# Low temperature transport in tunnel junction arrays: Cascade energy relaxation


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**ANL**

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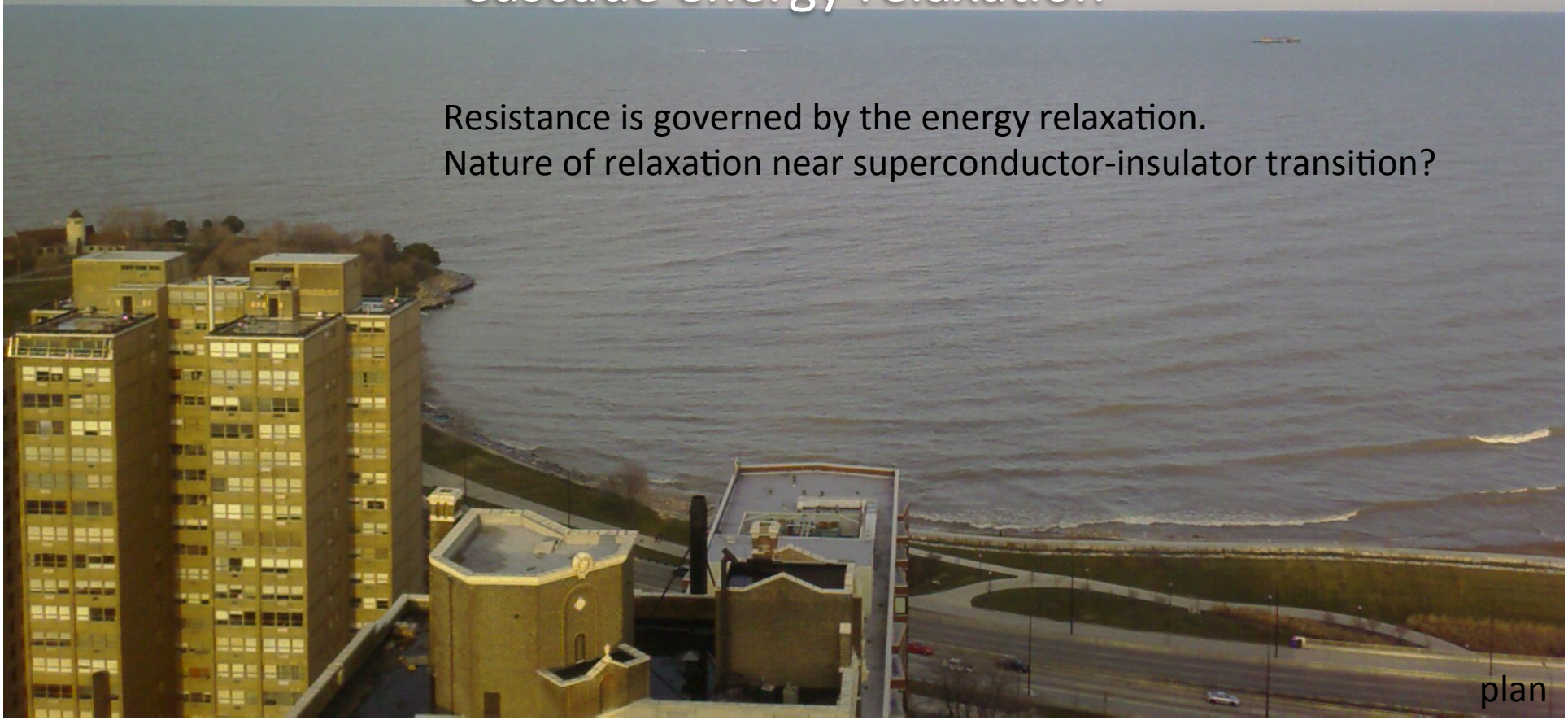
# Low temperature transport in tunnel junction arrays: Cascade energy relaxation

Resistance is governed by the energy relaxation.
Nature of relaxation near superconductor-insulator transition?

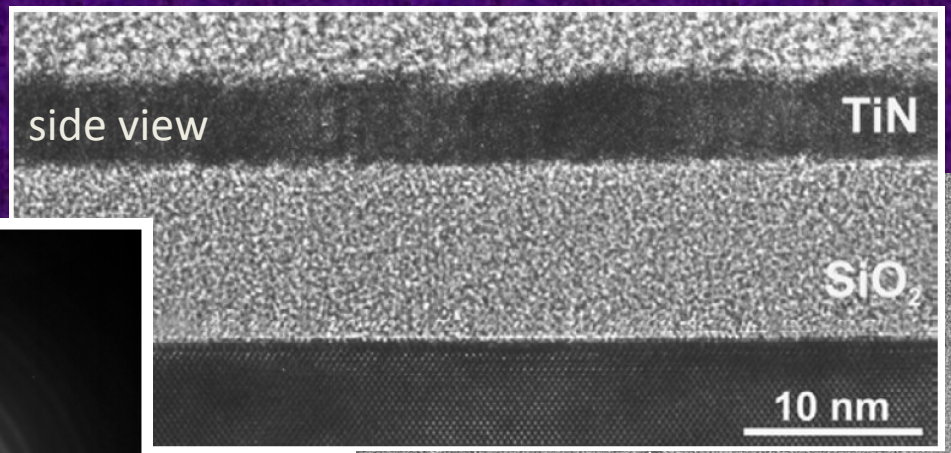


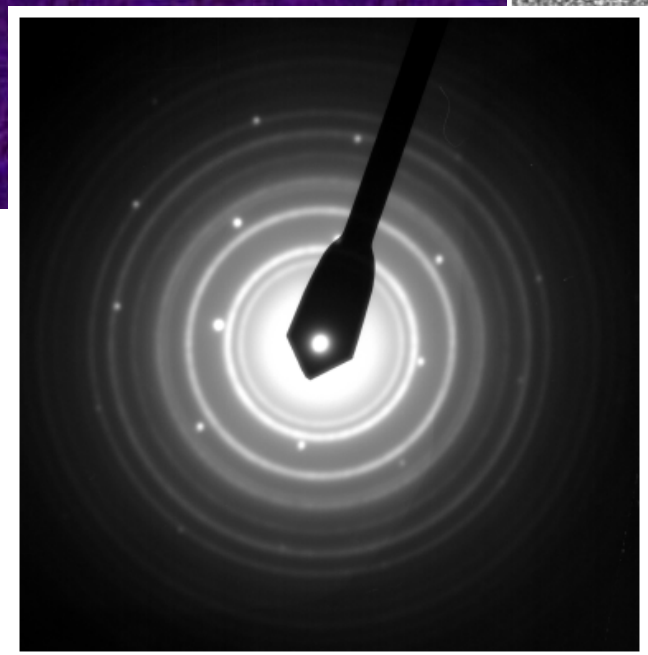
plan

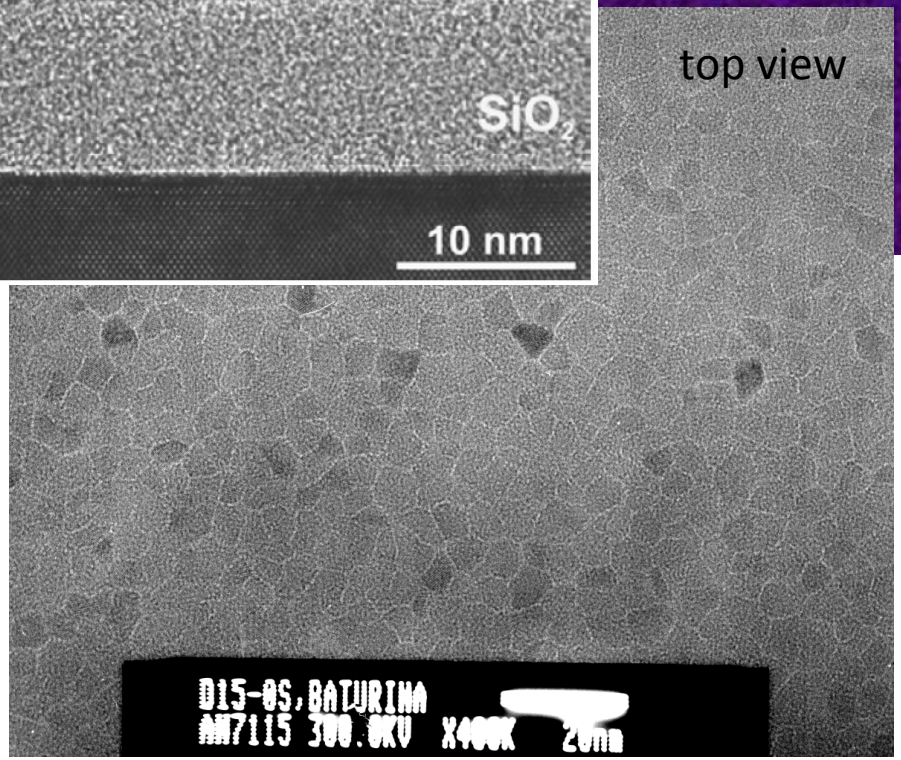
# Plan of the talk

- Environment assisted quantum tunneling
- Nonequilibrium environment
- Arrays of tunnel junctions. Cotunneling blockade by the self generated environment.
- Arrays of superconducting junctions. "Superinsulator" = gap of the environment?
- **Landau-Hopf** turbulence in arrays of superconducting junctions. Superinsulator.

# Experiment TiN films






✓ TiN films were formed by atomic layer chemical vapor deposition onto a $Si/SiO_2$ substrate at 350 $^0C$.

✓ Composition:

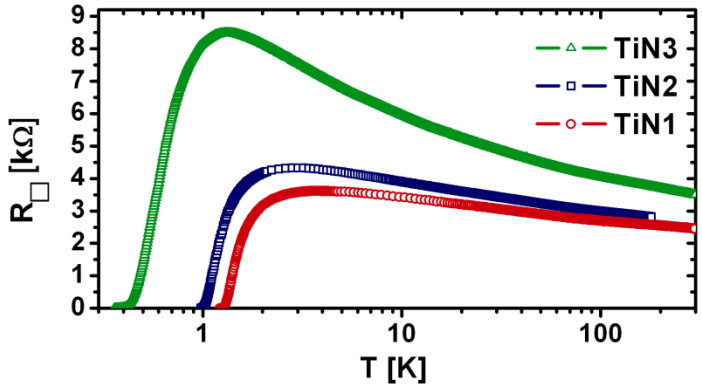
| Ti | N | Cl |
|---|---|---|
| 1 | 0.94 | 0.035 |

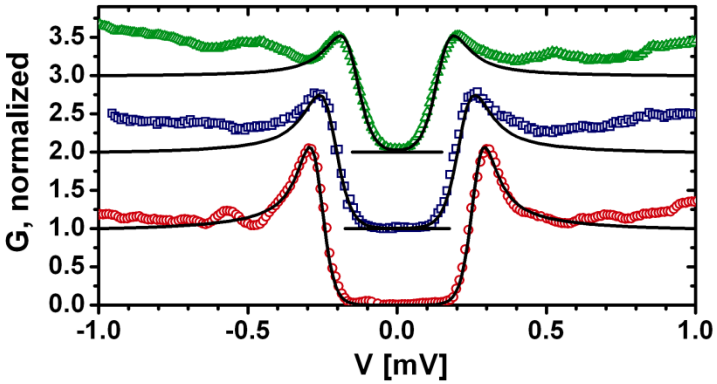
the thickness is 5 nm

$\xi_d$ = 9.3 nm - the superconducting coherence length

Samples are uniform on the material level! But nonuniform on electronic structure level…

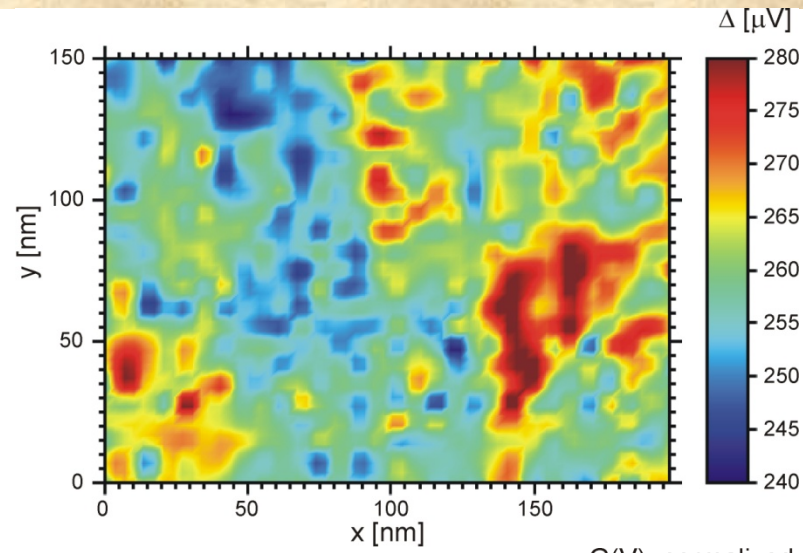
# STM measurements of LDOS
# inhomogeneous superconducting state (!)

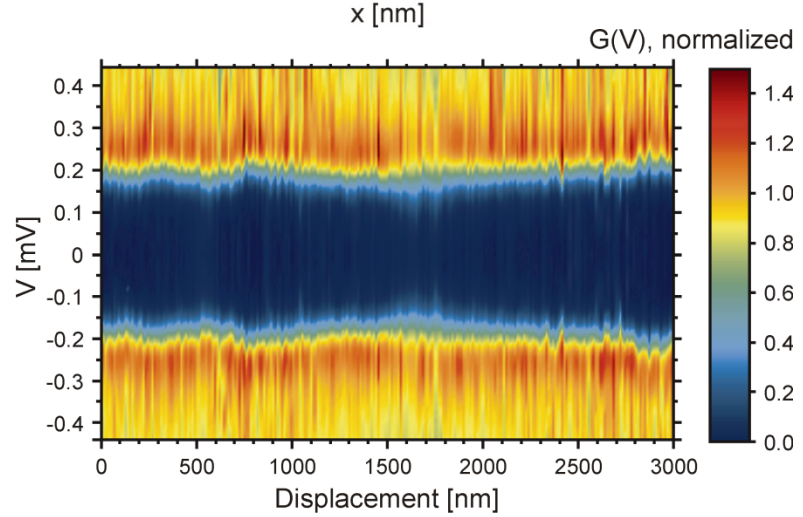



BCS fit

TiN1 – $\Delta = 260$ μeV, $T_{eff} = 0.25$ K
TiN2 – $\Delta = 225$ μeV, $T_{eff} = 0.32$ K
TiN3 – $\Delta = 154$ μeV, $T_{eff} = 0.35$ K



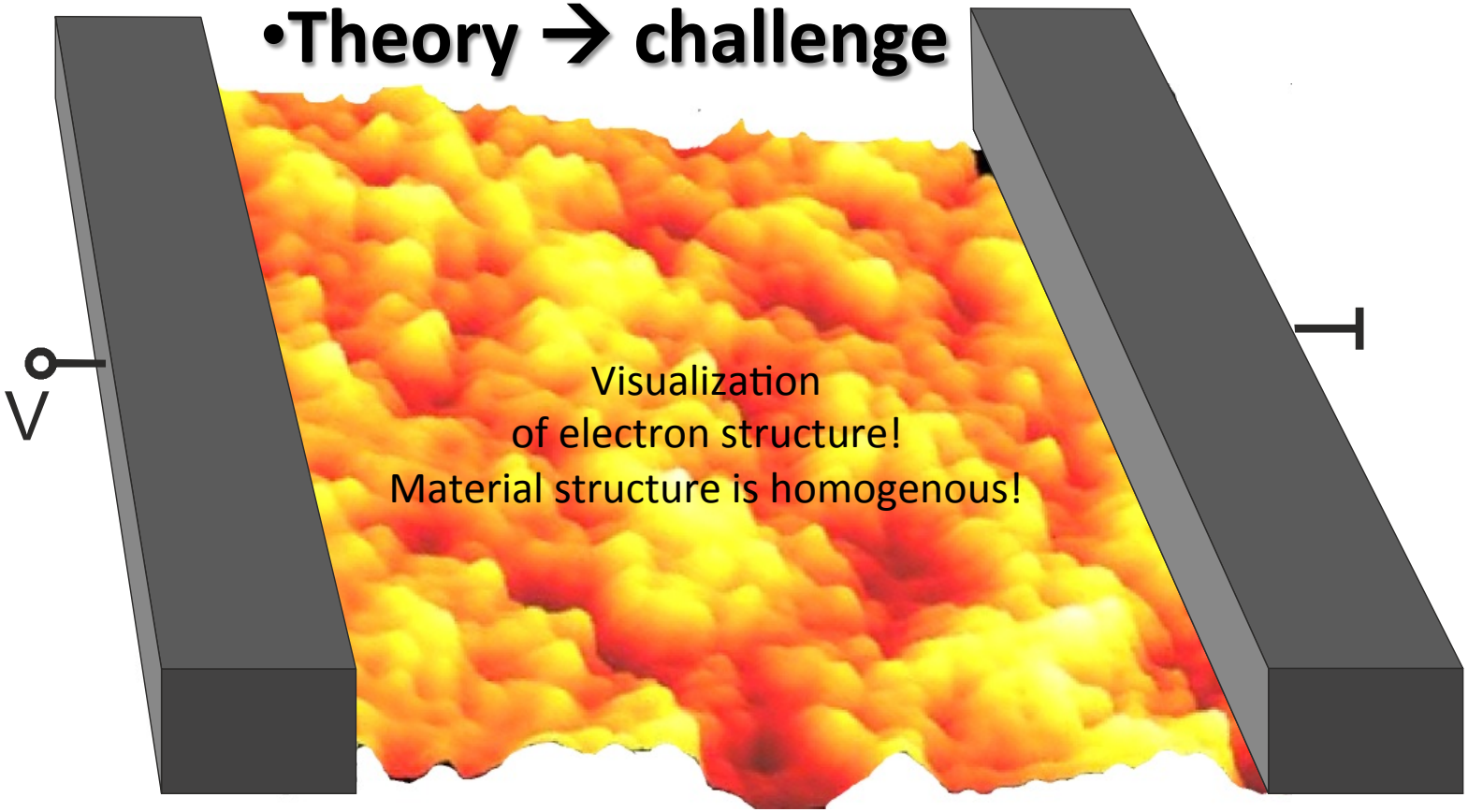

in TiN3 the magnitudes of Δ scattered in the interval from 125 μeV to 215 μeV

B. Sacépé, C. Chapelier, T. Baturina., V. Vinokur, M.R. Baklanov, M. Sanquer, PRL 2008

# Transport through a 2D film close to superconductor-insulator transition. What theory can tell?

**•Strong fluctuations •?percolation•Nonlinear I(V)**

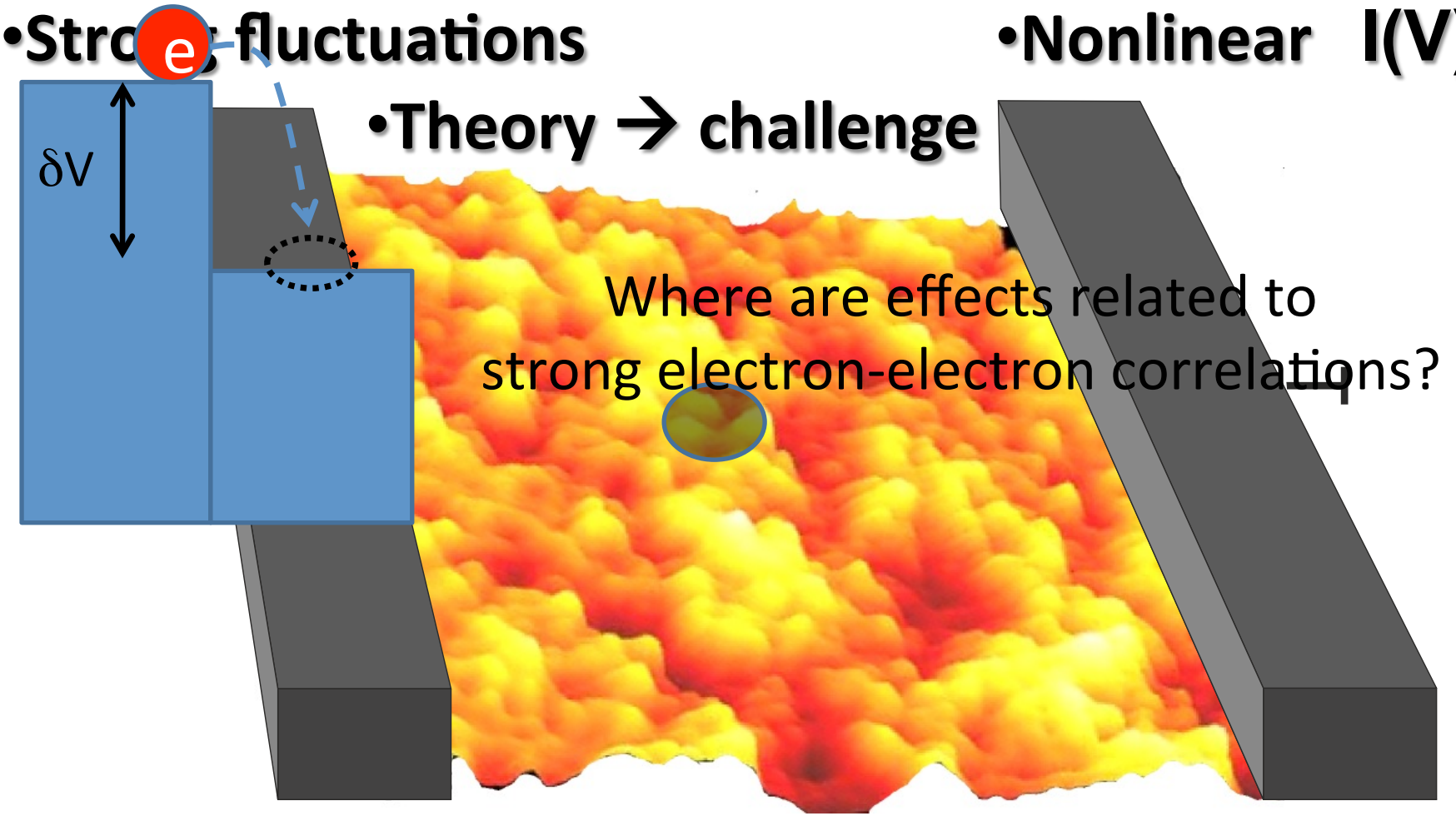
**•Theory → challenge**



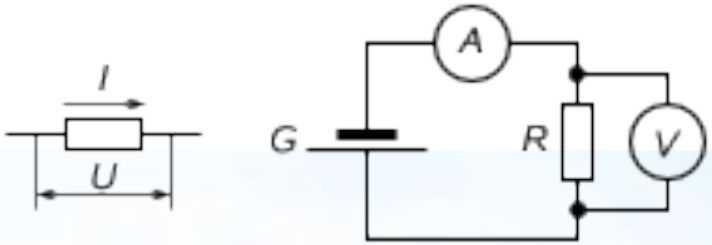
# Transport through a 2D film close to superconductor-insulator transition

superficial knowledge,
explains Ohm law.
nonlinear I(V)?

- **Strong fluctuations**
- **Nonlinear I(V)**
- **Theory → challenge**




Where are effects related to strong electron-electron correlations?




Water flow is the analog
of electron flow in circuits.

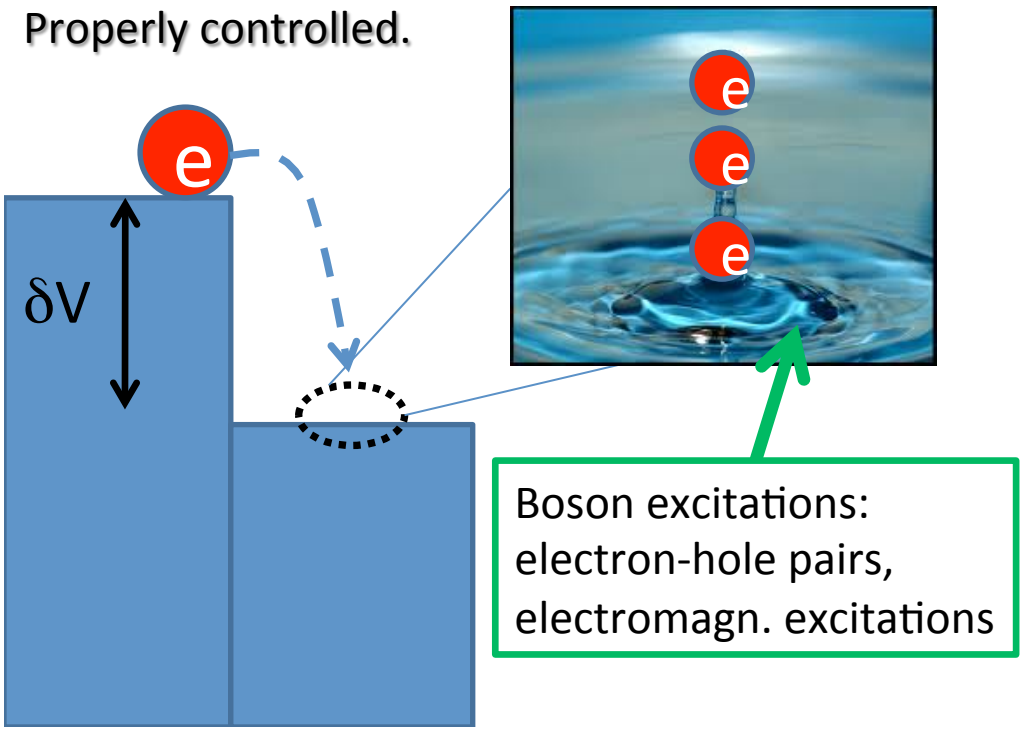
The losses on the **bubble** formation in the water flow are usually much smaller than the useful energy of the **translational motion**.

# environment in water flow

# Heating of the environment in the nano-junctions

Electric current in a nanostructure is accompanied by the excitation of the quantum liquid of the charge carriers (environment of quantum bubbles).

The environment heating phenomena should be Properly controlled.




The losses on the **bubble** formation in the water flow are usually much smaller than the useful energy of the **translational motion**.

