Statistics of Current Fluctuations and Coulomb Interaction in Diffusive Conductors

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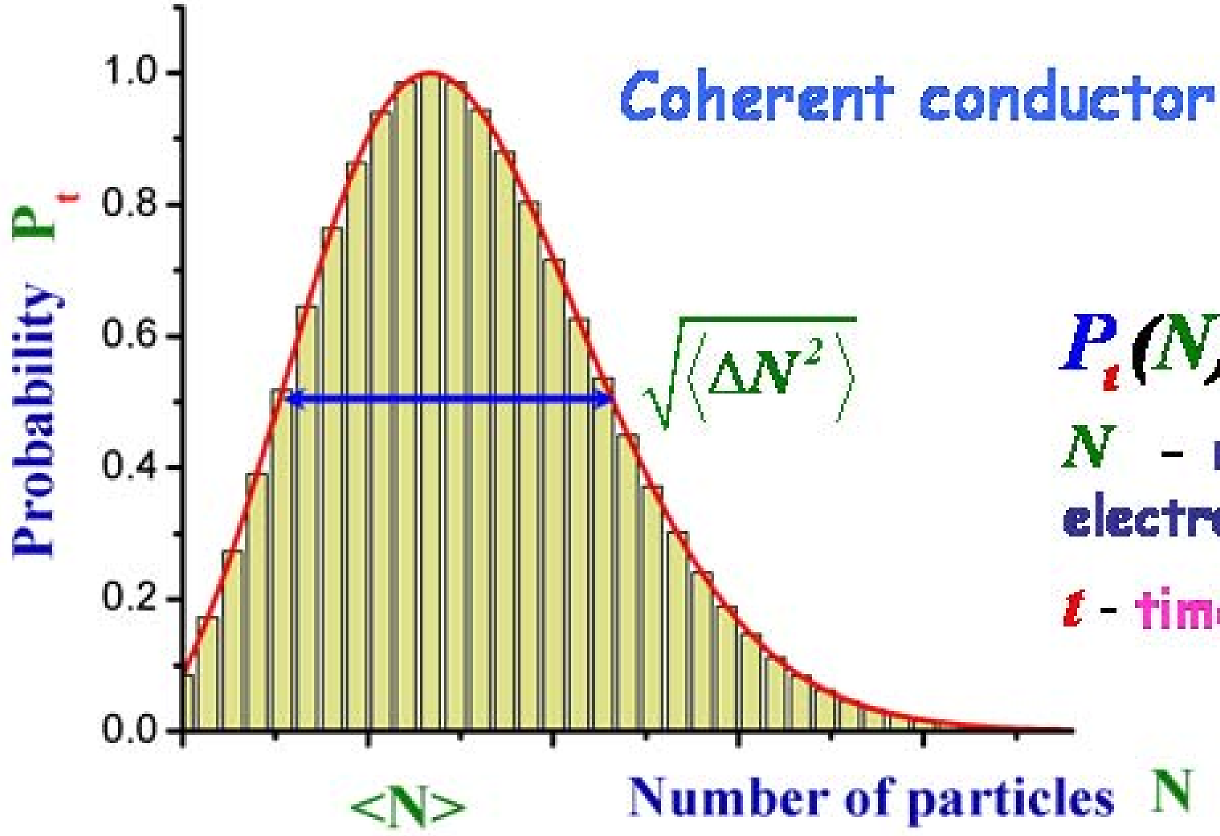
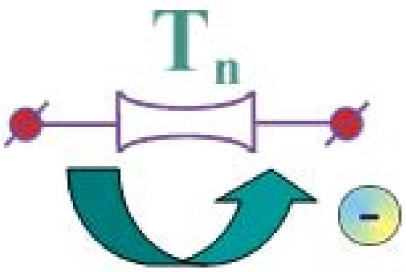
Outline

- Introduction to shot noise and Full Counting Statistics
  - photons (R. Glauber)
  - electrons (this talk)

- FCS in low-D diffusive conductors
  - Coherent and "hot electron" limits
  - Incoherent "cold electrons"
- Conclusions

Full Current Statistics





$$P_t(N) = \text{full info}$$

N - number of electrons transferred

t - time of the measurement

- What to measure?

(Tobiska, Nazarov '04)

Big fluctuations with threshold detectors !

