Multifractality and e-e interaction at MI, quantum Hall, and SI transitions

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Plan

- **Anderson localization and multifractality**
- **Dephasing and temperature scaling at MI and QH transitions**
- **Multifractality and interaction at SIT**

50 years of Anderson localization

Philip W. Anderson

1958 "Absence of diffusion
in certain random lattices"

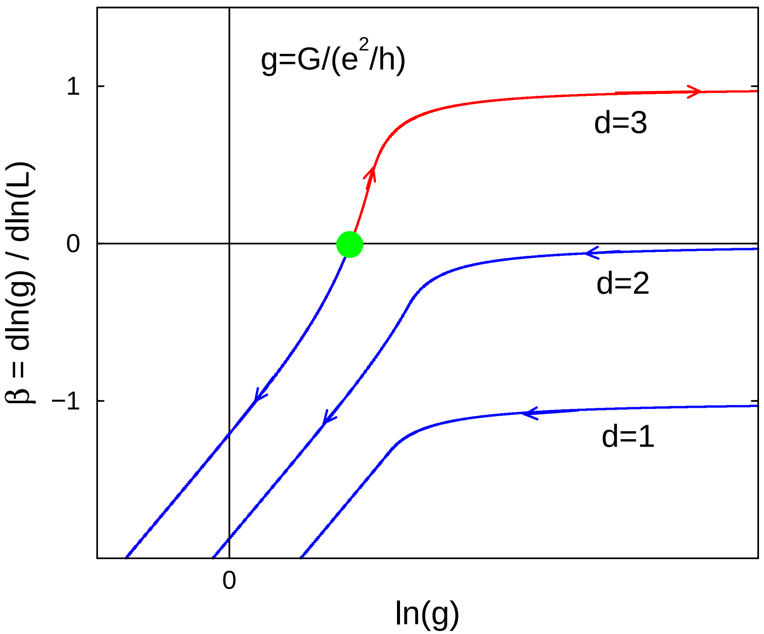
sufficiently strong disorder $\longrightarrow$ quantum localization

$\longrightarrow$ eigenstates exponentially localized, no diffusion

$\longrightarrow$ Anderson insulator

The Nobel Prize in Physics 1977

Anderson transition

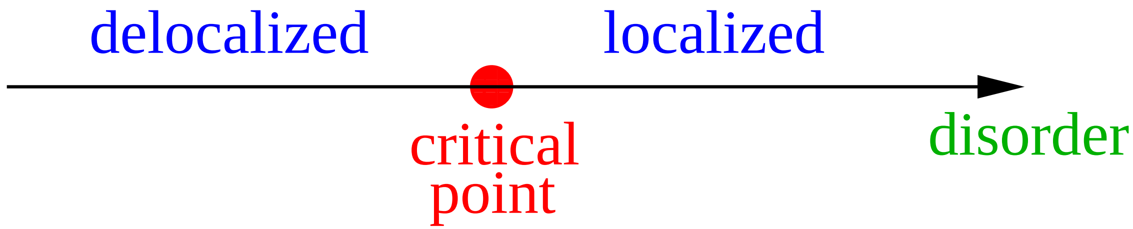


Scaling theory of localization:
Abrahams, Anderson, Licciardello, Ramakrishnan '79

Modern approach:
RG for field theory (σ-model)

quasi-1D, 2D: metallic $\longrightarrow$ localized crossover with decreasing g

$d > 2$: metal-insulator transition (sometimes also in $d = 2$)



Continuous phase transition with highly unconventional properties!

review: Evers, ADM, Rev. Mod. Phys.'08

Field theory: non-linear σ-model

$$S[Q] = \frac{\pi\nu}{4}\int d^d\mathrm{r}\,\mathrm{Str}\,[-D(\nabla Q)^2 - 2i\omega\Lambda Q], \qquad Q^2(\mathrm{r}) = 1$$

Wegner '79

σ-model manifold: symmetric space

e.g. for broken time-reversal invariance:

$\mathrm{U}(2n)/\mathrm{U}(n)\times\mathrm{U}(n)$, $\qquad n \to 0$

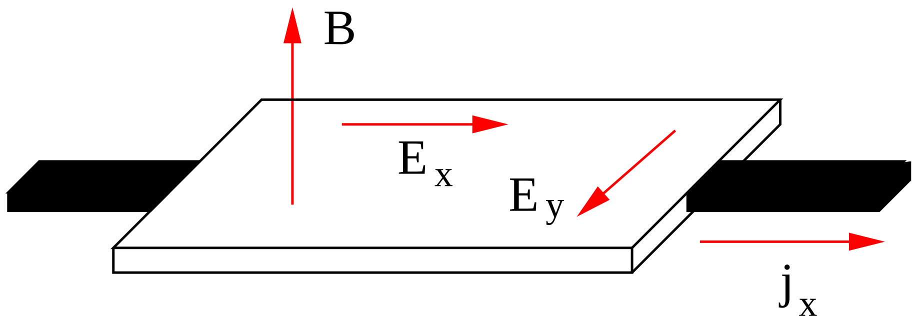
with Coulomb interaction: Finkelstein'83

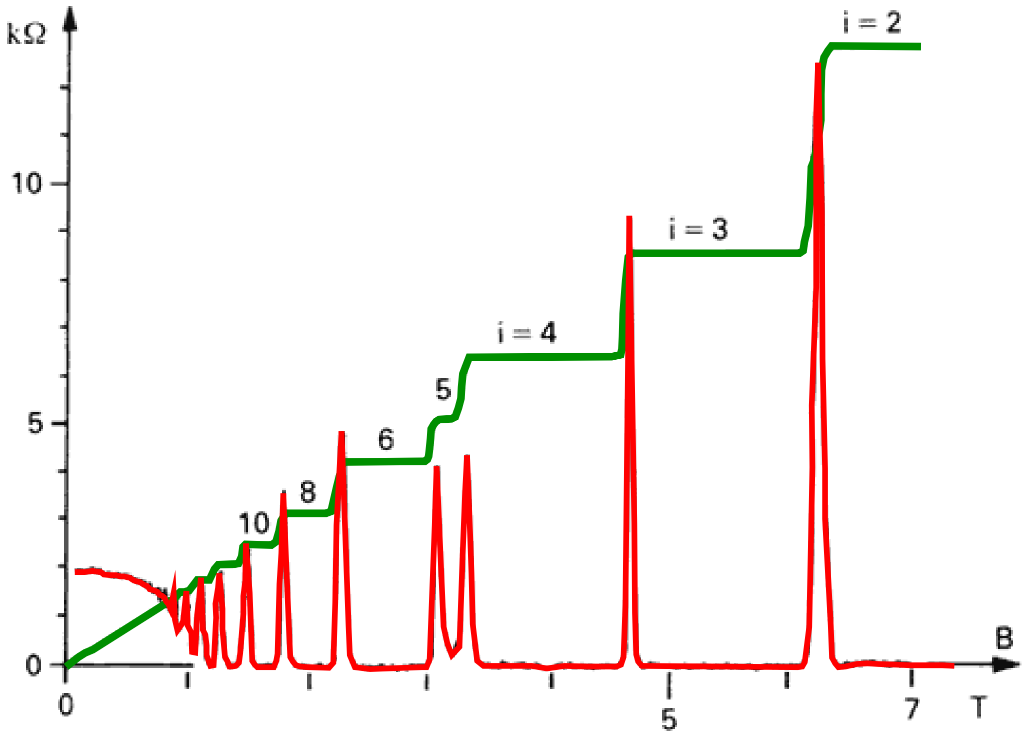
Magnetotransport in 2D: Integer Quantum Hall Effect

resistivity tensor:

$$\rho_{xx} = E_x / j_x$$

$$\rho_{yx} = E_y / j_x \quad \textbf{Hall resistivity}$$



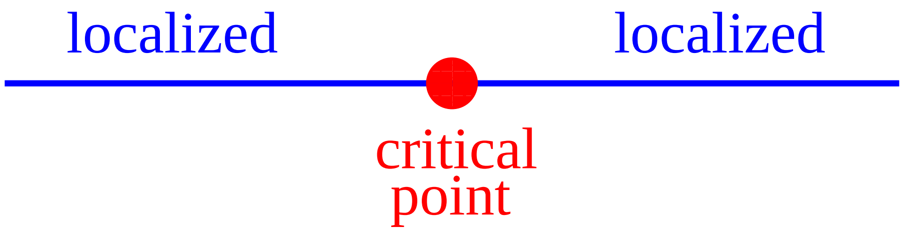


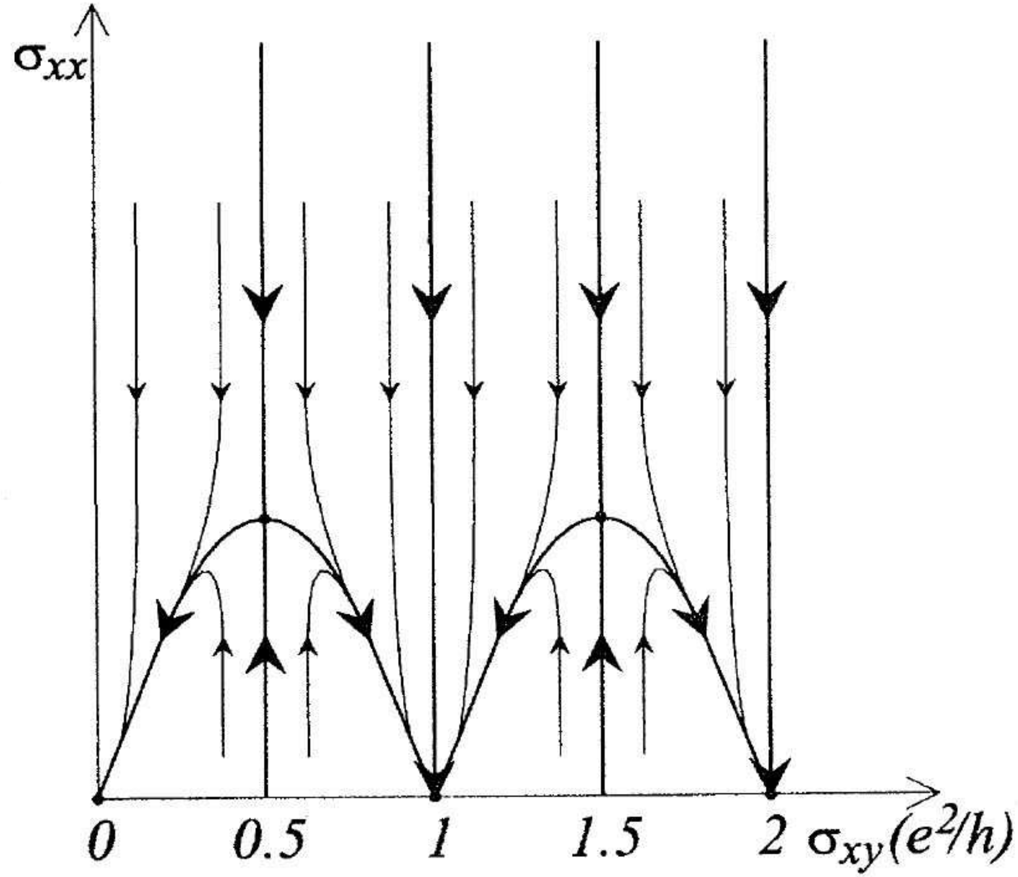
Klaus von Klitzing
Nobel Prize 1985

IQHE: $\mathbb{Z}$ topological insulator

IQHE flow diagram

Khmelnitskii' 83, Pruisken' 84





Field theory (Pruisken):

σ-model with topological term

$$S = \int d^2r \left\{ -\frac{\sigma_{xx}}{8} \mathrm{Tr}(\partial_\mu Q)^2 + \frac{\sigma_{xy}}{8} \mathrm{Tr}\epsilon_{\mu\nu} Q \partial_\mu Q \partial_\nu Q \right\}$$

QH insulators $\longrightarrow$ $n = \ldots, -2, -1, 0, 1, 2, \ldots$ edge states

$\longrightarrow$ $\mathbb{Z}$ topological insulator

Multifractality at the Anderson transition

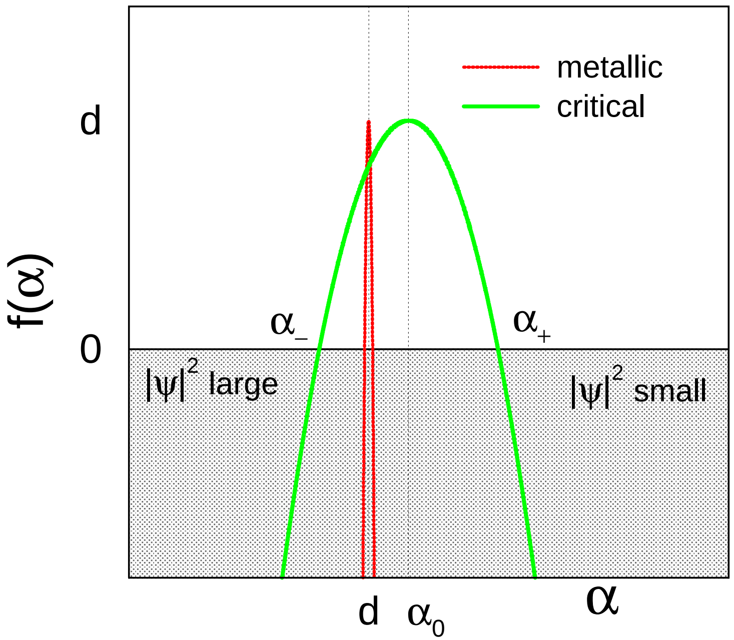
$P_q = \int d^d r |\psi(\mathrm{r})|^{2\mathrm{q}}$ inverse participation ratio

$$\langle P_q \rangle \sim \begin{cases} L^0 & \text{insulator} \\ L^{-\tau_q} & \text{critical} \\ L^{-d(q-1)} & \text{metal} \end{cases}$$

$$\tau_q = \underbrace{d(q-1)}_{\text{normal}} + \underbrace{\Delta_q}_{\text{anomalous}} \equiv D_q(q-1) \quad \text{multifractality}$$

$\tau_q \longrightarrow$ Legendre transformation

$\longrightarrow$ singularity spectrum $f(\alpha)$



wave function statistics:

$$\mathcal{P}(\ln|\psi^2|) \sim L^{-d+f(\ln|\psi^2|/\ln L)}$$

$L^{f(\alpha)}$ – measure of the set of points where $|\psi|^2 \sim L^{-\alpha}$

Multifractality and the field theory

- Δ_q – scaling dimensions of operators $\quad \mathcal{O}^{(q)} \sim (Q\Lambda)^q$

$d = 2 + \epsilon$: $\qquad \Delta_q = -q(q-1)\epsilon + O(\epsilon^4) \qquad$ Wegner '80

- Infinitely many operators with negative scaling dimensions
- wave function correlations $\longleftrightarrow$ Operator Product Expansion

Wegner 85 ; Duplantier, Ludwig 91

- $\Delta_1 = 0 \quad \longleftrightarrow \quad \langle Q \rangle = \Lambda \quad$ naive order parameter uncritical