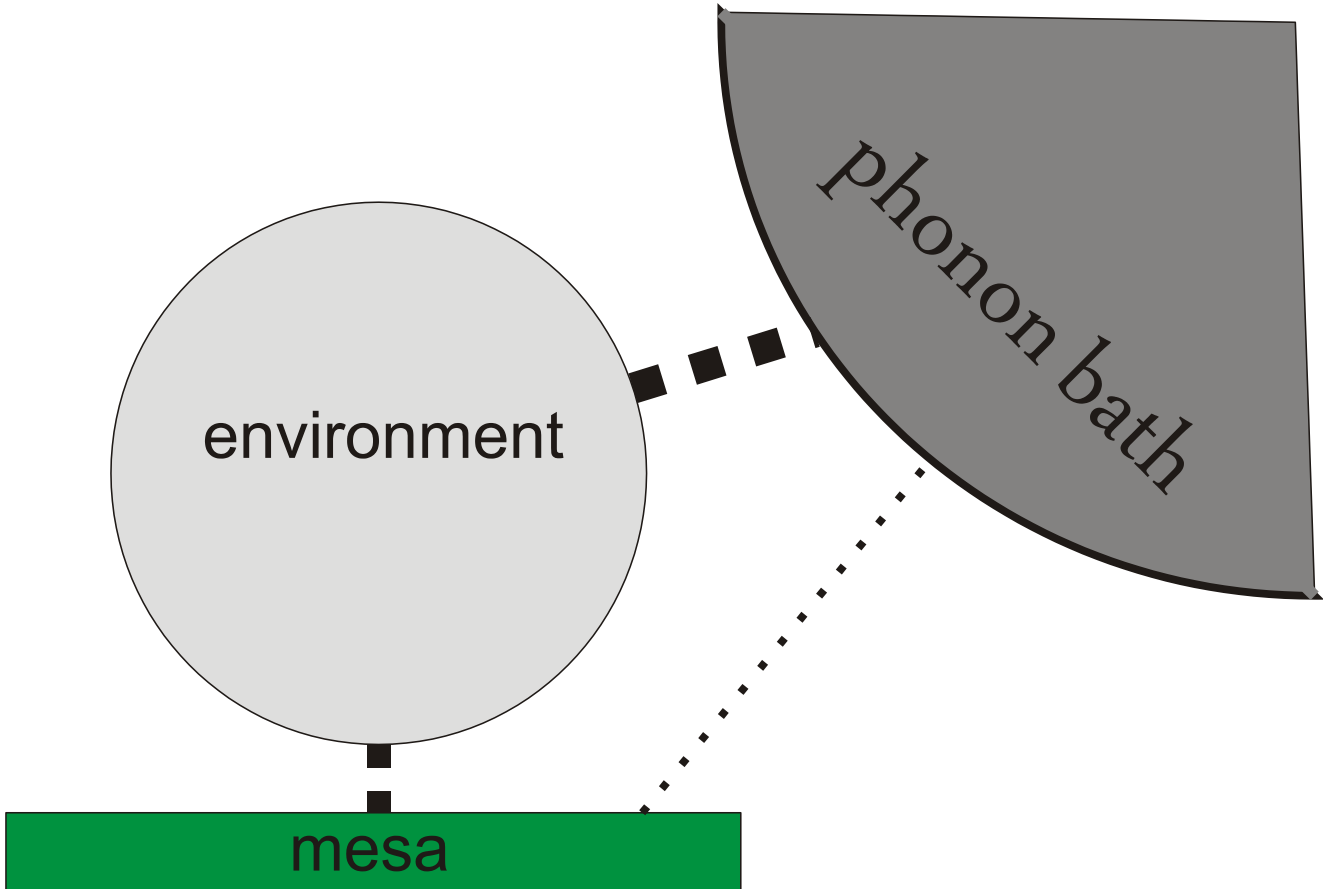
# Environment language

- Far from equilibrium transport phenomena in mesoscopic conductors are usually governed by the energy exchange between the transport agents (e.g., electrons) and the environment of the bosonic excitations represented, by phonons, photons, many-body excitations (e.g., electron-hole pairs), the electromagnetic modes in the leads…

- ❑heating
- ❑the Coulomb anomaly
- ❑the Coulomb drag
- ❑the hooping (cotunneling)
- ❑the many body localization
- ❑the resonance exciting of the environment modes
- ❑Spin resonance (e.g., NMR)
- ❑…

# Environment and bath

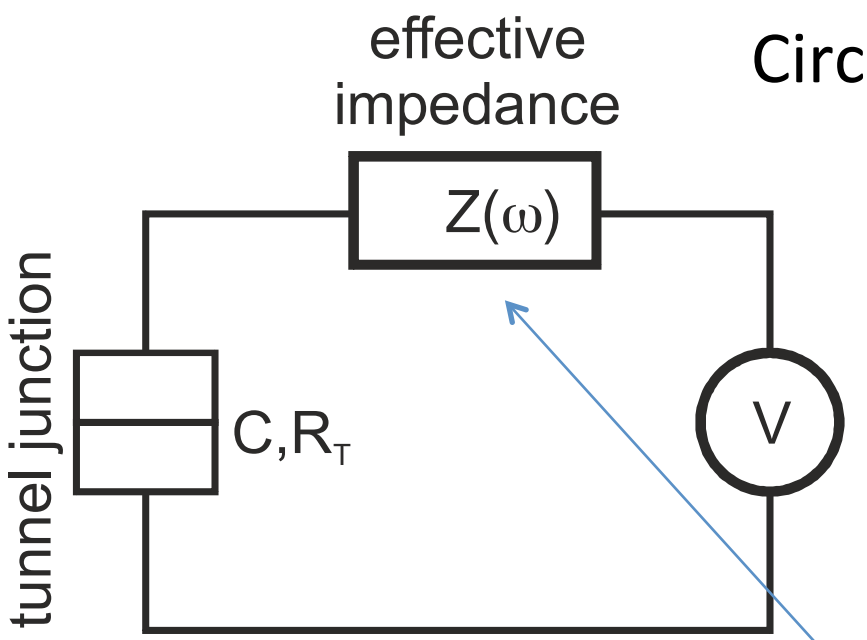
•N.M. Chtchelkatchev, V.M. Vinokur and T.I. Baturina,
•Hierarchical energy relaxation in mesoscopic tunnel junctions:
•Effect of nonequilibrium environment on low-temperature transport,
•Phys. Rev. Lett. **103**, 247003 (2009).

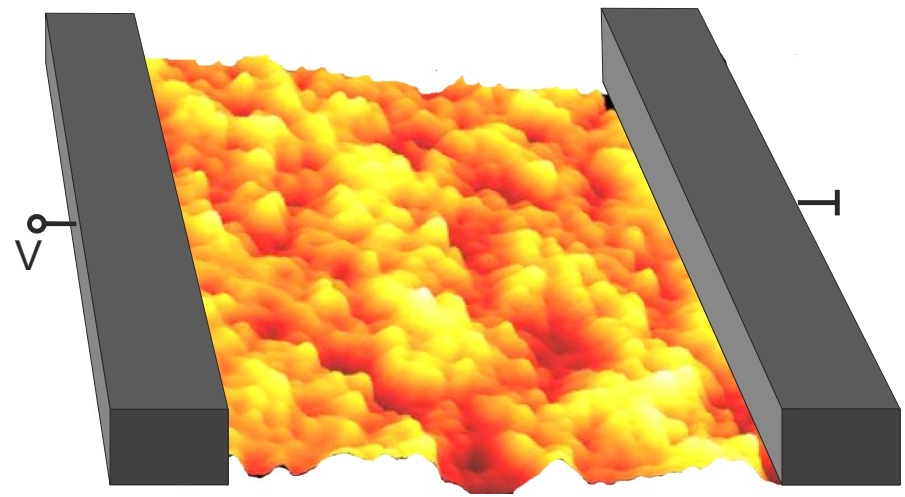


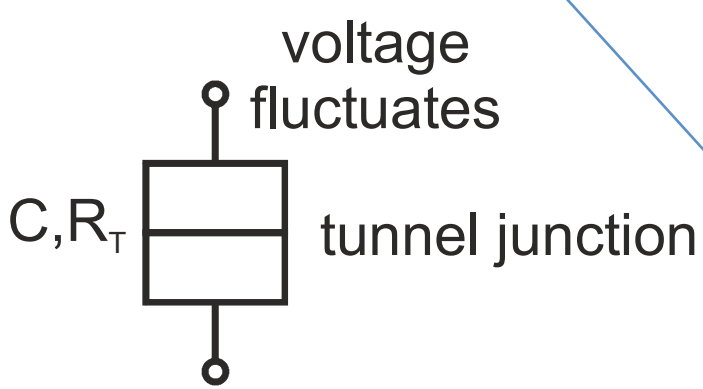
# Transport in 2D array --- "classical" Coulomb blockade picture

*Grabert and Devoret 1991*

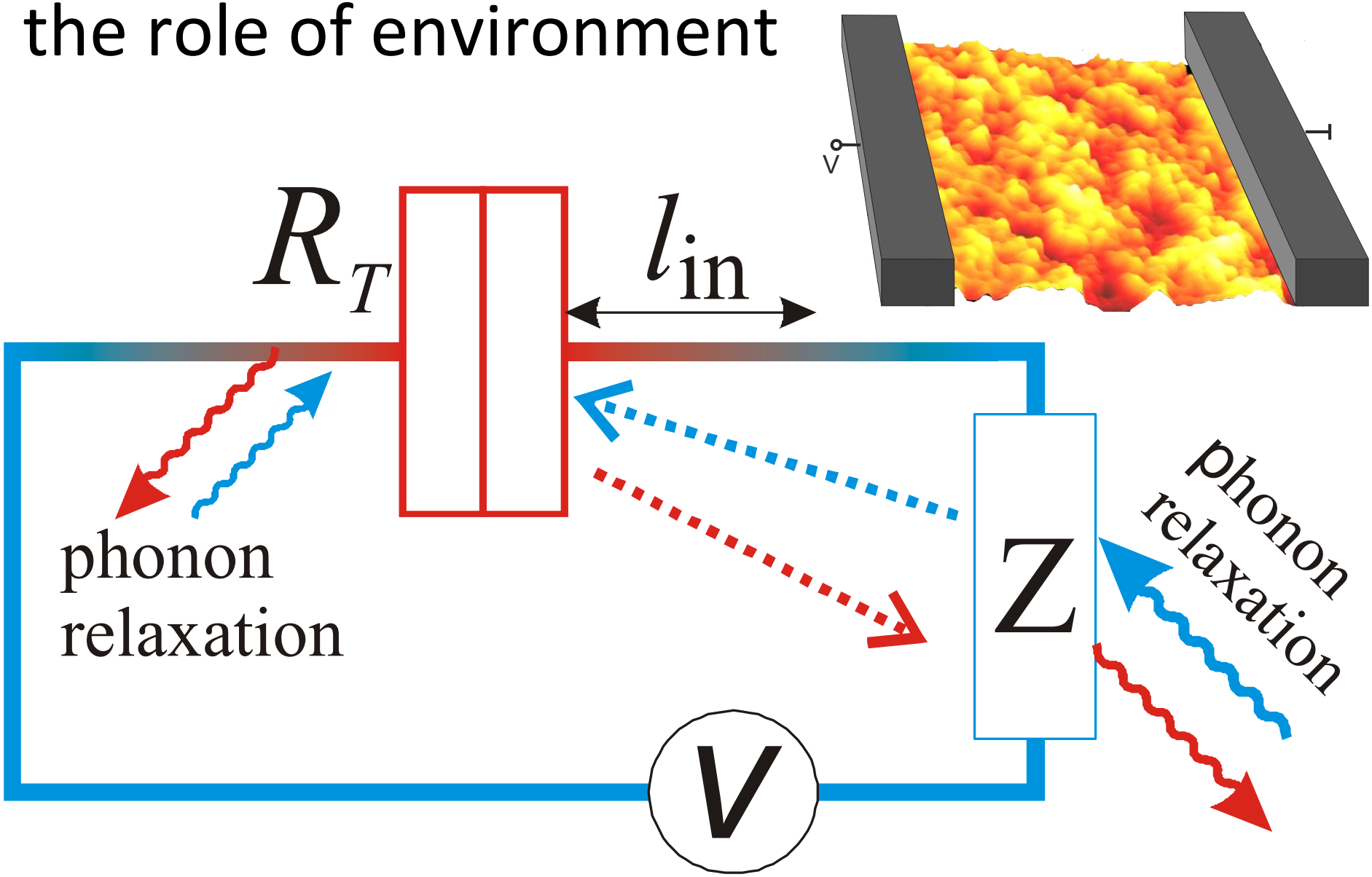
Circuit theory



# Energy relaxation and the role of environment



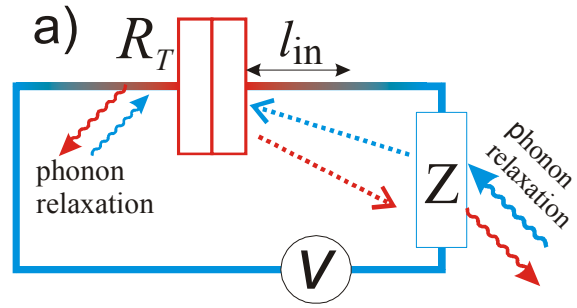
# General equations true for any environment

$$I = e\left(\overrightarrow{\Gamma} - \overleftarrow{\Gamma}\right)$$

where $\overrightarrow{\Gamma}$ is the tunneling rate from left to the right

$$\overrightarrow{\Gamma} = \frac{1}{R_{\rm T}} \int_{\epsilon\epsilon'} f_{\epsilon}^{(1)}(1 - f_{\epsilon'}^{(2)}) P^{<}(\epsilon - \epsilon')$$

$$P^{<}(E) = \int_{-\infty}^{\infty} dt \exp[J(t) + iEt]\,,$$




$$J(t) = 2\int_0^{\infty} \frac{d\omega}{\omega} \rho_{\alpha}(\omega) F_{\alpha}(\omega)\,,$$

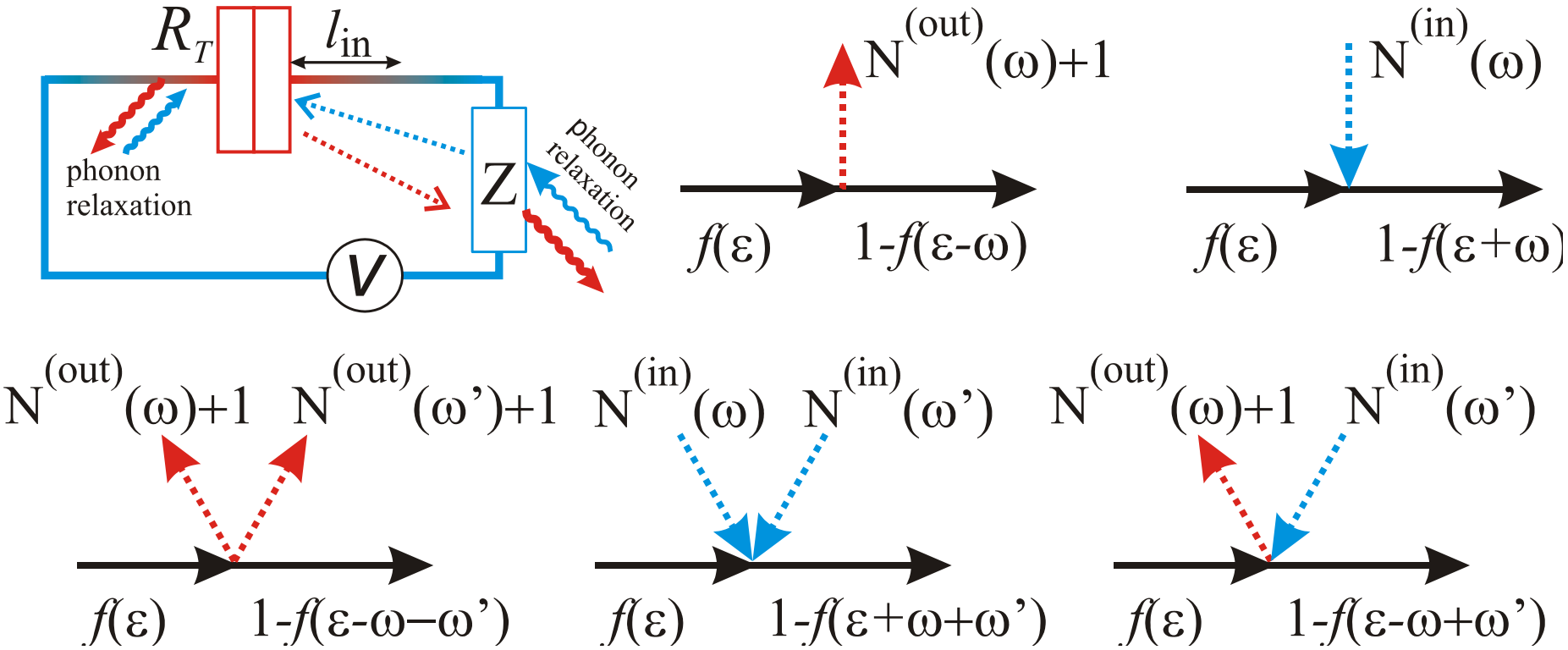
$$F_{\alpha}(\omega) = \left[N_{\omega}^{(\alpha)} e^{i\omega t} + (1 + N_{\omega}^{(\alpha)}) e^{-i\omega t} - B_{\omega}^{(\alpha)}\right],$$

$N_{\omega}^{(\alpha)}$ and $1 + N_{\omega}^{(\alpha)}$ combinations of the environmental excitations distribution functions describe absorbed and emitted environmental excitations and the index

$$B_{\omega}^{(\alpha)} = [1 + N_{\omega}^{(\alpha)}] + N_{\omega}^{(\alpha)}$$

Equilibrium: $B_{\omega} = \coth(\omega/2T)$

# Kinetic equations

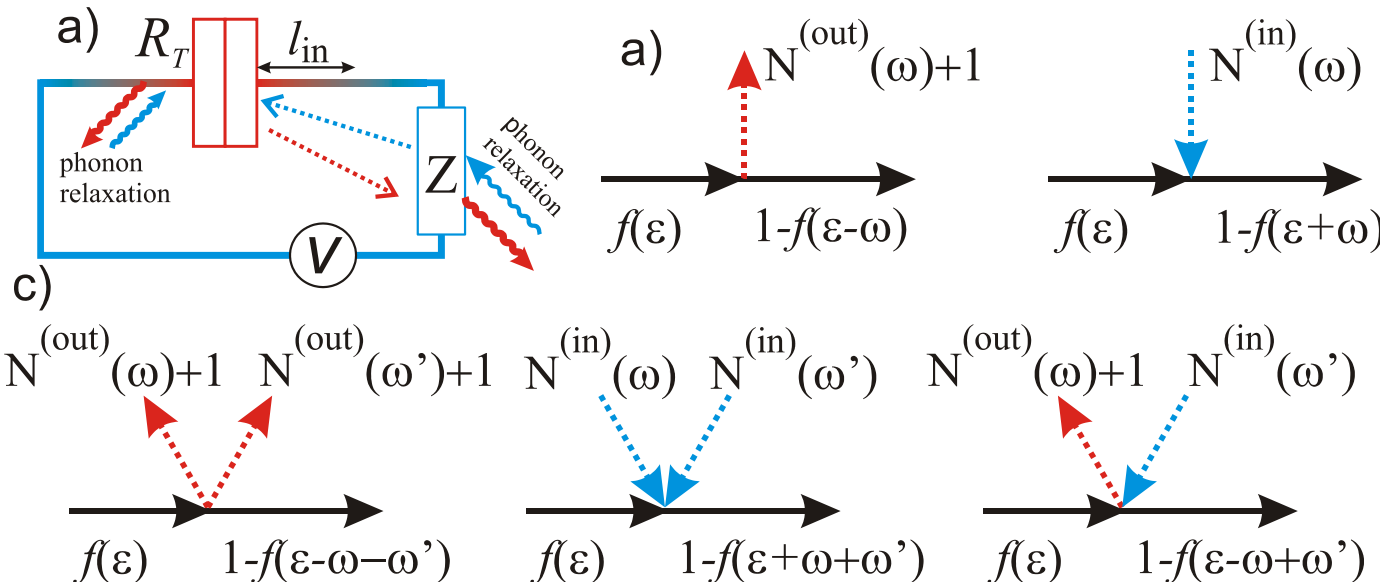



$$P^{<}(E) = \int_{-\infty}^{\infty} dt \exp[J(t) + iEt] \,,$$

$$J(t) = 2\int_0^{\infty} \frac{d\omega}{\omega} \rho_\alpha(\omega) F_\alpha(\omega) \,,$$

$$F_\alpha(\omega) = \left[ N_\omega^{(\alpha)} e^{i\omega t} + (1 + N_\omega^{(\alpha)}) e^{-i\omega t} - B_\omega^{(\alpha)} \right] ,$$

# Kinetic equations for environment bosonic excitations




$$\left(\frac{d}{dt}N_\omega^{(\alpha)}\right)_{\mathrm{e-env}} \sim -\frac{\rho^{(\alpha)}(\omega)}{\nu_\omega^{(\alpha)}R_{\mathrm{T}}}\times$$

$$\left[N_\omega^{(\alpha)}(1+n_\omega^{(\alpha)})-(1+N_\omega^{(\alpha)})n_\omega^{(\alpha)}\right]$$

## Solution of the kinetic equation for bosons

$$\left(\frac{d}{dt}N_\omega^{(\alpha)}\right)_{\mathrm{e-env}} \sim -\frac{\rho^{(\alpha)}(\omega)}{\nu_\omega^{(\alpha)}R_{\mathrm{T}}}\times \left[N_\omega^{(\alpha)}(1+n_\omega^{(\alpha)})-(1+N_\omega^{(\alpha)})n_\omega^{(\alpha)}\right]$$

Environment relaxation time

Electron-environment interaction: $1/\tau_{\mathrm{env-e}}^{(\alpha)} = \rho^{(\alpha)}(\omega)/\nu_\omega^{(\alpha)}R_{\mathrm{T}}$

$\rho_{\alpha=0}(\omega) = \mathrm{Re}[Z_{\mathrm{t}}(\omega)]/R_{\mathrm{Q}}$ Electromagnetic environment

$$\rho_{\alpha\in\{1,2\}}(\omega) = 2\,\mathrm{Im}\int_{\mathbf{q}}(D_\alpha q^2 - i\omega)^{-2}\tilde{U}_\alpha$$

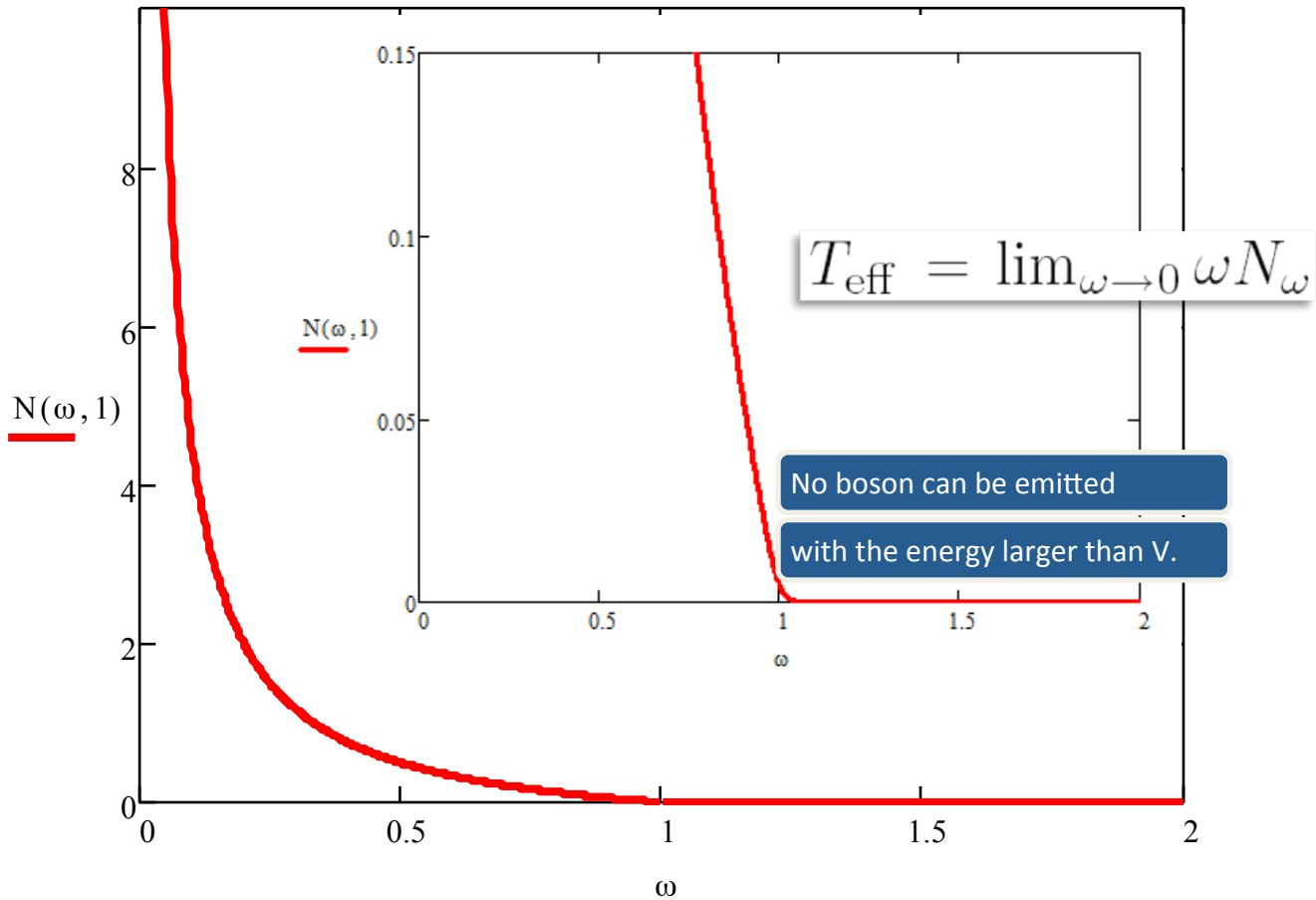
$\rho_3(\omega) = -2\,\mathrm{Im}\int_{\mathbf{q}}[(D_1q^2-i\omega)(D_2q^2-i\omega)]^{-1}\tilde{U}_3$ Electron-hole environment

$$\boxed{n_\omega^{(12)} = \frac{1}{2\omega}\int_\epsilon f_{\epsilon+\omega/2}^{(i)}\sigma_{ij}^{\mathrm{x}}(1-f_{\epsilon-\omega/2}^{(j)})}$$

$$n_B(\omega) := \frac{1}{\exp\left(\frac{\omega}{T}\right) - 1} \qquad T := 0.01$$

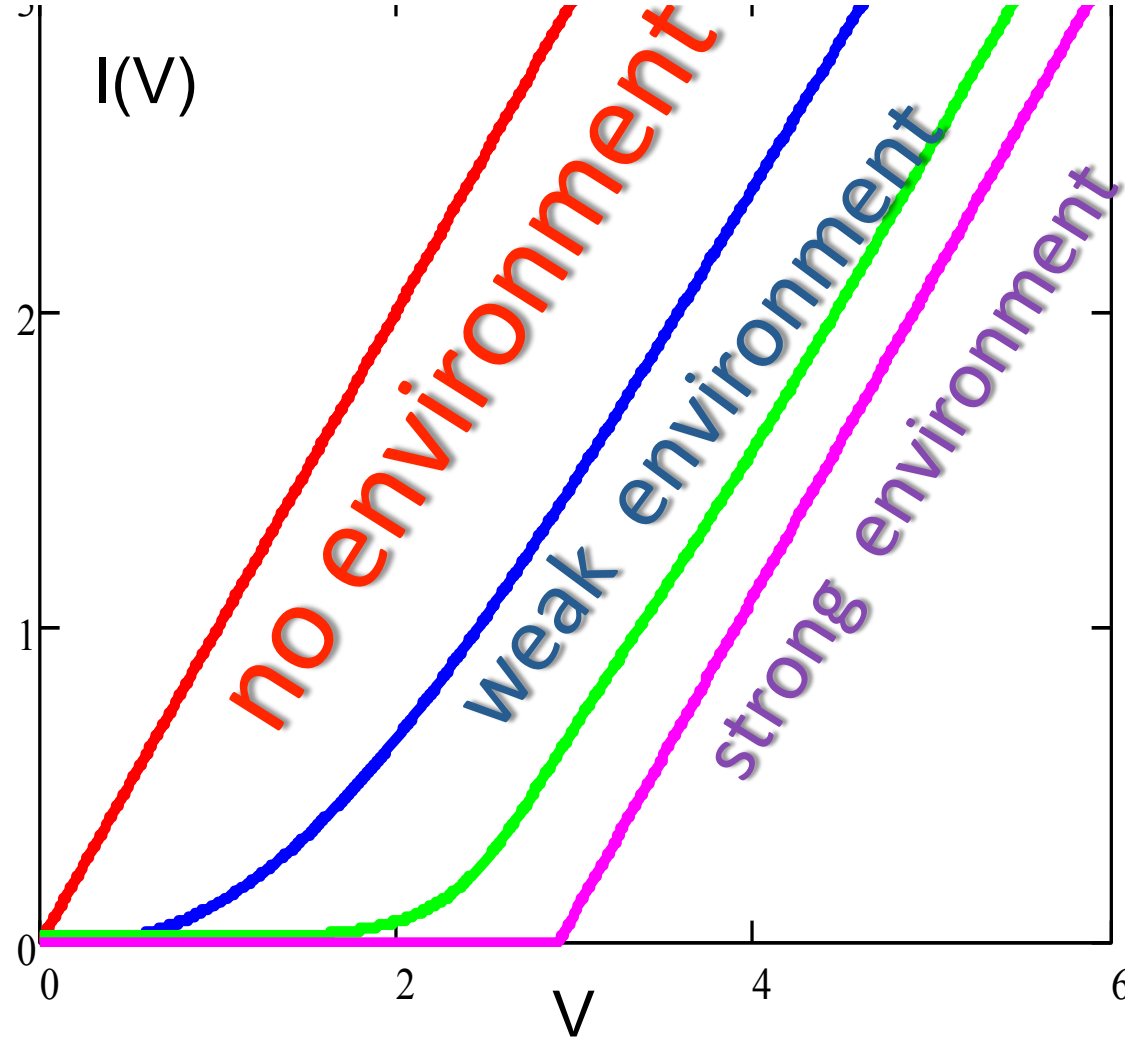
$$N(\omega, V) := \frac{1}{2\omega} \cdot \left[(\omega - V) \cdot n_B(\omega - V) + (\omega + V) \cdot n_B(\omega + V)\right]$$