FCS (fermions *vs.* bosons)

- Method: generating function

counting field

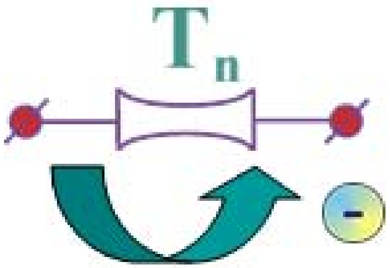
$$\exp[-S(\chi)] = \sum_N P_t(N)\exp(i N \chi)$$

$$\langle\langle n^k \rangle\rangle = i^k \frac{\partial^k}{\partial \chi^k} S(\chi)$$

- Counting electrons (fermions)

Levitov, Lee, Lesovik '95

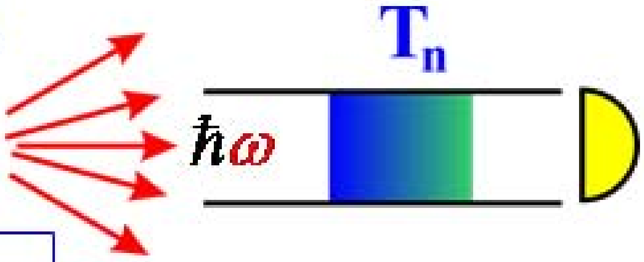
$$S(\chi) = -\frac{eVt_0}{2\pi}\sum_n \ln\{1+T_n[\exp(i\chi)-1]\}$$



- Counting photons (bosons)

Kindermann, Nazarov, Beenakker '02

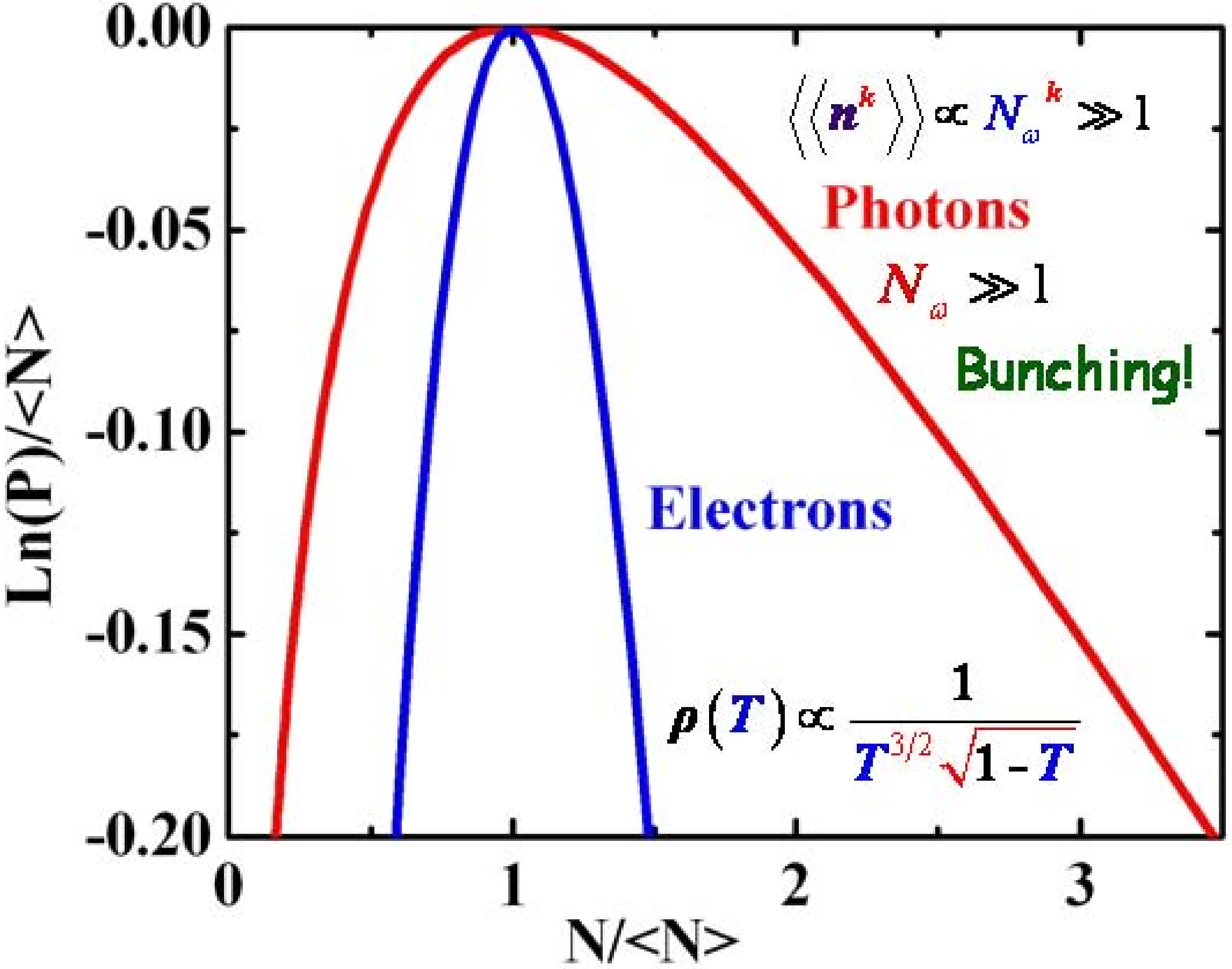
$$S(\chi) = +\frac{\delta\omega t_0}{2\pi}\sum_n \ln\{1-T_n N_\omega[\exp(i\chi)-1]\}$$



