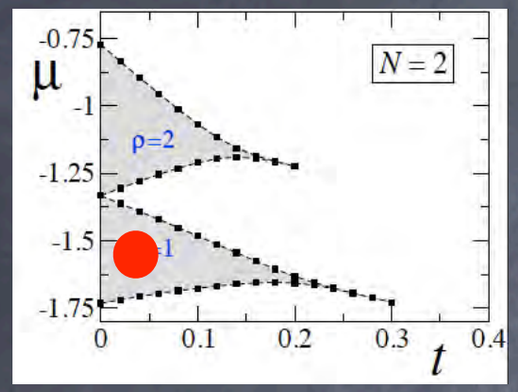


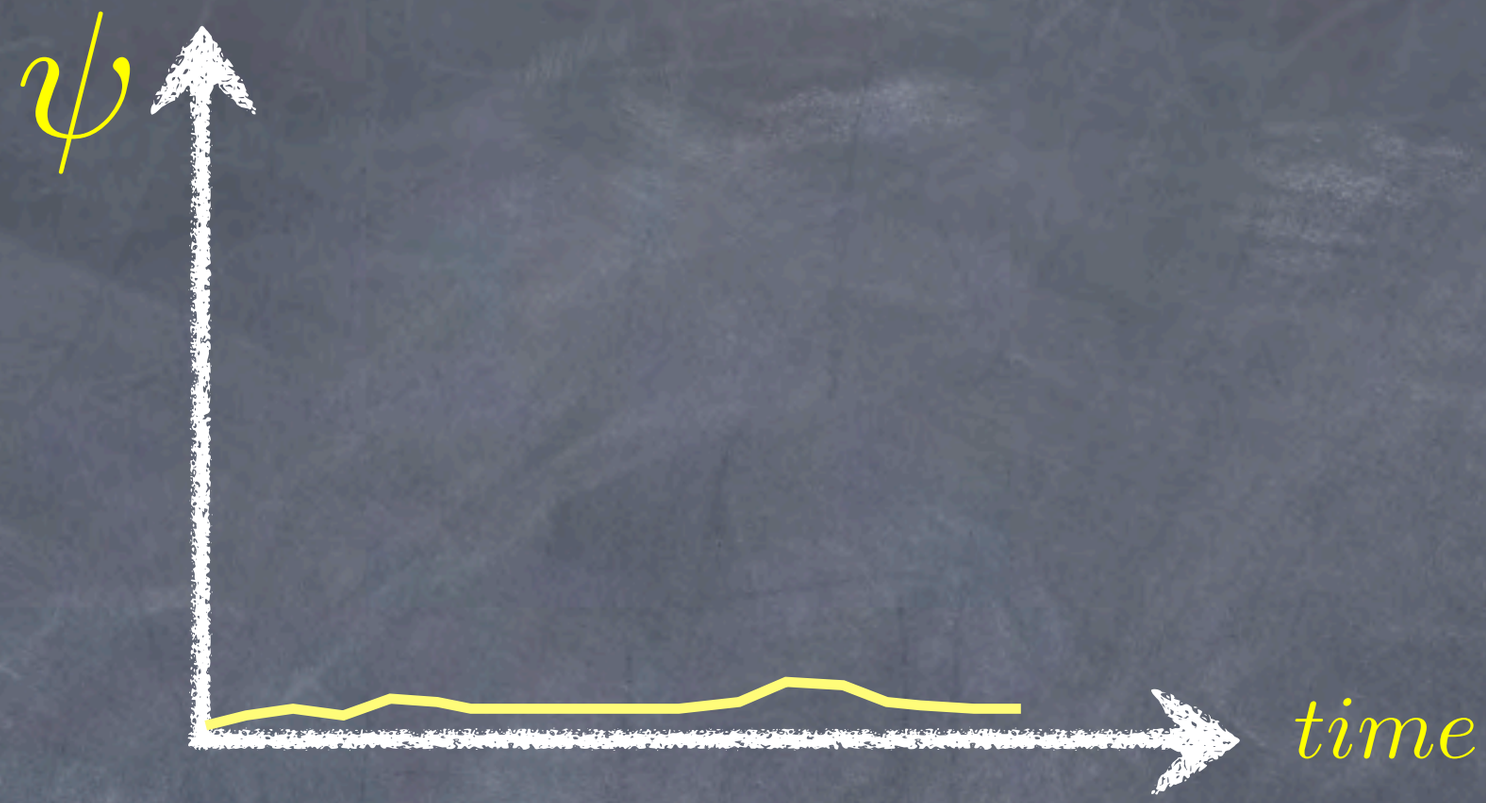

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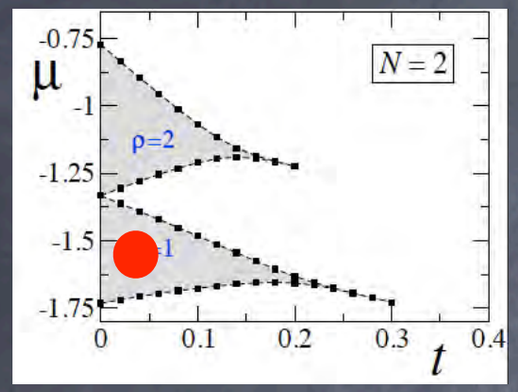
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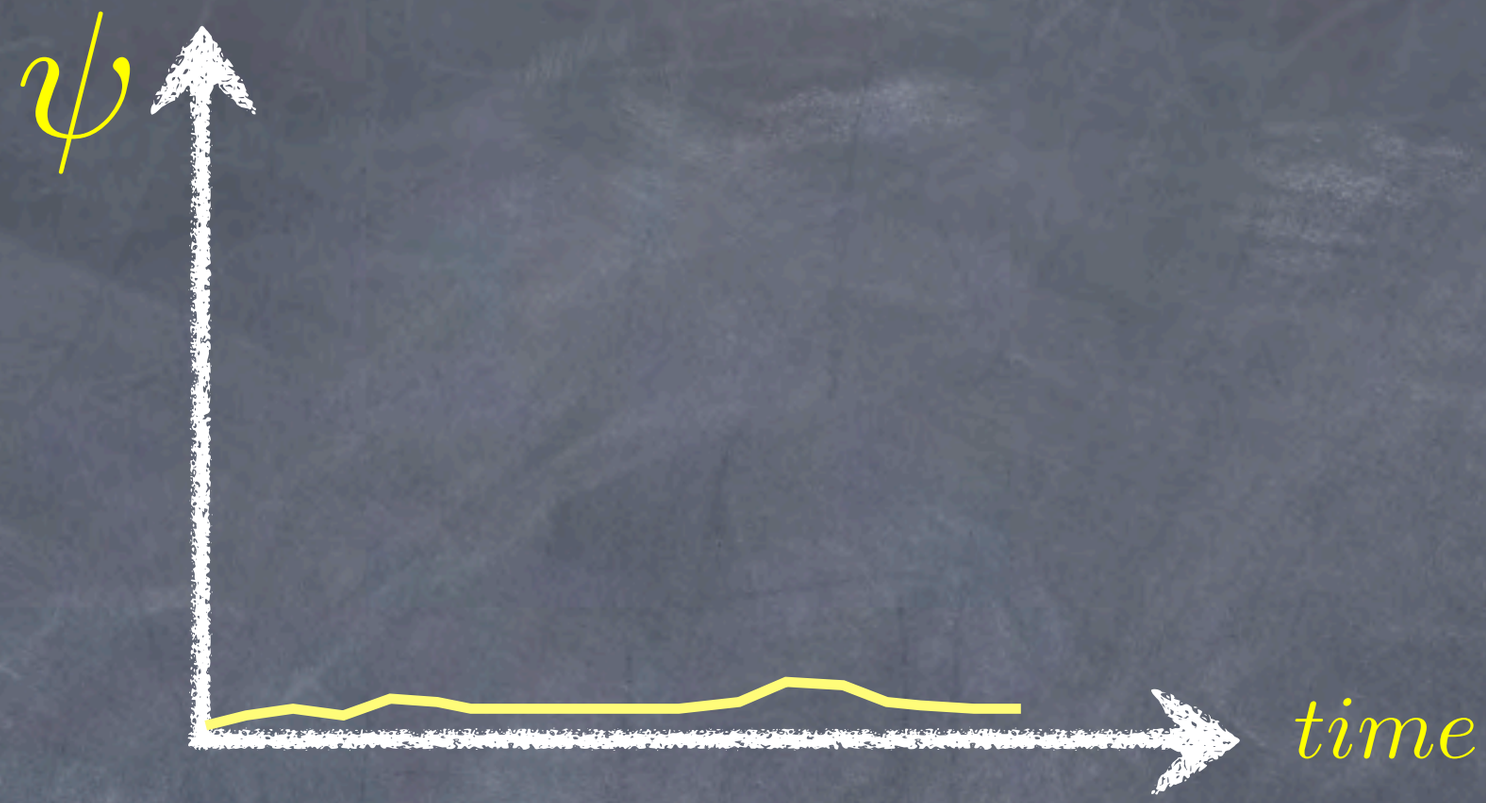
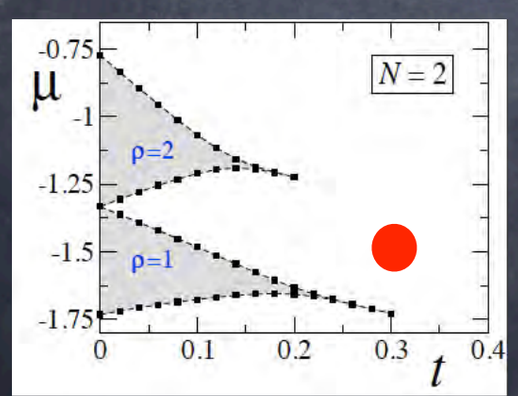
μ
-0.75
-1
-1.25
-1.5
-1.75
ρ=2
ρ=1
0
0.1
0.2
0.3
0.4
t