Transition described by an order parameter function $F(Q)$

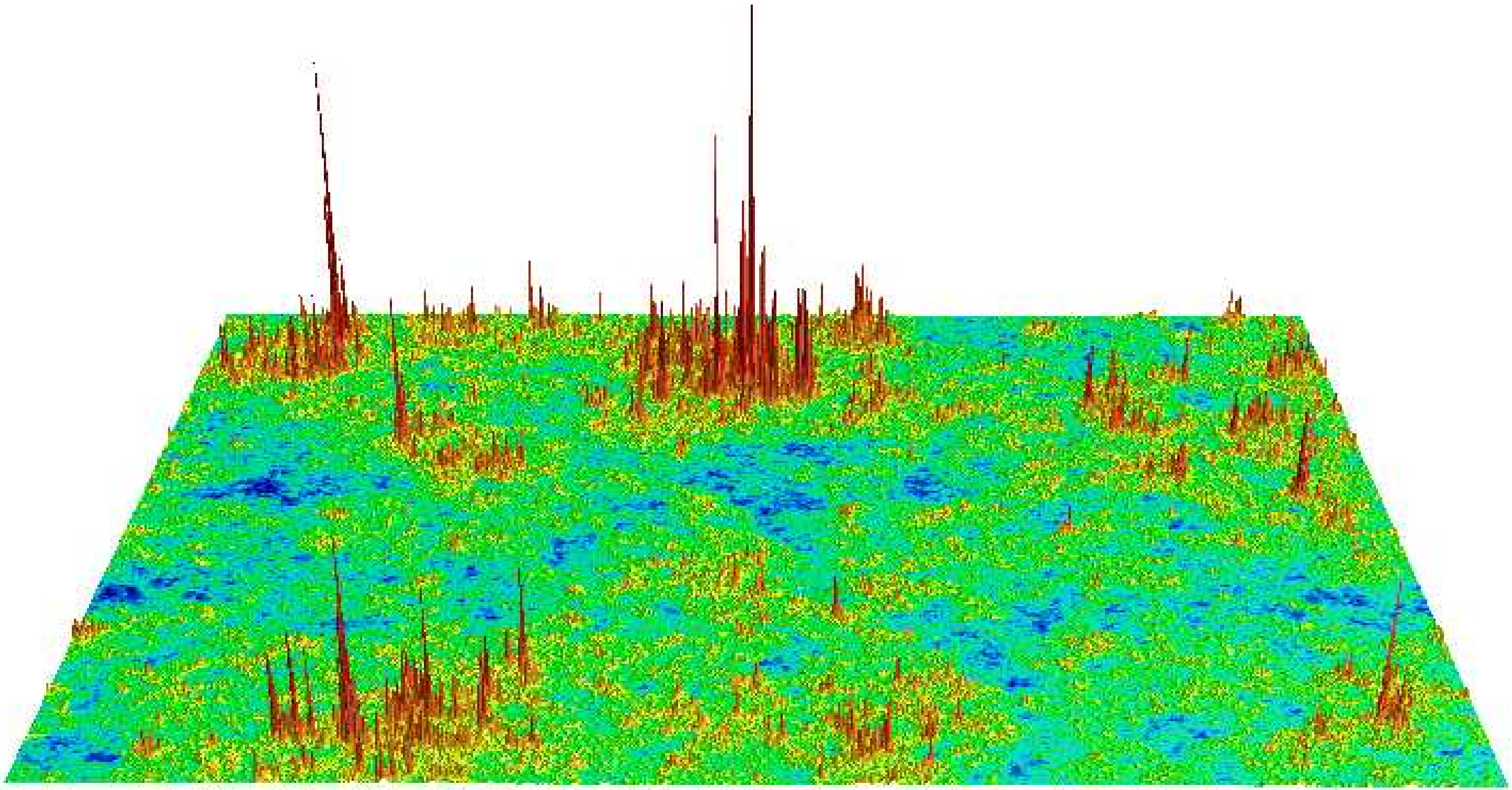
Zirnbauer 86, Efetov 87

$\longleftrightarrow$ distribution of local Green functions

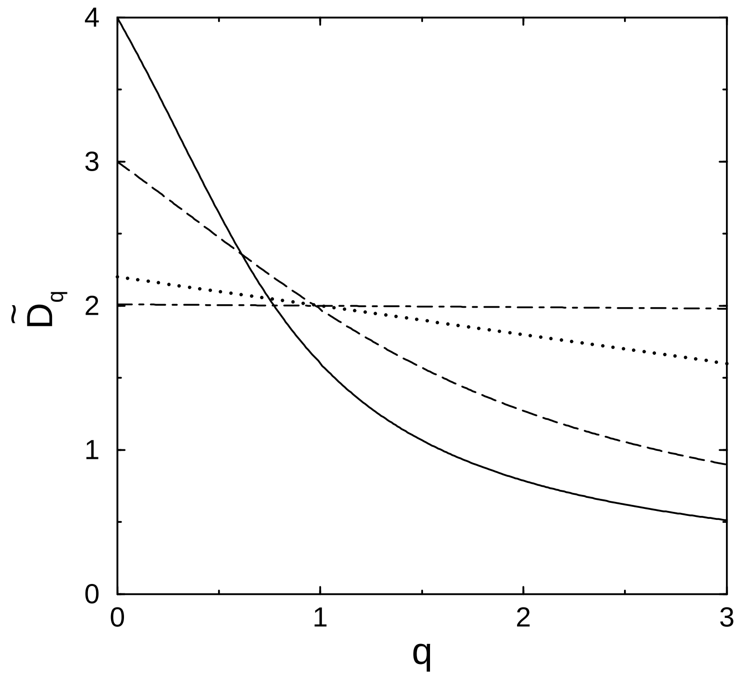
and wave function amplitudes

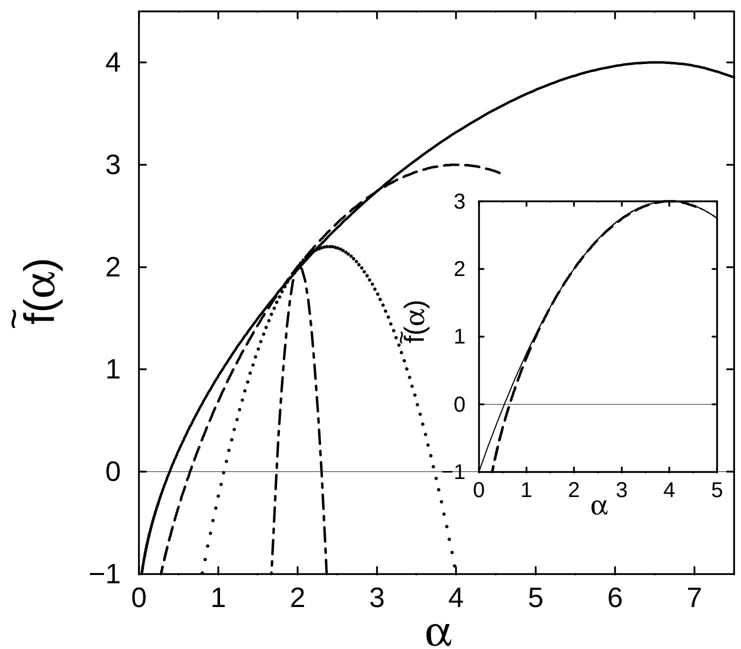
ADM, Fyodorov '91

Multifractal wave functions at the Quantum Hall transition



Dimensionality dependence of multifractality





Analytics $(2+\epsilon,$ one-loop$)$ and numerics

$$\tau_q = (q-1)d - q(q-1)\epsilon + O(\epsilon^4)$$

$$f(\alpha) = d - (d+\epsilon-\alpha)^2/4\epsilon + O(\epsilon^4)$$

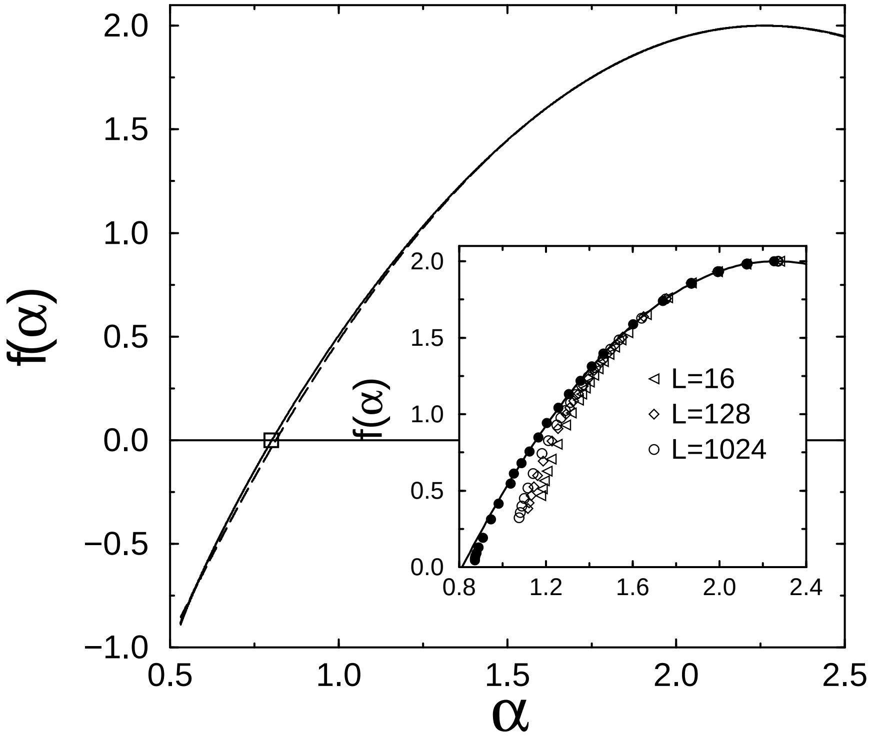
$d = 4$ (full)
$d = 3$ (dashed)
$d = 2+\epsilon, \quad \epsilon = 0.2$ (dotted)
$d = 2+\epsilon, \quad \epsilon = 0.01$ (dot-dashed)

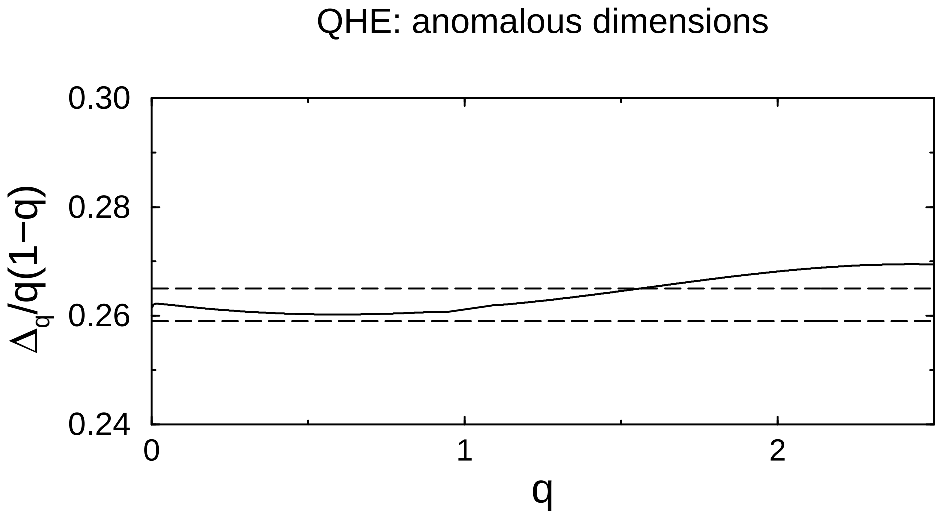
Inset: $d = 3$ (dashed)
vs. $d = 2+\epsilon, \ \epsilon = 1$ (full)

Mildenberger, Evers, ADM '02

Multifractality at the Quantum Hall critical point

Evers, Mildenberger, ADM '01





⟶ spectrum is close to parabolic (with small deviations):

$$f(\alpha) \simeq 2 - \frac{(\alpha - \alpha_0)^2}{4(\alpha_0 - 2)}, \qquad \Delta_q \simeq (\alpha_0 - 2)q(1-q)$$

with $\alpha_0 - 2 = 0.262 \pm 0.003$

Dephasing at metal-insulator and quantum Hall transitions

e-e interaction $\longrightarrow$ dephasing at finite T

$\longrightarrow$ smearing of the transition

local. length $\xi \propto |n - n_c|^{-\nu}$, dephasing length $L_\phi \propto T^{-1/z_T}$

$\longrightarrow$ transition width $\delta n \propto |n - n_c|^{\kappa}$, $\qquad \kappa = 1/\nu z_T$

