Non-equilibrium Fermi-Edge Singularity

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Details in

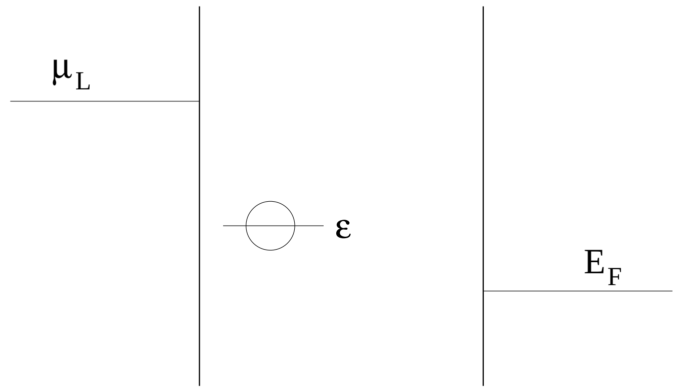
D.A. Abanin, L.S. Levitov, *PRL* **93**, 126802 (2004),

D.A. Abanin, L.S. Levitov, *PRL* **94**, 186803 (2005)

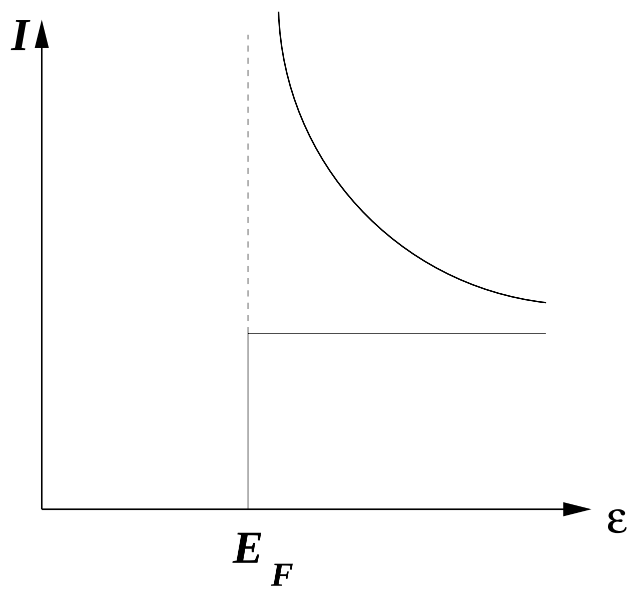
Plan:

- Introduction
- Experimental motivation and related work
- Method: functional determinants
- Discussion of the results
- Future directions

Fermi-Edge Singularity



Enhancement of tunneling current due to interaction:



Threshold singularity in the tunneling current

$$I \propto (\epsilon - E_F)^{-\alpha}\,\theta(\epsilon - E_F)$$

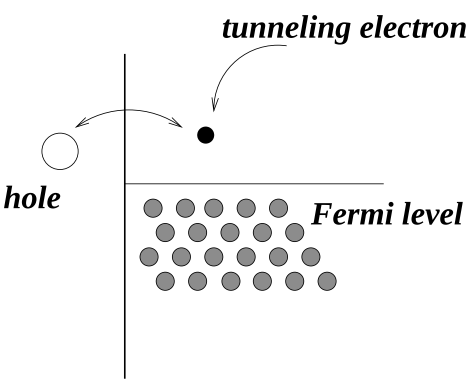
(*Mahan, Nozieres, DeDominicis '69; Matveev, Larkin'92*)

Two Competing Effects

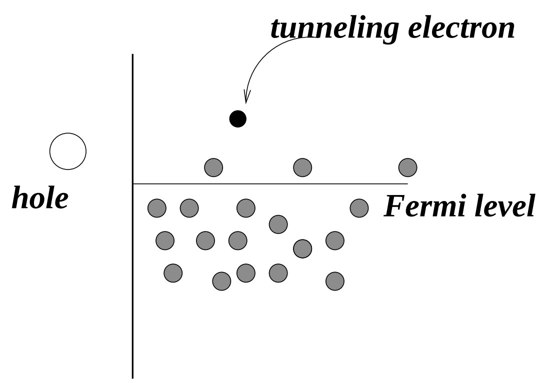
The exponent of the singularity is expressed via scattering phase δ on the hole potential

$$\alpha = 2\frac{\delta}{\pi} - \left(\frac{\delta}{\pi}\right)^2$$

The two terms correspond to two competing effects:



Attraction of the tunneling electron to the hole



Shake-up of the Fermi sea (Orthogonality catastrophe)

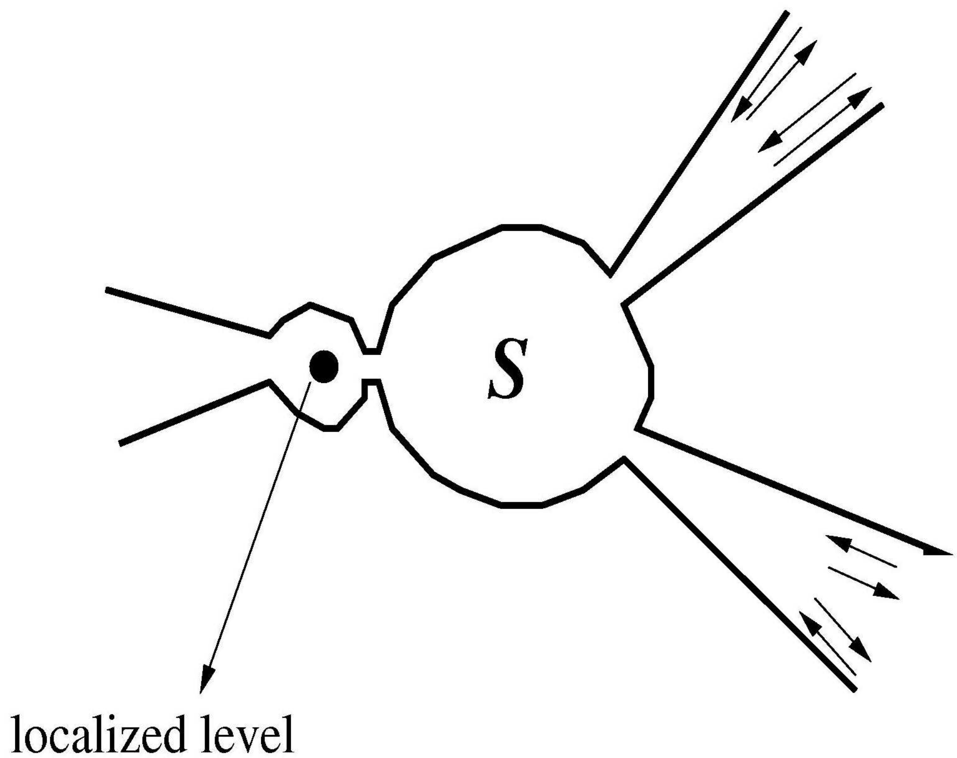
The quantity δ depends on microscopic interaction — a non-universal exponent!

For several scattering channels:

$$\alpha = 2\delta_0/\pi - \sum \delta_i^2/\pi^2$$

Experimental Motivation

Experiment by D.Zumbuhl and C.Marcus, 2005



Resonant tunneling into an open quantum dot:

- Is it possible to tune Fermi-Edge Singularity?
- Interplay of mesoscopic fluctuations and FES
- Non-equilibrium Fermi-Edge Singularity – TOPIC OF THIS TALK!