Environment distribution function, low temperatures of the phonon bath

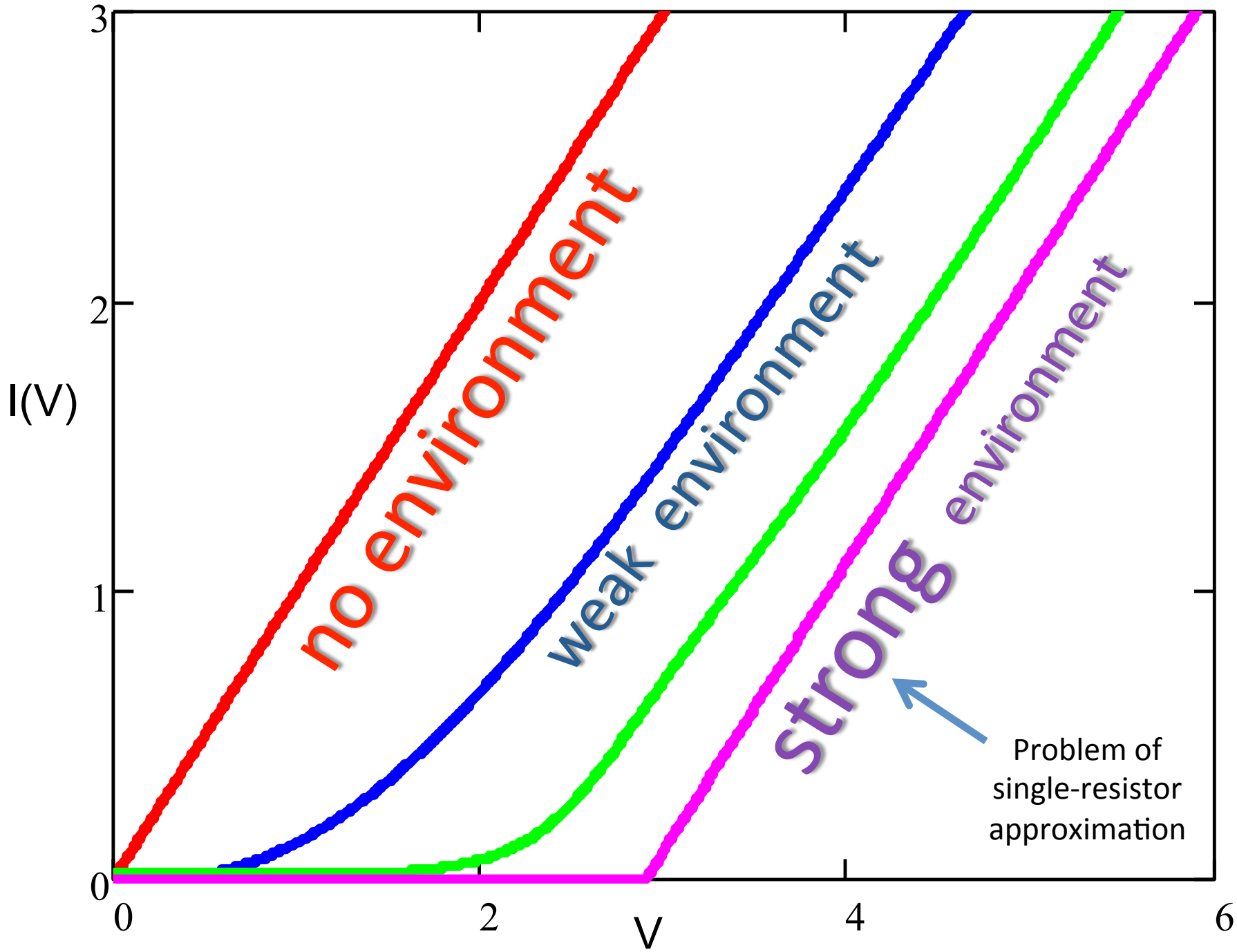


$$\Delta_\infty^{(0)} \sim \rho(0)\Lambda$$

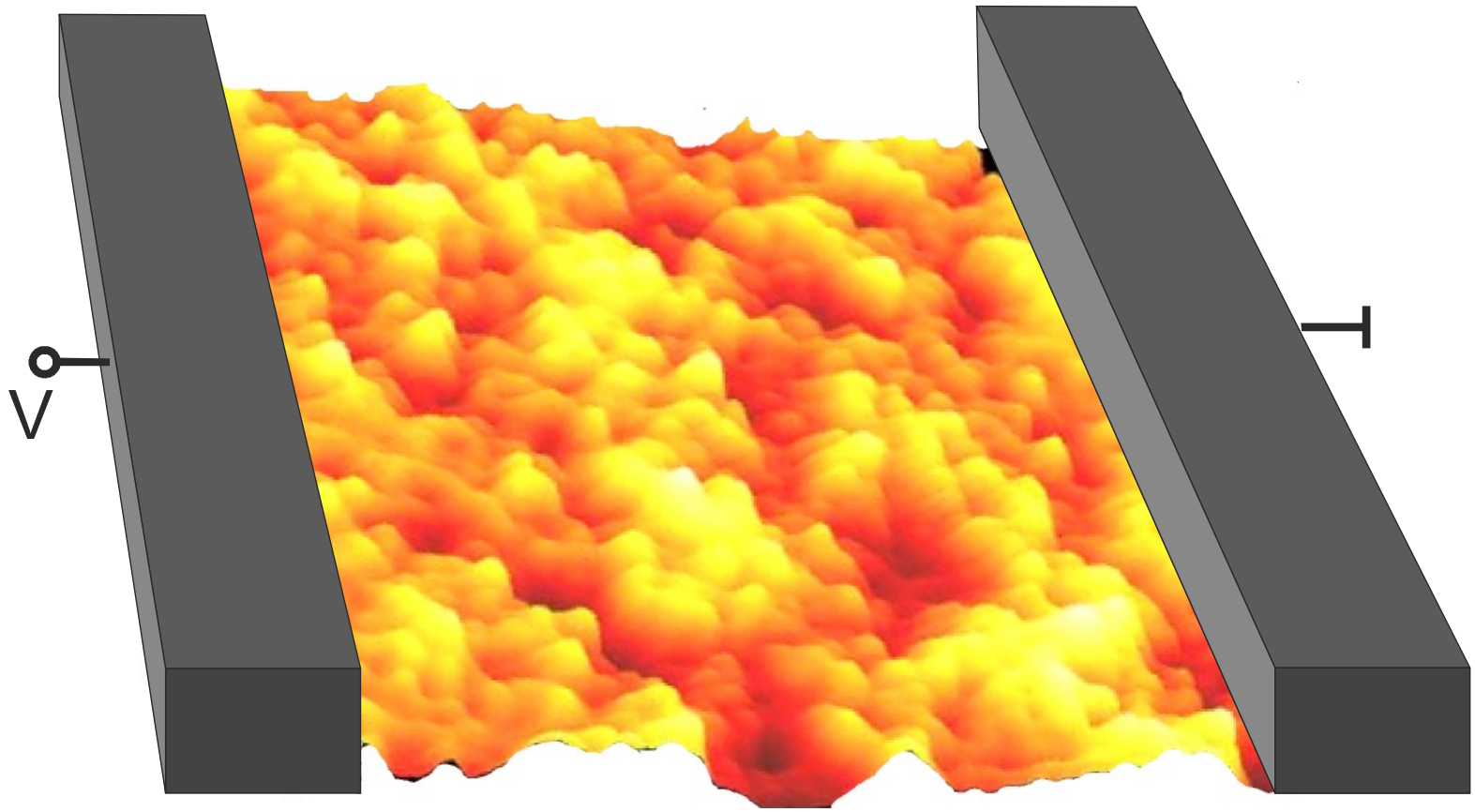
the cut-off (decay) frequency of ρ(ω)




$$\Delta_\infty = 2\int_0^\infty d\omega \rho_\omega [1 + N_\omega^{(\mathrm{out})} - N_\omega^{(\mathrm{in})}]$$
$$\simeq \Delta_\infty^{(0)} \ln(\Lambda / \min\{T_e, T_{\mathrm{env}}\})$$


3
2
1
0
I(V)
0
2
4
6
V
no environment
weak environment
strong environment
Problem of
single-resistor
approximation

# We should go beyond single junction approximation of the 2D-film near SI transition



Scale dependent superconductor-insulator transition

D. Kowal and Z. Ovadyahu

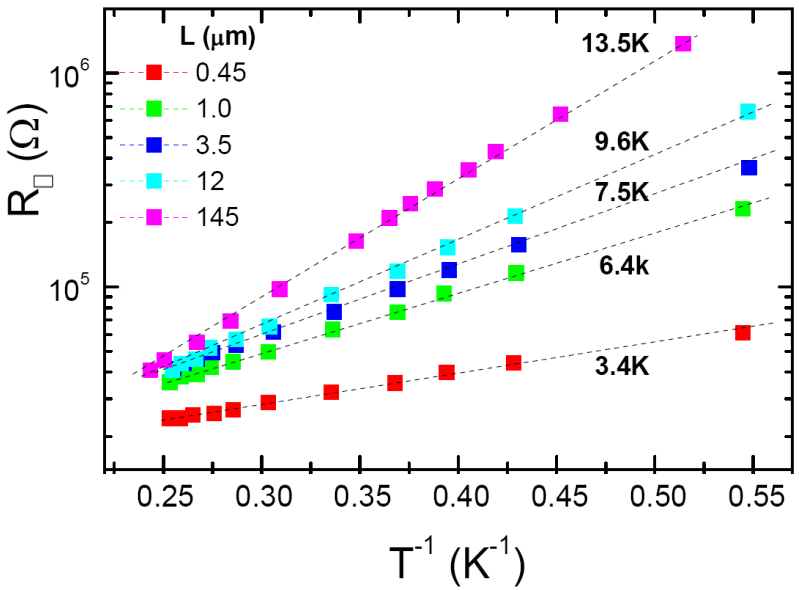
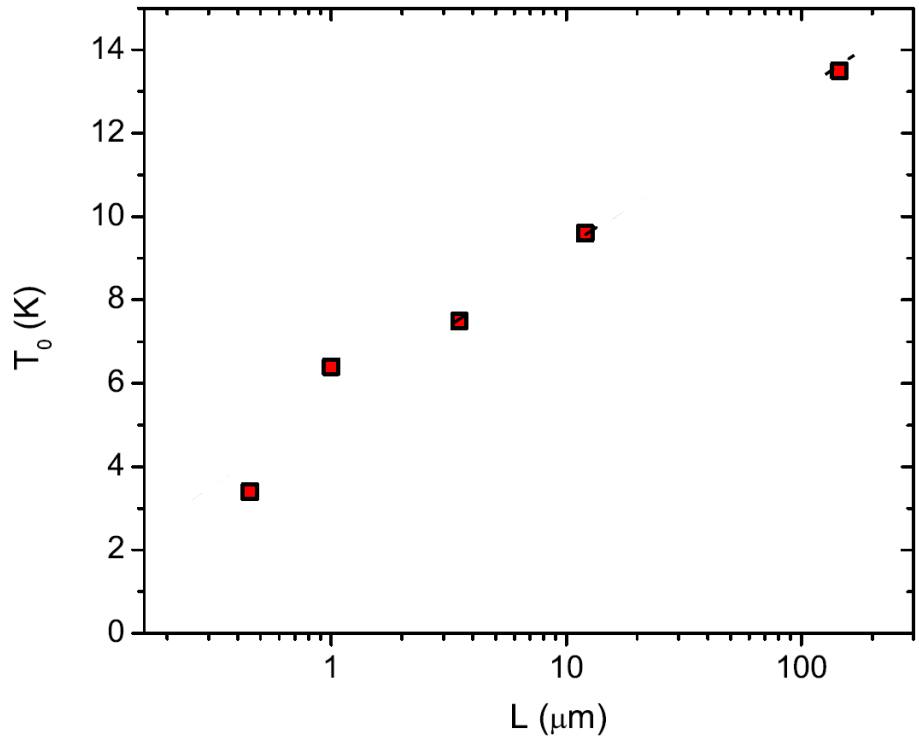
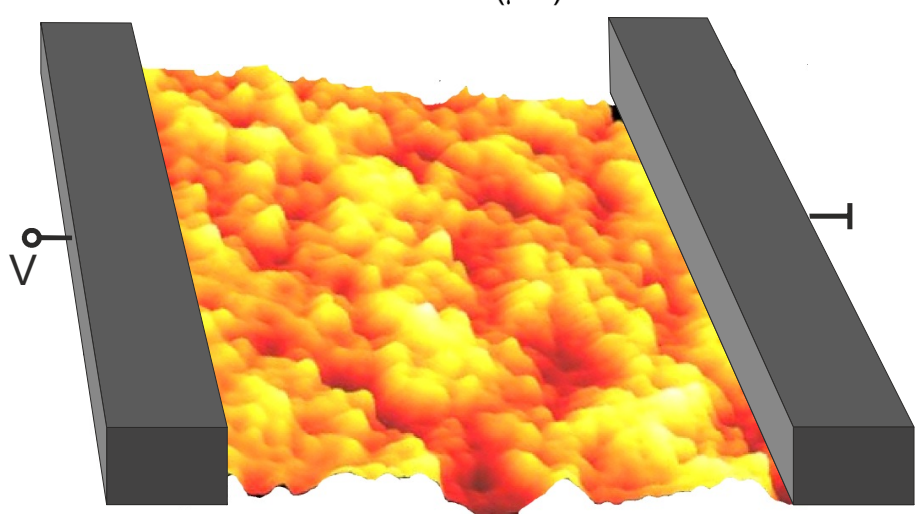



FIG. 4: (a) Resistance versus temperature for a single batch of $In_xO$ film with a common width of 500 $\mu$m and length as shown. Each sub-sample is labeled with its activation energy $T_0$ (see text).(b) Same batch of samples as in (a) after further thermal annealing.

*justification of the array approximation*

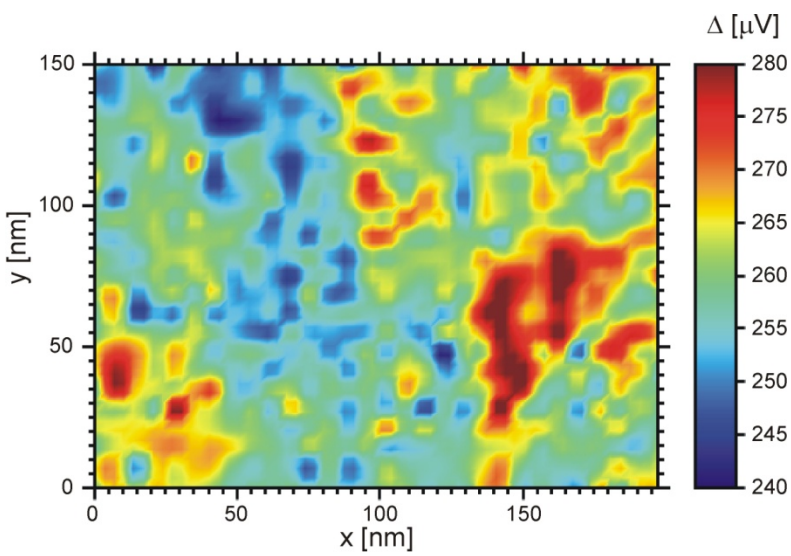


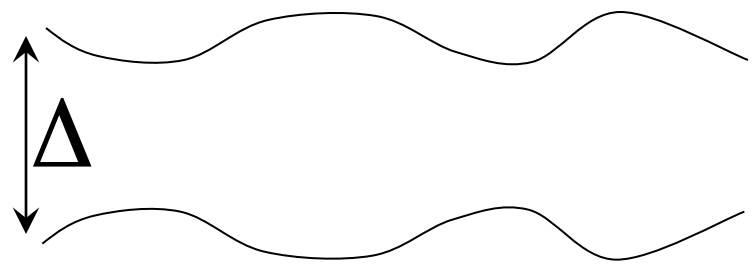



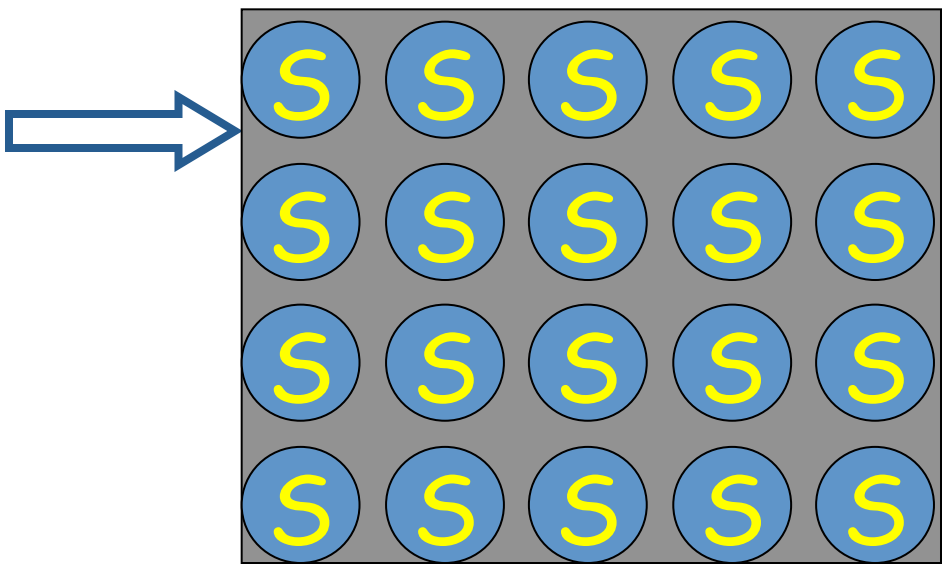
# "Island" structure of disordered superconductor

**L. B. Ioffe and A. I. Larkin, Zh. Eksp. Teor. Fiz. 81, 707 (1981) [Sov. Phys. JETP 54, 378 (1981)].**

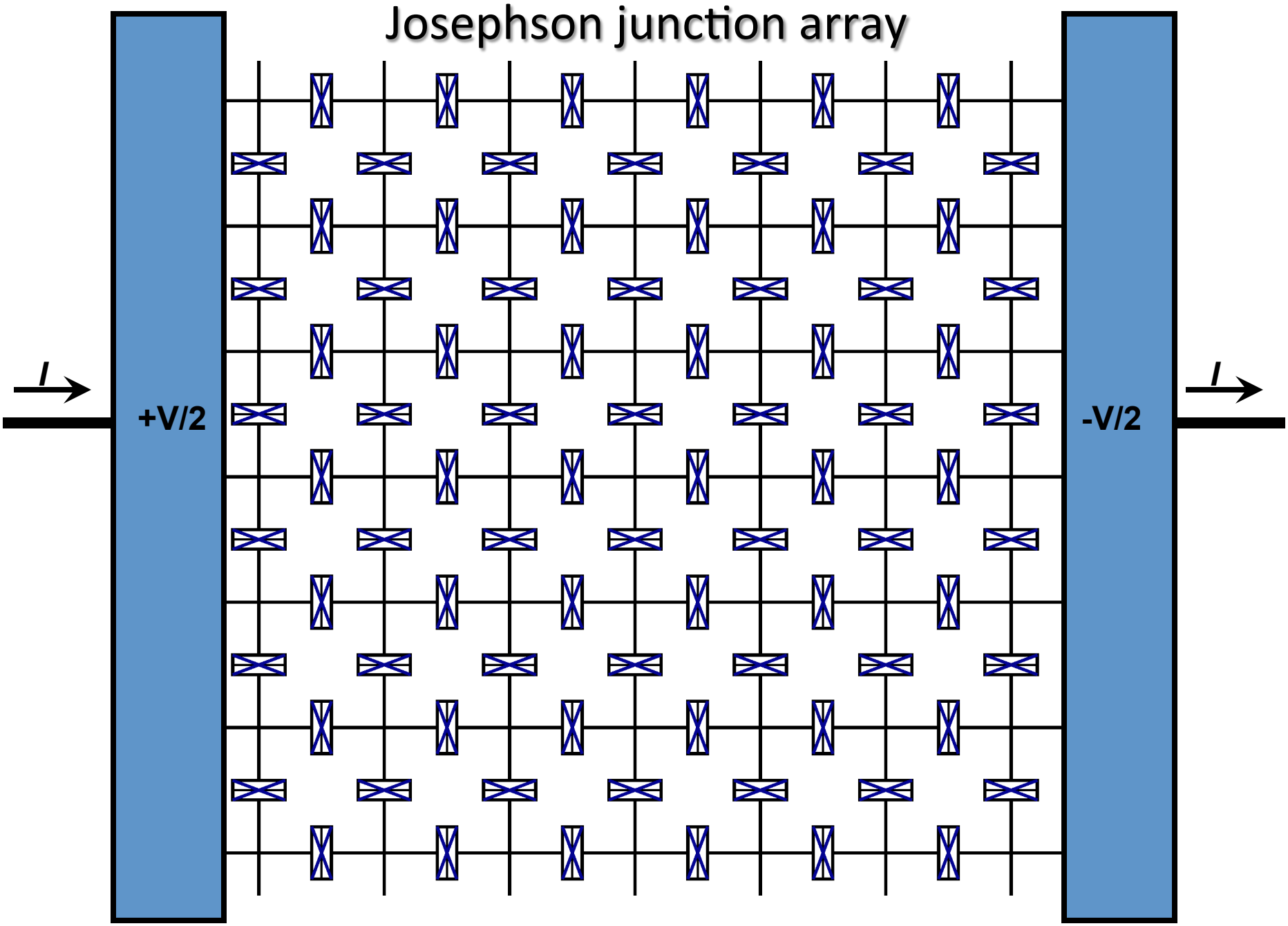
Fluctuations in disorder lead to appearance of regions with enhanced $T_c$, i.e. to formation of superconducting "islands" above the 'bulk' $T_c$.



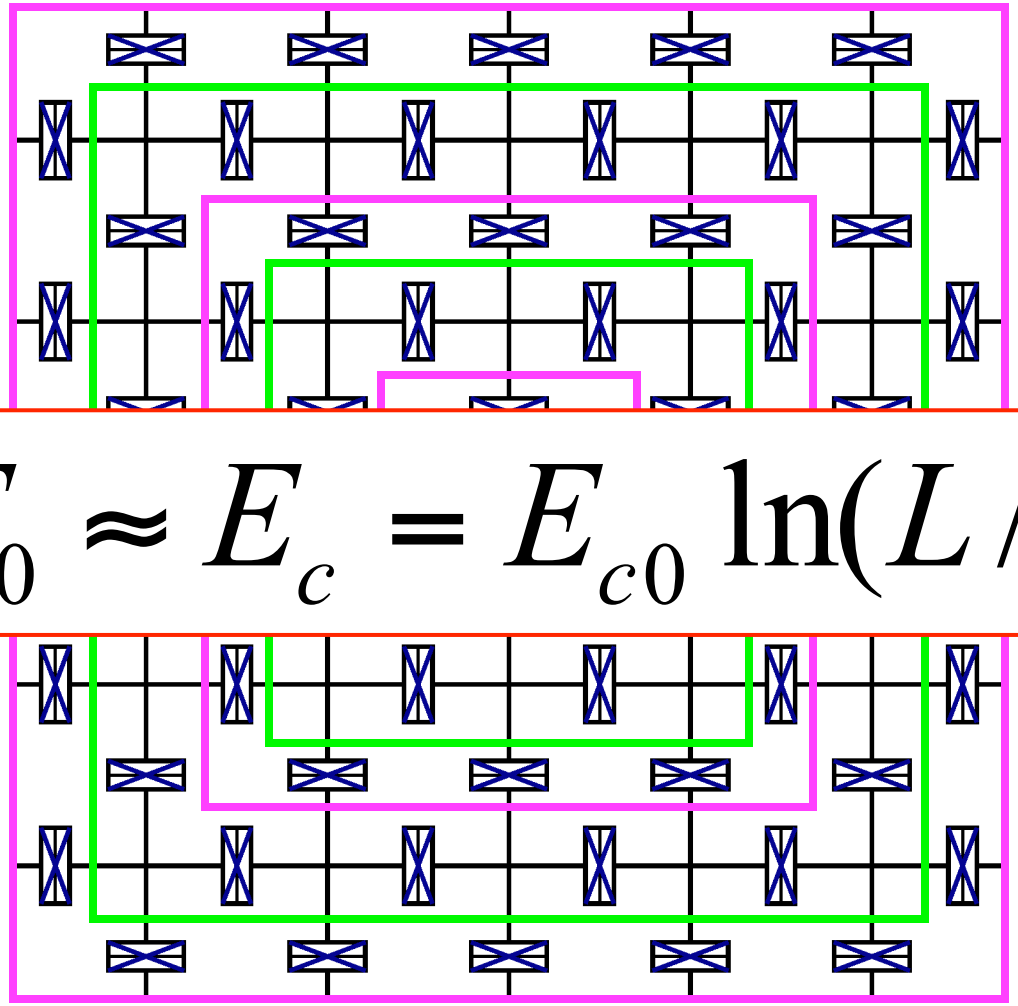



# Josephson junction array



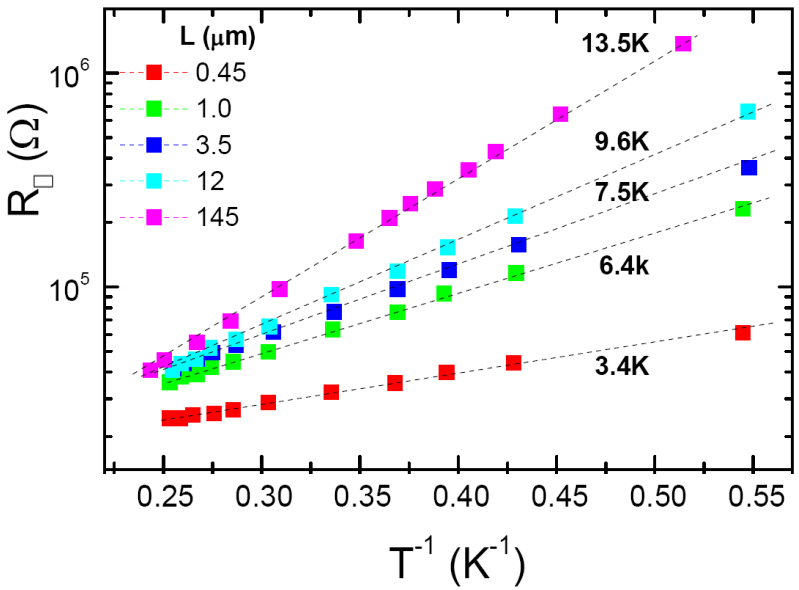
Capacitance of the array?
What is $T_0$?
How $T_0$ is related to $E_c$?

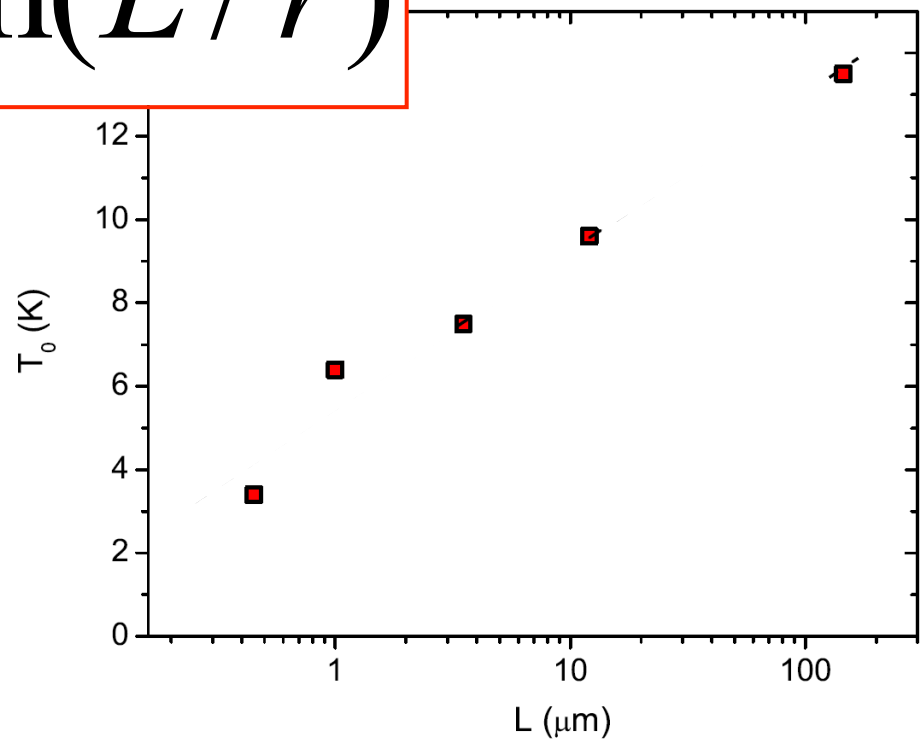


$$T_0 \approx E_c = E_{c0} \ln(L/r)$$

$$C \propto C_0 / \ln(R/r)$$

$$T_0 \approx E_c = E_{c0} \ln(L/r)$$

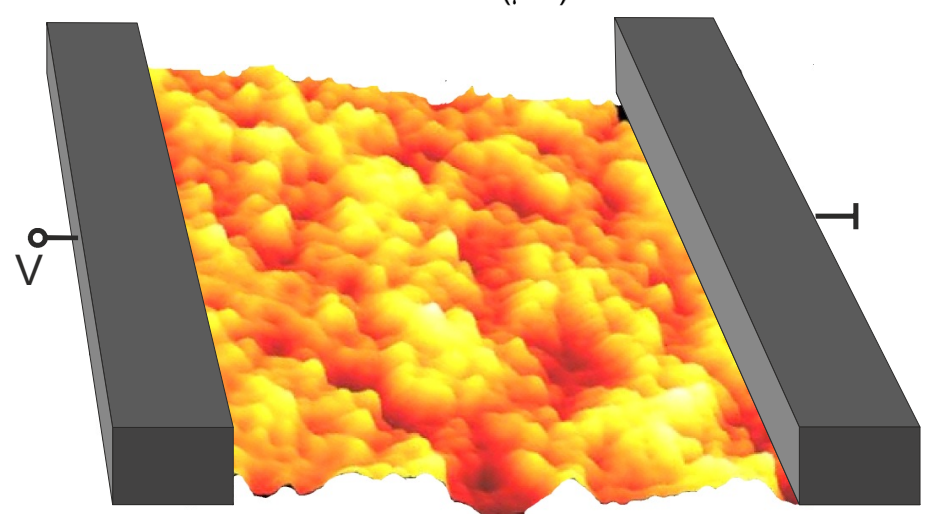



**Scale dependent superconductor-insulator transition**

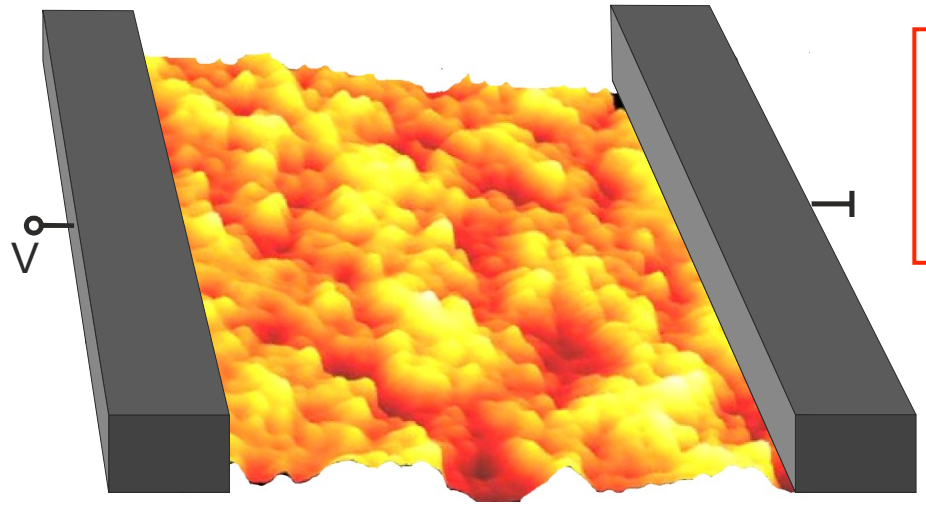
D. Kowal and Z. Ovadyahu

FIG. 4: (a) Resistance versus temperature for a single batch of $In_xO$ film with a common width of 500 $\mu$m and length as shown. Each sub-sample is labeled with its activation energy $T_0$ (see text).(b) Same batch of samples as in (a) after further thermal annealing.