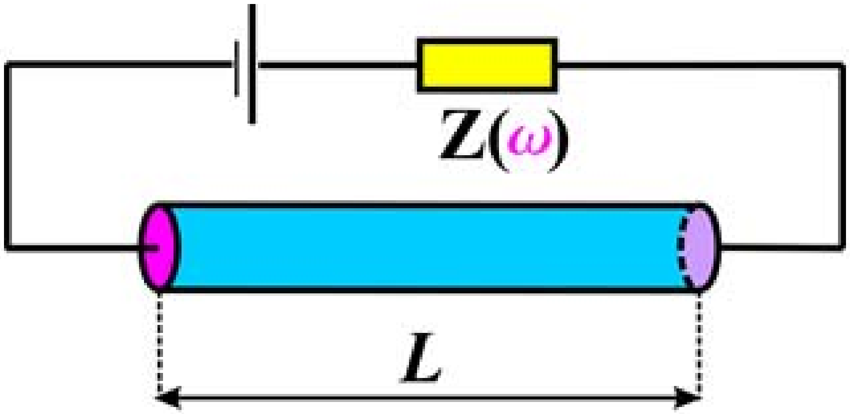
Incoherent radiation !

N_ω - (non)-equilibrium photon distribution function

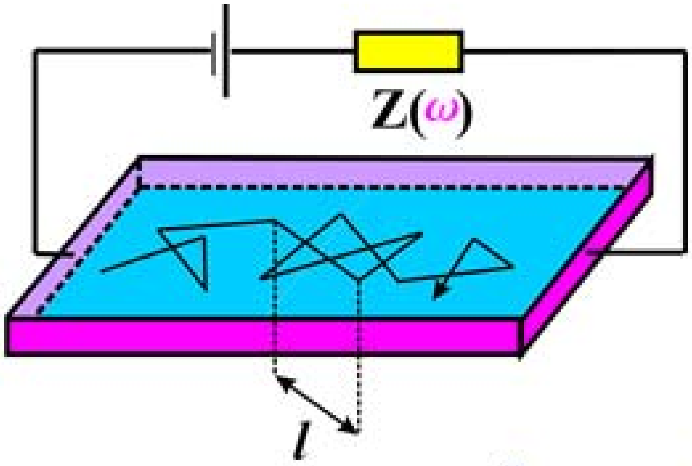
Double barrier geometry



Low-D disordered conductors



Quasi-1D wire



Quasi-2D film **mean free path**

$\boxed{1/\tau = \upsilon_F / l \gg \max\{eV, T\}}$ - **Diffusive regime of transport**

$D = \upsilon_F l / 3$ - **Diffusion constant** $\tau_D = L^2 / D$ - **Diffusion time**