Related work

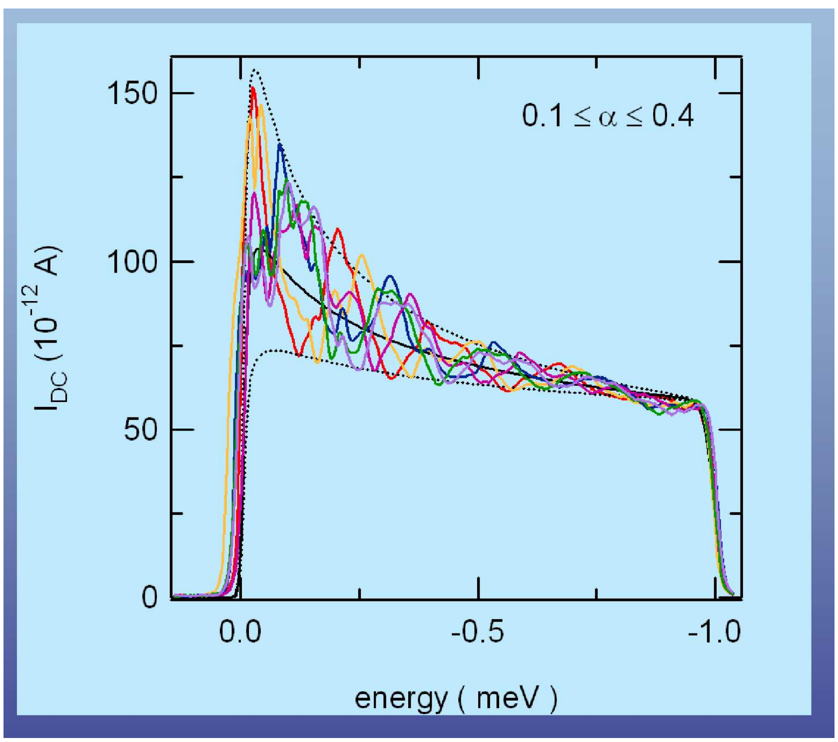
1. Tunable Fermi-Edge Singularity:

• Backscattering in the dot after tunneling changes FES exponent;

• It is possible to tune it to manipulate scattering in the dot $\longrightarrow$ TUNABLE FES

(details in D.Abanin, L.Levitov, PRL **93** 126802)

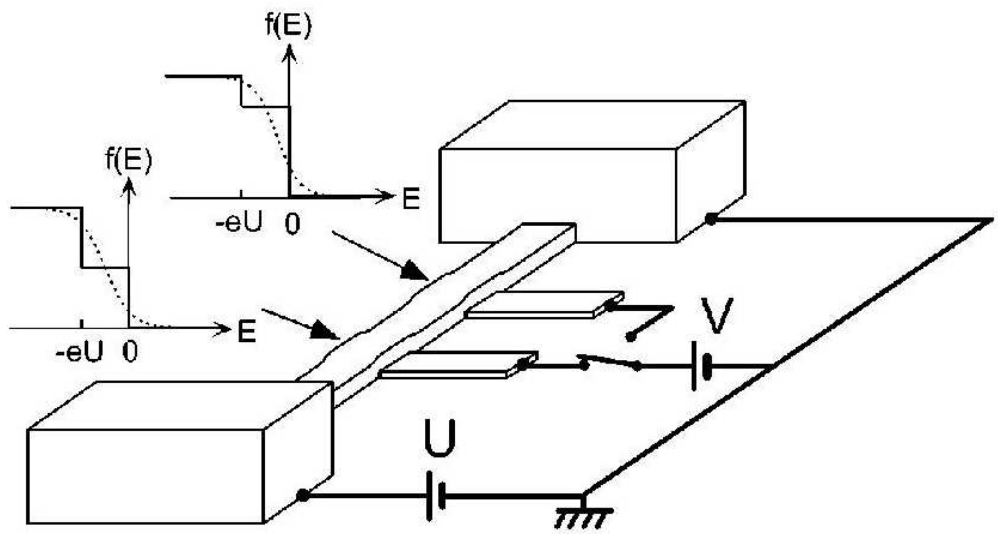
2. Mesoscopic FES:



• Mesoscopic fluctuations over FES have DIFFERENT power-law exponent;
• Possibility to separate excitonic and orthogonality contributions..