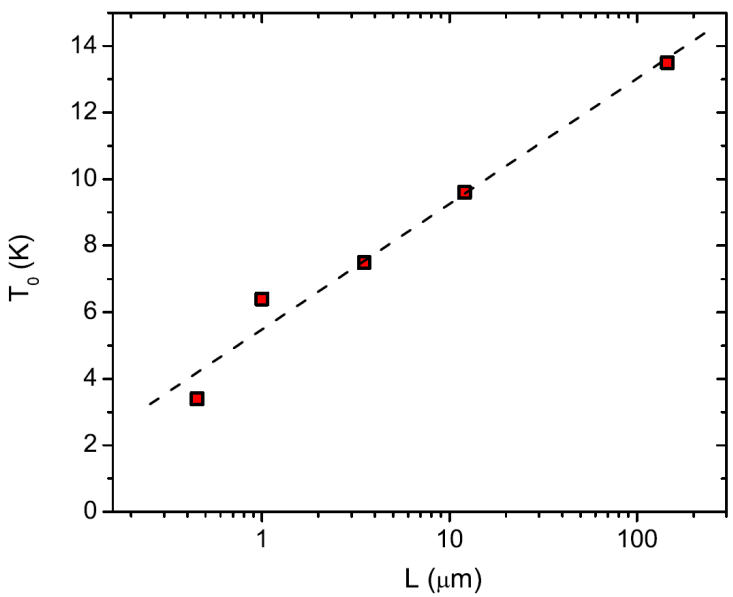

**Similar results: T. Baturina, C. Strunk et al**

# Array approximation, short summary

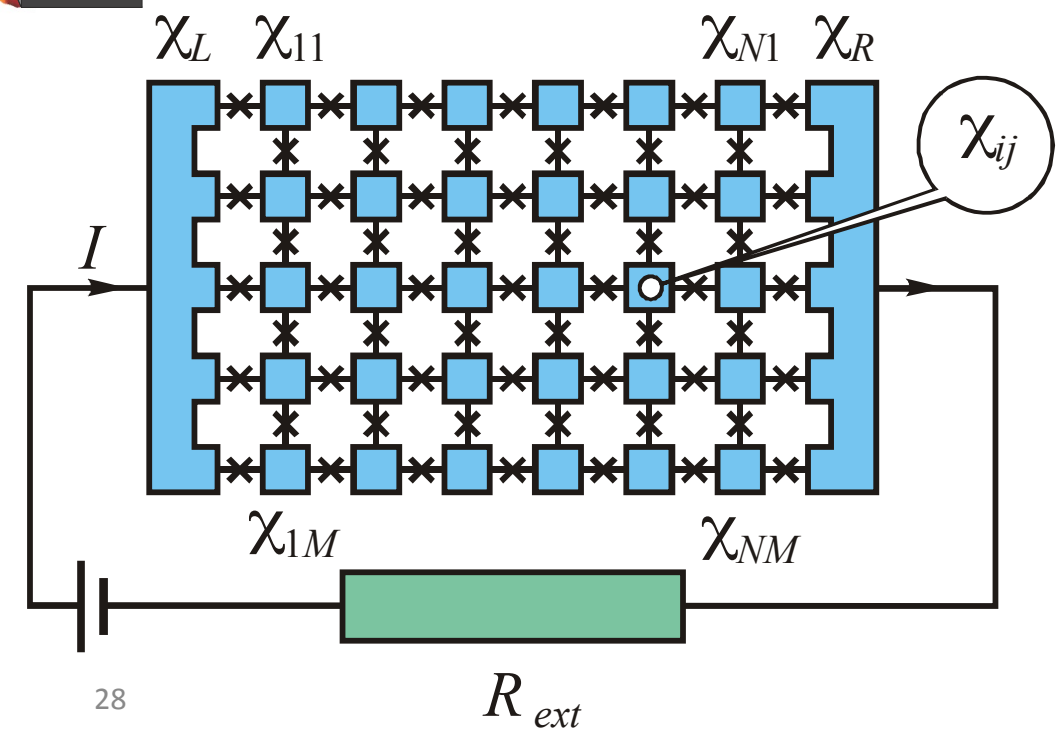



Activation energy:

$$T_0 \approx E_c = \begin{cases} E_{c0} \ln(L/r), \rightarrow 2D \\ E_{c0} L/r, \rightarrow 1D \end{cases}$$

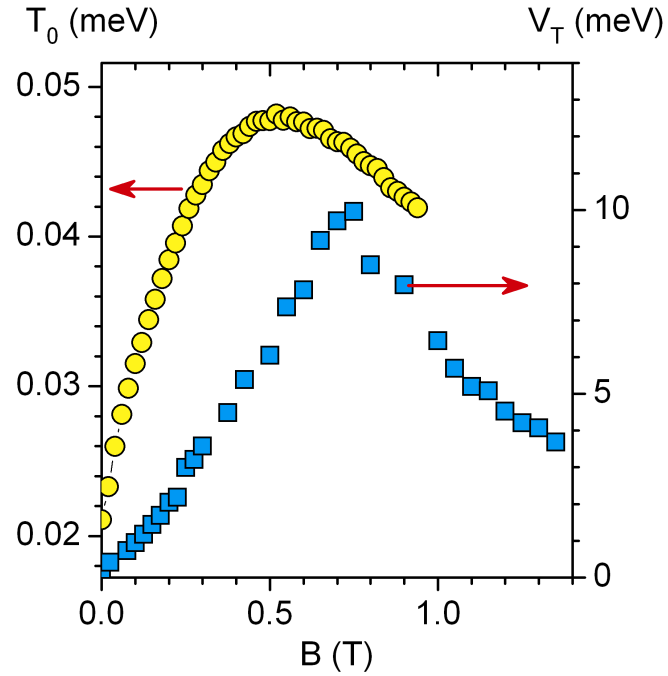


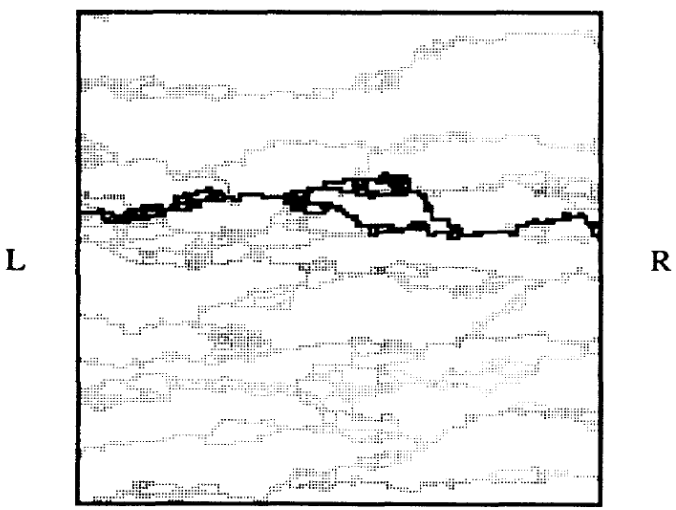

Array approximation:



- Transport in JJA is sensitive to **B**.
  Can we see this experimentally in films?
- Does the array model pass the magnetic field test?
- Can we see 1D array effects in films?

$$\frac{eV_T}{k_B T} \approx 220 \quad \text{at B=0.7T}$$




Voltage depinning threshold is actually **a dielectric breakdown** determined by the lowest resistance path. **1D!!**

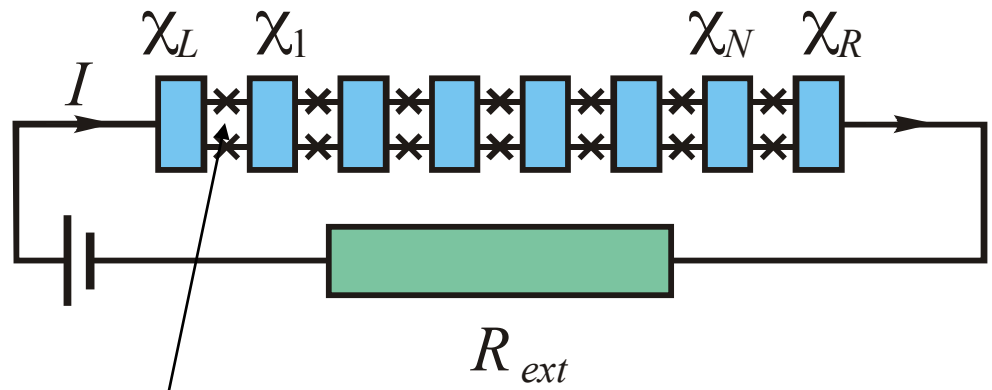
New scale:

$$\boxed{eV_T = (L/d)E_c}$$

Activation energy -> $k_B T_0 = E_c \ln(L/d)$

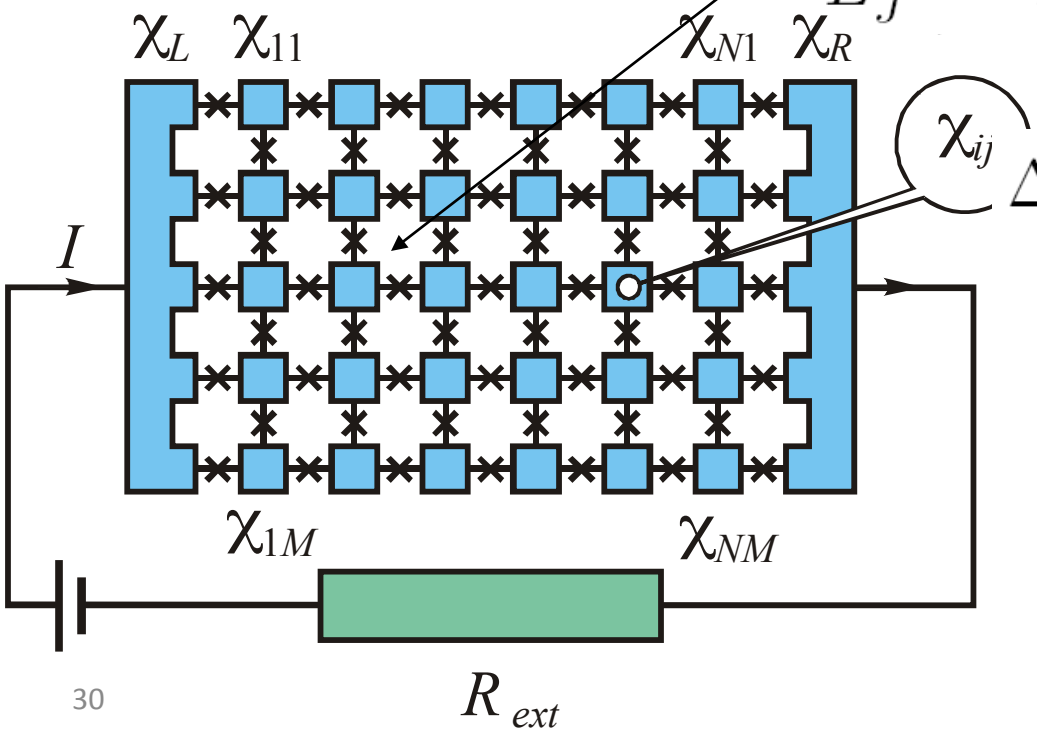
skip

$$E_J^{1D} = E_{J0}|\cos(\pi f)|$$




$$f = eBA_{loop}/\pi h$$

$$E_J^{2D} = E_{J0}\{1 - 4f\sin^2[\pi(1-f)/4]\}$$



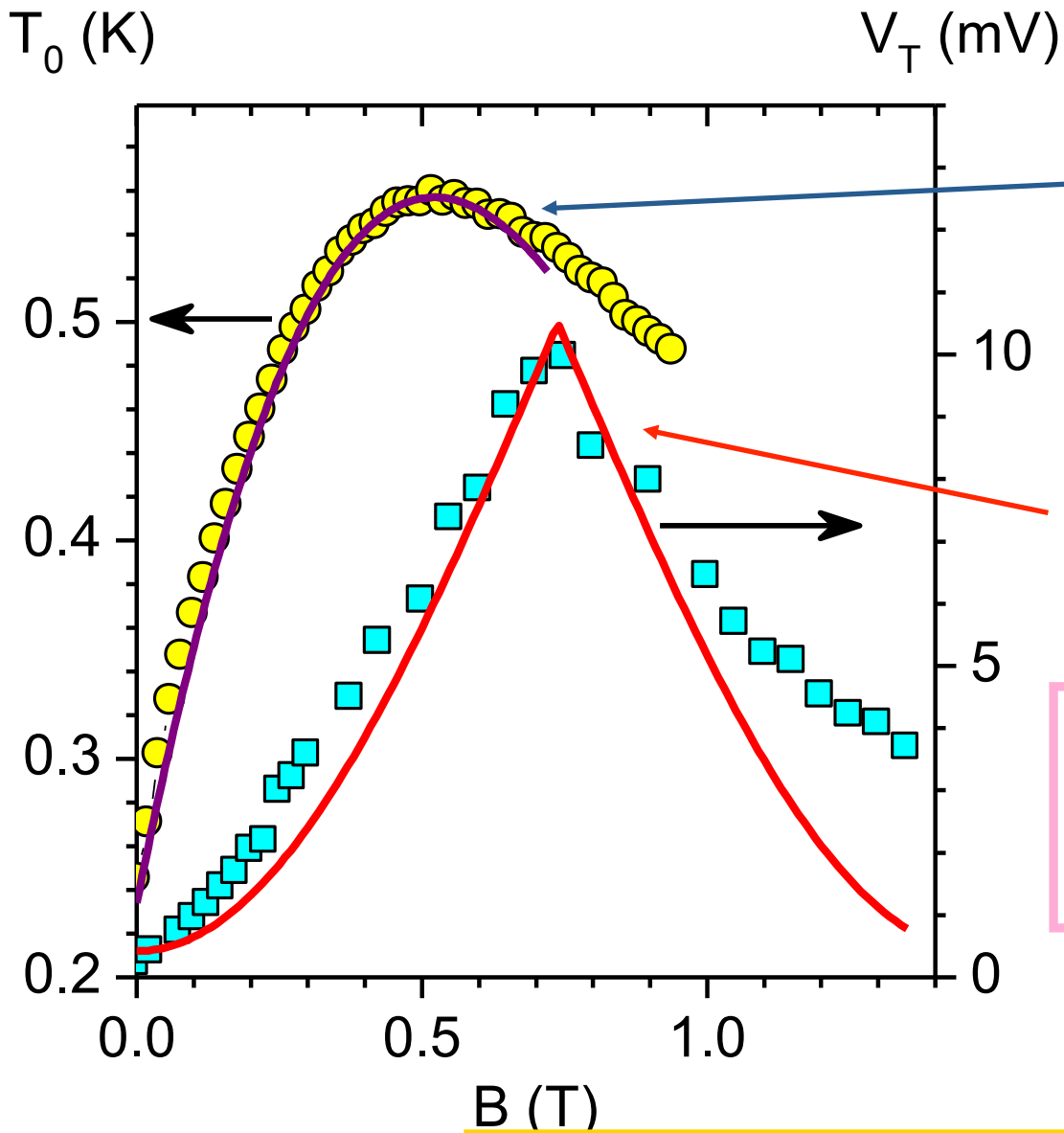

$$\Delta_c(B) = \Delta_c[1 - \alpha E_J(B)/E_c]$$

$$\alpha E_{J0}/E_c = 0.8$$

$$E_c \rightarrow \Delta \sim (\Delta_0/g)\ln(gE_C/\Delta_0)$$

Magnetic field dependence

$$\Delta_c(B) = \Delta_c[1 - \alpha E_J(B)/E_c]$$




2D fit using JJA approximation

1D fit, no fitting parameters

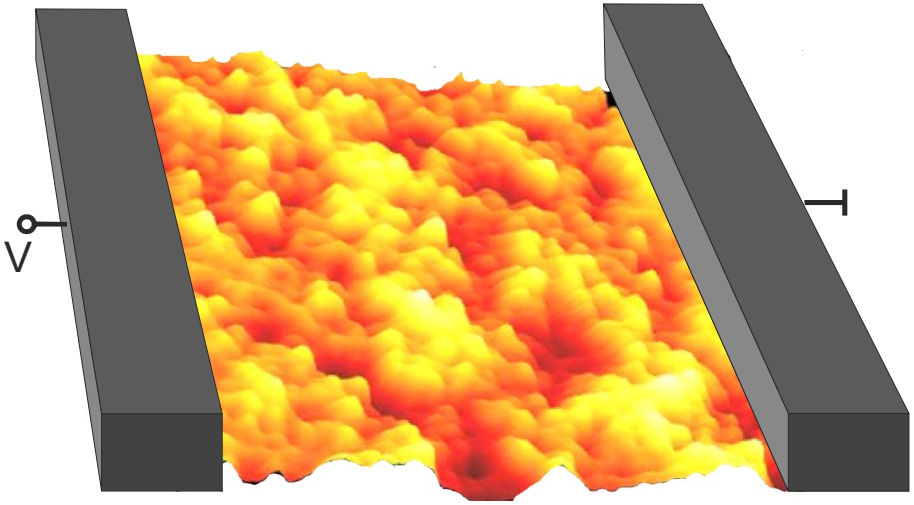
Voltage depinning threshold is actually a dielectric breakdown determined by the lowest resistance path. 1D!!

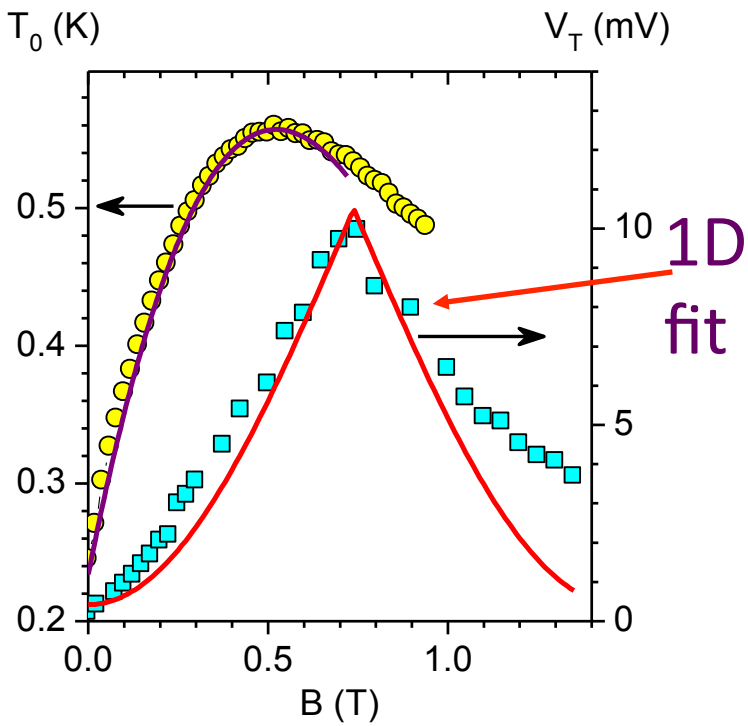
$$A_{loop} = 1.4 \cdot 10^{-3}\ \mu\mathrm{m}^2$$

$$\tilde{\alpha} E_{J0}/E_c = 0.96$$

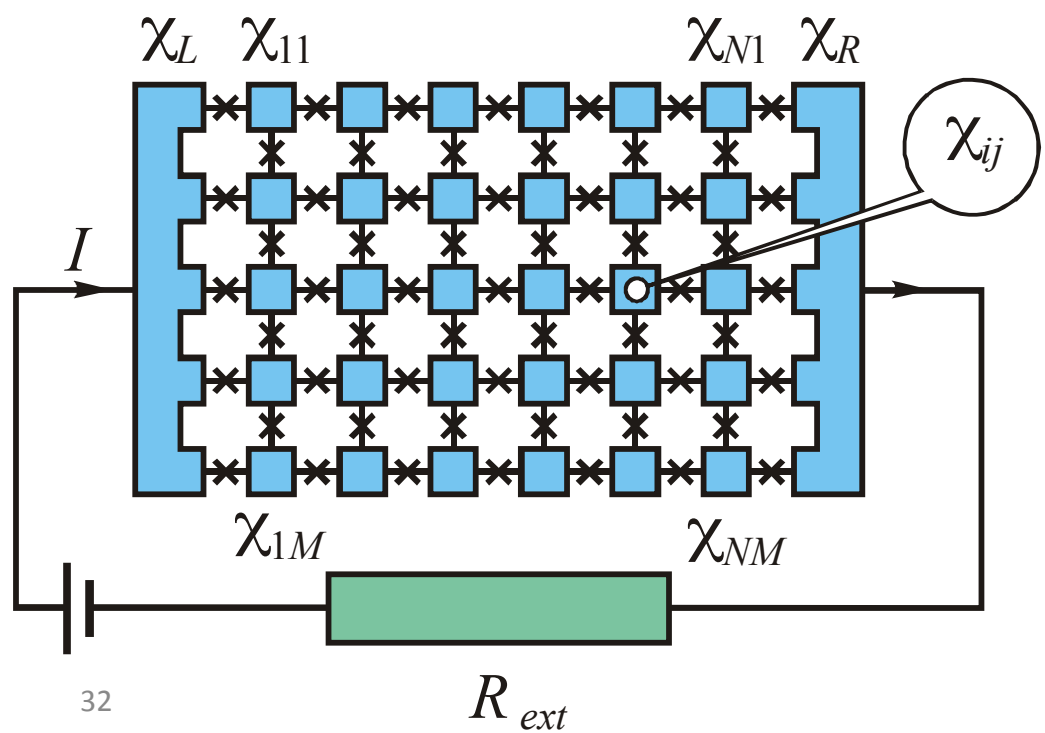
# Summary: array approximation works well…

$$\Delta_c(B) = \Delta_c[1 - \alpha E_J(B)/E_c]$$

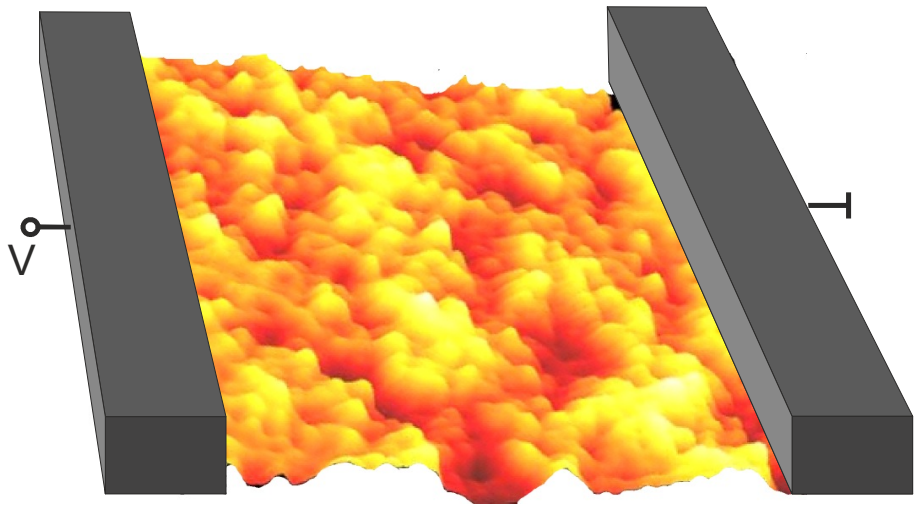


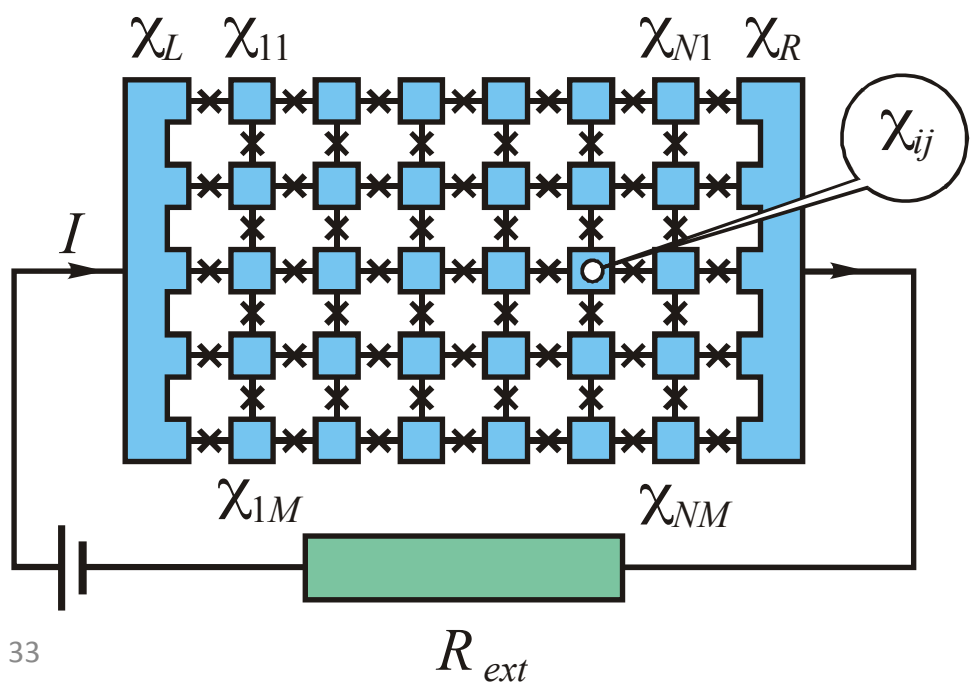

Array approximation:



# 2D-film near S-I transition. Environment

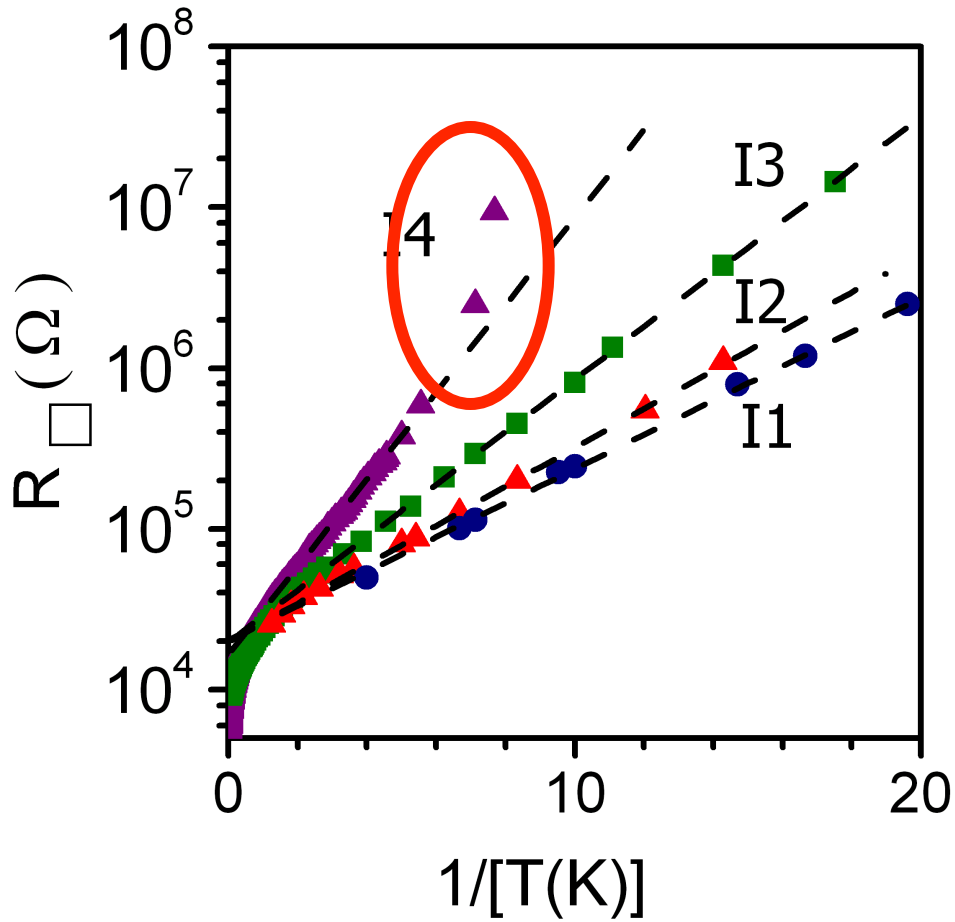


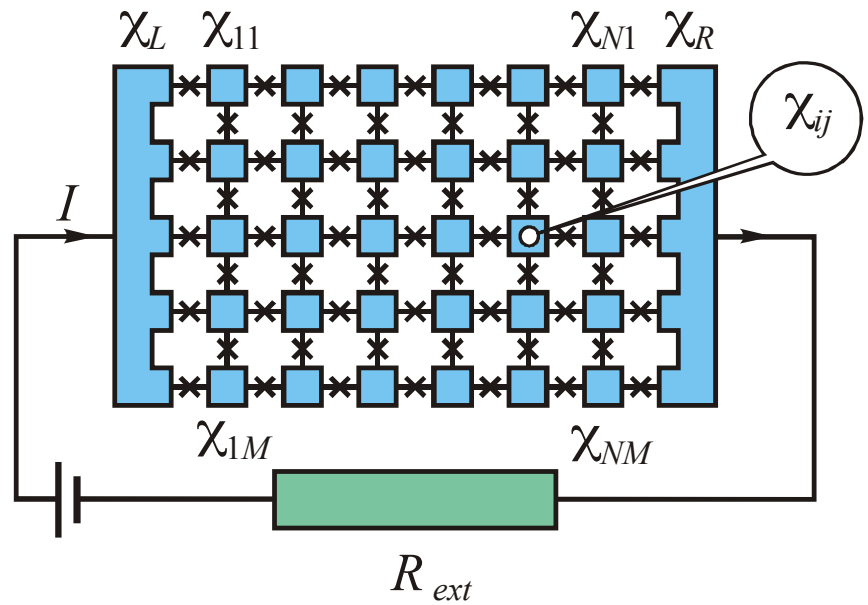

Array approximation:



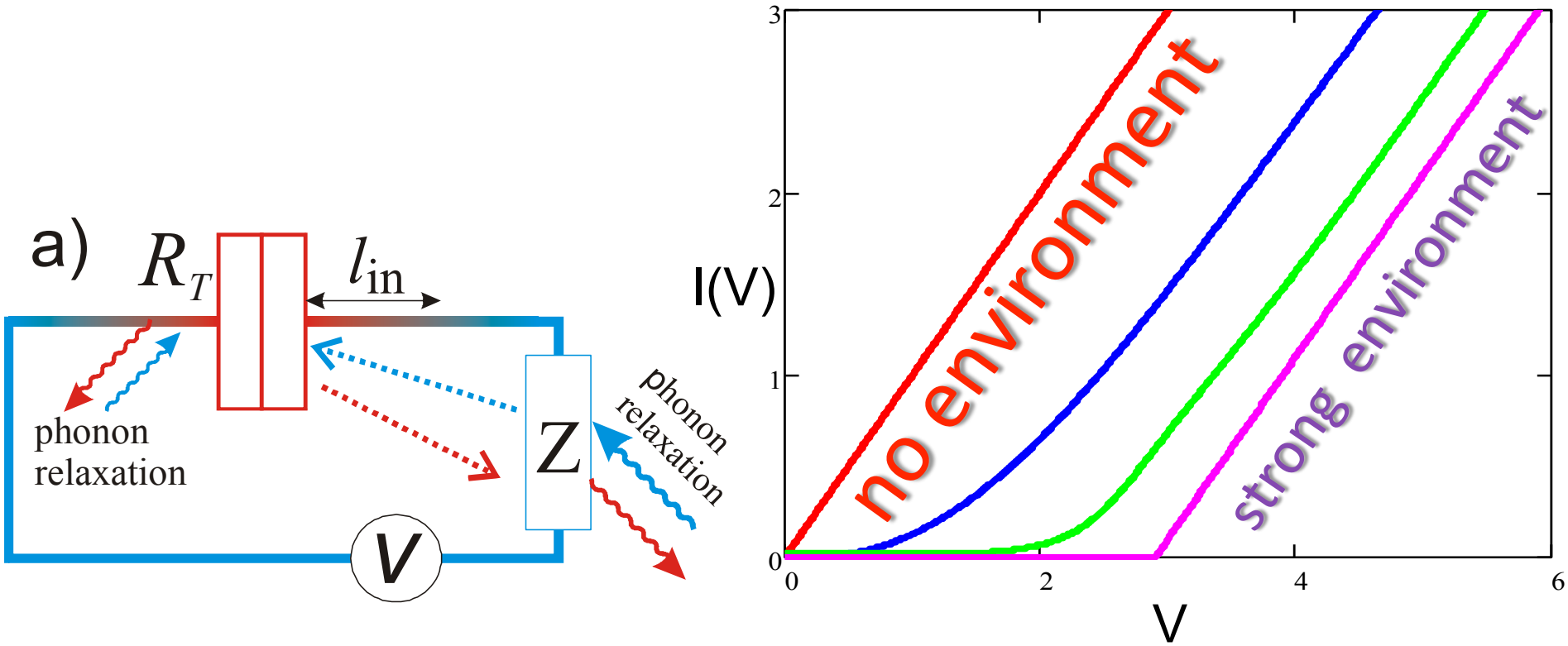

**Environment effects?**

# The nature of the high resistive state: superinsulator




Inelastic co-tunneling?




The pronounced gap in I(V) appears in a tunnel junction when the electron-environment interaction **ρ is large.**

**We can not expect that in films near the transition into insulating state. Then what?**

# Internal environment. Transport in mesoscopic conductors

How environment leaving inside the sample provides the relaxation illustrates the following simple example:
Tunneling between the neighboring potential wells

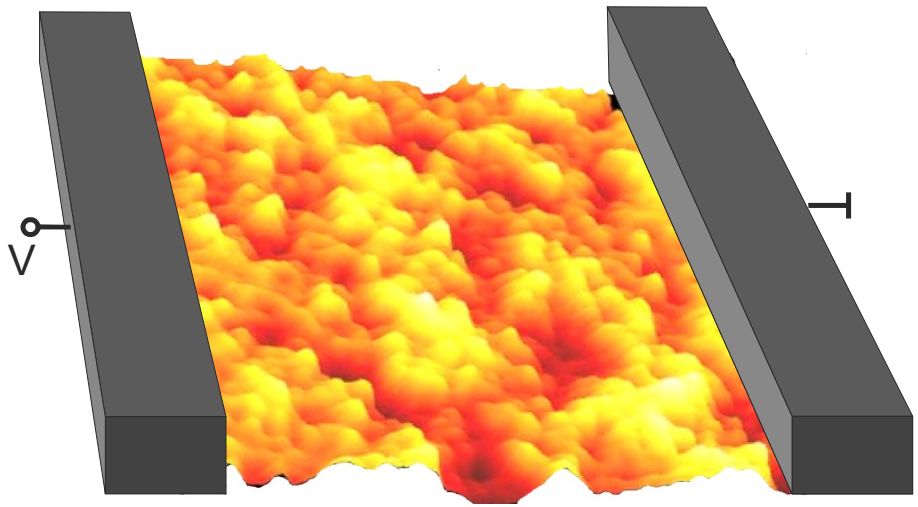
One needs relaxation mechanism (thermal bath) to ensure energy accommodation

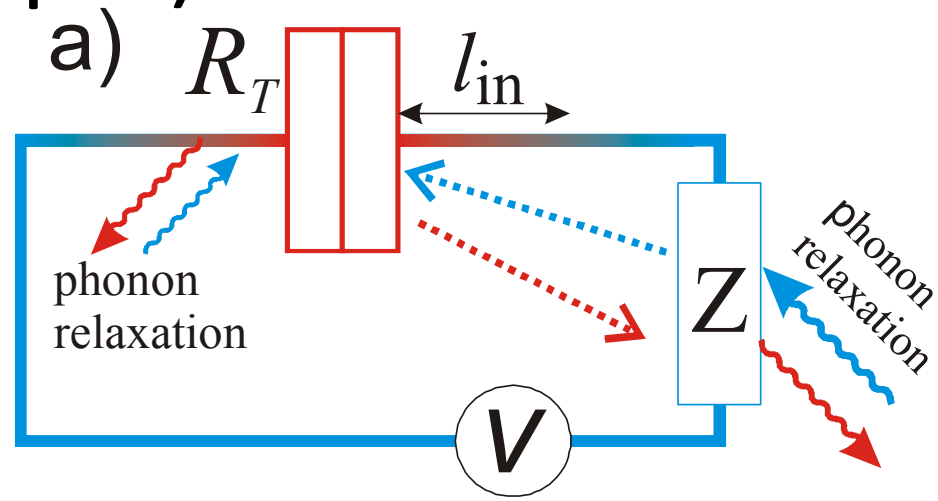
This relaxation mechanism governs the variable range hoping conductivity in semiconductors

$$\rho = \rho_0 \exp\left(\frac{T_M}{T}\right)^{1/4} \qquad \rho = \rho_0 \exp\left(\frac{T_{ES}}{T}\right)^{1/2}$$

(hopping conductivity: the relaxation mechanism still remains a puzzle)

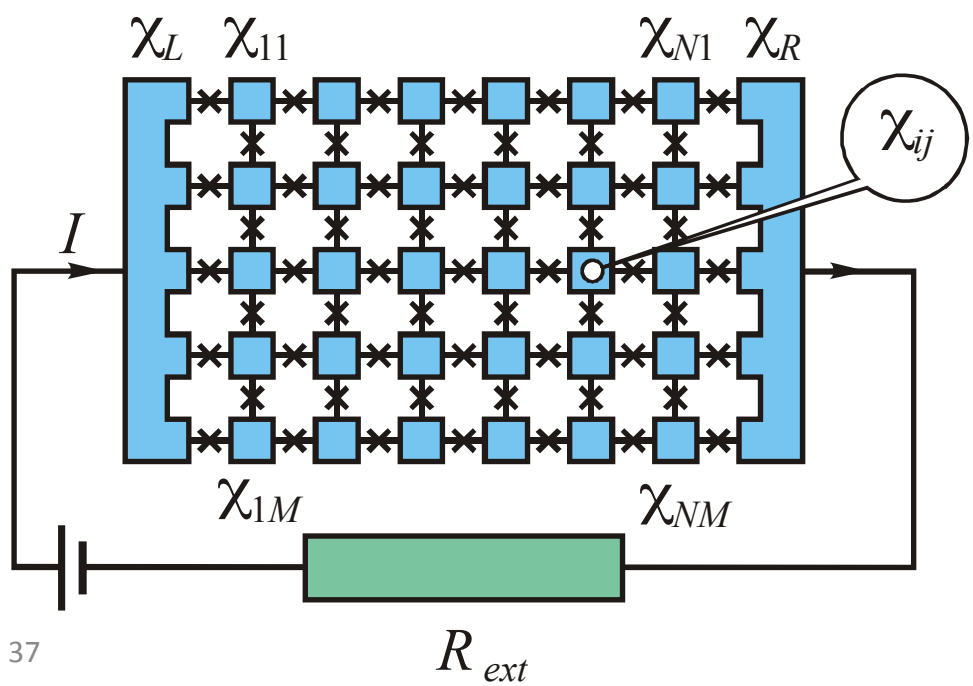
# 2D-film near (super)insulator state



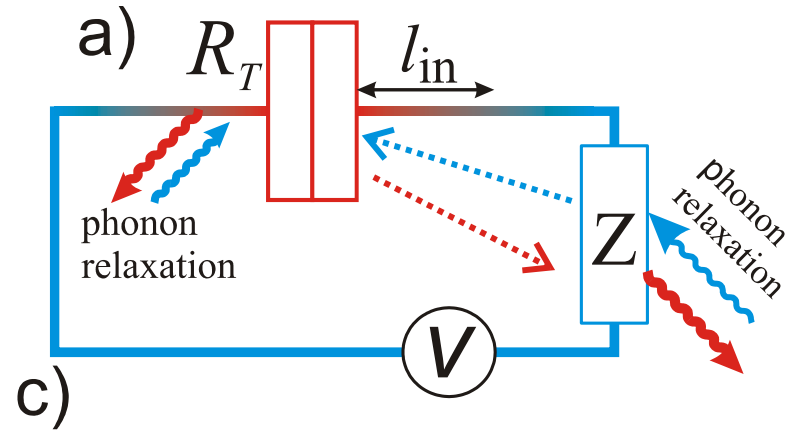



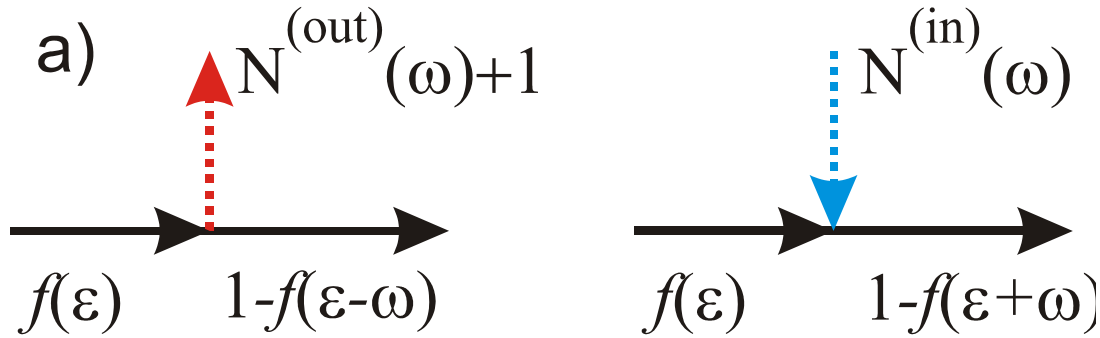

Environment is inside the film. Leads are attached to the surface and can not provide enough relaxation.

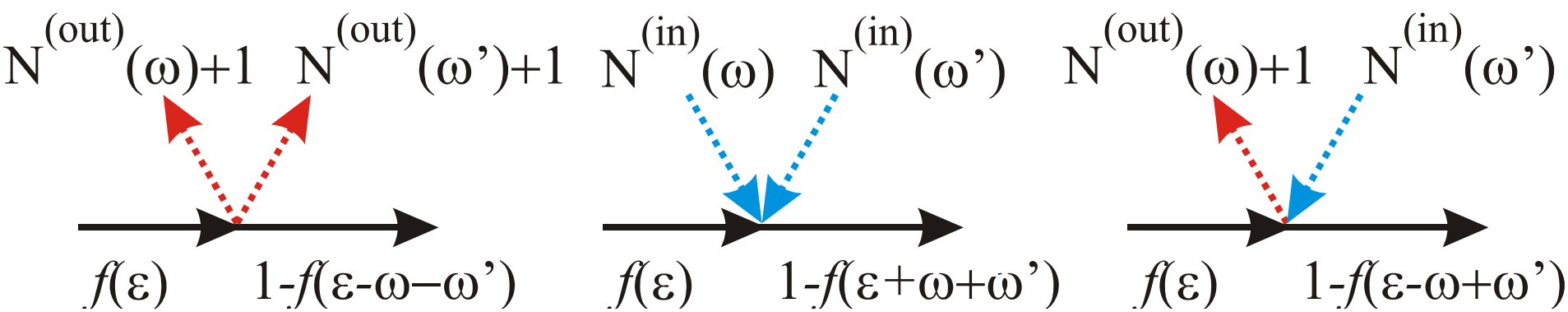
Array approximation:



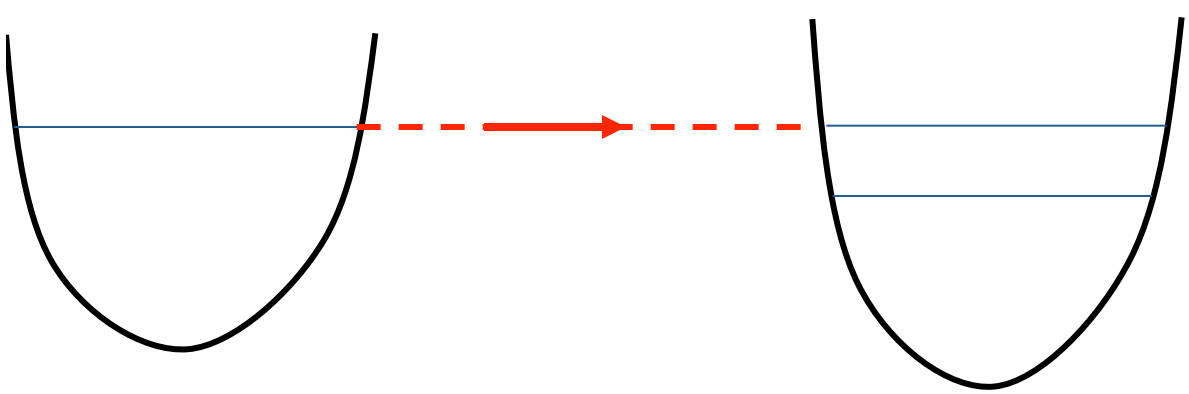
No tunneling without interaction with the environment bosons… Now I(V) is nonlinear always

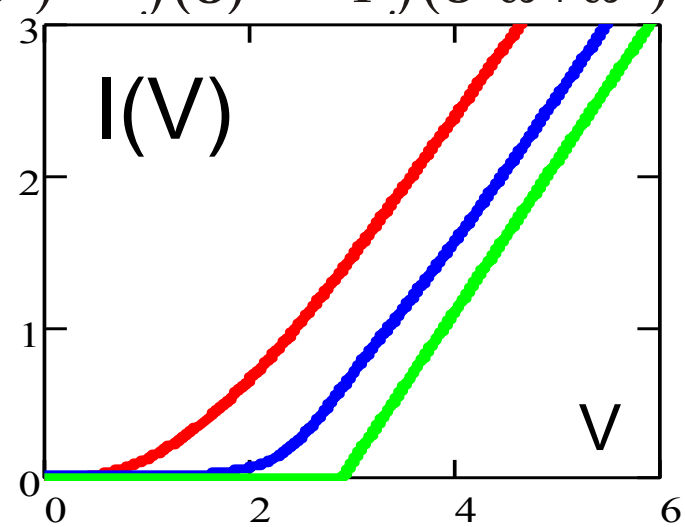



Tunneling between the neighboring potential wells





# Berezinskii-Kosterlitz-Thouless transition

Energy of interacting charges: $U = E_C \ln ( R / r_0 )$

Entropy due to arranging two (+) and (-) charges in the plane:

$$S = 2k_B \ln ( R / r_0 )$$

($r_0$ is some minimal characteristic size)

Free energy: $F = U - TS = E_C \ln ( R / r_0 ) - 2k_B \ln ( R / r_0 )$

**BKT transition** at $T = T_{BKT} = E_C / 2k_B$

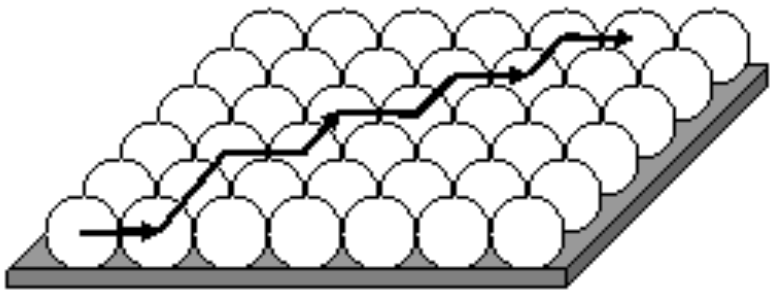
**BKT transition**

**is a consequence of the logarithmic interaction between charges!**

# Hopping conductivity in tunnel junction arrays

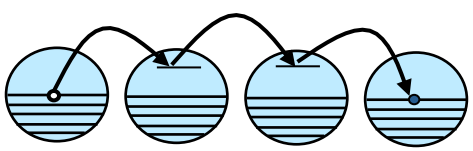
## Efros-Shklovskii (or Mott)-type VRH should not be seen!!

arrays of granules: electrons have to tunnel through other granules



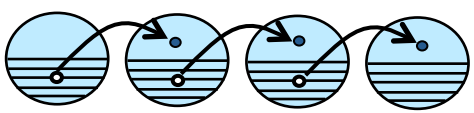
**Hopping between the nearest neighbors, and no room for the hopping distance optimization**

**tunneling via virtual states of intermediate grains**



Elastic cotunneling mechanism

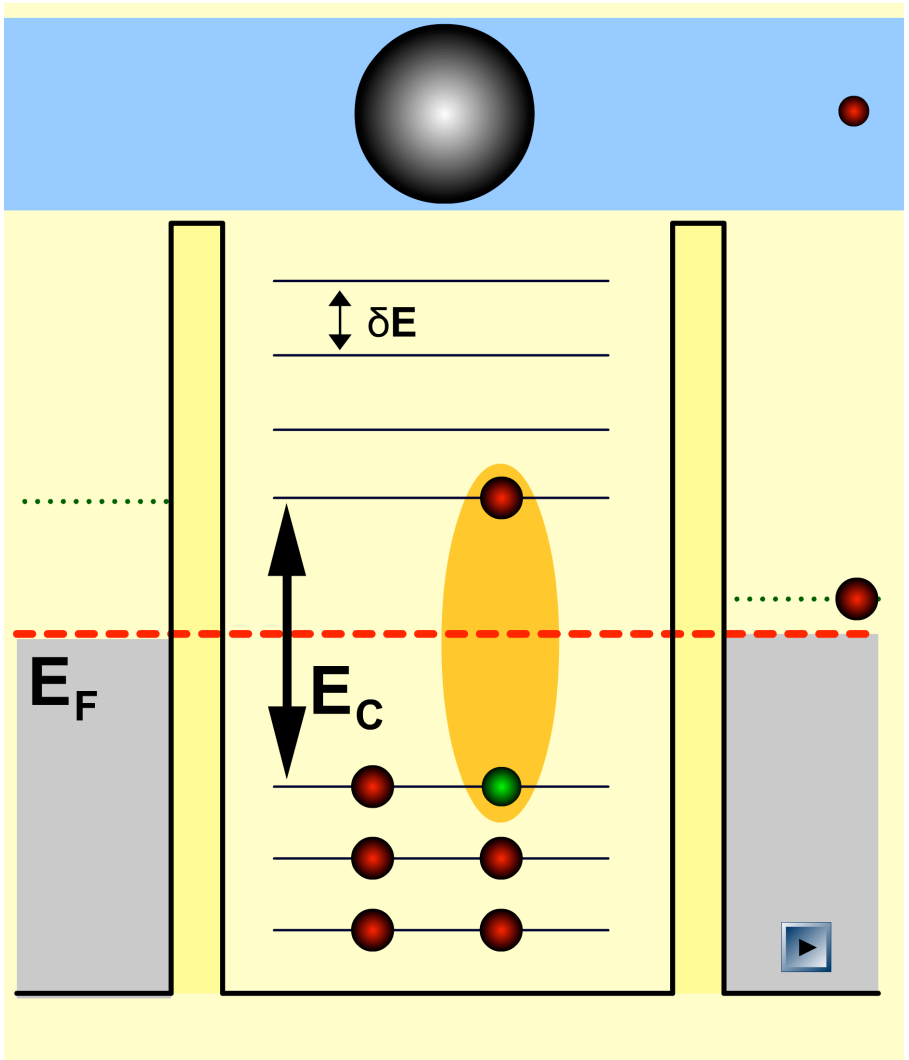
$T < \sqrt{E_0^c \delta}$



Inelastic cotunneling mechanism

$T > \sqrt{E_0^c \delta}$

# Transport in granular materials: cotunneling

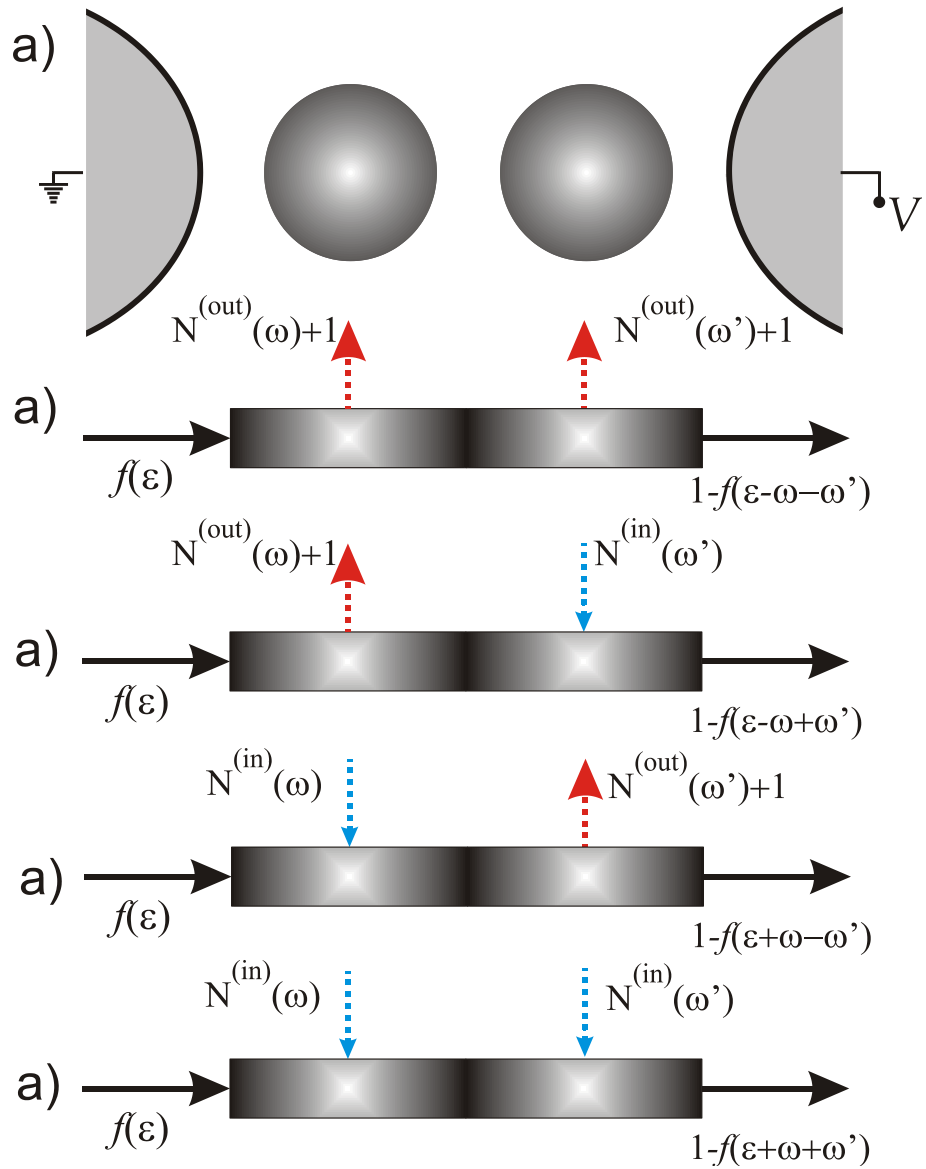



Relaxation mechanism:
creation of the string of the electron-hole pairs

I. S. Beloborodov, A. V. Lopatin, and V. M. Vinokur
Coulomb effects and hopping transport in granular metals
Phys. Rev. B 72 (12), 125121 - 125141 (2005)

The same mechanism was proposed in
"Many body localization"
I. Gorny, A. Mirlin, D. Polyakov, Phys. Rev. Lett. **95**, 206603 (2005)
D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Phys. Rev. B 76, 052203 (2007).
D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Ann. Phys. (N.Y.) 321, 1126 (2006).