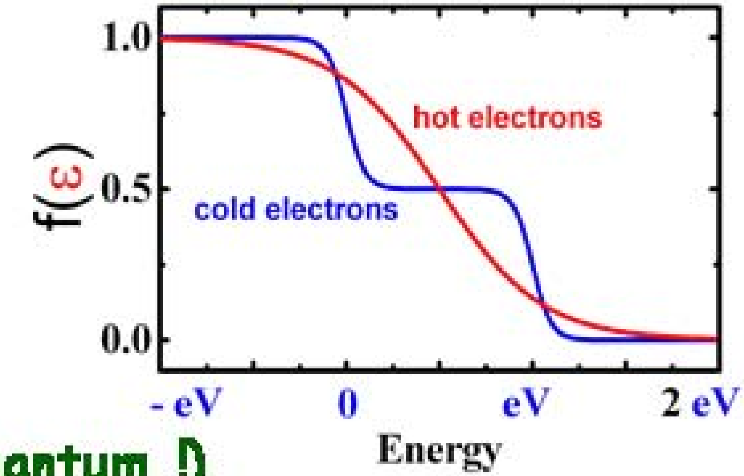
$\boxed{g = 4\pi \nu_d D L^{d-2} \gg 1}$ - **Metallic regime** - $\boxed{Z(\omega) \ll R_Q}$

Inelastic time scales

- **Energy relaxation time** τ_E

  (Classical !)

  Governs the relaxation of f(E)



- **Dephasing time** τ_φ

  Altshuler, Aronov, Khmelnitskii '82 (Quantum !)

  Governs the energy dependence of weak localization σ_{WL}

$$1/eV \lll \tau_\varphi \lll \tau_E$$

Time τ

d	$1/\tau_E$	$1/\tau_\varphi$
1	$(eV/D)^{1/2}\,\nu_1^{-1}$	$\left(eV^2/D\nu_1^{\,2}\right)^{1/3}$
2	eV/g	$(eV/g)\ln g$

Limiting cases

- Coherent regime - $\boxed{\tau_D \leq 1/eV}$ (Levitov et al., Nazarov '99)

  (a) Coulomb interaction is only due to environment, but $Z(\omega) \ll R_Q$

  (b) Generating function $S(\chi) = \frac{geV\,t_0}{8\pi}\,\mathrm{Arccosh}^2\left(2e^{i\chi}-1\right)$

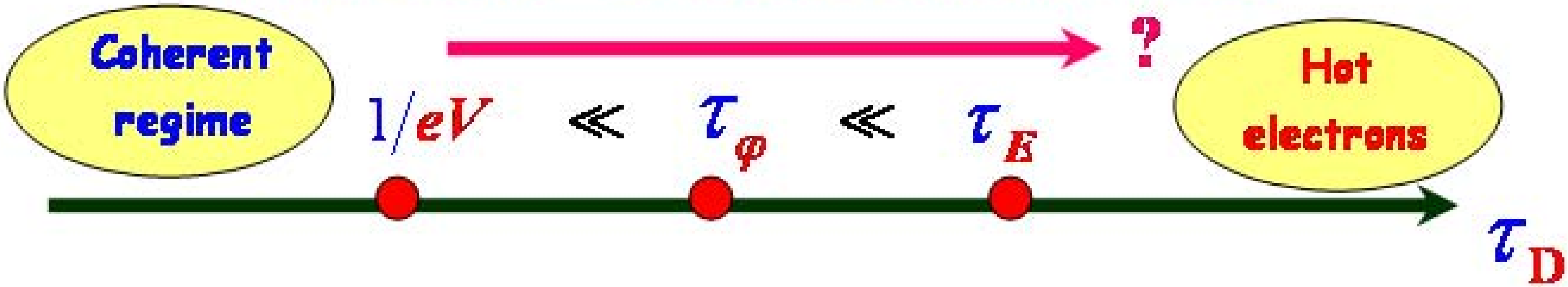
- "Hot electron" regime - $\boxed{\tau_D \gg \tau_E}$