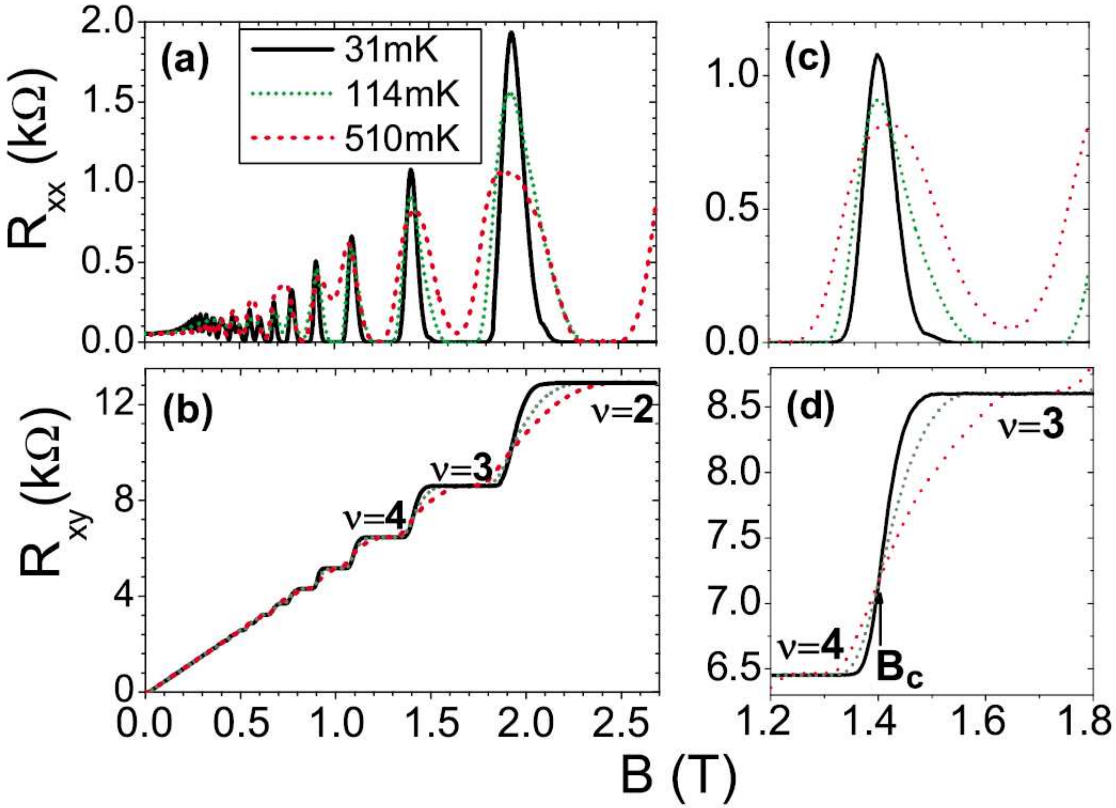
We focus on short-range e-e interaction:

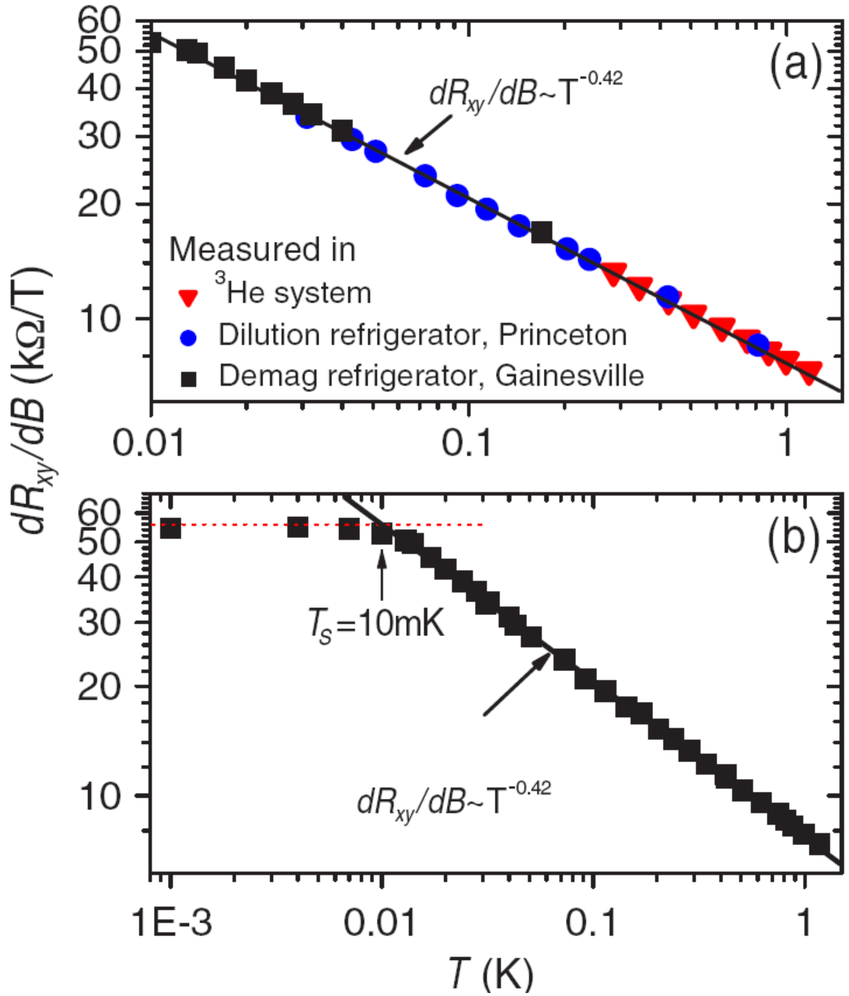
- long-range Coulomb interaction negligible because of large dielectric constant
- 2D: screening by metallic gate
- interacting neutral particles (e.g. cold atoms)

Earlier works:

Lee, Wang, PRL'96 ; Wang, Fisher, Girvin, Chalker, PRB '00

Temperature scaling of quantum Hall transition





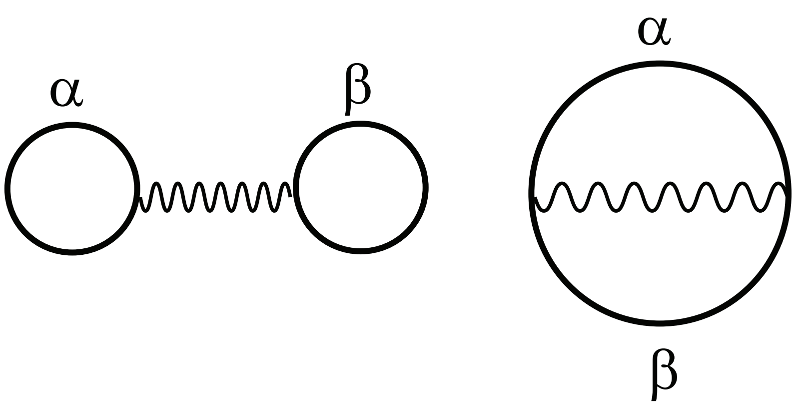
$$1/\nu z_T = 0.42 \pm 0.01$$

Wei, Tsui, Paalanen, Pruisken, PRL'88 ; Li et al., PRL'05, PRL'09

Interaction scaling at criticality

$$\mathcal{K}_1 = \frac{\Delta^2}{2} \sum_{\alpha\beta} \left\langle \left| \mathcal{B}_{\alpha\beta}(r_1, r_2) \right|^2 \delta(E + \omega - \epsilon_\alpha) \delta(E - \epsilon_\beta) \right\rangle$$

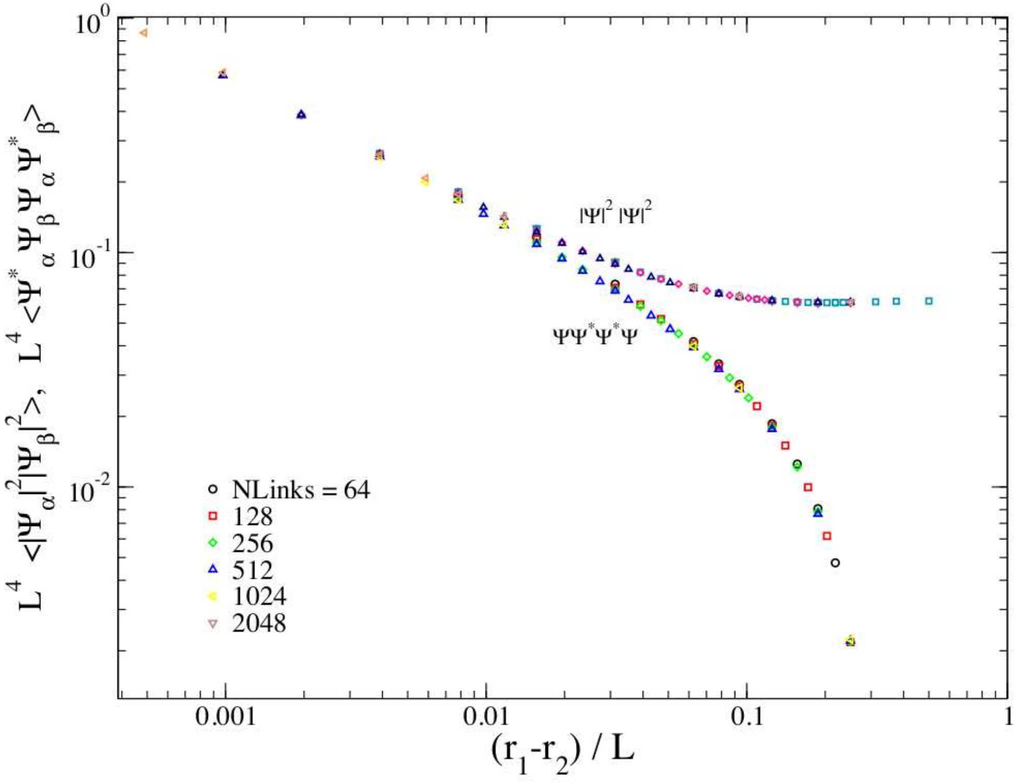
$$\mathcal{B}_{\alpha\beta}(r_1, r_2) = \phi_\alpha(r_1)\phi_\beta(r_2) - \phi_\alpha(r_2)\phi_\beta(r_1)$$