# COTUNNELING




PRL 2009

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**BKT transition** at $T = T_{BKT} = E_C / 2k_B$

**BKT transition**

**is a consequence of the logarithmic interaction between charges!**

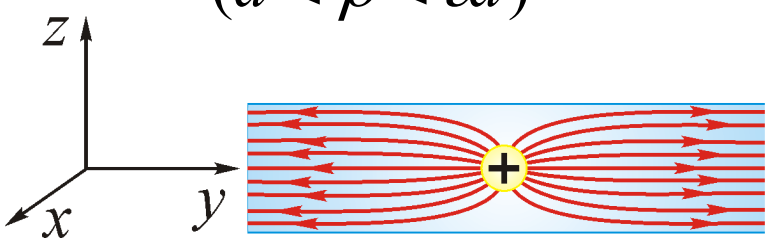
## 2D Coulomb interaction

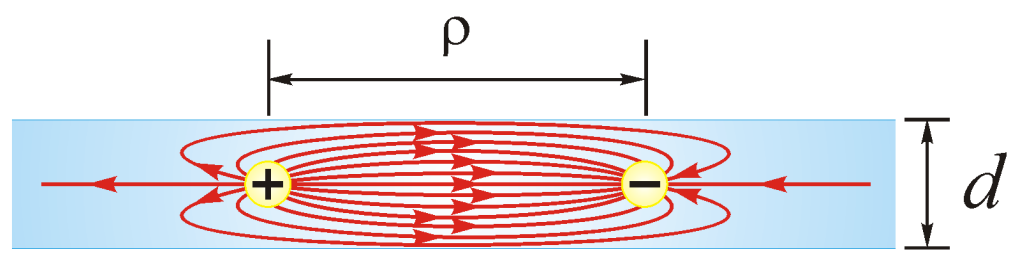
All electric force lines are trapped within the film

Energy of two interacting charges: $U = -\frac{q_1 q_2}{2\pi\varepsilon_0 \varepsilon d} \cdot \ln\frac{\rho}{d}$

**the logarithmic interaction between charges !**
**the interaction grows with the distance between charges !**

$(d < \rho < \varepsilon d)$




**3D world**

The larger the distance between the charges,
the smaller the interaction energy

$$U = -\frac{q_1 q_2}{4\pi\varepsilon_0 \varepsilon} \frac{1}{r}$$

**2D world**

The larger the distance between the charges,
the larger the interaction energy

$$U = -\frac{q_1 q_2}{2\pi\varepsilon_0 \varepsilon d} \cdot \ln\frac{\rho}{d}$$

## *2D Coulomb interaction*

All electric force lines are trapped within the film

Energy of two interacting charges:

$$U = -\frac{q_1 q_2}{2\pi\varepsilon_0 \varepsilon d} \cdot \ln\frac{\rho}{d}$$

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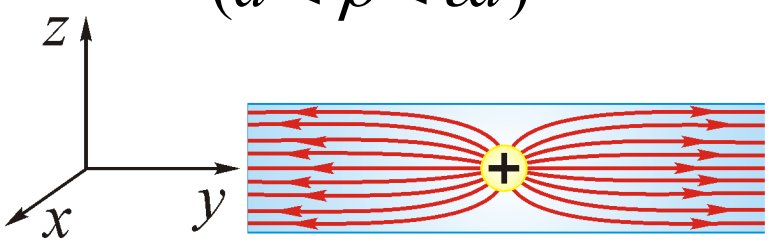
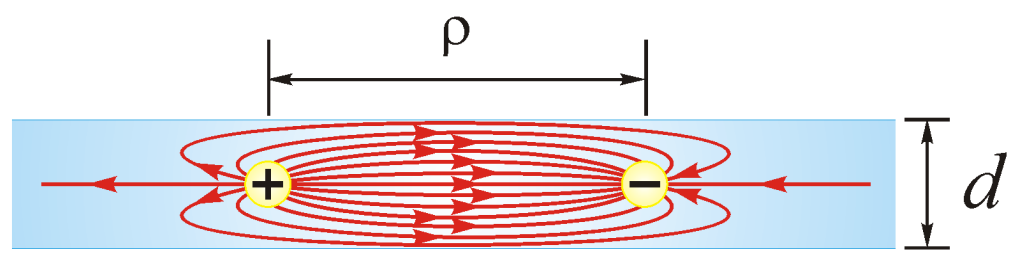



**BKT transition**

**is a consequence of the logarithmic interaction between vortices!**

❖ **Charge-vortex duality:**

**insulator-to-superinsulator transition as Berezinskii-Kosterlitz-Thouless transition in the 2D system of charges**

## *2D Coulomb interaction*

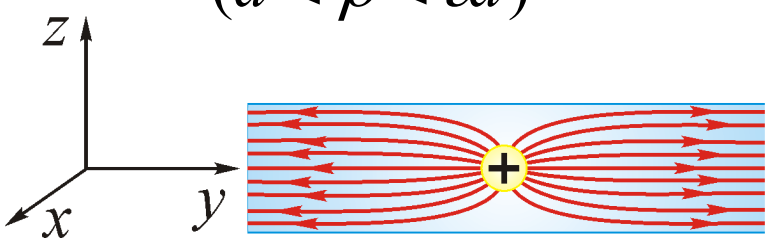
All electric force lines are trapped within the film

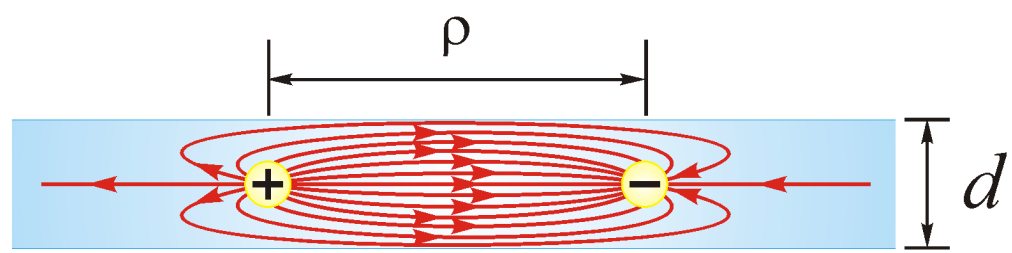
Energy of two interacting charges:

$$U = -\frac{q_1 q_2}{2\pi\varepsilon_0 \varepsilon d} \cdot \ln\frac{\rho}{d}$$

**the logarithmic interaction between charges !**
**the interaction grows with the distance between charges !**

$(d < \rho < \varepsilon d)$




**BKT transition**

**is a consequence of the logarithmic interaction between vortices!**

**Q:** How one can realize the 2D Coulomb interaction in our 3D world?

**A:** By arranging $\varepsilon >> 1.$

Near the SIT the dielectric constant of the film $\varepsilon \to \infty$

V.E. Dubrov, M.E. Levinshtein, and M.S. Shur,
ZhETP **70**, 2014 (1976)
[Sov. Phys. JETP **43**, 1050 (1976)].

## ANOMALY IN THE DIELECTRIC PERMEABILITY IN METAL-DIELECTRIC TRANSITIONS. THEORY AND SIMULATION

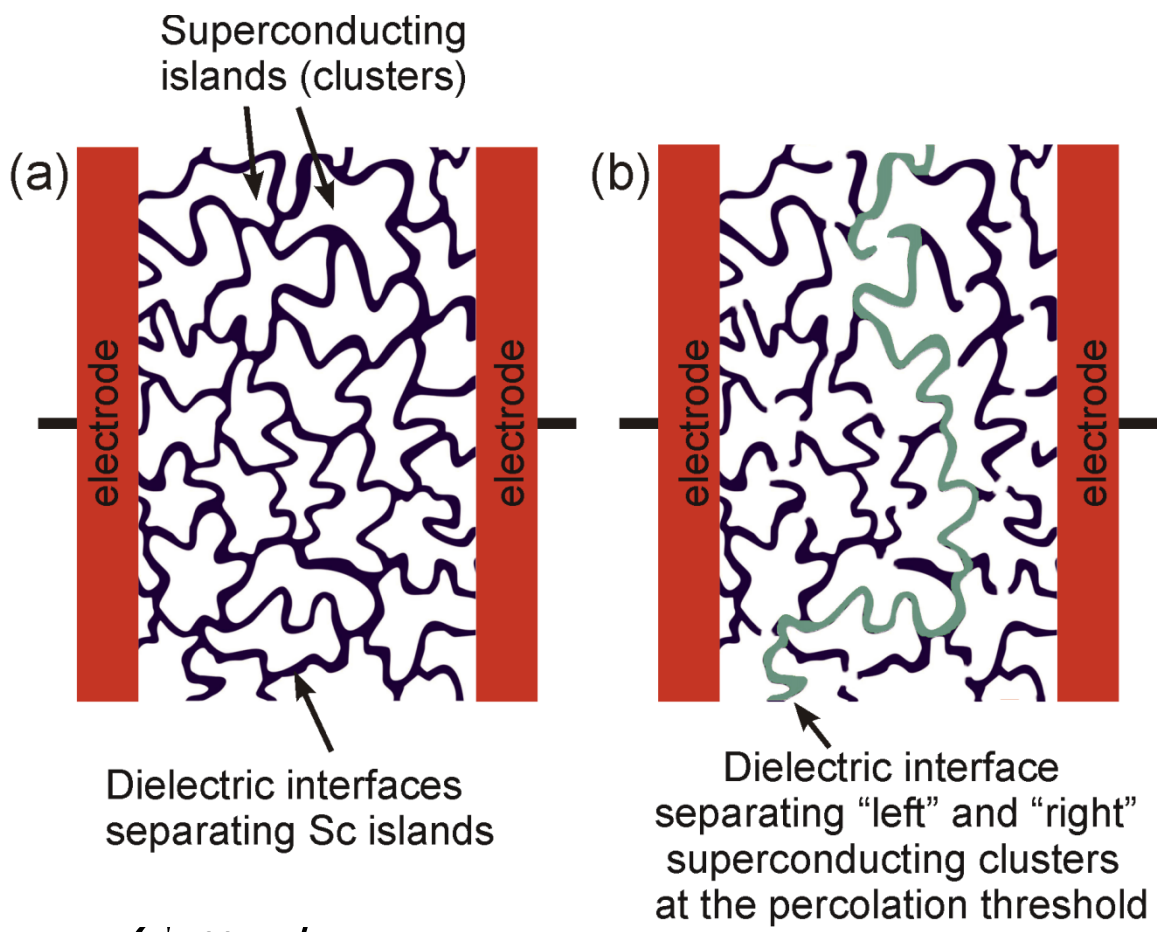
*V. E. Dubrov, M. E. Levinstein, M. S. Shur*

Some problems in the dynamic percolation theory are formulated whose solution may be of interest in studies of the dynamic properties of disordered systems such as strongly doped semiconductors, ferroelectric semiconductors with a diffuse phase transition, island films and other objects with a nonuniform conductivity and (or) inhomogeneous dielectric permeability. Some of the problems are simulated by means of networks consisting of condensors and resistors with randomly broken bonds and sites. The results obtained by such simulation and also the theory of effective media developed in the paper, as well as some considerations based on percolation theory indicate that the static dielectric permeability of the sample should become infinite for the metal-dielectric transition. This result is employed for interpreting qualitatively the «polarization catastrophe» in *n*-silicon observed in the metal-dielectric transition.

Near the SIT the dielectric constant of the film $\varepsilon \to \infty$

Superconducting film near superconductor-insulator transition

Percolation transition



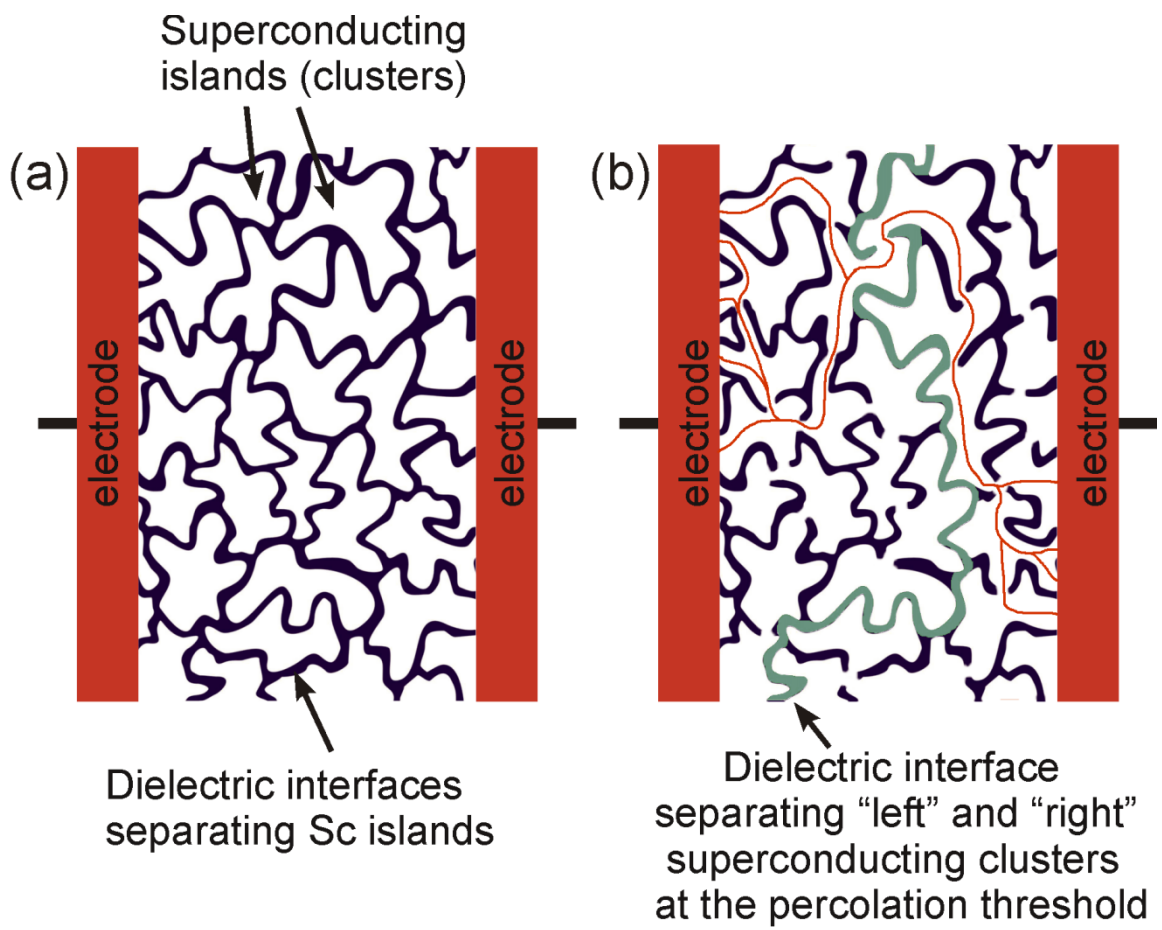

$C \propto \varepsilon l$

The capacitance between the two adjacent clusters is proportional to the length of the insulating layer separating them. Upon approaching the transition from the insulating side of the SIT, the length of this layer diverges infinitely. It results in the divergent growth of the effective capacitance of the system, implying the divergence of the dielectric constant $\varepsilon \to \infty$.

Near the SIT the dielectric constant of the film $\varepsilon \to \infty$

Superconducting film near superconductor-insulator transition

Percolation transition

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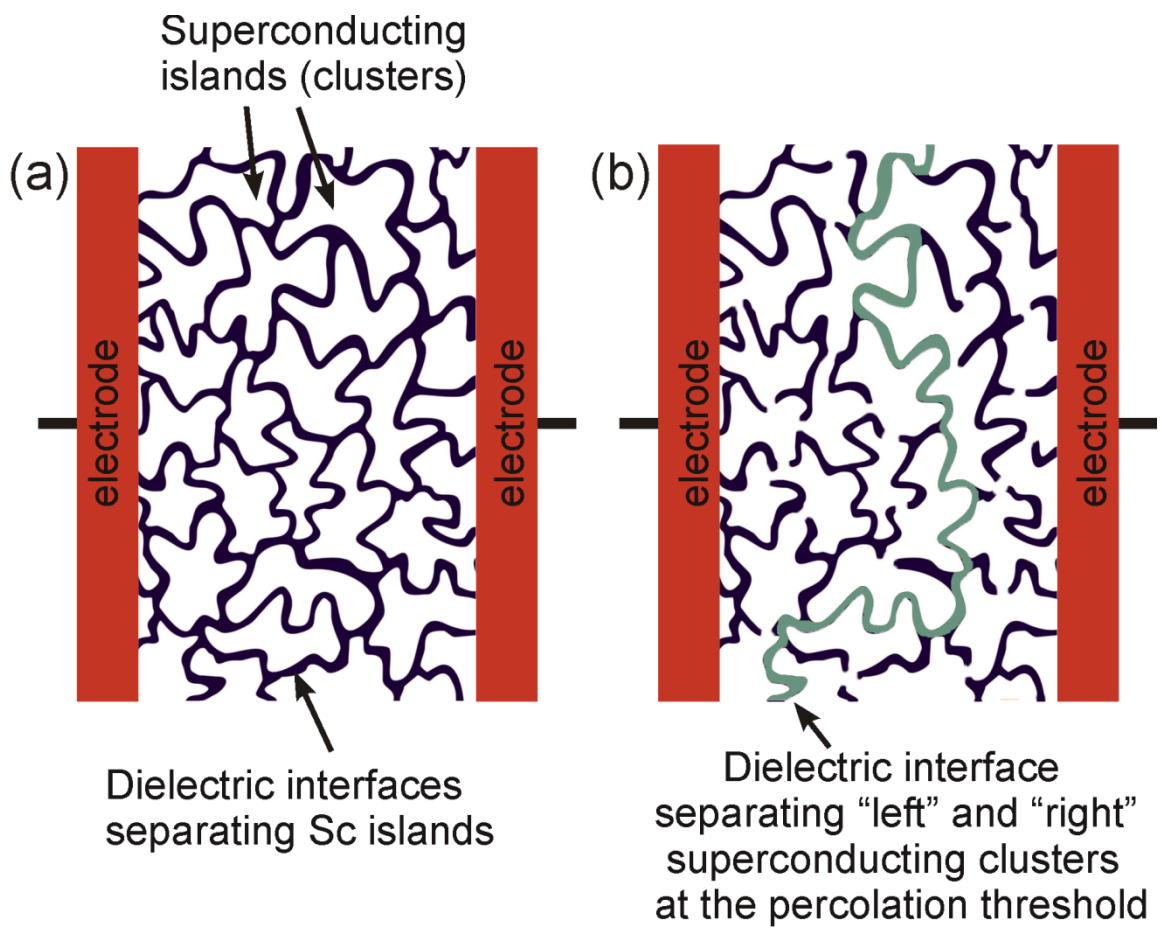


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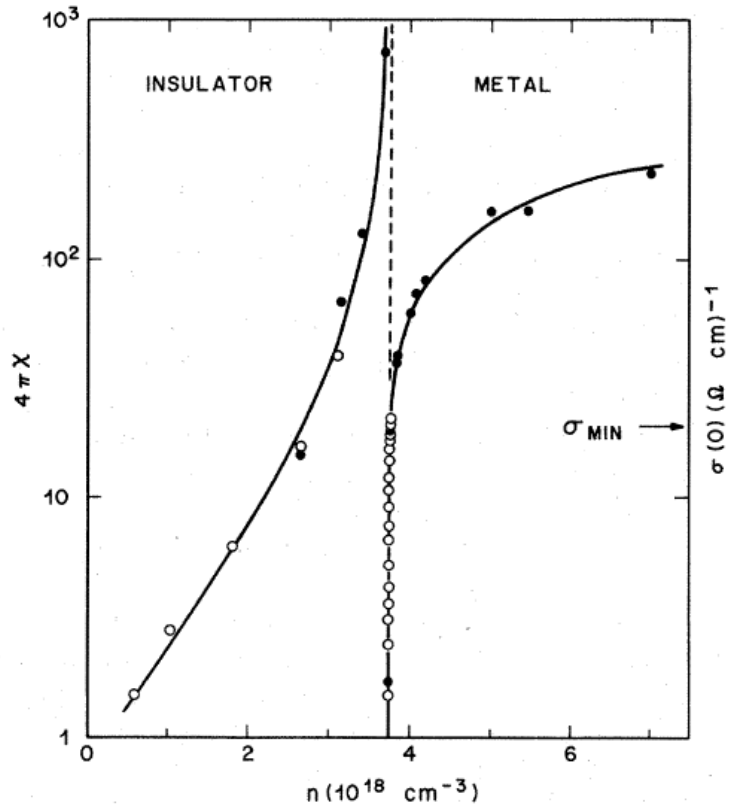
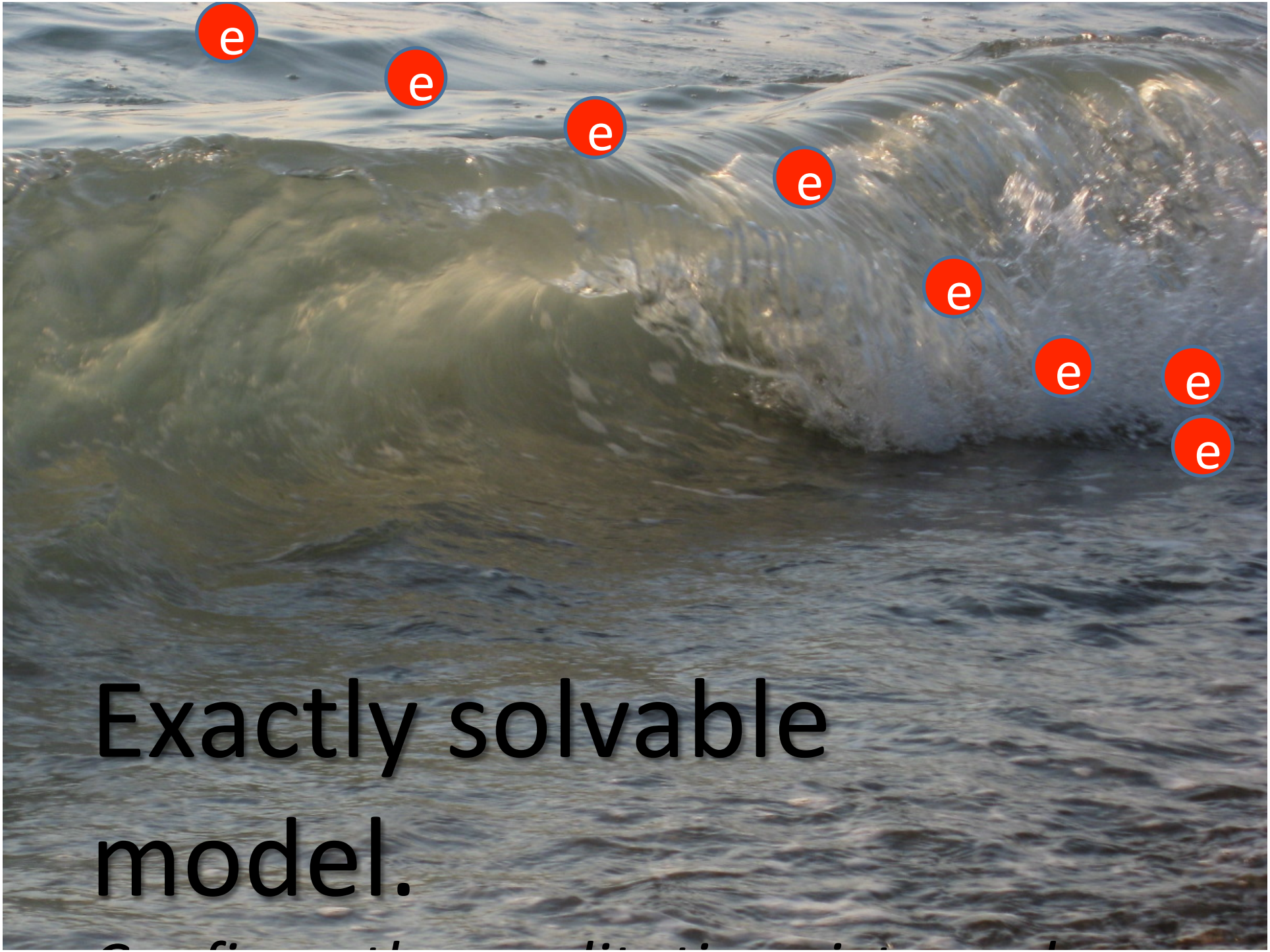
PHYSICAL REVIEW B VOLUME 25, NUMBER 8 15 APRIL 1982

# Giant dielectric constants at the approach to the insulator-metal transition

Harald F. Hess and Keith DeConde
*Joseph Henry Laboratory, Princeton, New Jersey 08544*

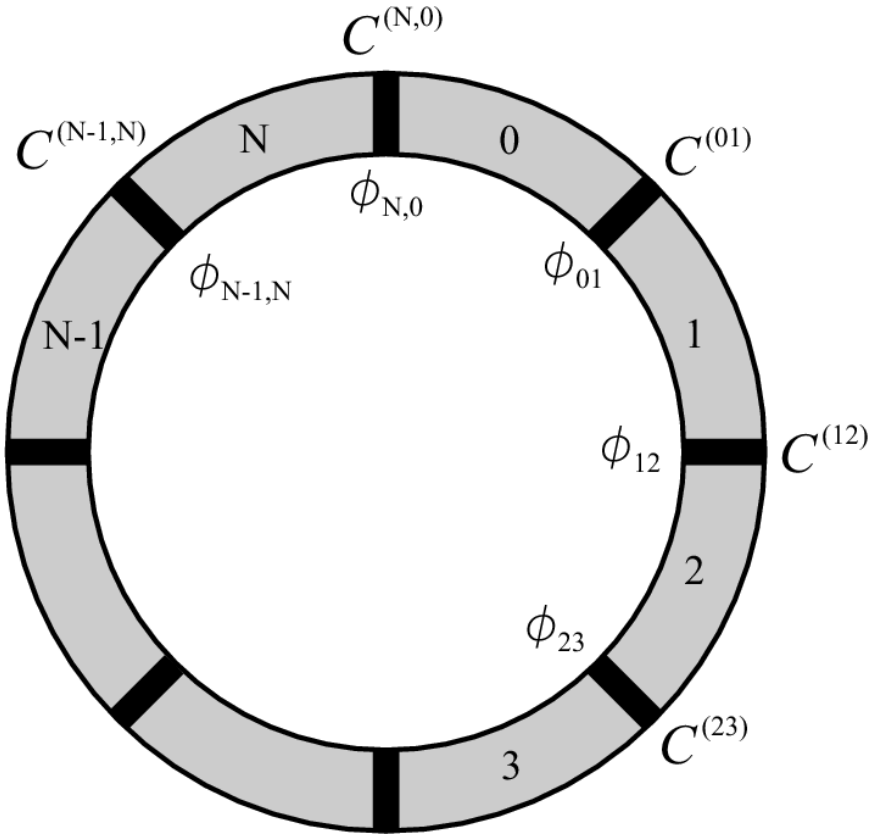
T. F. Rosenbaum* and G. A. Thomas
*Bell Laboratories, Murray Hill, New Jersey 07974*
(Received 25 February 1982)