  Kozub, Rudin '95 & Nagaev '95 $F = 1/3 \rightarrow \sqrt{3}/4$

  Gutman, Gefen, Mirlin '02 $C_3 = 1/15 \rightarrow 8/\pi^2 - 9/16$

  Pilgram '04, Gutman, Gefen, Mirlin '05

  - Effective action: numerical results for the CGF



Coherent regime

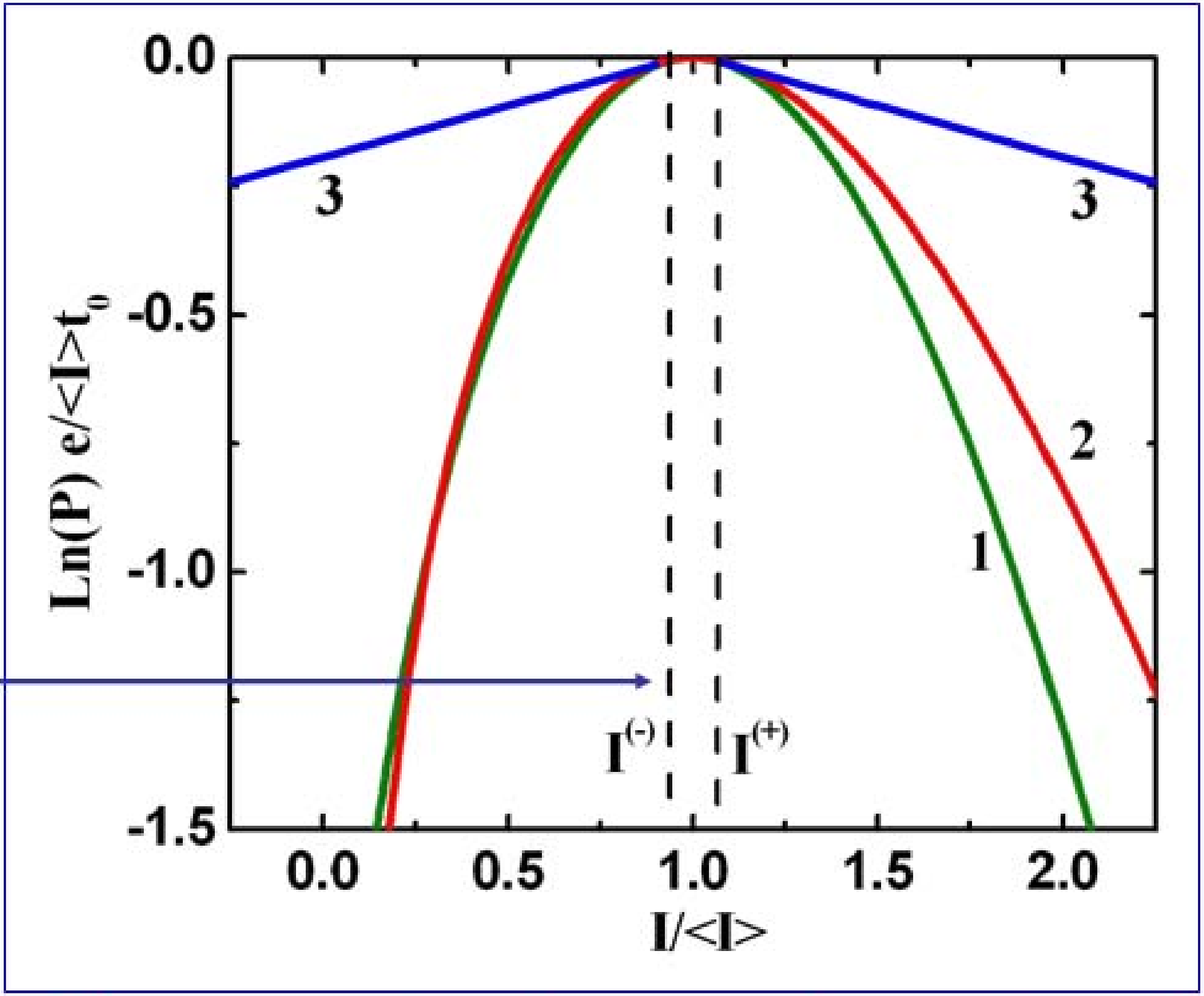
?

$1/eV \ll \tau_\varphi \ll \tau_E$

Hot electrons

τ_{D}

Huge current fluctuations



1-coherent regime

2-hot electrons

3-cold electrons

Threshold currents

Photon-assisted current fluctuations !