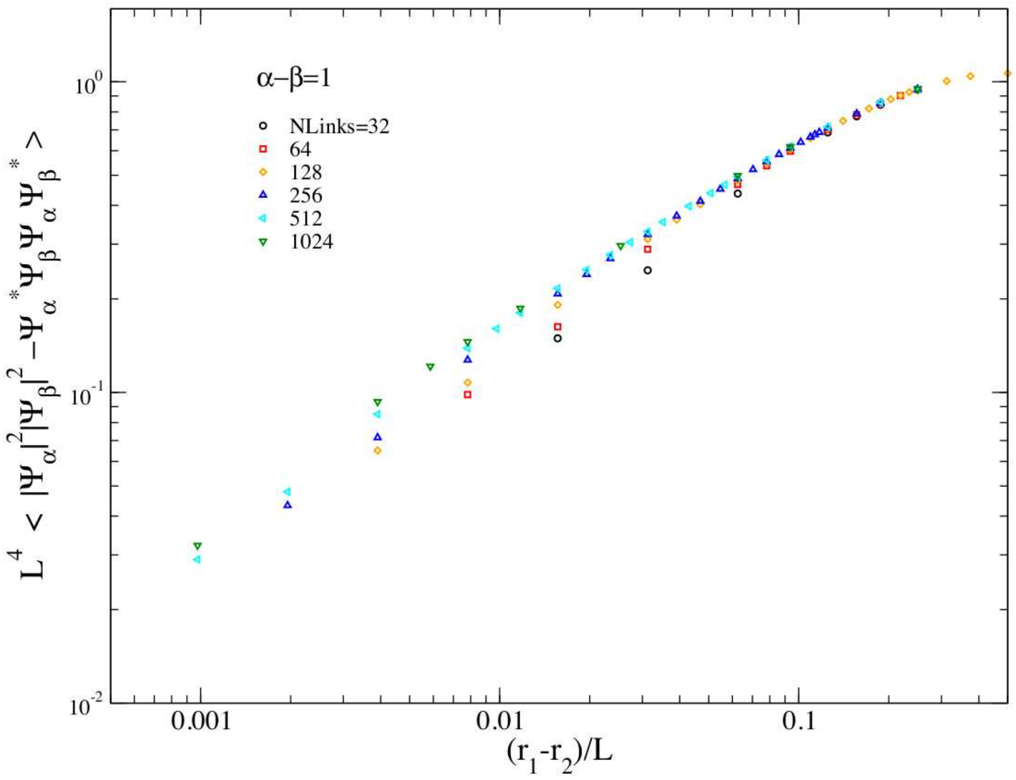


$$\mathcal{K}_1(r_1, r_2, E, \omega) = L^{-2d} \left(\frac{|r_1 - r_2|}{L_\omega} \right)^{\mu_2}, \qquad |r_1 - r_2| \ll L_\omega$$

$L_\omega = L(\Delta/|\omega|)^{1/d}$ length scale set by frequency ω

Interaction scaling at quantum Hall critical point





Hartree, Fock

enhanced by multifractality

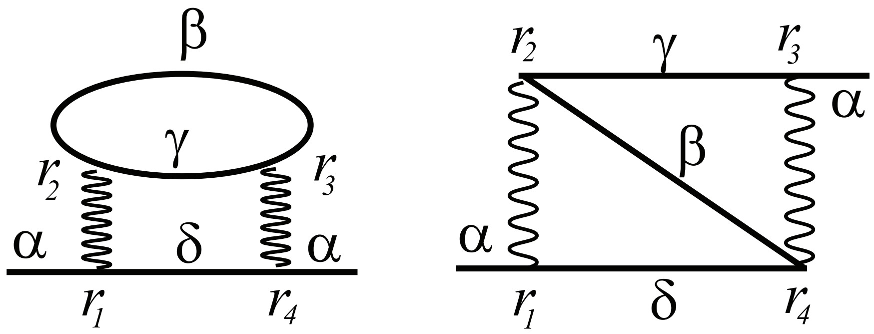
exponent $\Delta_2 \simeq -0.52 < 0$

Hartree − Fock

suppressed by multifractality

exponent $\mu_2 \simeq 0.62 > 0$

Interaction-induced dephasing



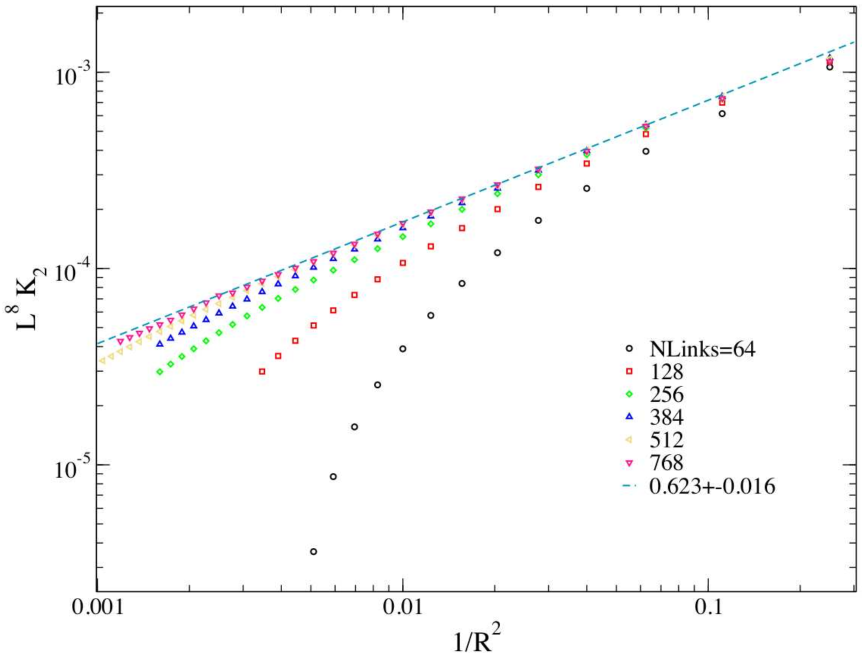
$$\mathrm{Im}\Sigma^R(0,0) \sim -\frac{1}{2\Delta^3}\left(\prod_{j=1}^{4}\int d\boldsymbol{r}_j\right) U(\boldsymbol{r}_1-\boldsymbol{r}_2)U(\boldsymbol{r}_3-\boldsymbol{r}_4)\int d\Omega\,\Omega$$
$$\times\left\{\coth\frac{\Omega}{2T}-\tanh\frac{\Omega}{2T}\right\}\mathcal{K}_2(\{\boldsymbol{r}_j\},0,0,\varepsilon'\sim T,\Omega)$$

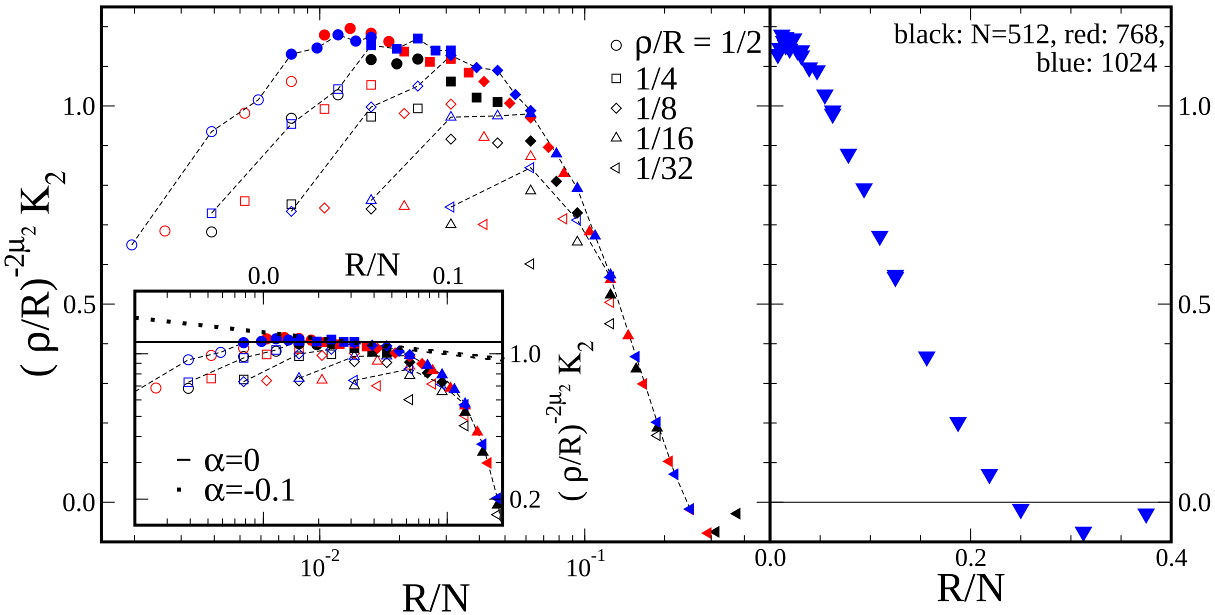
$$\mathcal{K}_2(\{\boldsymbol{r}_j\},E,\varepsilon,\varepsilon',\Omega)=\frac{\Delta^4}{8}\Big\langle\sum_{\alpha\beta\gamma\delta}\mathcal{B}^*_{\alpha\beta}(\boldsymbol{r}_1,\boldsymbol{r}_2)\mathcal{B}_{\delta\gamma}(\boldsymbol{r}_1,\boldsymbol{r}_2)\mathcal{B}^*_{\gamma\delta}(\boldsymbol{r}_3,\boldsymbol{r}_4)\mathcal{B}_{\beta\alpha}(\boldsymbol{r}_3,\boldsymbol{r}_4)$$
$$\times\delta(E-\epsilon_\alpha)\delta(\varepsilon'+\Omega-\epsilon_\beta)\delta(\varepsilon'-\epsilon_\gamma)\delta(\varepsilon+\Omega-\epsilon_\delta)\Big\rangle.$$

$$\mathcal{K}_2(\{\boldsymbol{r}_j\},0,0,\varepsilon'\sim\Omega,\Omega)=L^{-4d}\left(\frac{|\boldsymbol{r}_1-\boldsymbol{r}_2|}{R}\frac{|\boldsymbol{r}_3-\boldsymbol{r}_4|}{R}\right)^{\mu_2}\left(\frac{R}{L_\Omega}\right)^{\alpha}$$

$$R=(\boldsymbol{r}_1+\boldsymbol{r}_2-\boldsymbol{r}_3-\boldsymbol{r}_4)/2$$

Interaction scaling at quantum Hall critical point: Second order





$\mu = 0.62 \pm 0.05$ in agreement with scaling of first order

$\alpha = -0.05 \pm 0.1$ (possibly exactly zero)