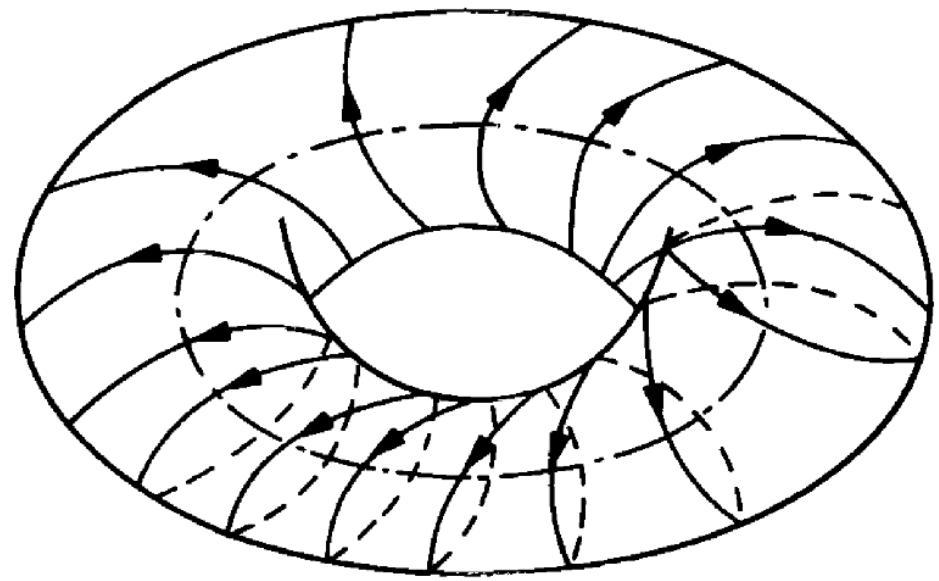



# Exactly solvable model.

# 1D array of Josephson junctions.

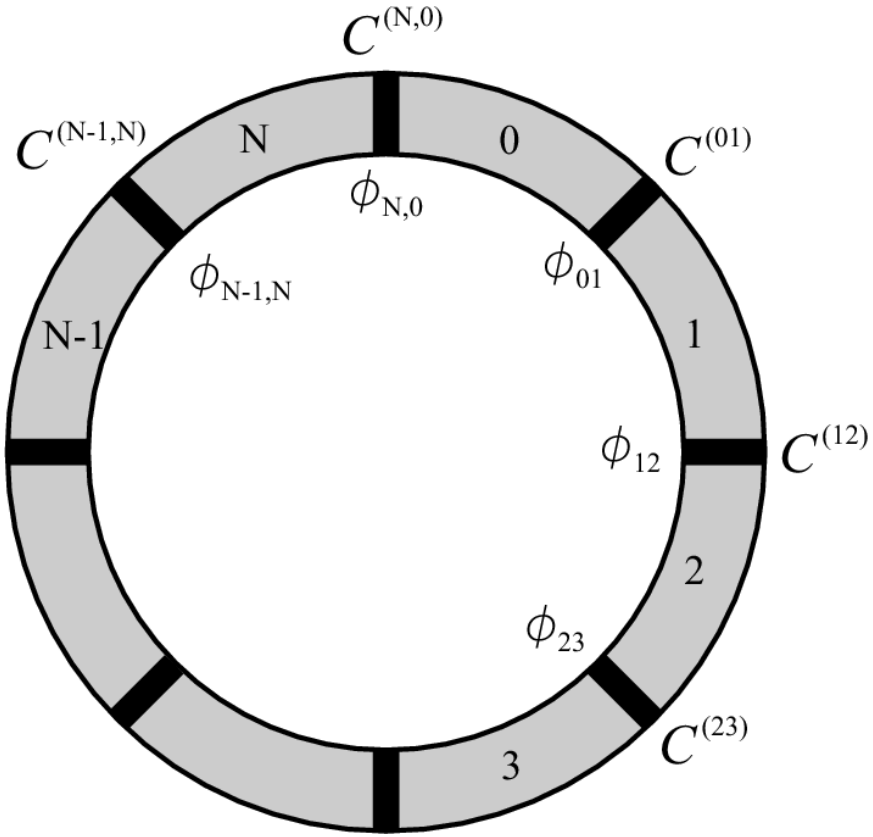



Time-dependent flux
generates electric field

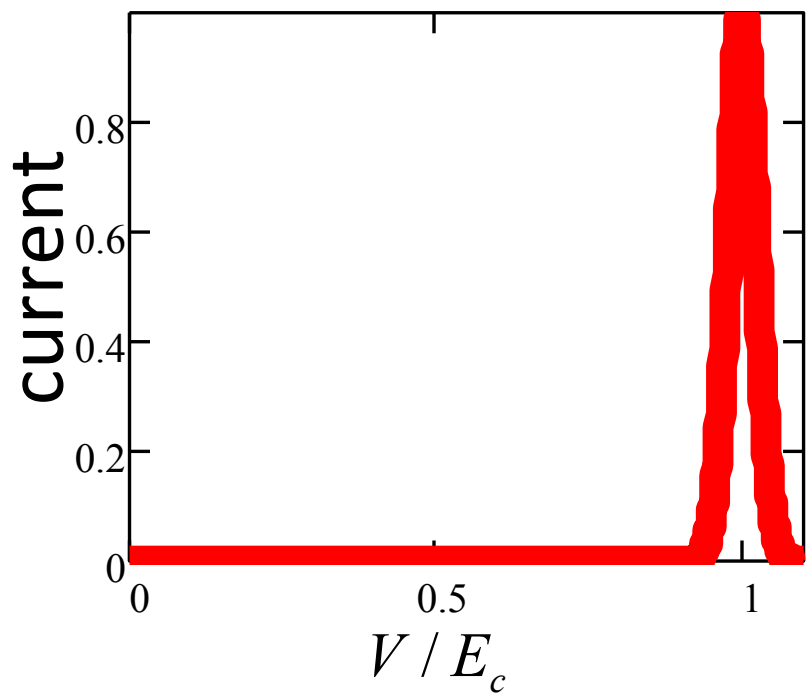


Landau-Hopf turbulence

# 1D array of Josephson junctions.



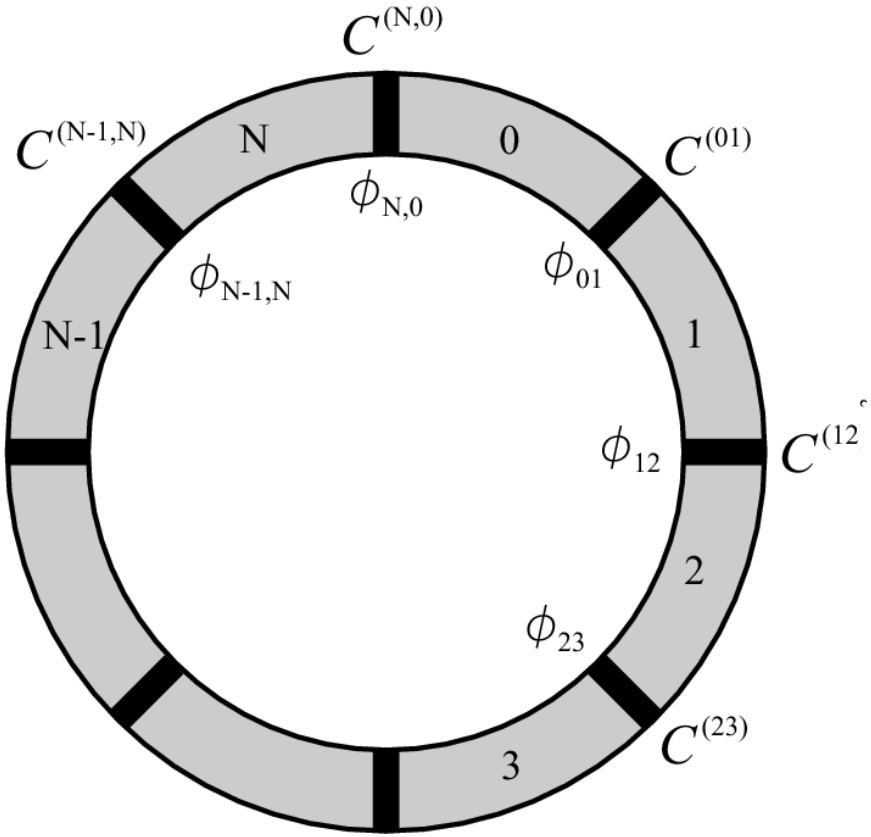

K. Efetov



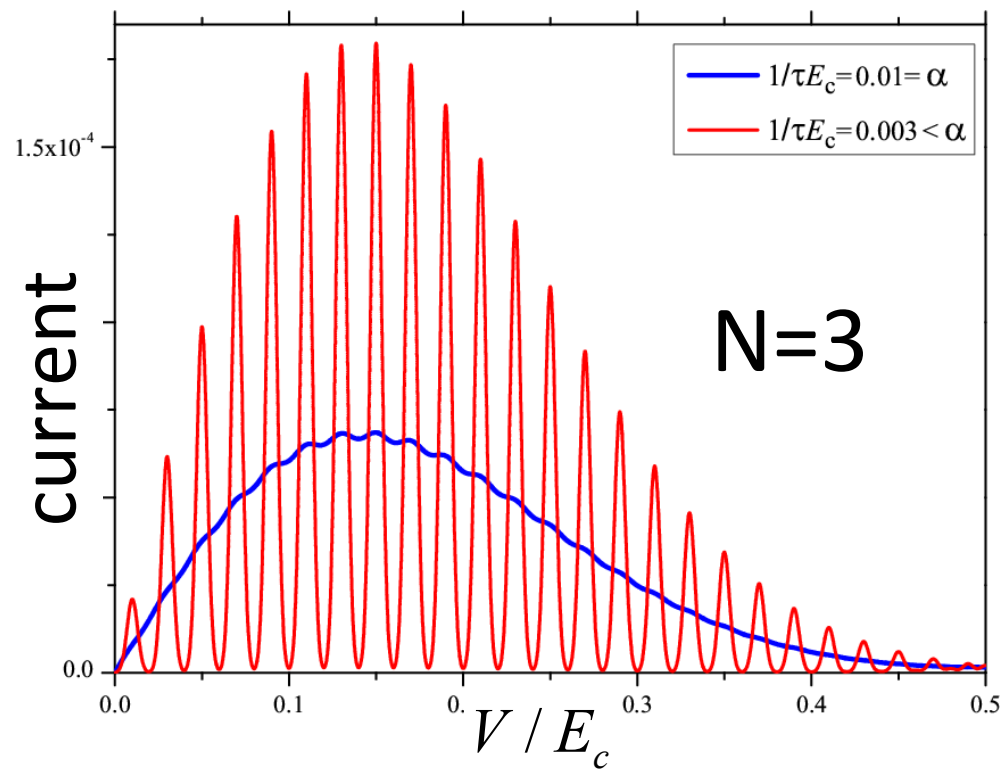

$$E_c >> E_J$$

Time-dependent flux
generates electric field

# 1D array of Josephson junctions.

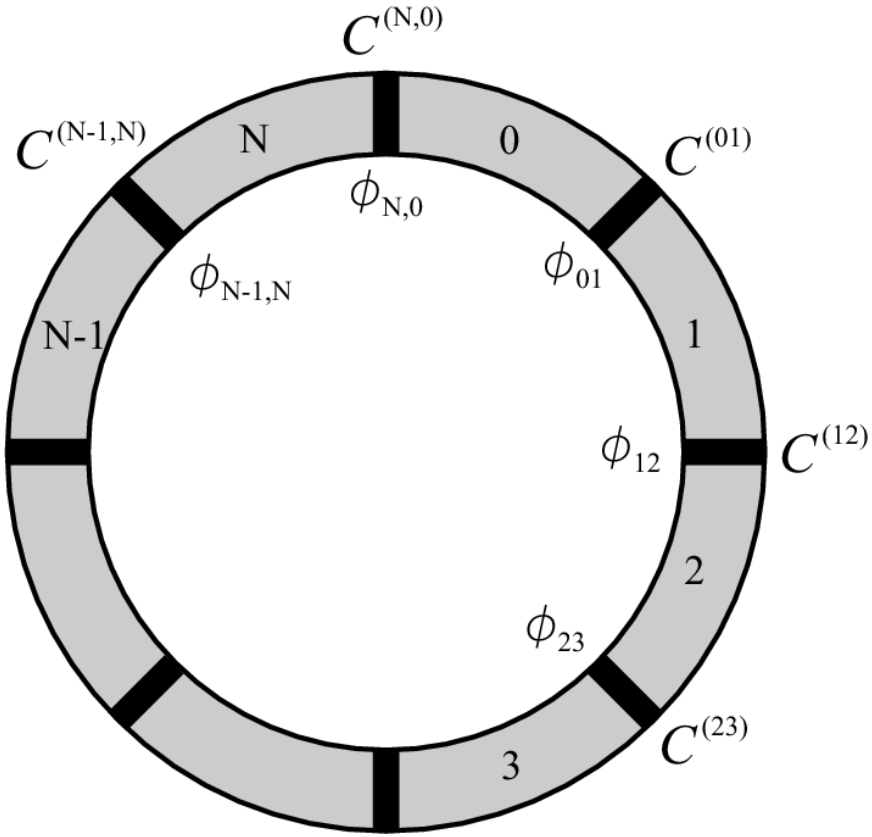



## The I(V) is smooth!!!




Time-dependent flux generates electric field $E_c >> E_J$

# 1D array of Josephson junctions.




The I(V) is smooth!!!
Why???

- The model is exactly solvable
- At arbitrary disorder
- Solution --- special functions

Time-dependent flux generates electric field $E_c >> E_J$

$$I_{\rm qp}(V) = \frac{1}{eR_T}\int_{-\infty}^{\infty} dE dE' N_S(E) N_S(E') P(E-E') \times$$
$$\{f_i(E)[1-f_{i+1}(E')] - f_{i+1}(E)[1-f_i(E')]\},$$

$$I_s = -\frac{2E_J^2}{\hbar^2}\int_{-\infty}^{\infty} dt \sin[2eVt]\,\mathrm{Im}[\mathcal{K}(t)]$$

$$\mathcal{K}(\omega) = \sum_{\{n\},\{m\}} P_{\{n\}} \left|\langle\{n\}|e^{i2\phi_{i,i+1}}|\{m\}\rangle\right|^2 \times$$
$$2\pi\delta(E_{\{n\}} - E_{\{m\}} + \omega)$$

$$P_{\{n\}} = Z^{-1} e^{-\sum_{i>0} \frac{E_C^{(i,i+1)} n_{i,i+1}^2}{2T}}$$

The quantum states of the annular JJAs have the wave function

$$|\{n\}\rangle = \prod_{i=0}^{N} \psi_{n_{i,i+1}}(\phi_{i,i+1}) \ ,$$

where $\sum_{i=0}^{N} \phi_{i,i+1} = 0$ and $\psi_n(\phi) = \exp\{in\phi\}/\sqrt{2\pi}$ is the wave function of the quantum rotator.

$$I_s = D\left\{e^{-\frac{(V+N\bar{E}_c)^2}{2TN\bar{E}_c}} - e^{-\frac{(V-N\bar{E}_c)^2}{2TN\bar{E}_c}}\right\}\vartheta\left(z,\mathcal{T}\right)$$

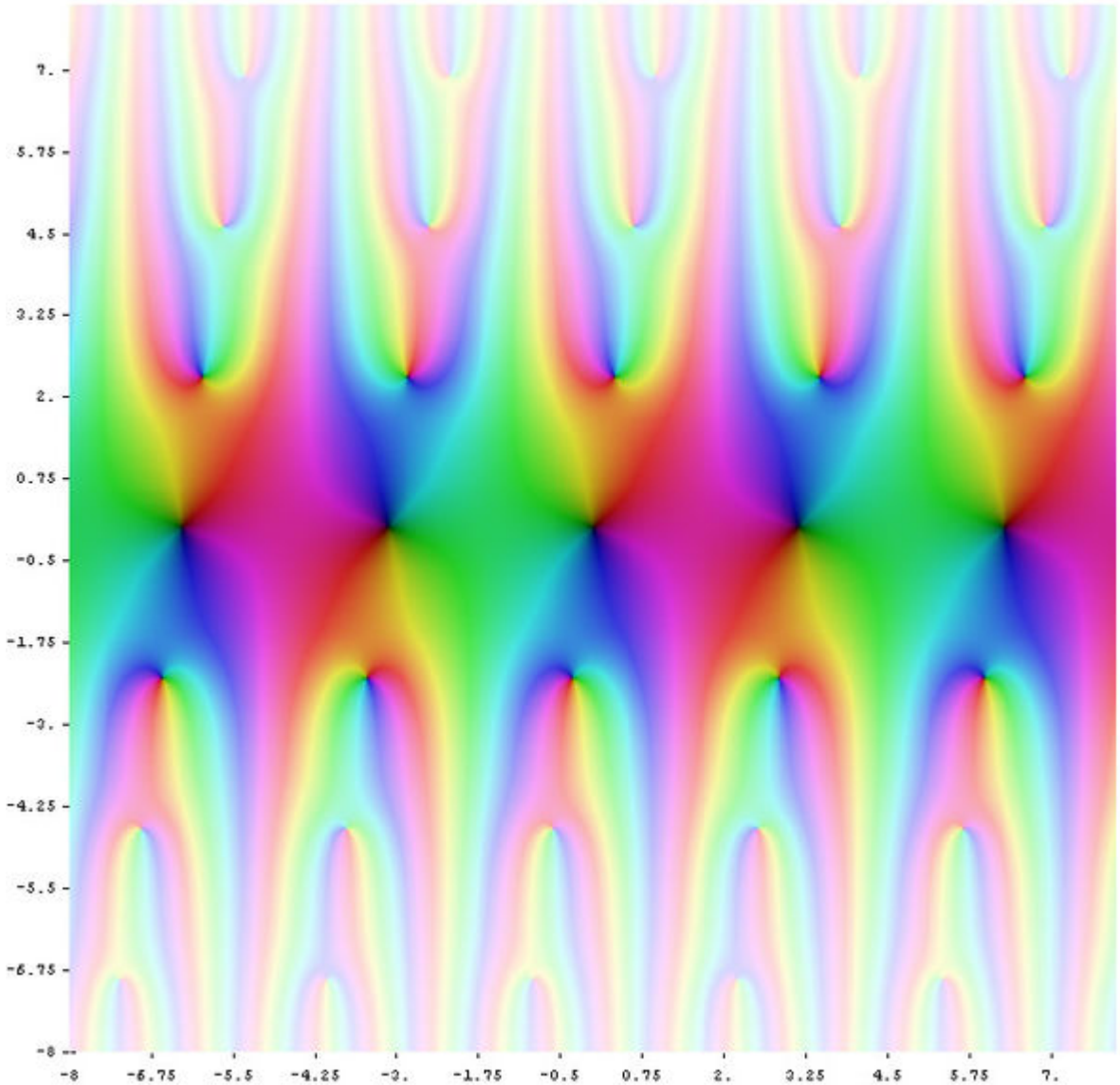
$$\text{where } D = \frac{\pi E_J^2\sqrt{(2\pi T)^{N-1}}}{Z\sqrt{N\bar{E}_c^{N+1}}}, \text{ and } z = V/\bar{E}_c$$

Jacoby generalized Theta-function

$$\vartheta(z,\mathcal{T}) = \sum_{\vec{m}} e^{i2\pi z\vec{m}\cdot\vec{a} - \pi\vec{m}^\tau\mathcal{T}\vec{m}}$$

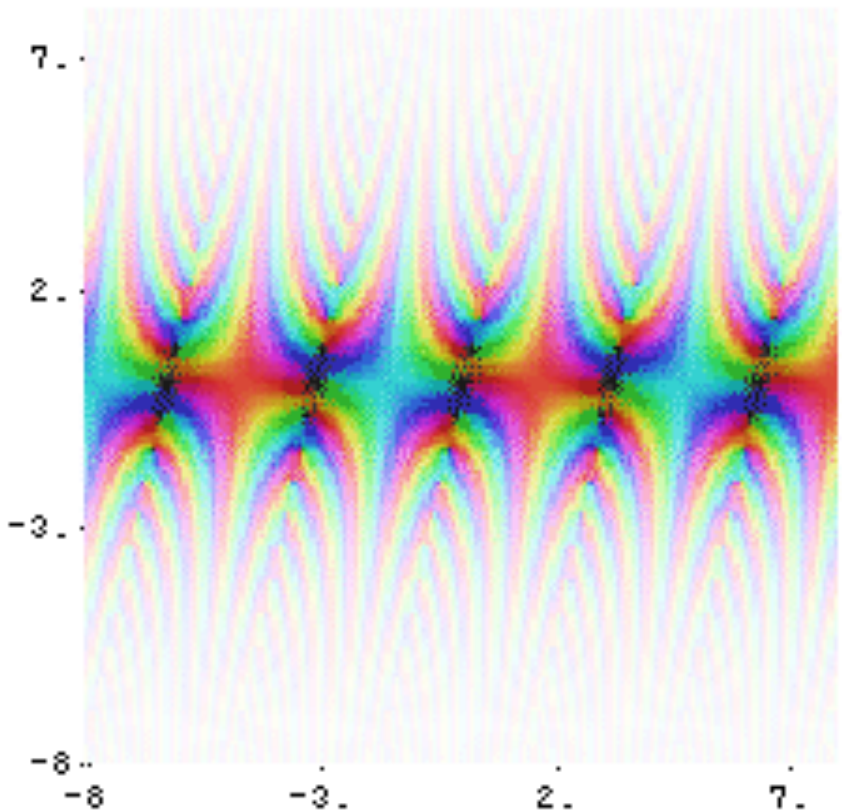
$$\mathcal{T}_{ij} = \frac{2\pi T}{\bar{E}_c}H_{ij} + \frac{\pi\sigma^2}{2(N\bar{E}_c)^2}, \quad H_{ij} = \frac{1}{N}\left(\frac{\sum e}{e_i}\delta_{ij} - 1\right)$$

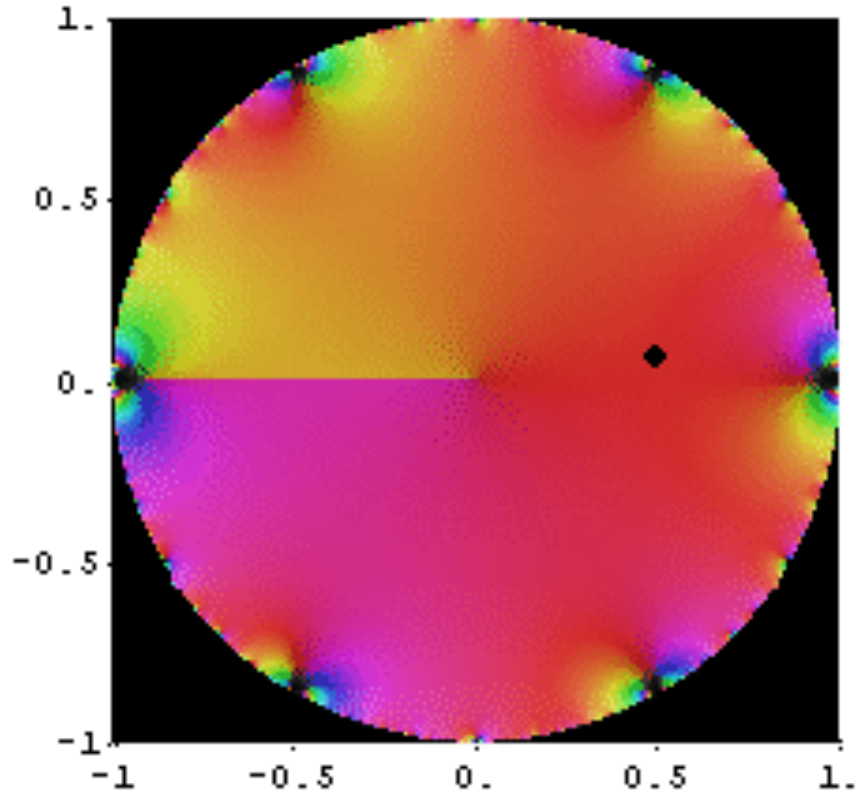
Here $\sum e = \sum_k e_k$.




$$\vartheta(z;\tau) = \sum_{n=-\infty}^{\infty} \exp(\pi i n^2 \tau + 2\pi i n z) = 1 + 2\sum_{n=1}^{\infty} \left(e^{\pi i \tau}\right)^{n^2} \cos(2\pi n z) = \sum_{n=-\infty}^{\infty} q^{n^2} \zeta^n$$

$$\vartheta(z + a + b\tau; \tau) = \exp(-\pi i b^2 \tau - 2\pi i b z)\, \vartheta(z;\tau)$$


7.
2.
-3.
-8.
-8
-3.
2.
7.


1.
0.5
0.
-0.5
-1.
-1
-0.5
0.
0.5
1.

$$I_s = D\left\{e^{-\frac{(V+N\bar{E}_c)^2}{2TN\bar{E}_c}} - e^{-\frac{(V-N\bar{E}_c)^2}{2TN\bar{E}_c}}\right\}\vartheta\left(z,\mathcal{T}\right)$$

$$\text{where } D = \frac{\pi E_J^2\sqrt{(2\pi T)^{N-1}}}{Z\sqrt{N\bar{E}_c^{N+1}}}, \text{ and } z = V/\bar{E}_c$$

# Jacoby generalized matrix Theta-function

$$\vartheta(z,\mathcal{T}) = \sum_{\vec{m}} e^{i2\pi z\vec{m}\cdot\vec{a} - \pi\vec{m}^\tau\mathcal{T}\vec{m}}$$

$$\mathcal{T}_{ij} = \frac{2\pi T}{\bar{E}_c}H_{ij} + \frac{\pi\sigma^2}{2(N\bar{E}_c)^2}, \quad H_{ij} = \frac{1}{N}\left(\frac{\sum e}{e_i}\delta_{ij} - 1\right)$$

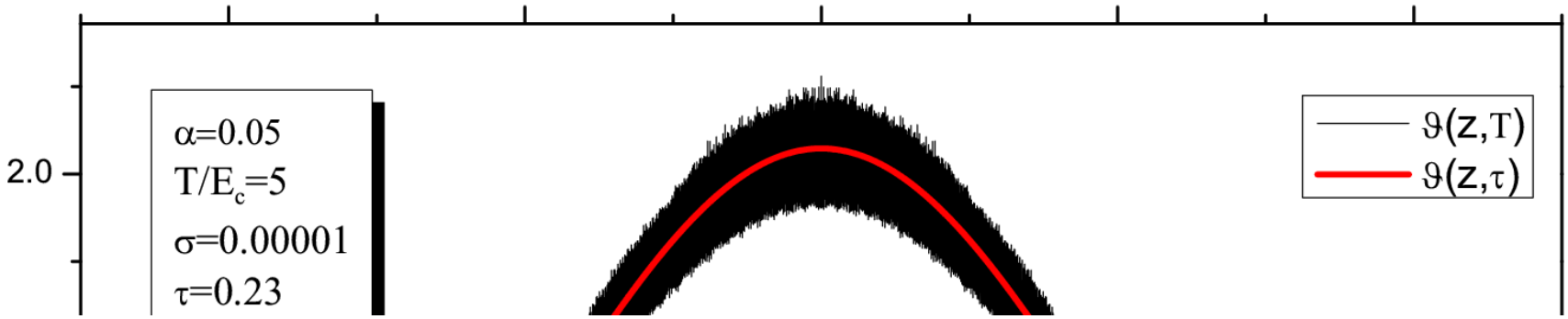
Here $\sum e = \sum_k e_k$.

Regularization term

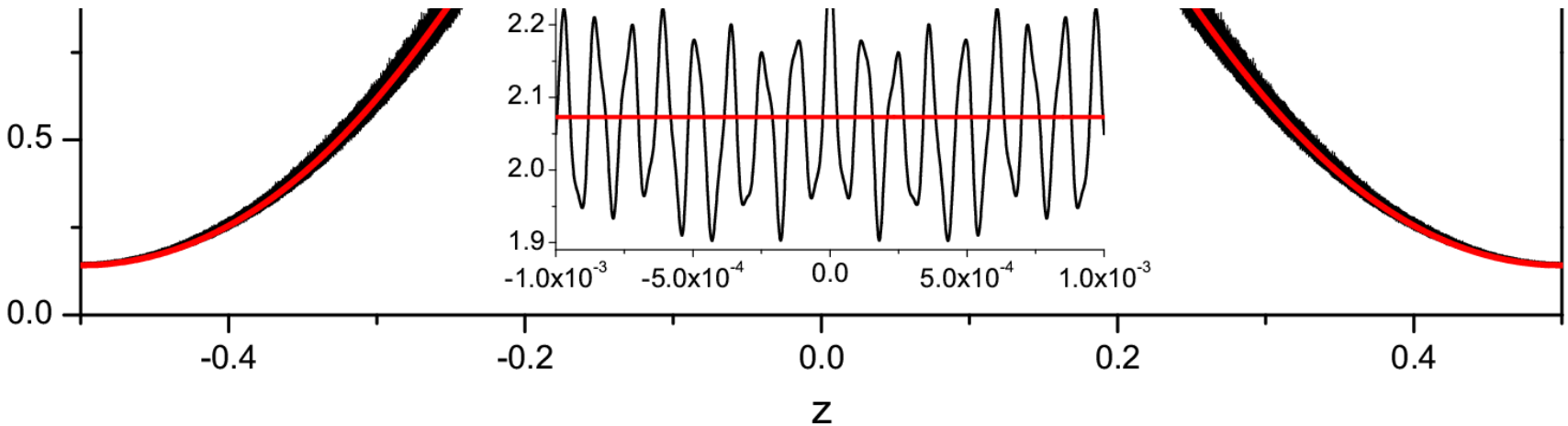
One zero eigenvalue.