Higher-order cumulants (1D)

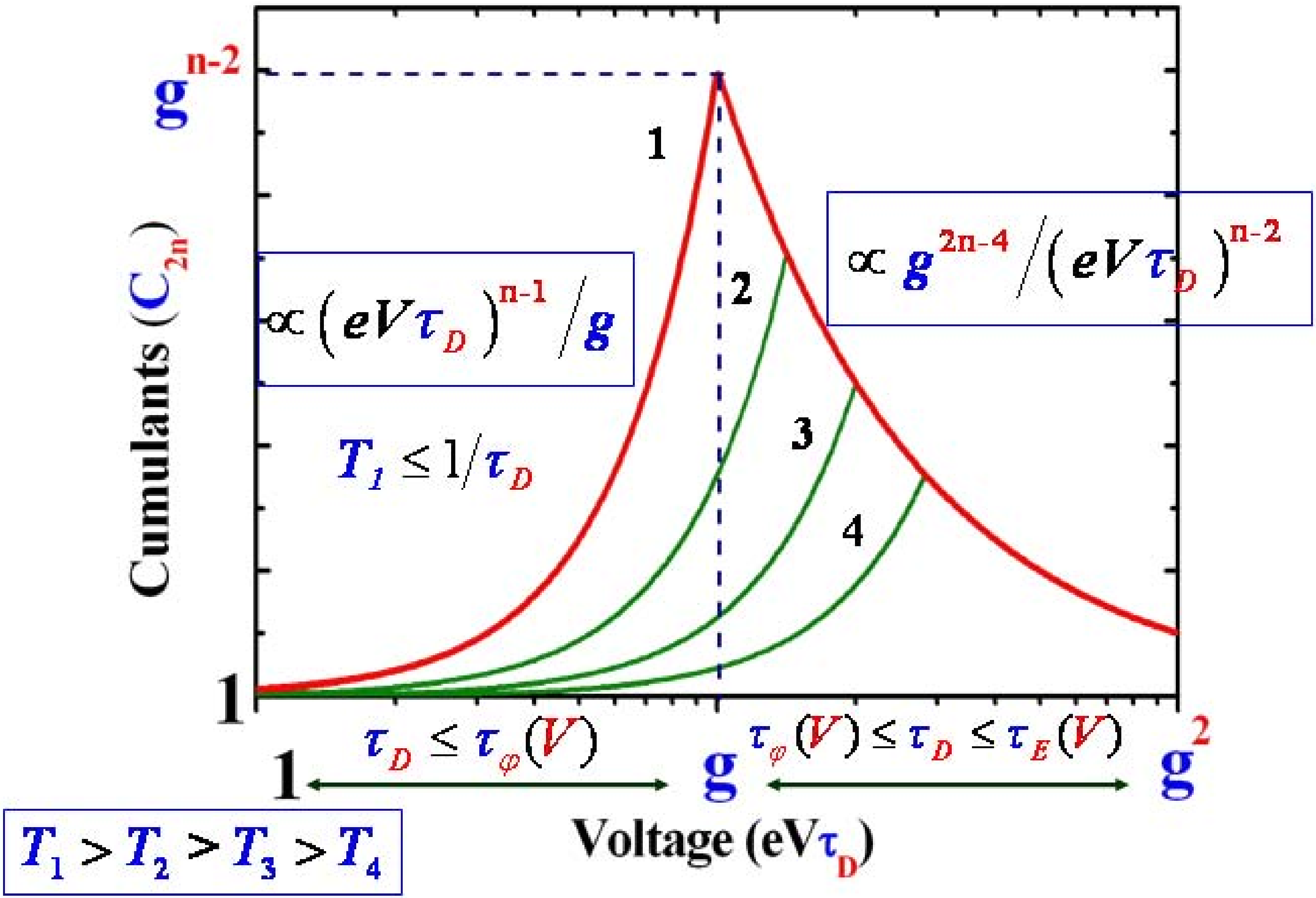
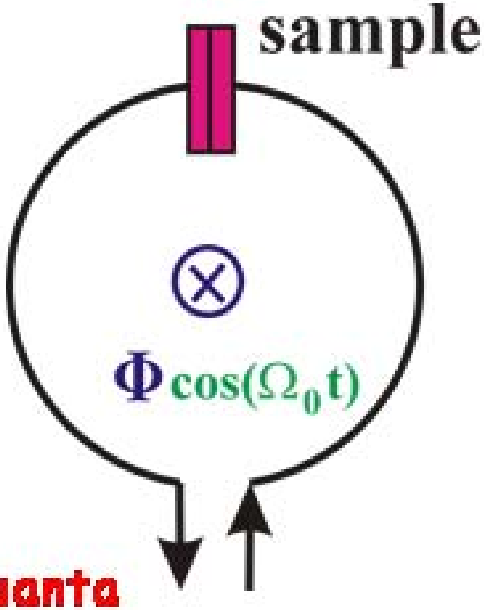