Experiments with electrons out of equilibrium

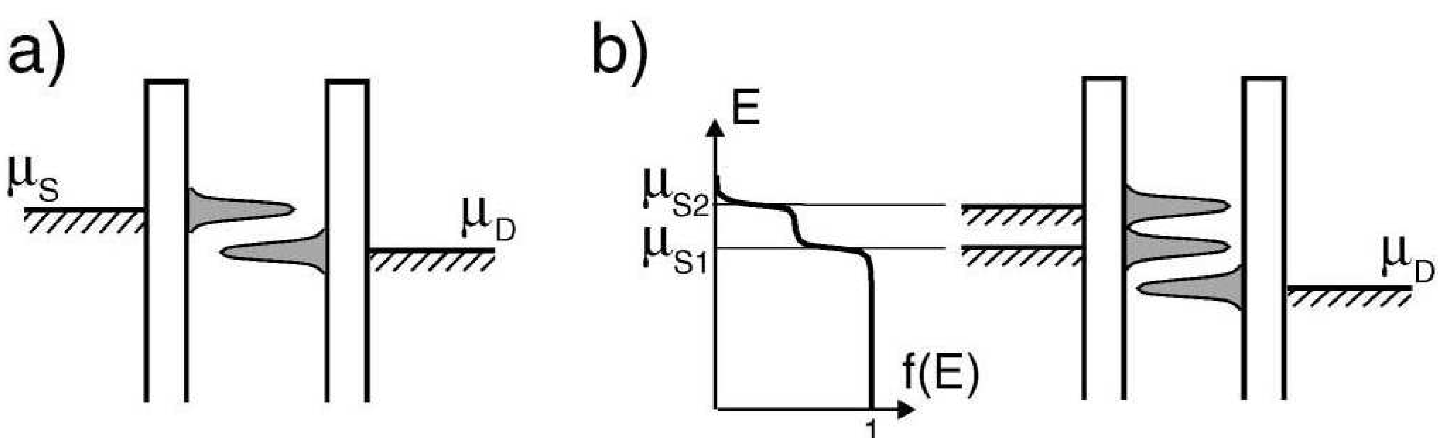
1. Energy relaxation in wires (Saclay, 1997)



2. Non-equilibrium Kondo effect (Delft, 2002)

Out-of-Equilibrium Kondo Effect in a Mesoscopic Device

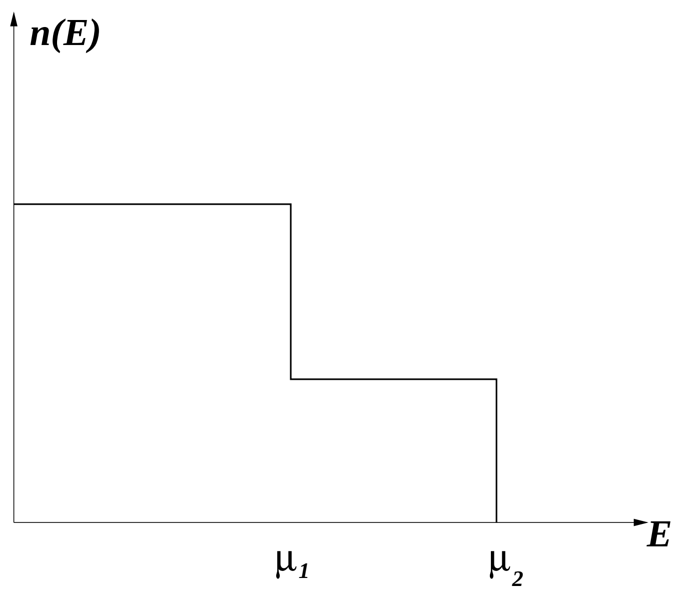
We study the nonequilibrium regime of the Kondo effect in a quantum dot laterally coupled to a narrow wire. We observe a split Kondo resonance when a finite bias voltage is imposed across the wire. The splitting is attributed to the creation of a double-step Fermi distribution function in the wire. Kondo



Non-equilibrium Fermi-Edge Singularity

Resonant tunneling into electron gas out of equilibrium:

$$n(E) = (1-x)n(E-\mu_1) + xn(E-\mu_2)$$



- Method?(known methods fail)
- Splitting of the FES? New exponents?
- Broadening?