The other eigenvalues are of the order of unity

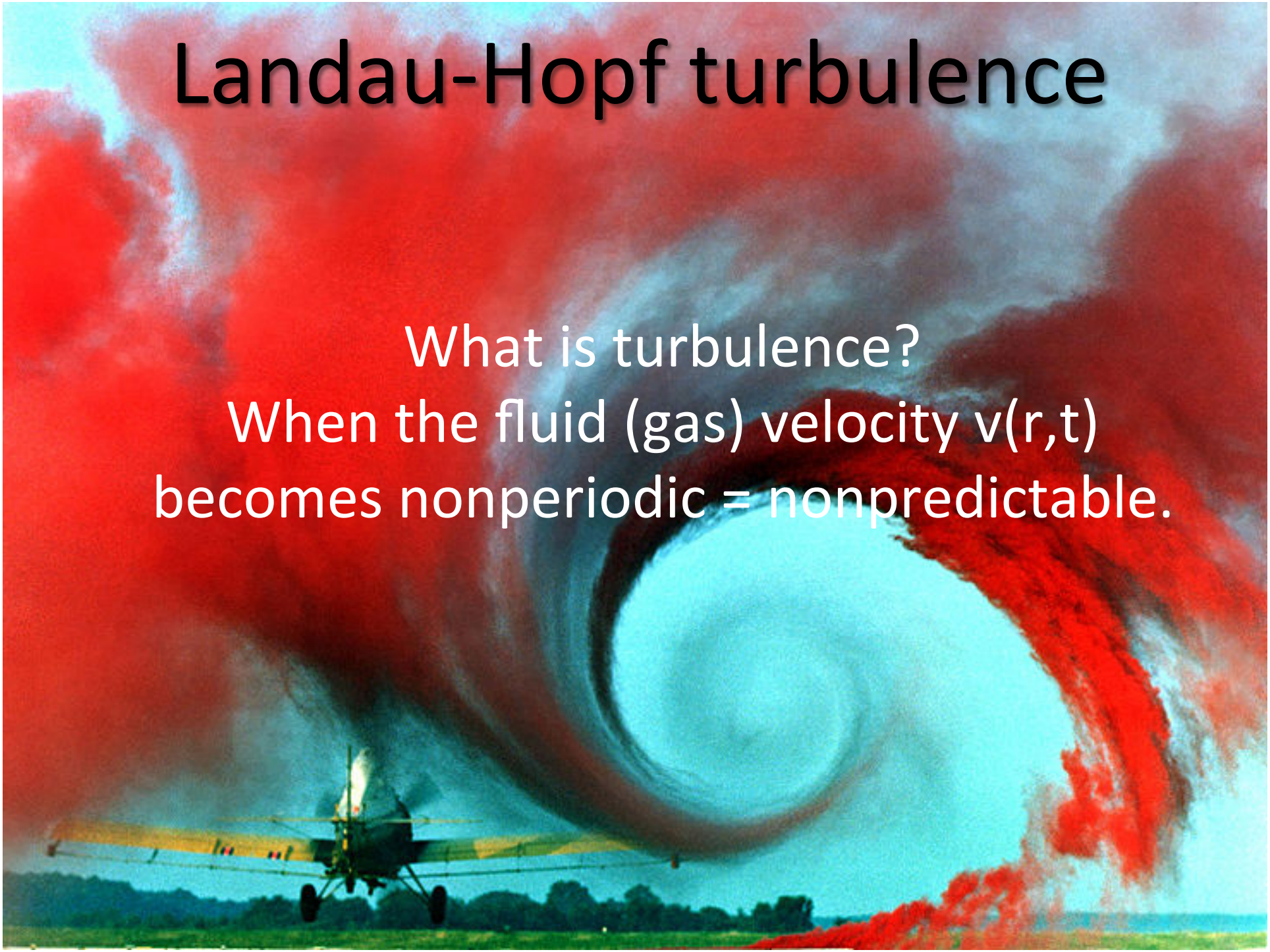
$$\vartheta(z,\tau) = \sum_{n=-\infty}^{\infty} e^{i2\pi nz - \pi n^2 \tau}, \quad \tau = 2\pi\left[\frac{\sigma^2}{4\bar{E}_c^2} + \frac{T\alpha^2 N}{\bar{E}_c}\right]$$




$$\alpha\bar{E}_c \lesssim \sigma e^{S}, \qquad S \approx YN\ln N.$$



# Landau-Hopf turbulence



What is turbulence?
When the fluid (gas) velocity v(r,t)
becomes nonperiodic = nonpredictable.

# Landau-Hopf turbulence

What is turbulence?
When the fluid (gas) velocity v(r,t) becomes nonperiodic = nonpredictable.

L.D. Landau, Doklady Akademii Nauk SSSR **44**, 339 (1944).
E. Hopf, Communications on Pure and Applied Mathematics **1**, 303 (1948).
A. N. Kolmogorov, Dokl. Akad. Nauk. SSSR **30**, 9 (1941); **32**, 16 (1941), reproduced in Proc. R. Soc. London, Ser. A **434**, 9 (1991).

ТЕОРЕТИЧЕСКАЯ ФИЗИКА

VI

Л.Д. ЛАНДАУ
Е.М. ЛИФШИЦ

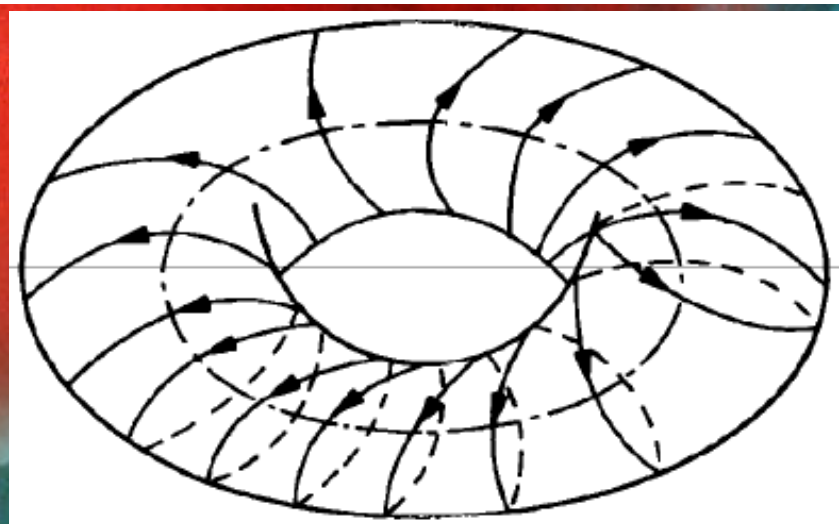
# ГИДРОДИНАМИКА

# Landau-Hopf turbulence

Assuming now that this way or *scenario* of development of turbulence is in fact possible,‡ we write the general form of the function $\mathbf{v}(\mathbf{r}, t)$, whose time dependence is governed by some number $N$ of different frequencies $\omega_i$. It may be regarded as a function of $N$ different phases $\phi_i = \omega_i t + \beta_i$ (and of the coordinates), periodic in each with period $2\pi$. Such a function may be expressed as a series

$$\mathbf{v}(\mathbf{r}, t) = \sum \mathbf{A}_{p_1 p_2 \ldots p_N}(\mathbf{r}) \exp\left\{-i \sum_{i=1}^{n} p_i \phi_i\right\}, \tag{30.2}$$

which is a generalization of (26.13), the summation being over all integers $p_1, p_2, \ldots, p_N$. The flow described by this formula involves $N$ arbitrary initial phases $\beta_i$ and has $N$ degrees of freedom.††



Entropy **S ~N log N**

$$t \sim const \exp(S)$$

v(t) – nonperiodic function!

# Landau-Hopf turbulence

$$\mathcal{K}(t) = \langle \exp[i2\phi_{i,i+1}(t)] \exp[-i2\phi_{i,i+1}(0)] \rangle_{H_C}$$

$$\mathcal{K}(t) = e^{i\Omega t} \sum A_{p_1 p_2 \ldots p_N} \exp\left\{ i \sum_{i=1}^{N} p_i \, \varphi_i(t) \right\}$$

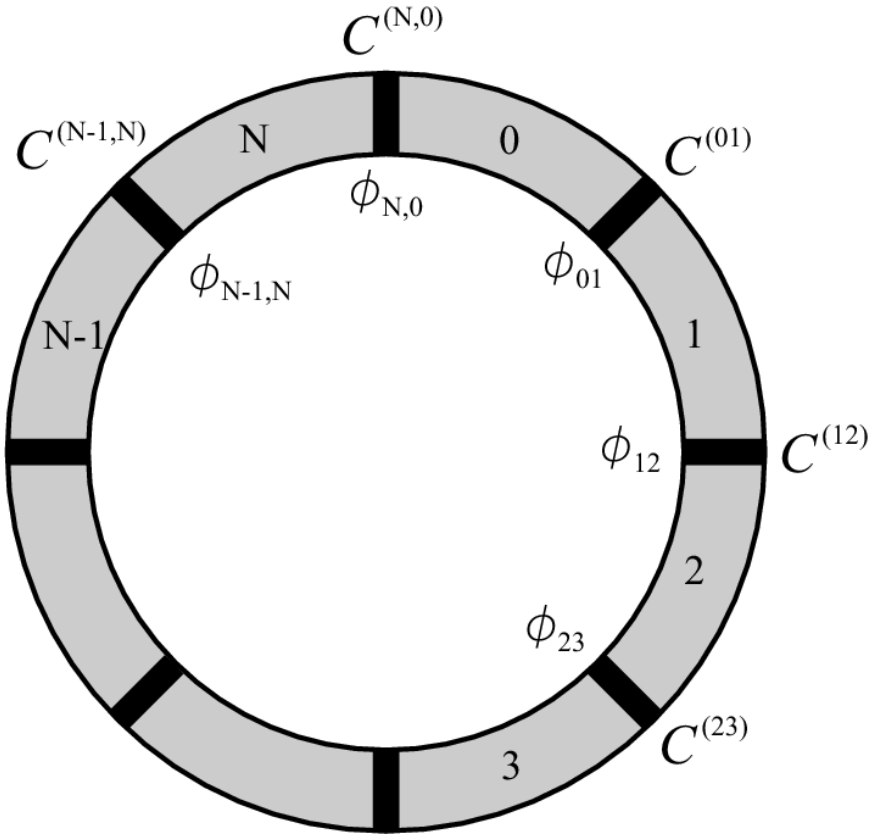
$$\mathbf{v}(\mathbf{r}, t) = \sum \mathbf{A}_{p_1 p_2 \ldots p_N}(\mathbf{r}) \exp\left\{ -i \sum_{i=1}^{n} p_i \phi_i \right\}$$

$$\varphi_i = \omega_i t + \beta_i$$

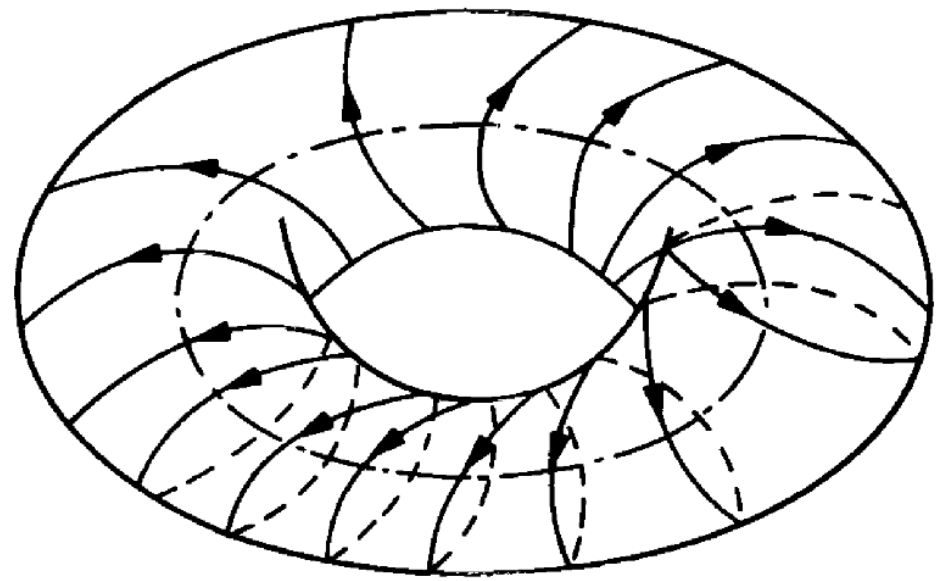
$$\omega_i = E_c^{(i,i+1)}$$

$$A_{p_1 p_2 \ldots p_N} = P_{\{p\}} = Z^{-1} e^{-E_c^{(i,i+1)} p_i^2 / 2T}$$

# 1D array of Josephson junctions.




Time-dependent flux
generates electric field



Landau-Hopf turbulence

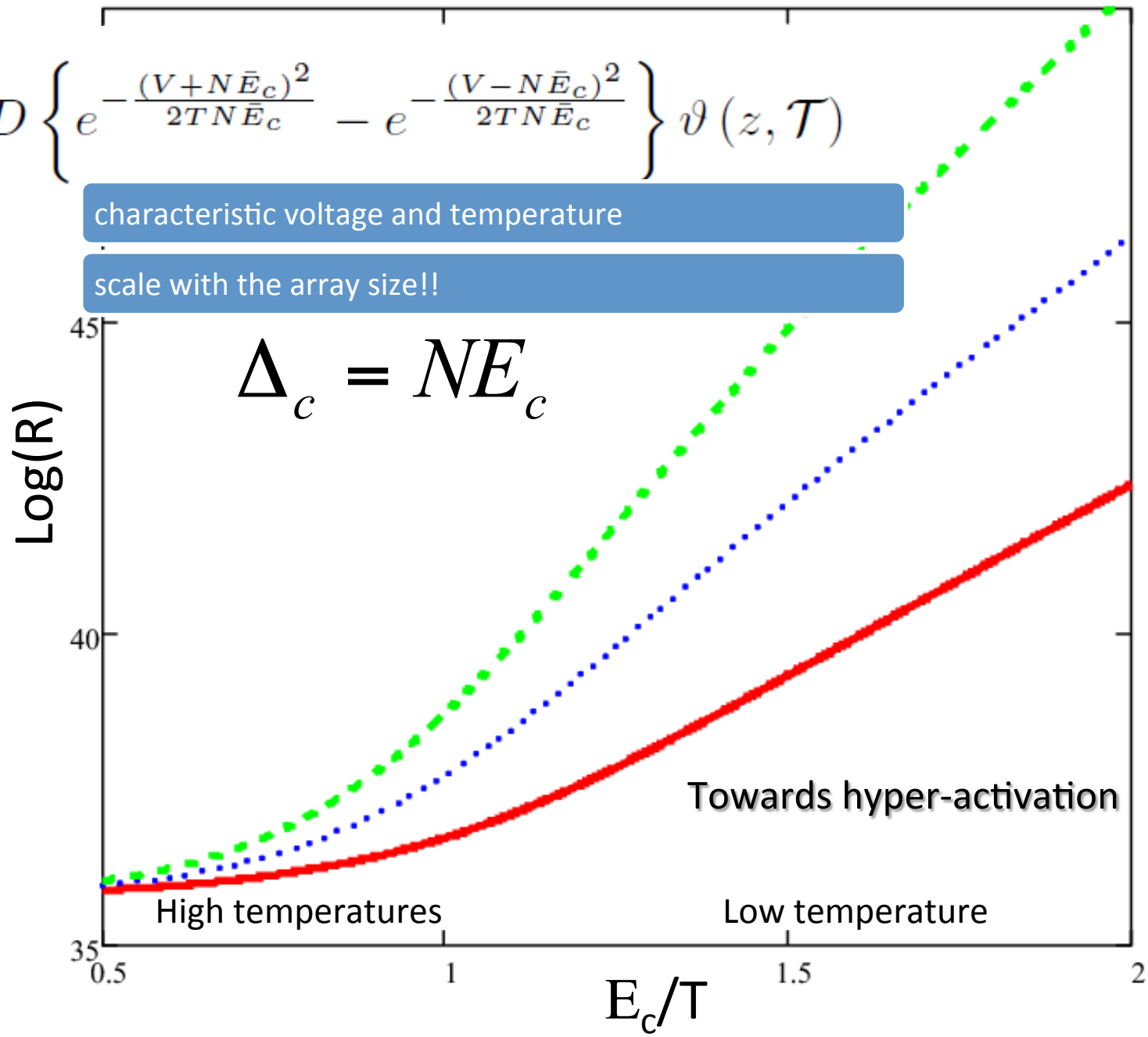
Turbulence short summary

$$I_s = D\left\{e^{-\frac{(V+N\bar{E}_c)^2}{2TN\bar{E}_c}} - e^{-\frac{(V-N\bar{E}_c)^2}{2TN\bar{E}_c}}\right\}\vartheta\left(z,\mathcal{T}\right)$$

characteristic voltage and temperature

scale with the array size!!

$$\Delta_c = NE_c$$



# Thank you for your attention!

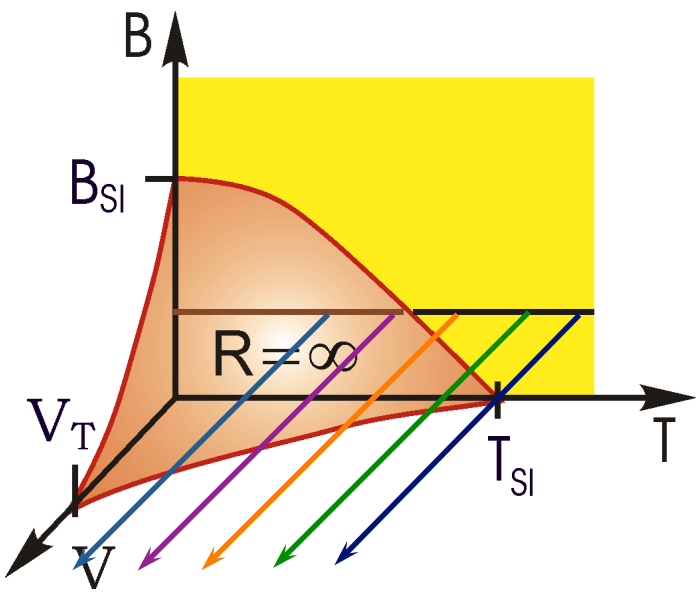
•We show that chaos deblocades the tunneling current in the arrays of Josephson junctions in the quantum limit.
•The chaos sources are mesoscopic fluctuations of the junction capacitances.

•When the chaotic regime is fully developed the environment of the low lying neutral excitations self-generates in the array and deblocks the transport; in the opposite case the environment is frozen (gaped) and so the transport is blocked.

•There is a map between the Landau-Hopf scenario of turbulence development in the liquid and the scenario of the chaos deblocade in Josephson junction arrays.

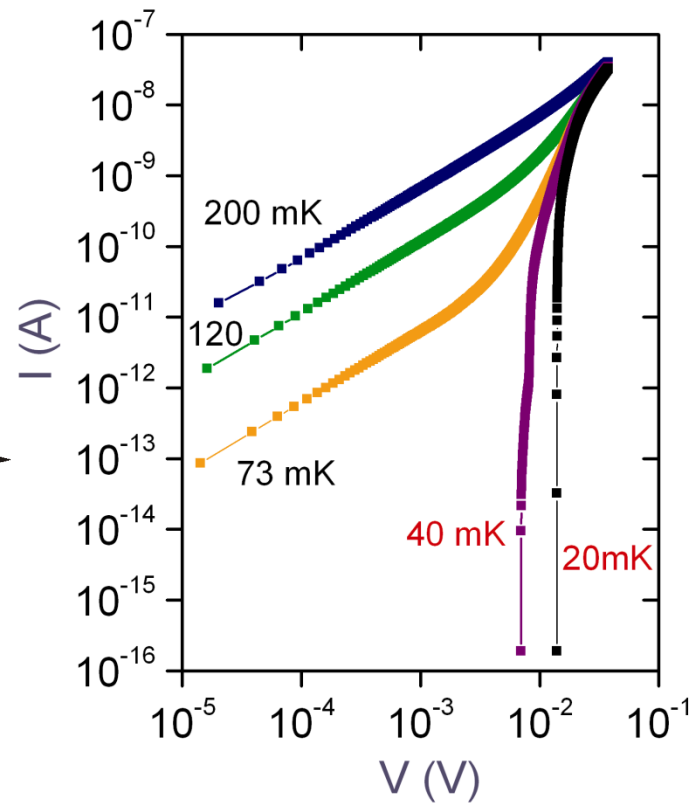
# BKT I-V curves in TiN films on insulating side

Phase diagram for insulating side

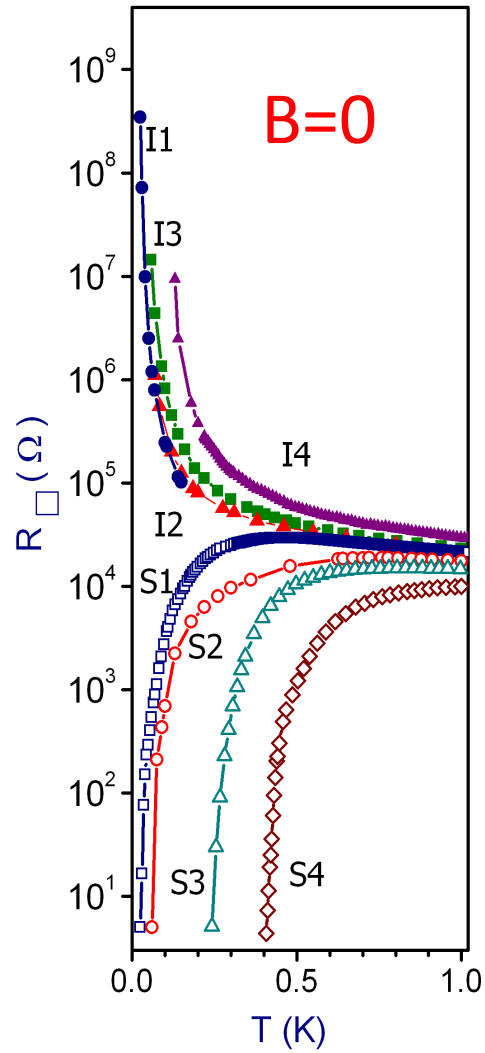



I2 at B = 0.37 T

I vs V

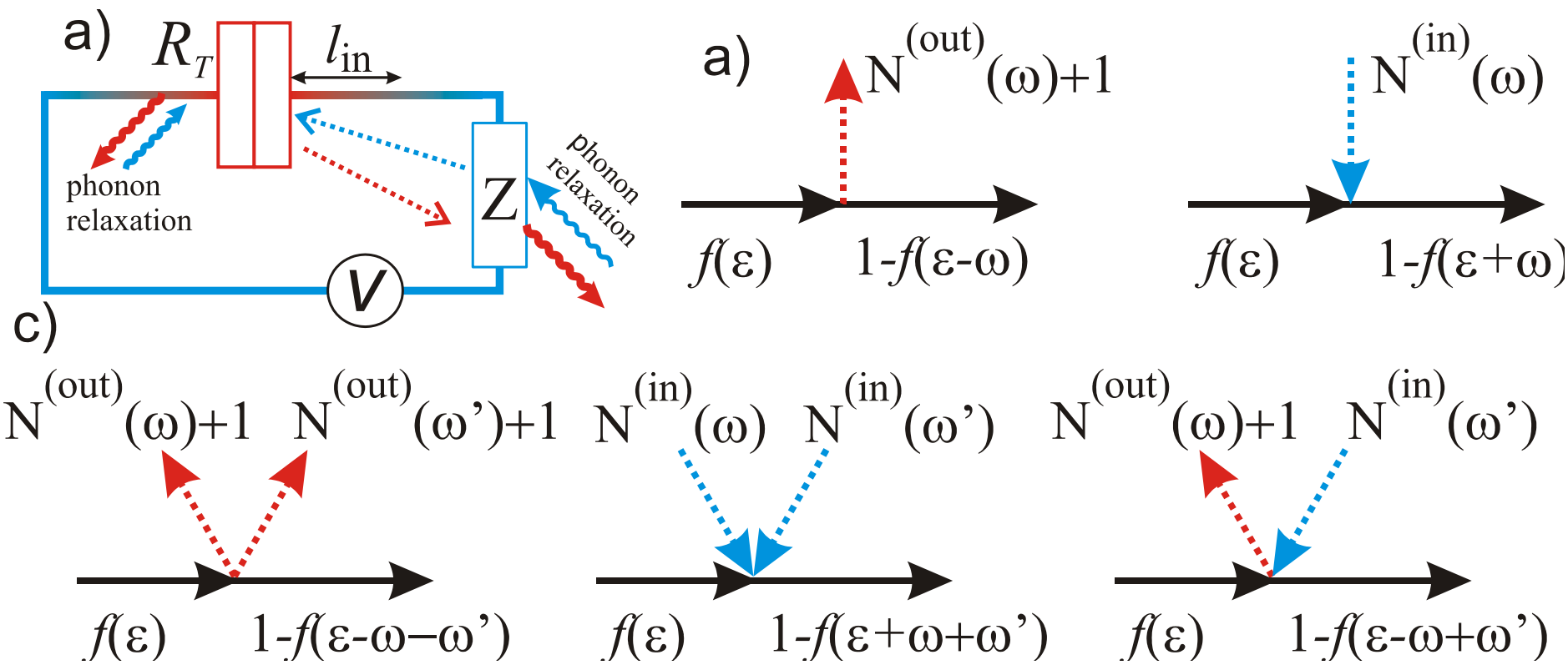



TiN films

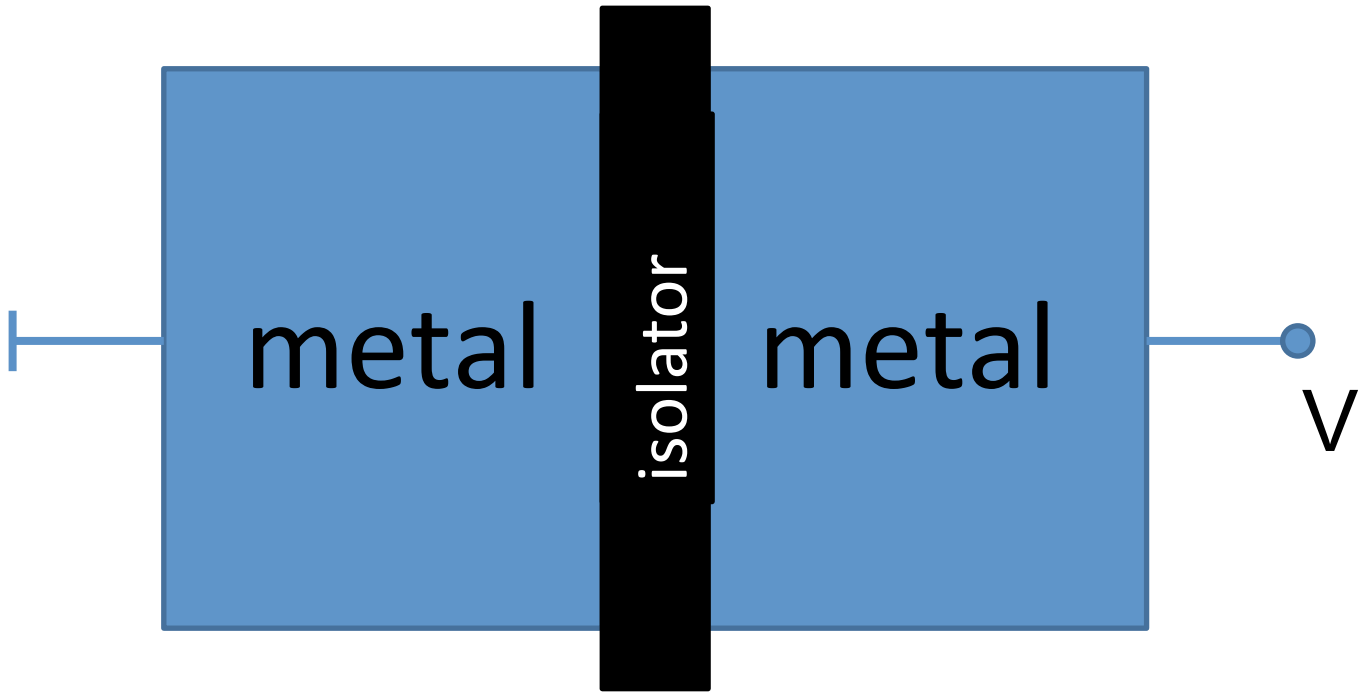



$T_{SI}$ is magnetic field dependent

- ❑Environment and bath are different objects.
- ❑Environment can become nonequilibrium.
- ❑Environment can be tuned.



# Tunnel junction




$$\vec{\Gamma}(V) = \frac{1}{e^2 R_T} \int_{-\infty}^{+\infty} \mathrm{d}E \mathrm{d}E' \, f(E)[1 - f(E' + eV)]P(E - E')$$

$$I(V) = V/R_T$$