ψ
time

$J > J_{cr}$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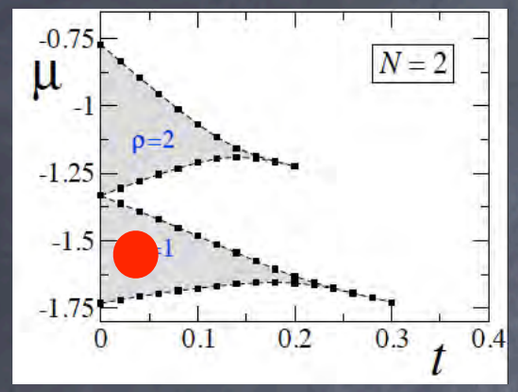
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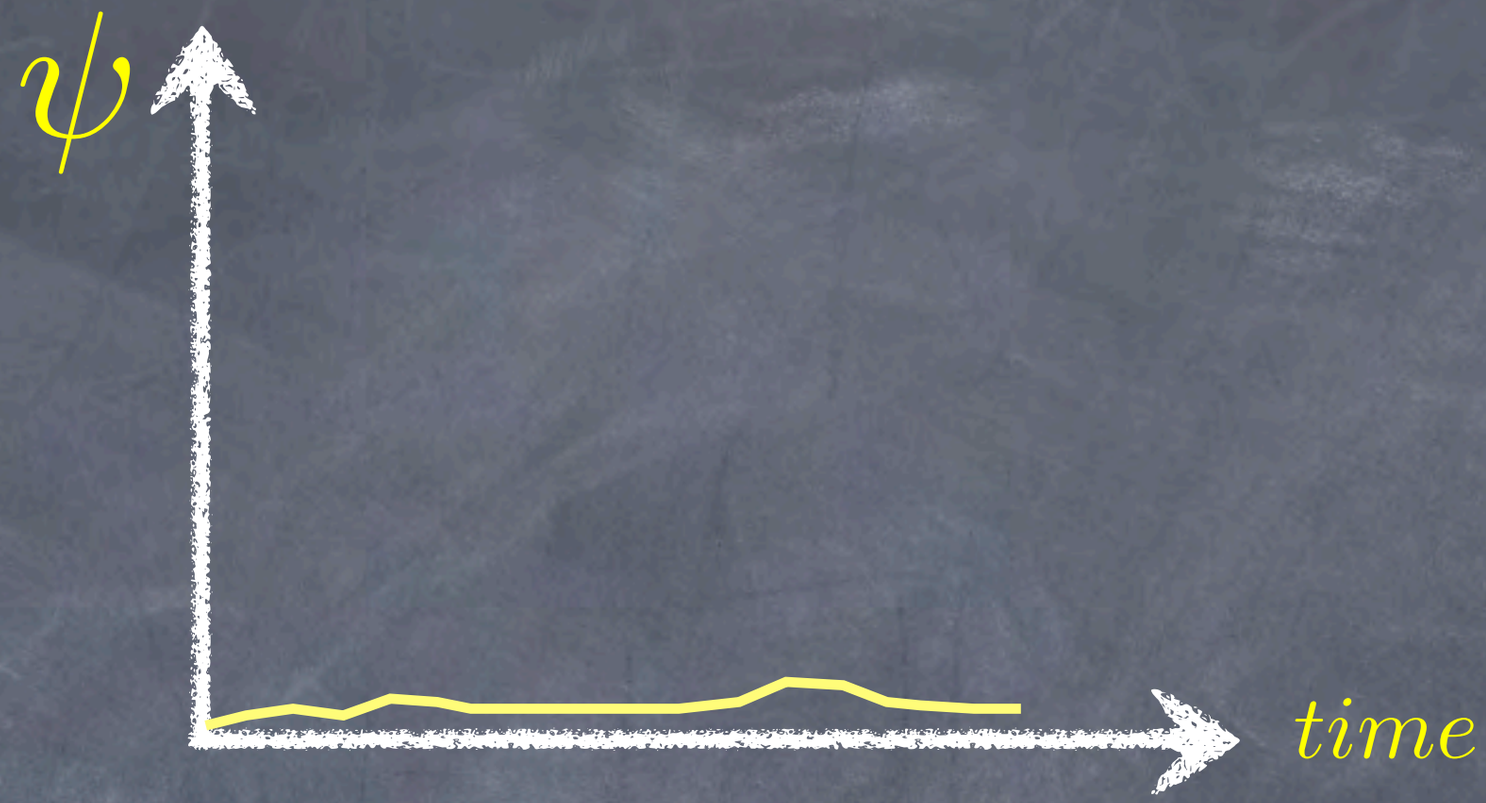
ψ
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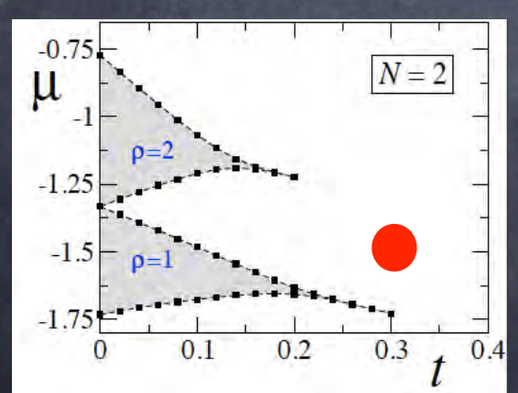