Dephasing rate at criticality and transition width

- **short-range interaction:** decays faster than $r^{-d-\mu_2}$
- $\alpha > 2\mu_2 - d$

$\longrightarrow$ **dephasing rate** $\tau_\phi^{-1} \propto T^p$ with $p = 1 + 2\mu_2/d$

dephasing length

$$L_\phi \propto T^{-1/z_T} \qquad\qquad z_T = \frac{d}{p} = \frac{d}{1 + 2\mu_2/d}$$

z_T – **dynamical critical exponent for scaling with** T

ω-scaling: another dynam. exponent; in the present case $z = d$

Transition width exponent $\quad \kappa = \dfrac{1}{z_T \nu} = \dfrac{1 + 2\mu_2/d}{\nu d}$

$\mu \simeq 0.62$, $\alpha \simeq -0.05 \quad \longrightarrow \quad p \simeq 1.62 \quad \longrightarrow \quad z_T \simeq 1.23$

$\nu \simeq 2.35$ (Huckestein et al '92, . . .) $\longrightarrow \quad \kappa \simeq 0.346$

$\nu \simeq 2.59$ (Ohtsuki, Slevin '09) $\longrightarrow \quad \kappa \simeq 0.314$

Scaling at QH transition: Theory and Experiment

Experiment (**long-range** $1/r$ **Coulomb interaction):**

$\kappa = 0.42 \pm 0.01$

Reported results $\nu \simeq 2.3 \pm 0.1, \quad z = z_T \simeq 1.0 \pm 0.1$
may be strongly affected by systematic errors.

Theory (**short-range interaction):**

$z = 2, \quad z_T \simeq 1.62, \quad \nu = 2.35 \div 2.59, \quad \kappa = 0.314 \div 0.346$

No controllable theoretical results for long-range interaction

Difference in κ **fully consistent with short-range and Coulomb** $(1/r)$ **problems being in different universality classes**
$\longrightarrow$ **all exponents expected to be different**

Experimental realization of short-range interaction would be very interesting!

Anderson transition: $2+\epsilon$ dimensions, short-range interaction

$$-dt/d\ln L \equiv \beta(t) = \epsilon t - 2t^3 - 6t^5 + O(t^7) \qquad \text{(Wegner '89)}$$

$t = 1/2\pi g$ $\qquad$ g – dimensionless conductance

Metal-insulator transition at $\quad t_* = \left(\frac{\epsilon}{2}\right)^{1/2} - \frac{3}{2}\left(\frac{\epsilon}{2}\right)^{3/2} + O(\epsilon^{5/2})$

Localization length index $\quad \nu = -1/\beta'(t_*) = \frac{1}{2\epsilon} - \frac{3}{4} + O(\epsilon)$

Exponents controlling scaling of interaction:

$$\mu_2 = \sqrt{2\epsilon} - \frac{3}{2}\zeta(3)\epsilon^2 + O(\epsilon^{5/2}) \qquad \alpha = O(\epsilon^{5/2})$$

Temperature scaling of transition:

$$z_T = 2 - 2\sqrt{2}\epsilon^{1/2} + 5\epsilon - 4\sqrt{2}\epsilon^{3/2} + O(\epsilon^2)$$

$$\kappa = \epsilon + \sqrt{2}\epsilon^{3/2} + \epsilon^2 + \epsilon^{5/2}/\sqrt{2} + O(\epsilon^3)$$

Anderson transition: $2+\epsilon$ dimensions, Coulomb interaction

Broken time-reversal symmetry (unitary class), 2-loop calculation

Baranov, Burmistrov, Pruisken, PRB '02

$$\beta(t) = \epsilon t - 2t^2 - 4At^3 \; ; \qquad\qquad A \simeq 1.64$$

$$t_* = \frac{\epsilon}{2} - \frac{A}{2}\epsilon^2 + O(\epsilon^3)$$

$$\nu = \frac{1}{\epsilon} - A + O(\epsilon)$$

$$z = z_T = 2 + \frac{\epsilon}{2} + \left(\frac{A}{2} - \frac{\pi^2}{24} - \frac{3}{4}\right)\epsilon^2 + O(\epsilon^3)$$

$$\kappa = \frac{\epsilon}{2} + \left(\frac{A}{2} - \frac{1}{8}\right)\epsilon^2 + O(\epsilon^3)$$

Exponents for short-range and Coulomb interaction are different!

Anderson transition in 3D: Short-range vs Coulomb

Theory, short-range interaction:

$\nu = 1.57 \pm 0.02$ (Slevin, Othsuki '99)

μ_2 and α remain to be calculated $\longrightarrow$ z_T, κ

Coulomb interaction: no controllable theory for exponents

Experiment (Coulomb):

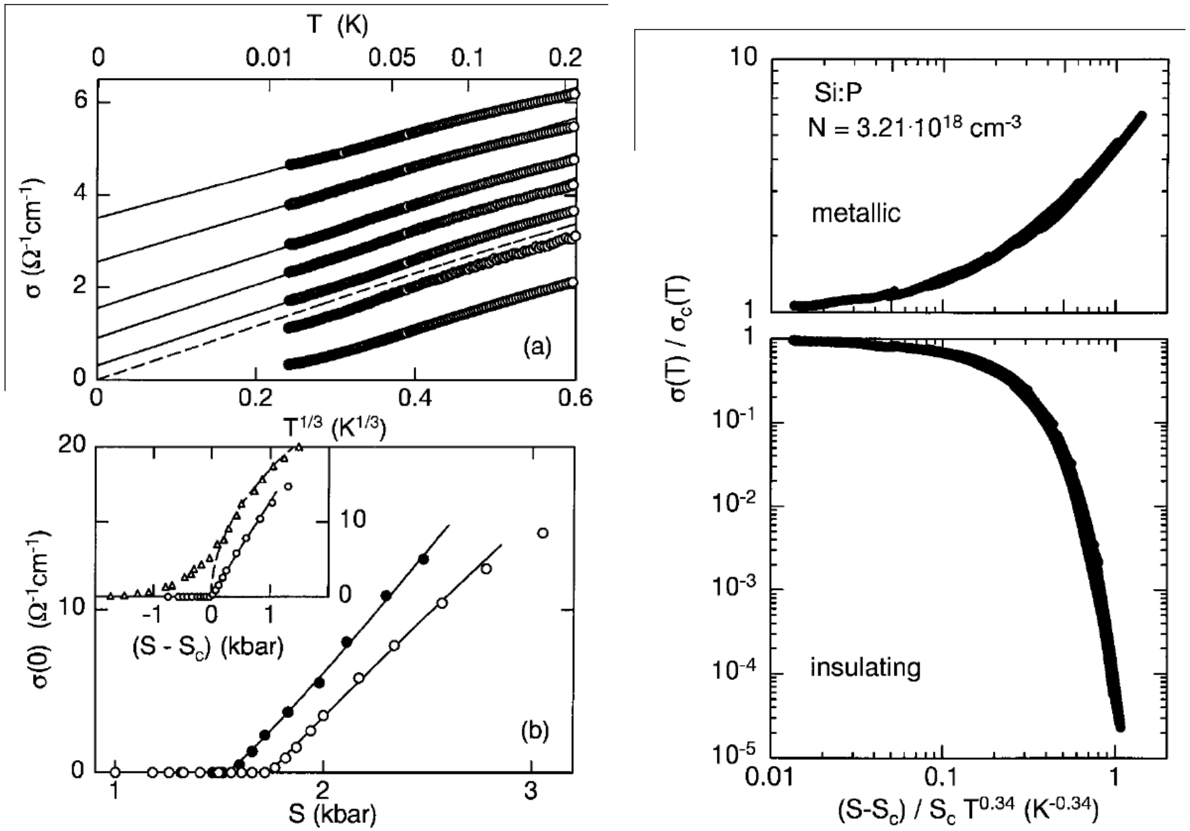
$s = \nu \simeq 1.0 \pm 0.1$

z_T in the range from 2 to 3

Experiment, short-range:

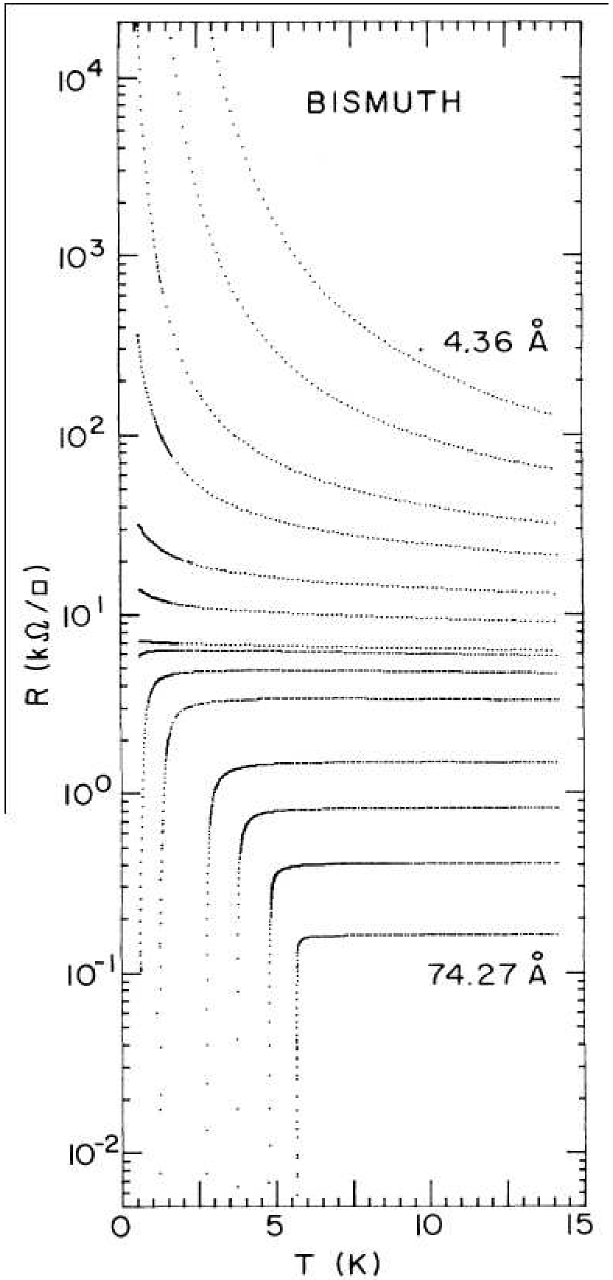
not available

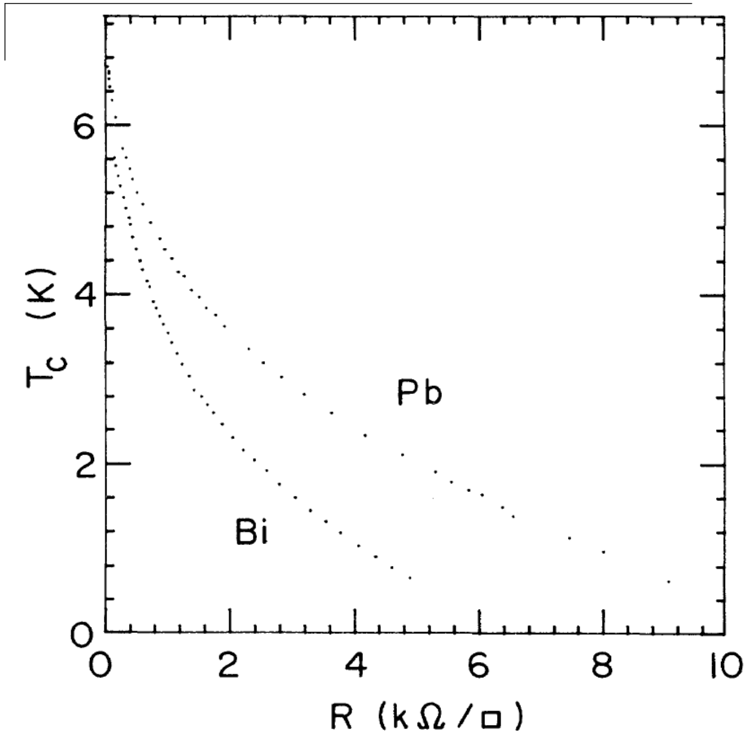
Cold atom systems?



Waffenschmidt, Pfleiderer, v. Löhneysen, PRL'99

Superconductor-Insulator Transition





Haviland, Liu, Goldman, PRL'89

Suppression of T_c by disorder

RG theory: Finkelstein '87:
combined effect of disorder
and Coulomb interaction

What is the effect of disorder on T_c
if the interaction is short-range ?

Enhancement of superconductivity by multifractality

short-range interaction

Feigelman et al, PRL '07, Ann. Phys.'10 :

multifractality of wave functions near MIT in 3D

$\longrightarrow$ **enhancement of Cooper-interaction matrix elements**

$\longrightarrow$ **enhancement of T_c**

- **what are predictions of RG ?**
- **effect of disorder on T_c in 2D systems ?**

SIT in disordered 2D system: Orthogonal symmetry class

Funkelstein's σ-model RG but with short-range (rather than Coulomb) spin-singlet interaction γ_s:

$$\frac{dt}{dy} = t^2 - \left(\frac{\gamma_s}{2} + 3\frac{\gamma_t}{2} + \gamma_c\right)t^2$$

$$\frac{d\gamma_s}{dy} = -\frac{t}{2}(\gamma_s + 3\gamma_t + 2\gamma_c)$$

$$\frac{d\gamma_t}{dy} = -\frac{t}{2}(\gamma_s - \gamma_t - 2\gamma_c)$$

$$\frac{d\gamma_c}{dy} = -\frac{t}{2}(\gamma_s - 3\gamma_t) - \gamma_c^2 \qquad y \equiv \ln L$$

Interactions: singlet γ_s , triplet γ_t , Cooper γ_c

Disorder: dimensionless resistivity $t = 1/G$

Assume small bare values: t_0 , $\gamma_{i,0} \ll 1$

SIT in disordered 2D system: Orthogonal class (cont'd)

Weak interaction $\longrightarrow$ discard $\gamma_i t^2$ contributions to $dt/d\ln L$

$$\frac{d}{dy}\begin{pmatrix}\gamma_s\\ \gamma_t\\ \gamma_c\end{pmatrix} = -\frac{t}{2}\begin{pmatrix}1 & 3 & 2\\ 1 & -1 & -2\\ 1 & -3 & 0\end{pmatrix}\begin{pmatrix}\gamma_s\\ \gamma_t\\ \gamma_c\end{pmatrix} - \begin{pmatrix}0\\ 0\\ \gamma_c^2\end{pmatrix}; \qquad \frac{dt}{dy} = t^2$$

Eigenvalues and -vectors of linear problem (without BCS term γ_c^2):

$$\lambda_1 = 2t \;:\; \begin{pmatrix}-1\\ 1\\ 1\end{pmatrix}; \qquad \lambda_{2,3} = -t \;:\; \begin{pmatrix}1\\ 1\\ -1\end{pmatrix} \text{ and } \begin{pmatrix}1\\ -1\\ 2\end{pmatrix}$$

2D system is "weakly critical" (on scales shorter than ξ)

The eigenvalues λ_i are exactly multifractal exponents:

$\lambda_1 \equiv -\Delta_2 > 0$ (RG relevant), $\quad \lambda_{2,3} = -\mu_2 < 0$ (RG irrelevant)

SIT in disordered 2D system: Orthogonal class (cont'd)

Couplings that diagonalize the linear system:

$$\begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} = \begin{pmatrix} -\frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \gamma_s \\ \gamma_t \\ \gamma_c \end{pmatrix}$$

Upon RG γ_1 **increases, whereas** $\gamma_{2,3}$ **decrease.**

Solution approaches the $\boldsymbol{\lambda}_1$**–eigenvector, i.e..** $\gamma_s = -\gamma_t = -\gamma_c$

$\longrightarrow$ **neglect** $\gamma_{2,3}$ **and keep** γ_1 **only:**

$$\frac{d\gamma_1}{dy} = 2t\gamma_1 - \frac{1}{3}\gamma_1^2 \qquad\qquad t(y) = \frac{t_0}{1 - t_0 y}$$

Superconductivity may develop if the starting value

$$\gamma_{1,0} = \frac{1}{6}(-\gamma_{s,0} + 3\gamma_{t,0} + 2\gamma_{c,0}) < 0$$

SIT in disordered 2D system, orthogonal class: Results

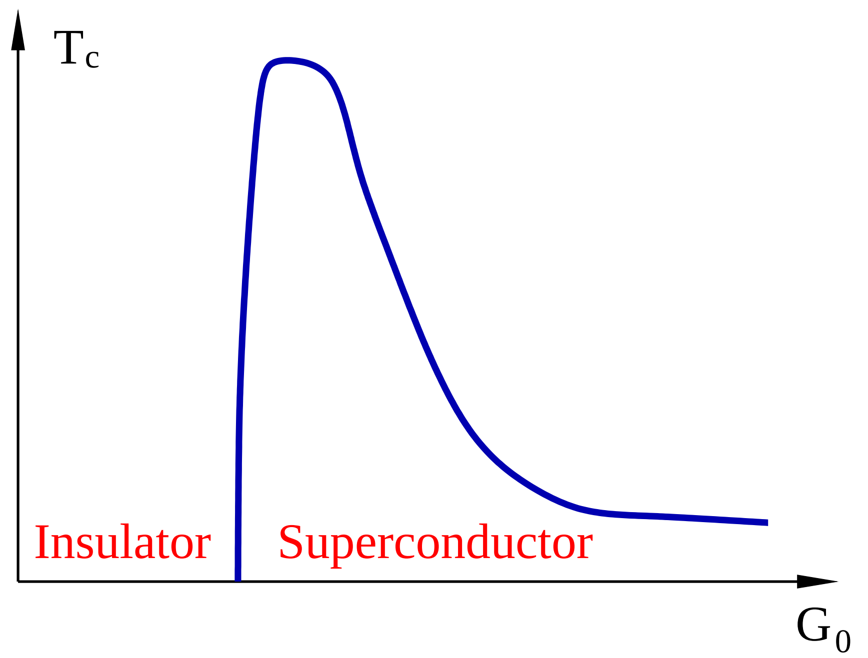
$$T_c \sim \exp\{-2/|\gamma_{c,0}|\} \quad \text{(BCS)}\ , \qquad G_0 \gtrsim |\gamma_{1,0}|^{-1}$$

$$T_c \sim \exp\{-2G_0\}\ , \qquad |\gamma_{1,0}|^{-1/2} \lesssim G_0 \lesssim |\gamma_{1,0}|^{-1}$$

$$\text{insulator}\ , \qquad G_0 \lesssim |\gamma_{1,0}|^{-1/2}$$

Non-monotonous dependence
of T_c on disorder (G_0)

Exponentially strong enhancement of superconductivity by multifractality
in the intermediate disorder range,
$|\gamma_{1,0}|^{-1/2} \lesssim G_0 \lesssim |\gamma_{1,0}|^{-1}$



2D: BCS and BKT

Calculating T_c, we treated superconductivity on the BCS level.