Method: functional determinants

Consider orthogonality catastrophe contribution:

$$e^{C(\tau)} = \mathrm{tr}\left(e^{-iH_1\tau} e^{iH_0\tau} \hat{\rho}_e\right)$$

Using the determinant formula, write $e^{C(\tau)}$ in terms of the one-particle evoultion operator

$$e^{C(\tau)} = \det\left(1 - \hat{n}(\epsilon) + R(t)\hat{n}(\epsilon)\right),$$

where $n(\epsilon)$ is distribution function and $R(\tau)$ is a time-dependent **single-particle** scattering matrix:

$$R(t) = \begin{cases} e^{2i\delta}, & 0 < t < \tau \\ 1, & \text{else} \end{cases}.$$

For the equilibrium case, $\hat{n}$ is a *projector* onto the subspace of functions analytic in the upper half-plane, which allows to calculate the determinant.

DIFFICULTY: how to analyze determinant in the NON-EQUILIBRIUM case??

Energy-time duality

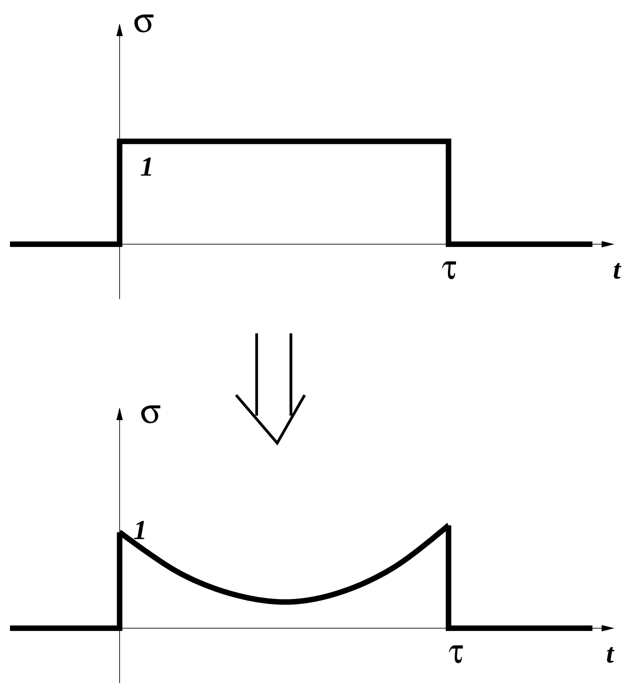
Rewrite orthogonality determinant:

$$e^{C(\tau)} = \det\left(1 - \sigma(t) + (A\hat{n(\epsilon)}) + 1)\sigma(t)\right),$$

where

$$\sigma(t) = \theta(t) - \theta(t - \tau)$$

has two *well-separated (for long times) steps in the time domain*:



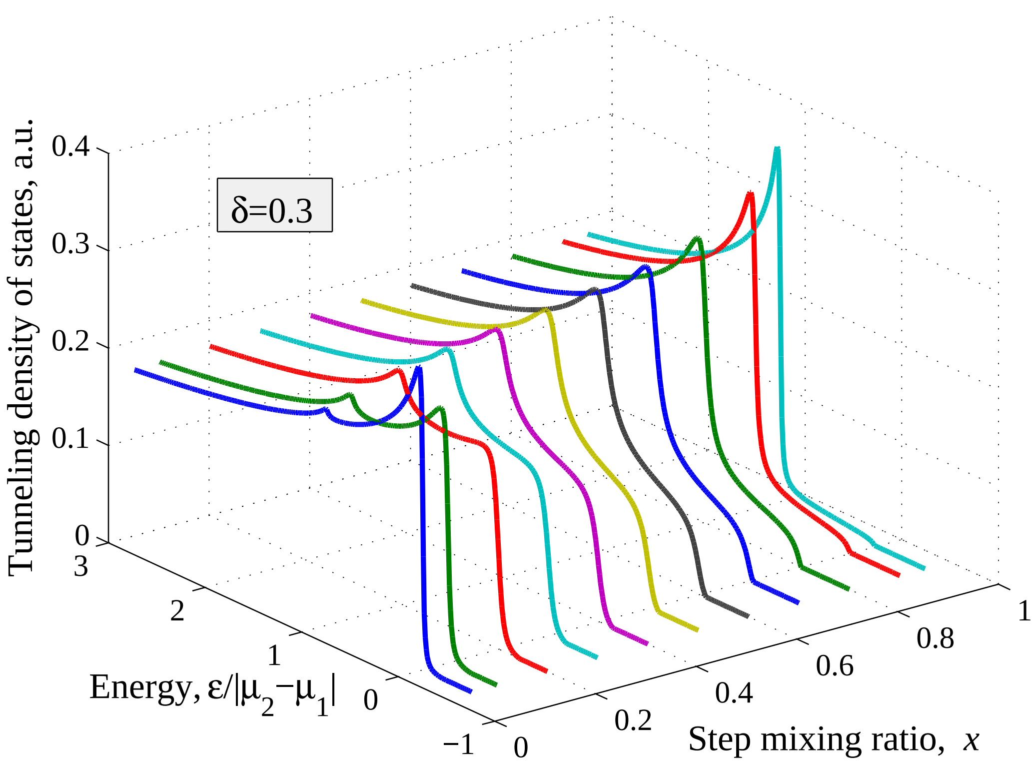
Contributions of the two steps can be treated independently. For a single step, the determinant is dual to the equilibrium one!

Results

- FES splitting
- FES exponent partition: $\alpha_1 + \alpha_2 = \alpha$
- Peaks broadened: Effective temperature

$$T \sim \int n(1-n) \propto x(1-x)$$

- Non-zero tunneling current *under the threshold*



A Remark: anomaly in OC

Broadening:

$$\gamma = |\mu| \ln \left(1 - 4x(1-x)\sin^2\delta\right),$$

where x is a step height, μ is a difference of the chemical potentials of two steps.

Anomaly: at $x = 1/2$, $\delta = \pi/2$ broadening becomes infinite! That means that orthogonality catastrophe is *faster than exponential*!

Future directions

- Non-equilibrium Kondo problem
- Mesoscopic fluctuations of the LDOS – a way to get rid of broadening?
- Other non-equilibrium mesoscopic systems ...

Phys. Rev. Lett. **93**, 126802, Phys. Rev. Lett. **94** 186803

Formulas

Energy distribution:

$$n(\epsilon) = (1-x)n_F(\epsilon-\mu_1) + xn_F(\epsilon-\mu_2).$$

Found: splitting of FES resonance $N(\epsilon) = \mathrm{Im}G(\epsilon)$,

$$G(\epsilon) \propto \int \frac{1-n(\epsilon')}{(\epsilon'-\mu_1)^{\alpha_1}(\epsilon'-\mu_2)^{\alpha_2}} \times D(\epsilon-\epsilon')d\epsilon',$$

with *complex exponents* $\alpha_1 = 2(\delta-\tilde{\delta})/\pi$, $\alpha_2 = 2\tilde{\delta}/\pi$ and

$$\tilde{\delta} = \frac{1}{2i}\ln(1-x+e^{2i\delta}x)$$

For small $\delta \ll 1$, the exponents are expressed through partial density of states $\alpha_1 = (1-x) \times 2\delta/\pi$, $\alpha_2 = x \times 2\delta/\pi$

The broadening function $D(\epsilon) = \int e^{i\epsilon\tau}D(\tau)d\tau$ with

$$D(\tau) = \frac{(1-i\mu\tau)^{\delta\tilde{\delta}/\pi^2}}{(1+\mu^2\tau^2)^{\tilde{\delta}^2}/2\pi^2}\left(-i\tau\xi_0\right)^{-\delta^2/2\pi^2}\exp(-\gamma\tau)$$