$J > J_{cr}$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ρ=1
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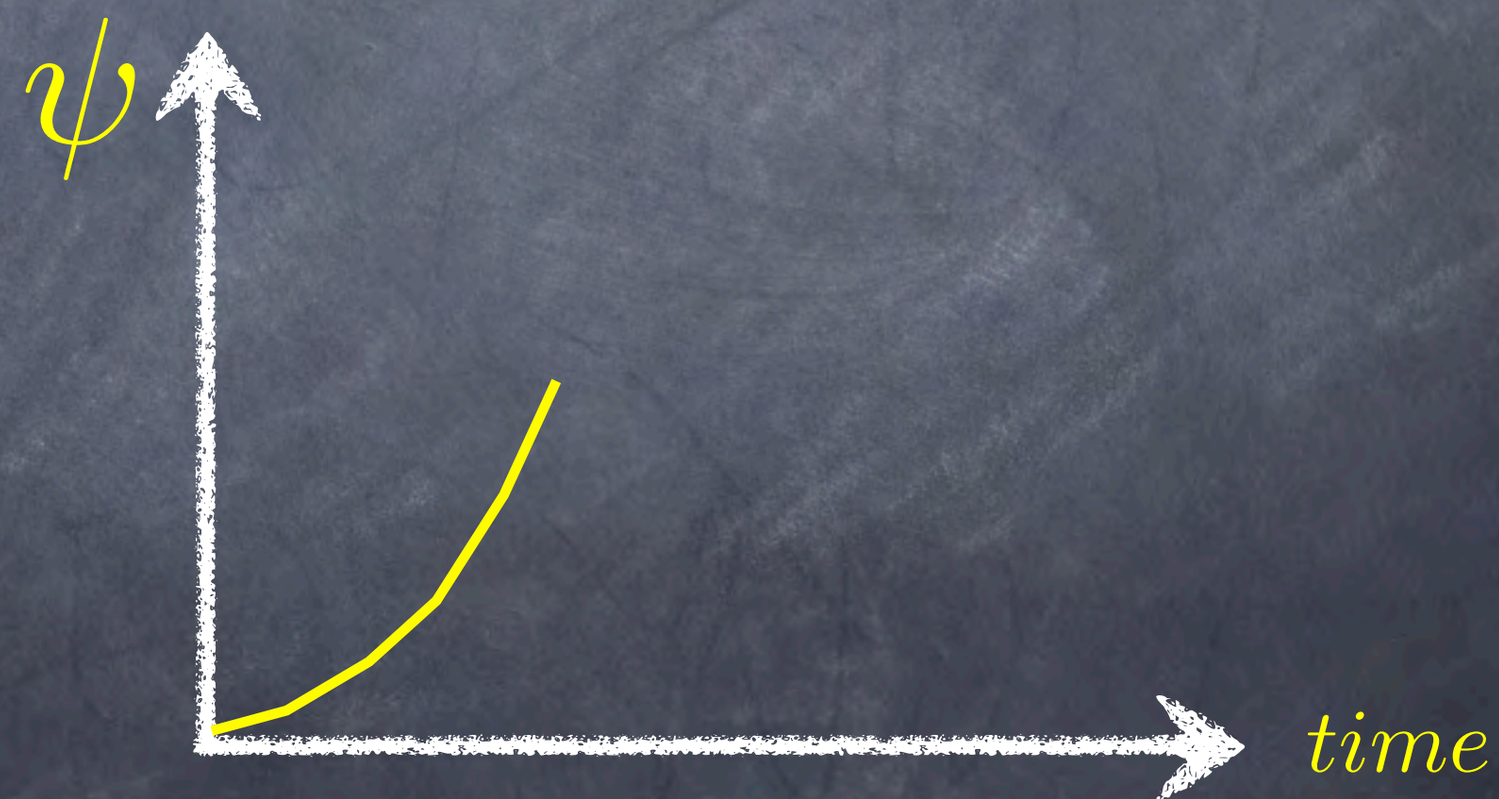
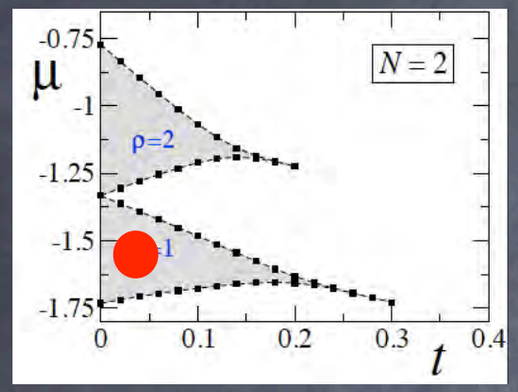
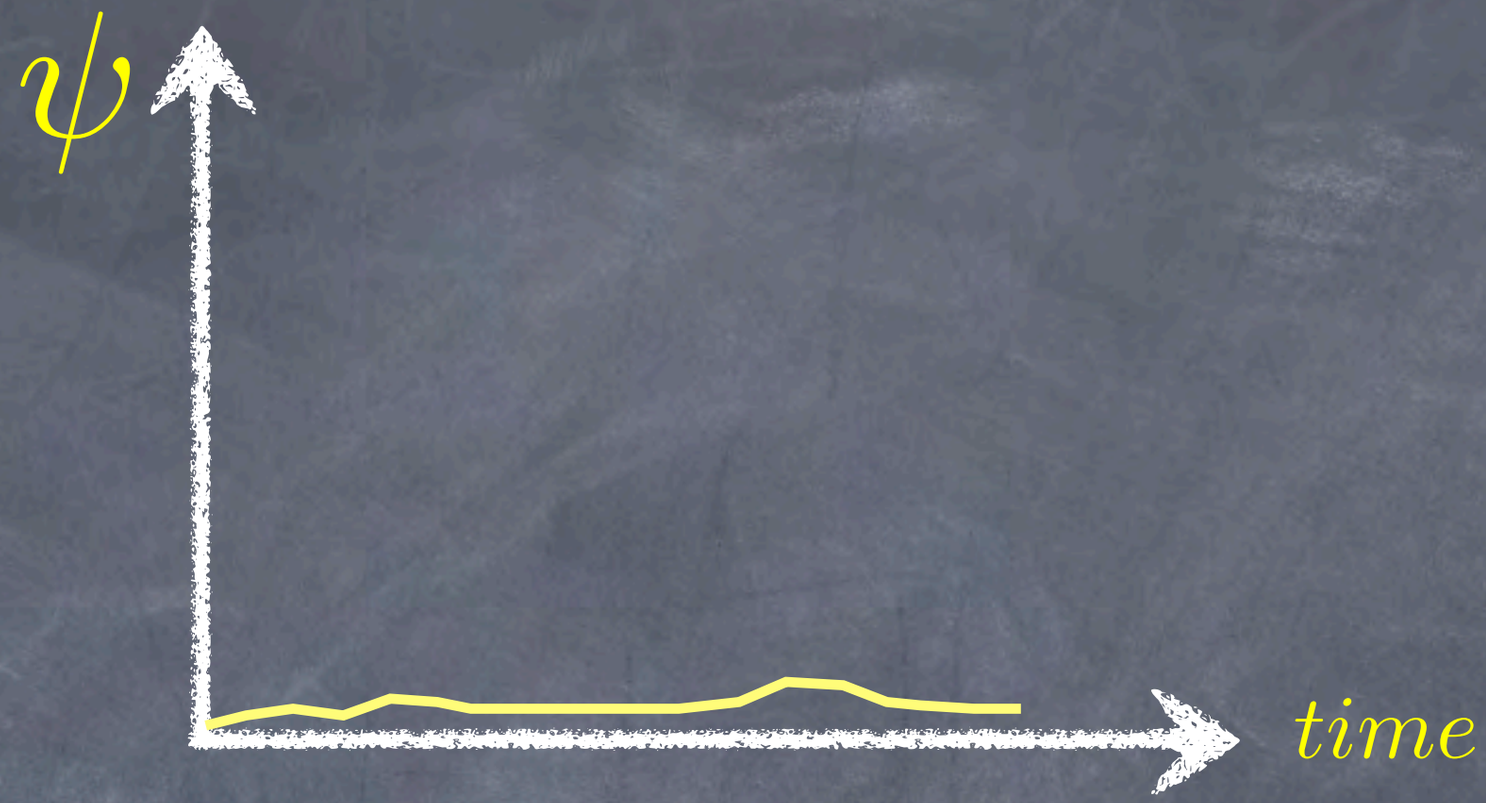
0.3
0.4
t