The actual transition in 2D is of the BKT character.

However, the temperatures are close, $T_{\mathrm{BKT}} \simeq T_c$

Beasley, Mooij, Orlando, PRL '79
Halperin, Nelson, JLTP'79
Kadin, Epstein, Goldman, PRB'83
Benfatto, Castellani, Giamarchi, PRB'09

Since the obtained enhancement of T_c is large,
it should be equally applicable to T_{BKT}

Disordered 2D system: Symplectic symmetry class

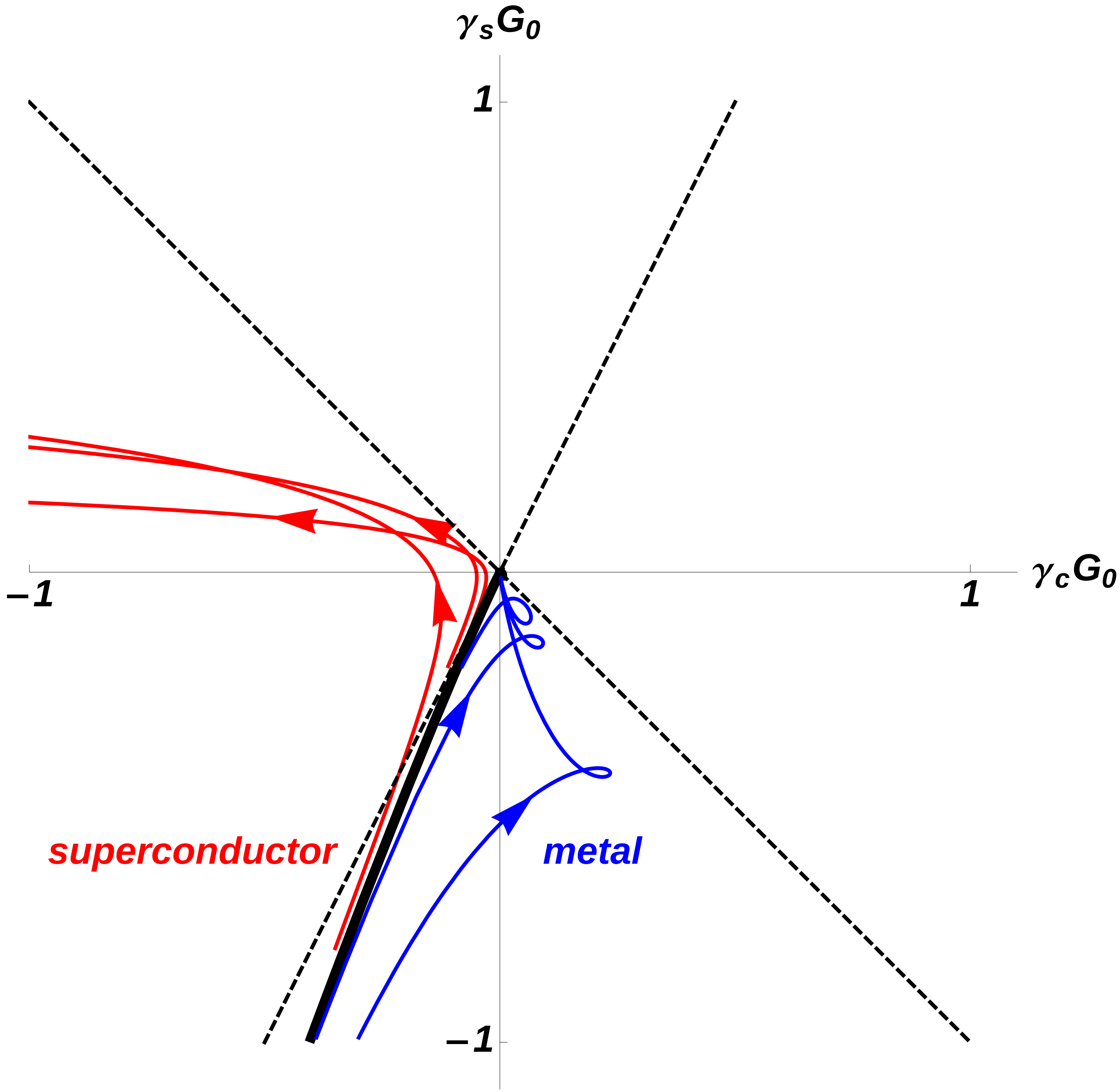
Strong spin-orbit interaction $\longrightarrow$ **only spin-singlet modes survive**

$$\frac{dt}{dy} = -\frac{1}{2}t^2 - (\frac{\gamma_s}{2} + \gamma_c)t^2$$
$$\frac{d\gamma_s}{dy} = -\frac{t}{2}(\gamma_s + 2\gamma_c)$$
$$\frac{d\gamma_c}{dy} = -\frac{t}{2}\gamma_s - \gamma_c^2$$

$$t(y) = \frac{t_0}{1 + yt_0/2}$$ **antilocalization**

$$\frac{d}{dy}\begin{pmatrix}\gamma_s \\ \gamma_c\end{pmatrix} = -\frac{t}{2}\begin{pmatrix}1 & 2 \\ 1 & 0\end{pmatrix}\begin{pmatrix}\gamma_s \\ \gamma_c\end{pmatrix} - \begin{pmatrix}0 \\ \gamma_c^2\end{pmatrix}$$

Disordered 2D system: Symplectic symmetry class



Disordered 2D system: Symplectic class (cont'd)

Eigenvalues and -vectors of linear system:

$$\lambda_1 = \frac{t}{2} \;:\; \begin{pmatrix} -1 \\ 1 \end{pmatrix} \;; \qquad \lambda_2 = -t \;:\; \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Again λ_i are multifractal exponents:

$\lambda_1 \equiv -\Delta_2 > 0$ (RG relevant), $\lambda_{2,3} = -\mu_2 < 0$ (RG irrelevant)

Couplings that diagonalize the linear system:

$$\begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \gamma_s \\ \gamma_c \end{pmatrix}$$

Upon RG γ_1 increases, whereas γ_2 decreases.

Solution approaches the λ_1–eigenvector, i.e.. $\gamma_s = -\gamma_c$

$\longrightarrow$ neglect γ_2 and keep γ_1 only:

$$\frac{d\gamma_1}{dy} = \frac{t}{2}\gamma_1 - \frac{2}{3}\gamma_1^2 \qquad\qquad t(y) = \frac{t_0}{1 + t_0 y/2}$$

Disordered 2D system, symplectic class: Results

Superconductivity if $\gamma_{1,0} = \frac{1}{3}(-\gamma_{s,0} + 2\gamma_{c,0}) < 0$

$$T_c \sim \exp\left\{-2/|\gamma_{c,0}|\right\} \quad \text{(BCS)}\ , \qquad G_0 \gtrsim |\gamma_{1,0}|^{-1}$$

$$T_c \sim \exp\left\{-c(G_0/|\gamma_{1,0}|)^{1/2}\right\}\ , \qquad 1 \lesssim G_0 \lesssim |\gamma_{1,0}|^{-1}$$

Exponentially strong enhancement of superconductivity

by multifractality at $1 \lesssim G_0 \lesssim |\gamma_{1,0}|^{-1}$

Before the system becomes insulator at $G_0 < G_c \sim 1$,
there is a further enhancement of T_c
near the Anderson transition point G_c

SIT near Anderson transition

Consider system at Anderson localization transition

in 2D (symplectic symmetry class) or 3D

$$\frac{d\gamma_1}{dy} = -\Delta_2\gamma_1 - \gamma_1^2$$

Superconductivity if $\gamma_{1,0} < 0$

$\Delta_2 < 0$ – multifractal exponent at Anderson transition point

$$T_c \sim |\gamma_{c,0}|^{d/|\Delta_2|}$$

Strong enhancement of superconductivity:

Power-law instead of exponential dependence of T_c on interaction!

Agrees with Feigelman et al.

Summary

- Multifractality of wave functions – remarkable property of Anderson localization transitions
- Strongly affects interaction-induced physics in problems with short-range interaction
- Scaling of dephasing rate and transition width for MI and quantum Hall transitions is controlled by multifractality
- Non-monotonous dependence of T_c on resistivity; exponential enhancement of superconductivity by multifractality in dirty 2D systems and near Anderson transitions
- Possible experimental realizations:
  - systems with strongly screened Coulomb interaction (large ϵ, metallic gate)
  - interacting neutral fermions (cold atoms, . . .)