Photo-assisted noise

Mesoscopic conductor under AC bias :

(Lesovik, Levitov '94)

$$eV = \Phi\,\Omega_0$$



$$S_0(\omega \to 0) = 4\hbar\Omega_0 G_Q D(1-D)\sum_{n=1}^{+\infty} n J_n^2(\Phi)$$

Probability to absorb n quanta

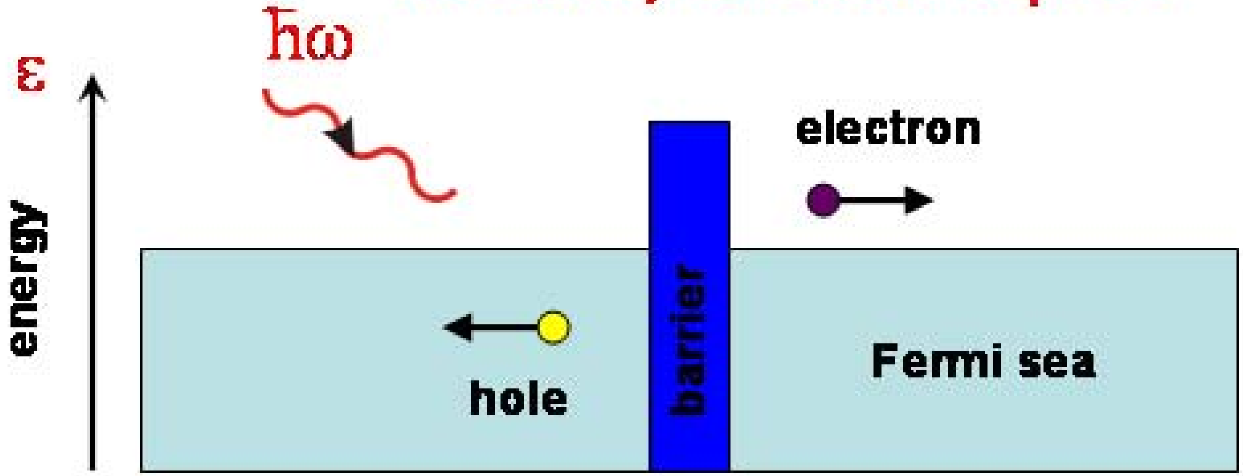


Photo-assisted FCS

- **FCS** in a fluctuating classical electro-magnetic field

$$F(\chi) \propto \frac{g\,\omega^2}{\left(Dq^2\right)^2+\omega^2}\int_0^L dz\,\omega\Pi(\chi,z)\left|A_{\omega,q}\right|^2$$