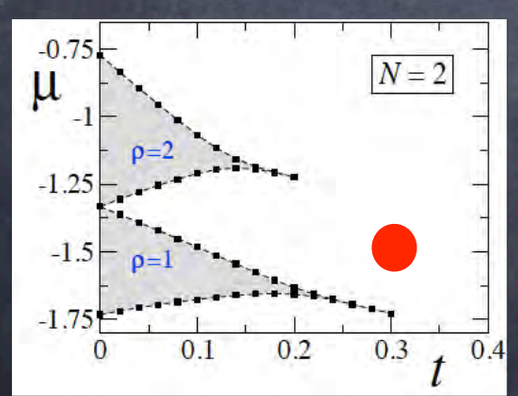

ψ
time

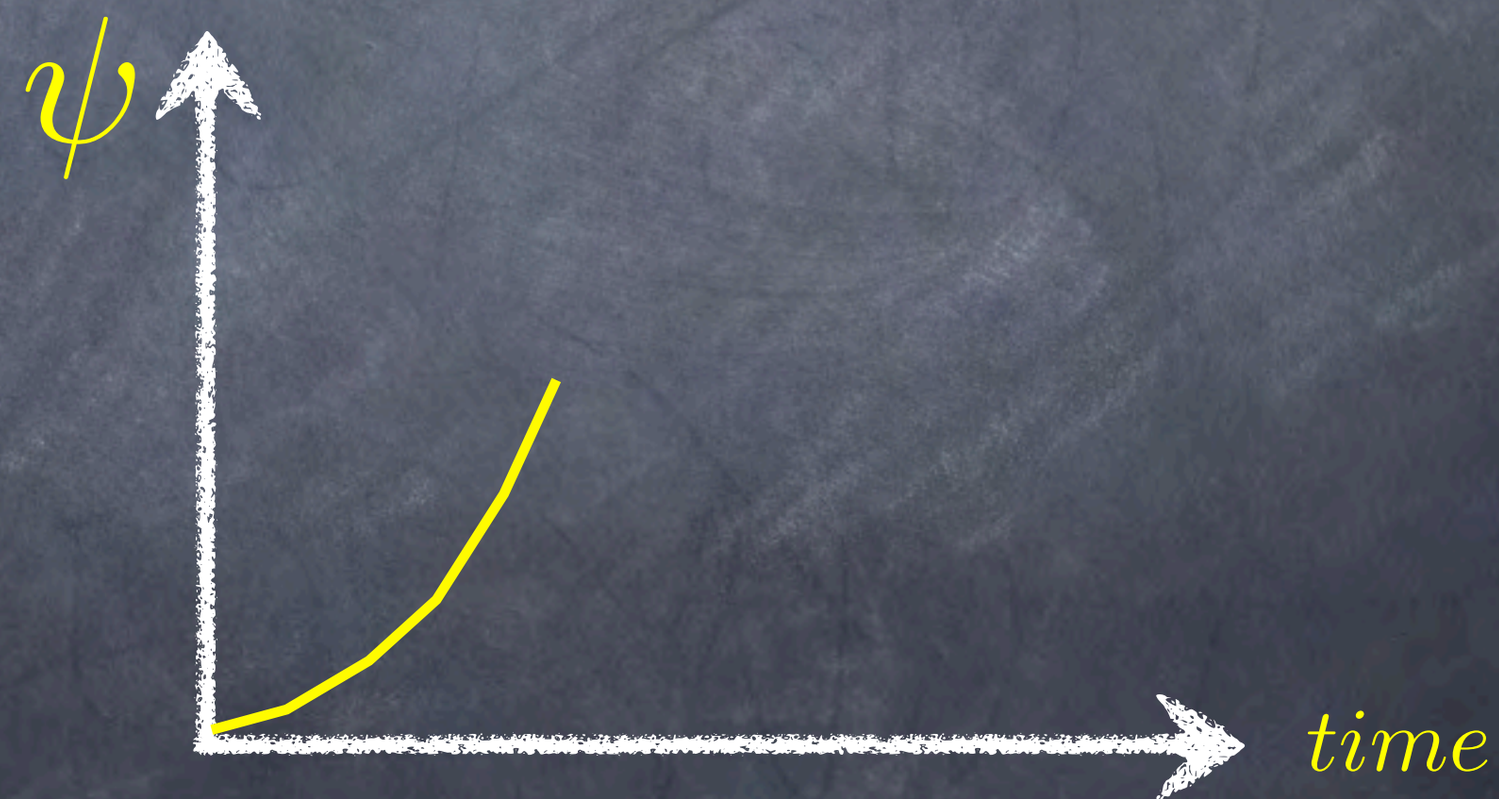
$J < J_{cr}$




$J > J_{cr}$




see Altman & Auerbach, PRL (2002)

# Mean FIeld Approximation

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$$\hat{\mathcal{H}}_{\mathrm{MF}} = \sum_i [\mathcal{H}_i^{(U)} - zJ(\psi^* a_i + h.c.)]$$

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$$\psi(t) = \mathrm{Tr}[a\rho(t)]$$

# Photon population

$$n(t) = n(0)e^{-2\kappa t}$$

Photon population has no feautures

# Observables

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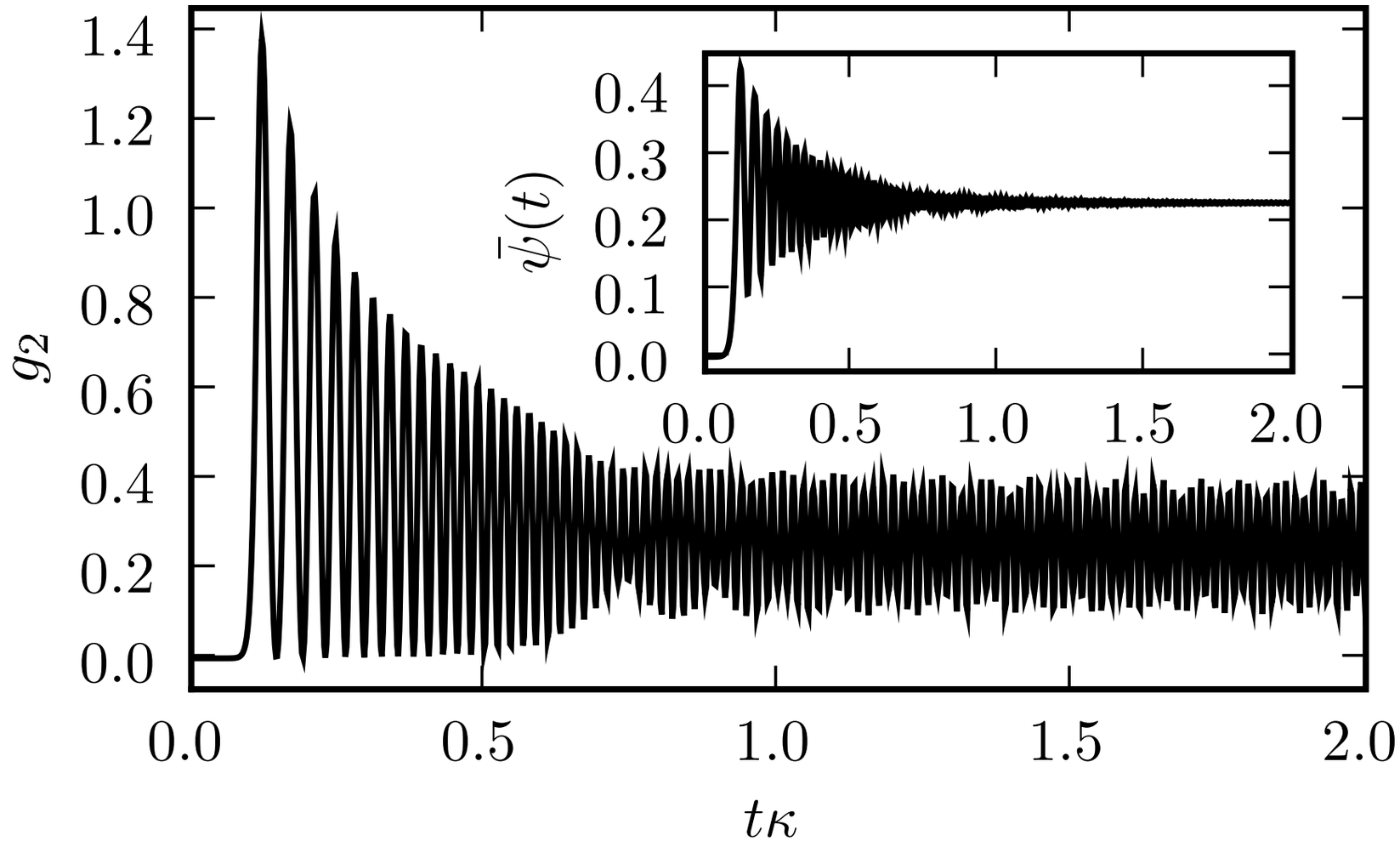
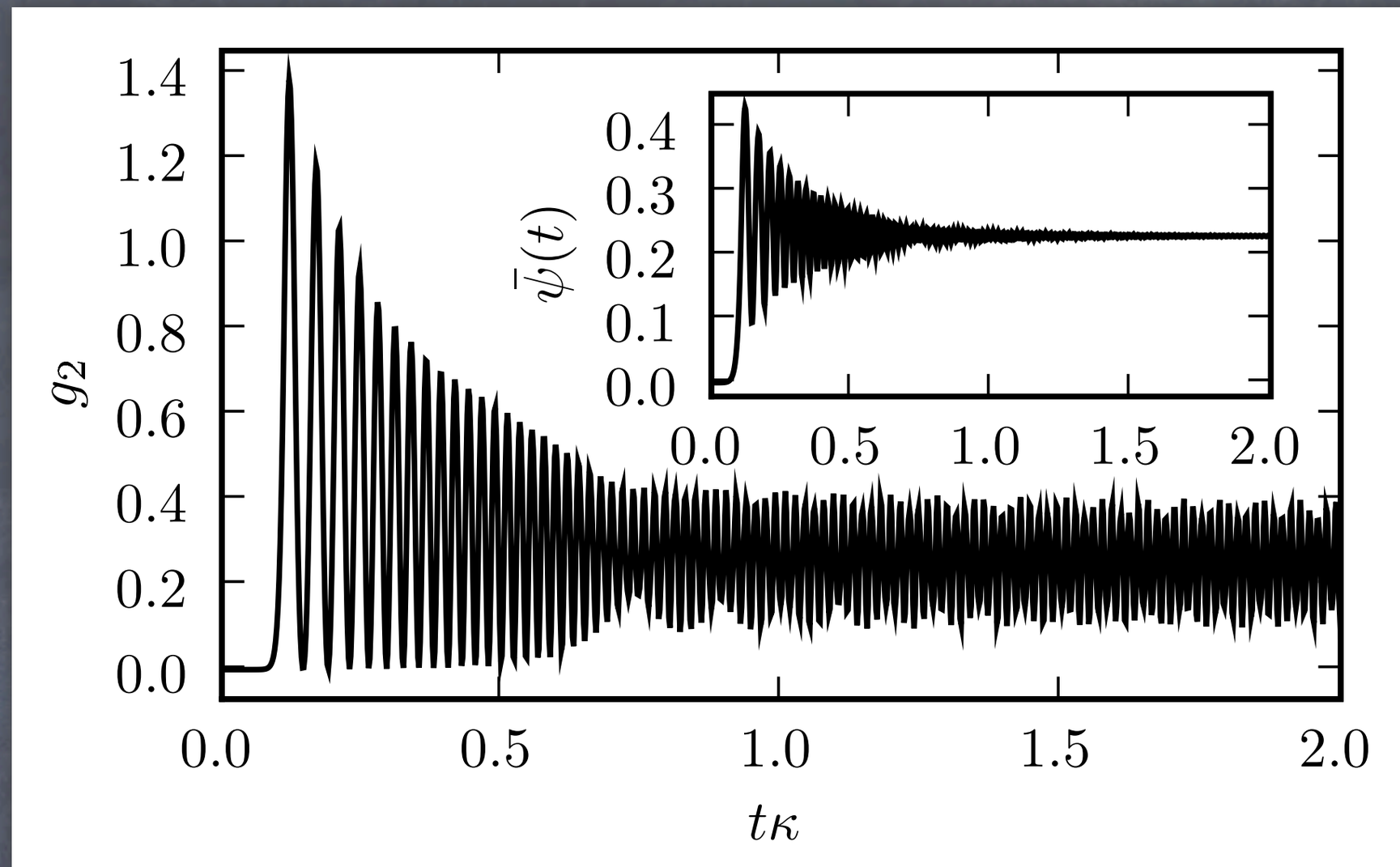
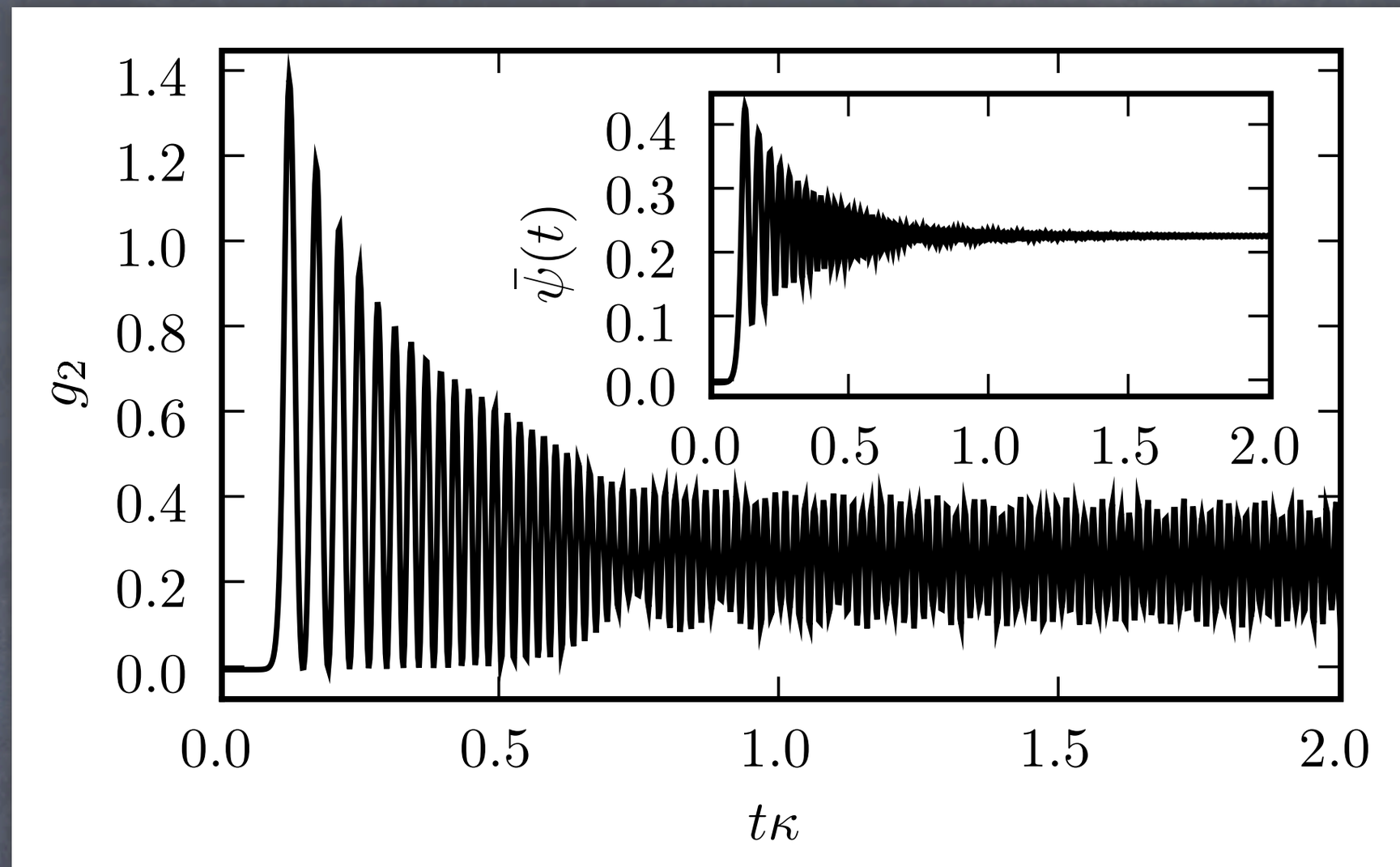
$$g_i^{(2)} = \langle \hat{n}_i^2 - \hat{n}_i \rangle / \langle \hat{n}_i \rangle^2$$