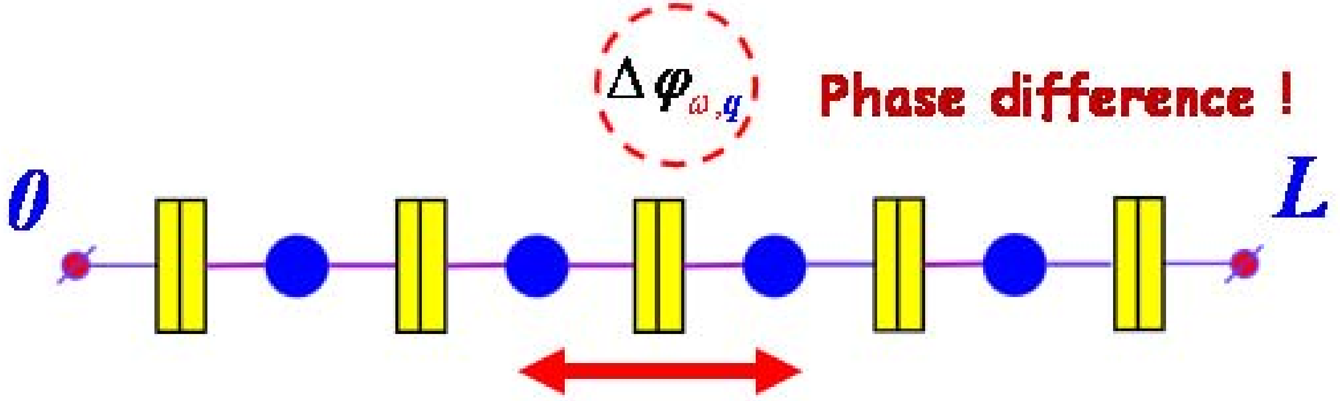
$$\left\langle \left|A_{\omega,q}\right|^2 \right\rangle = \frac{N_\omega}{\nu D\omega}$$

- **Generating function** (E.m. fluctuations + electron-hole pairs!)

$$S(\chi) \propto \sum_q \int_0^L dz \int_{\omega^*}^{eV} \frac{d\omega}{2\pi} \ln\left[1-\frac{\omega^2 N_\omega \Pi(\chi,z)}{\left(Dq^2\right)^2+\omega^2}\right]$$

$$N_\omega \simeq eV/\omega \gg 1$$

Photon bunching !



Phase difference !

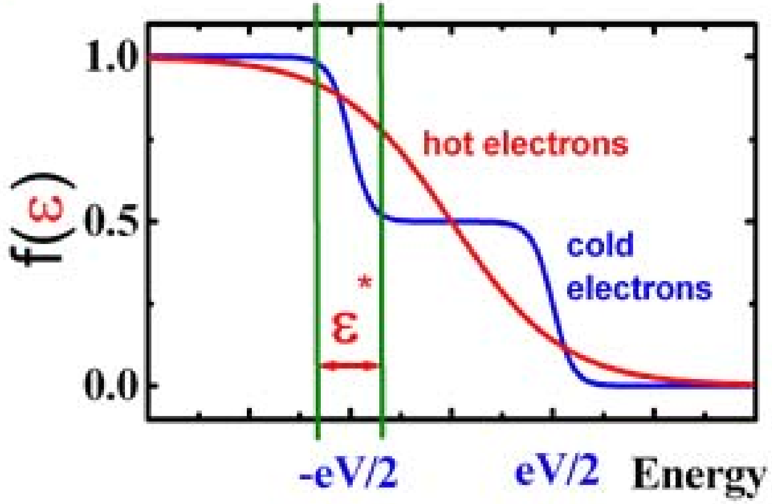
Regularization

Cut-off $\omega^* = \max\left\{\varepsilon^*, T, 1/\tau_D\right\}$ **= Effective temperature**

- **1st order correction to f(E) at** $\varepsilon_\pm = \left|\varepsilon \pm eV/2\right| \ll 1$

$$\delta f_{(1)}(\varepsilon_\pm) \sim \frac{\tau_D}{\tau_E(V)} \int_{\varepsilon_\pm}^{eV} \frac{d\omega}{\omega} \left(\frac{eV}{\omega}\right)^{(2-d)/2}$$

Danger !



- **We find the scale** $\varepsilon^*(eV)$ **so that** $\delta f_{(1)}(\varepsilon^*) \sim 1$

$$\varepsilon^* \tau_D = \begin{cases} \left(eV\tau_D/g\right)^2, & 1\text{-D} \\ eV\tau_D \exp\left(-g/eV\tau_D\right), & 2\text{-D} \end{cases}$$

Conclusions

- **FCS** in low 1 & 2-D diffusive conductors

  + Coulomb interaction

- Long exponential tails in P(N) for $1/eV \ll \tau_D \ll \tau_E$

  Photo-assisted current fluctuations !

- No smooth crossover in **FCS** from the coherent to hot electron regime

- Higher order cumulants (n>3) are sensitive to **AAK** scale $\tau_\varphi(V)$!

  Maximal fluctuations at low $T\tau_\varphi(V) \leq 1$ and $\tau_D \simeq \tau_\varphi(V)$

Thank you !