1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$g_2$
0.0
0.5
1.0
1.5
2.0
$t\kappa$
0.4
0.3
0.2
0.1
0.0
$\bar{\psi}(t)$
0.0
0.5
1.0
1.5
2.0




- At short times, a linear instability sets in and both quantities increase exponentially

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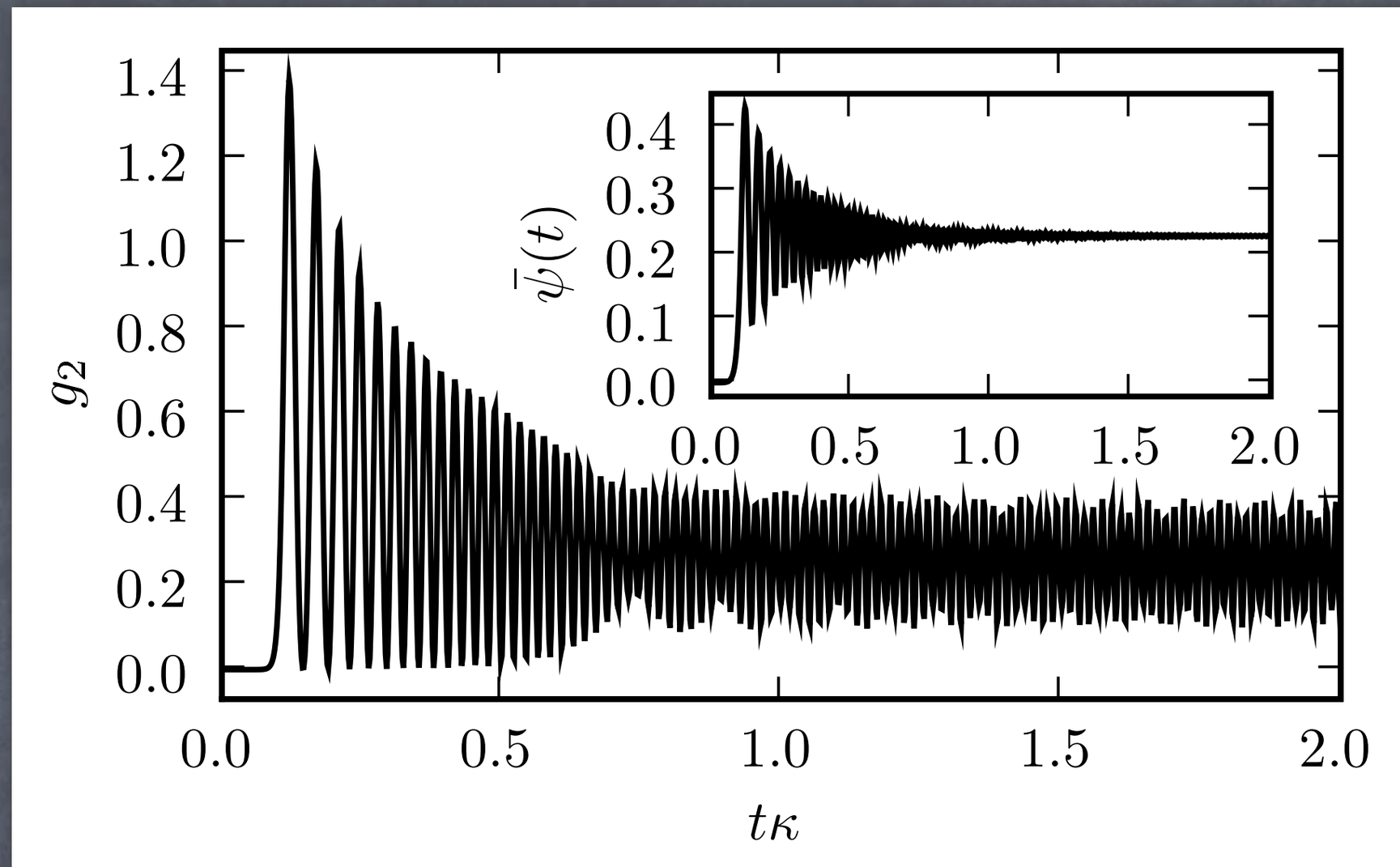


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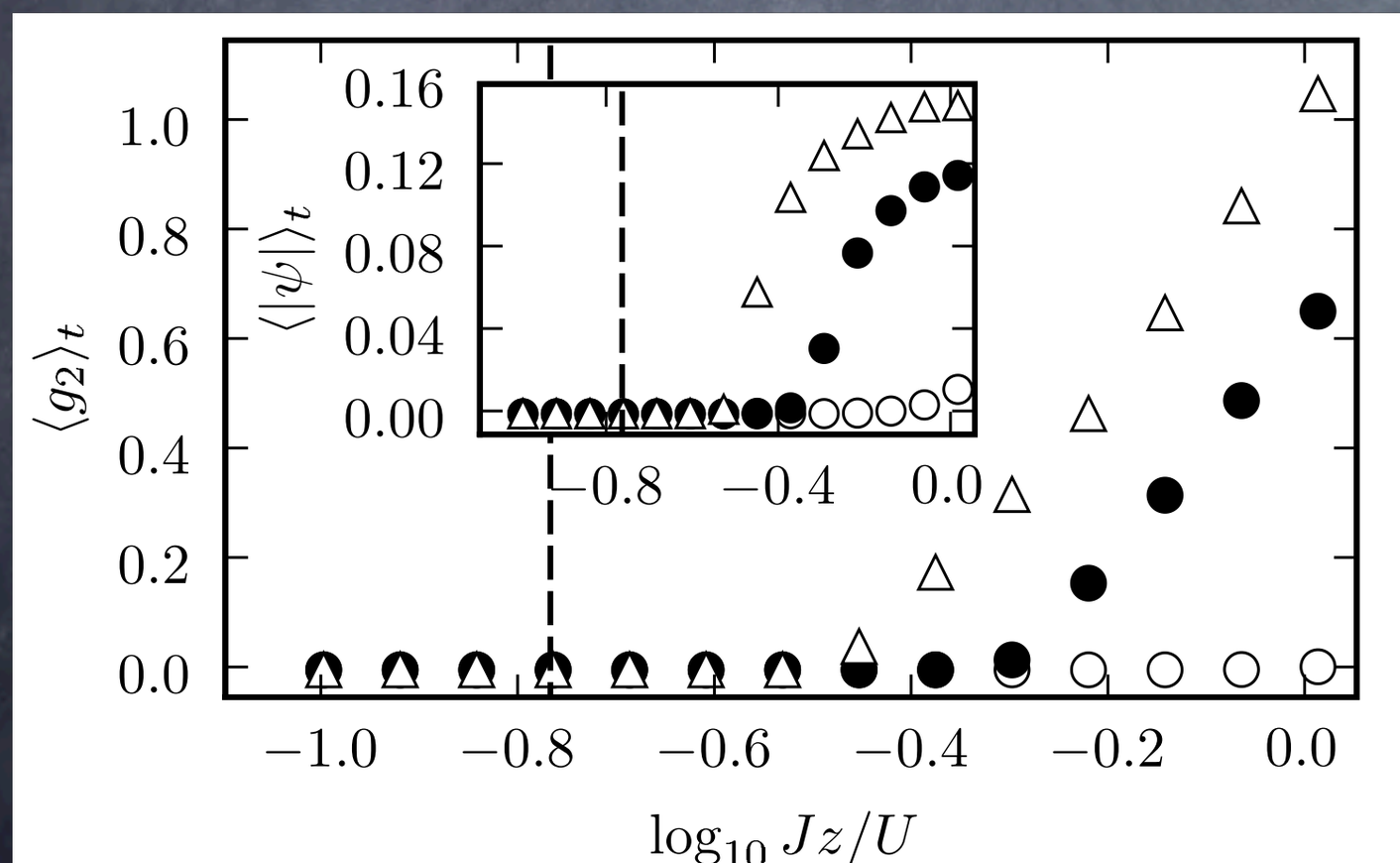
- At later times, the collective (non-linear) dynamics of the array leads to oscillations
- Damping at times larger than the photon lifetime

# ZERO-TIME DELAY SECOND-ORDER CORRELATION FUNCTION AVERAGED OVER A CERTAIN INTERVAL OF TIME

$$0.1\kappa^{-1} < t < 2.0\kappa^{-1}$$



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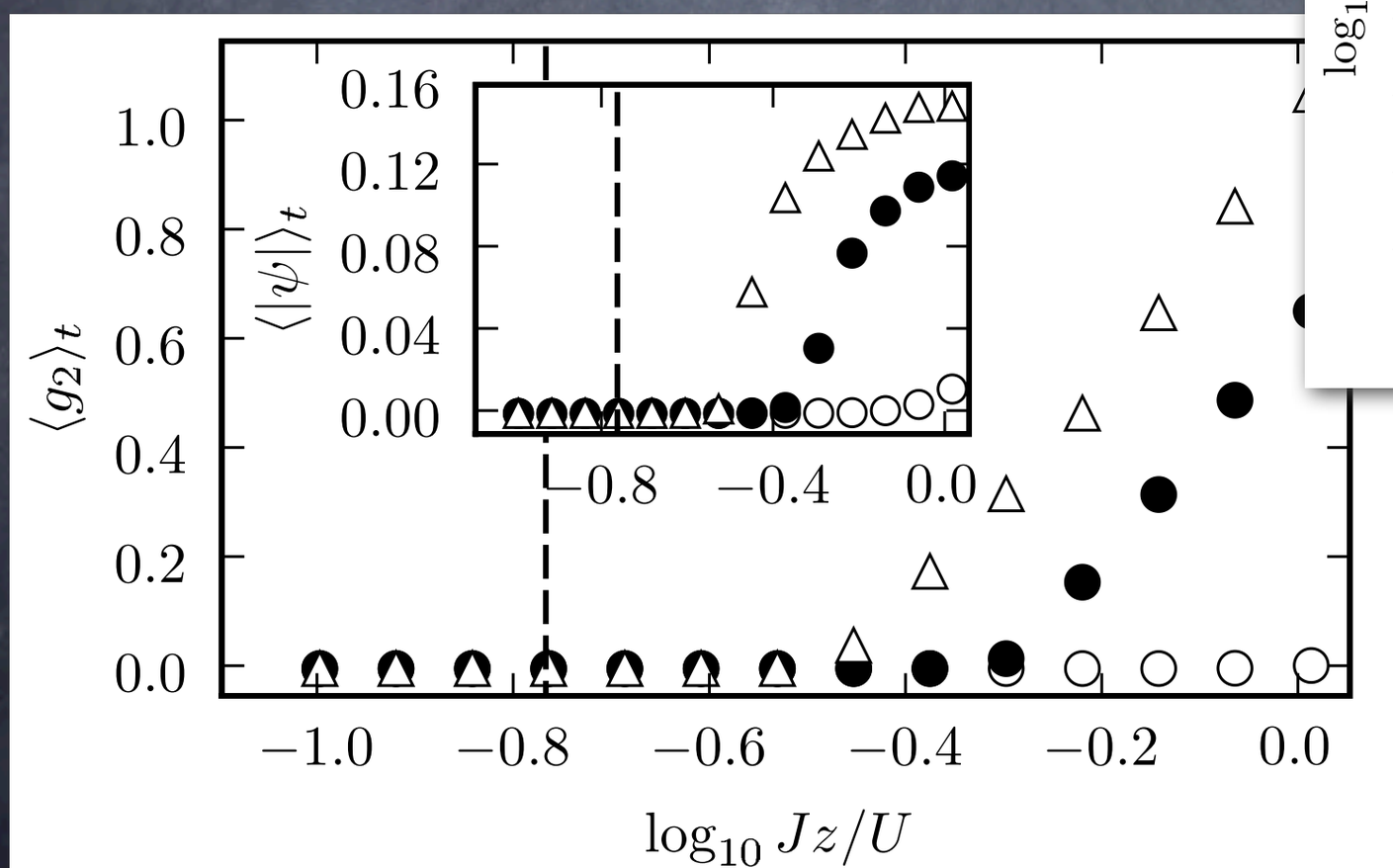
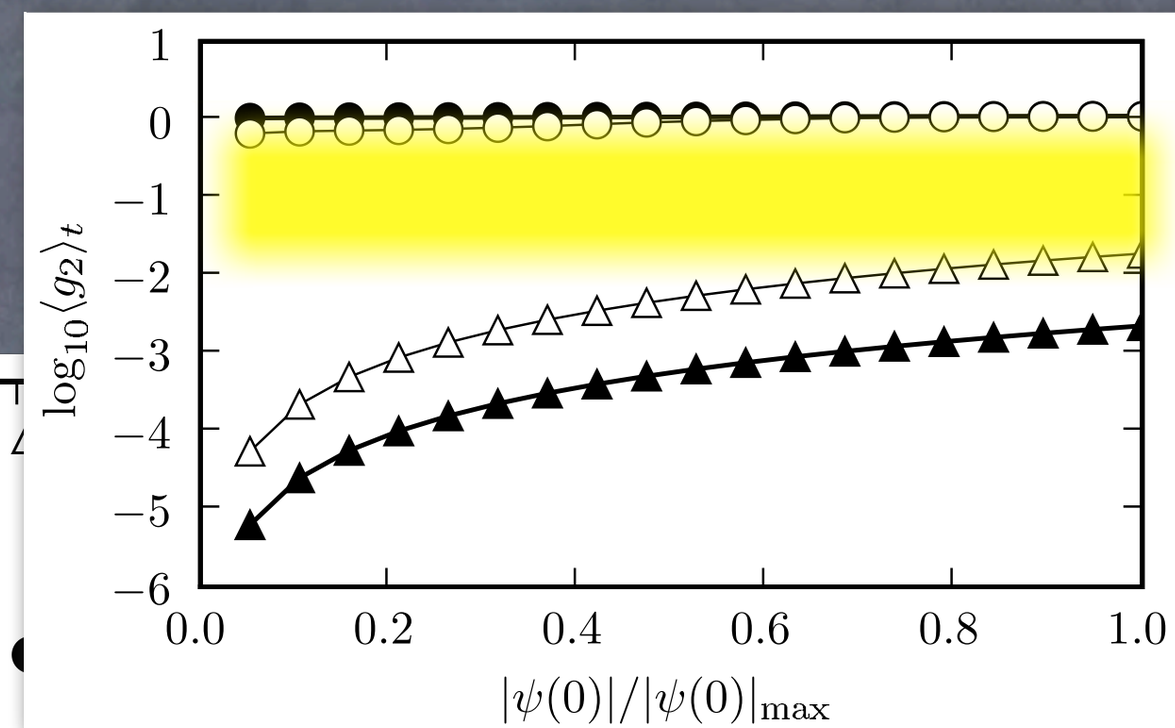
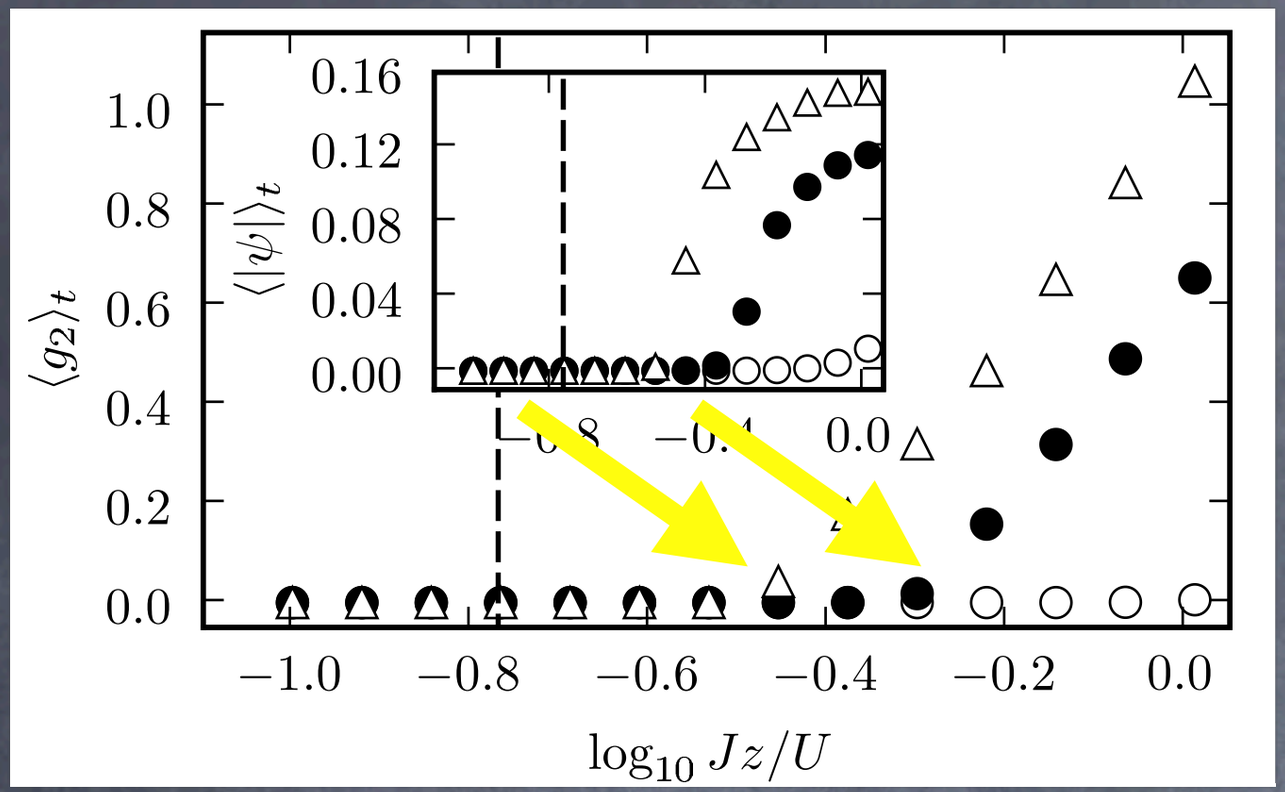





$\kappa_{\triangle} > \kappa_{\bullet}$

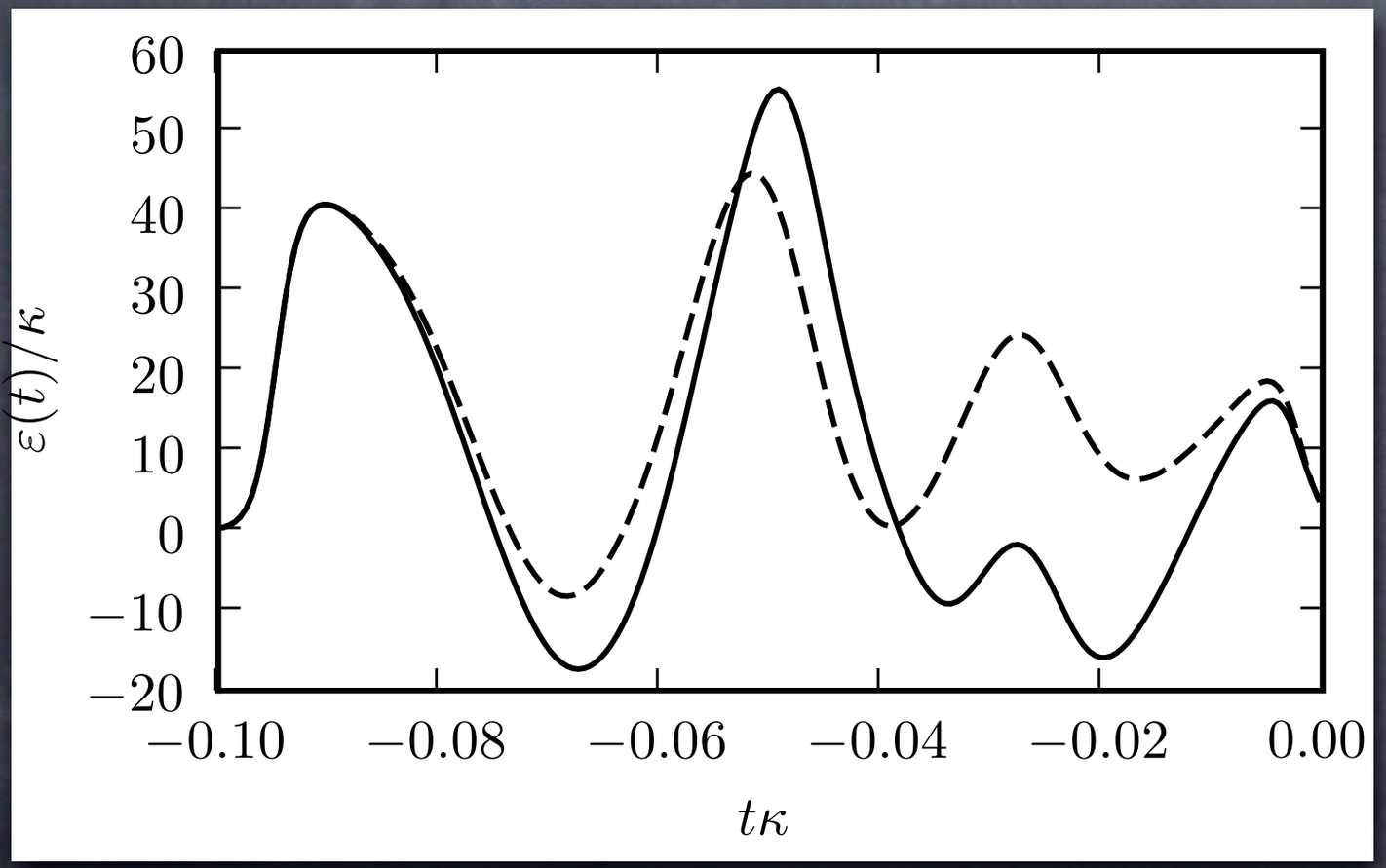
The onset of the instability shifts with the damping rate

$$\delta(J/U)_c \sim \kappa^2/2U^2$$

U is the scale of the non-linearity

# Preparation of the Initial state

Important to have short pulses for preparation




$$\mathcal{F} \sim 0.99$$

- dashed line J=0
- continous line zJ/U = 0.5

# Conclusions

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- Networks based QED-cavities?

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- Non-equilibrium quantum phase transitions?