

# Superconductivity on the localization threshold

**from quantum corrections**

**to magnetic-field-tuned  
superconductor-insulator  
quantum phase transition**

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International Argonne Fall Workshop on Nanophysics V:  
Nanoscale Superconductivity and Magnetism

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15 November 2005

Tatyana Baturina

# Concrete address

✓ Disordered

$$T\tau = 1$$

✓ Superconducting

✓ Two-dimensional systems

quasi-2D  $\rightarrow$  electronic spectrum is 3D

$$l, \lambda_F < d < \xi, l_T$$

# Evolution

weak disorder

metal

Drude conductivity  
+  
quantum corrections

strong disorder

insulator

activated conductivity

+

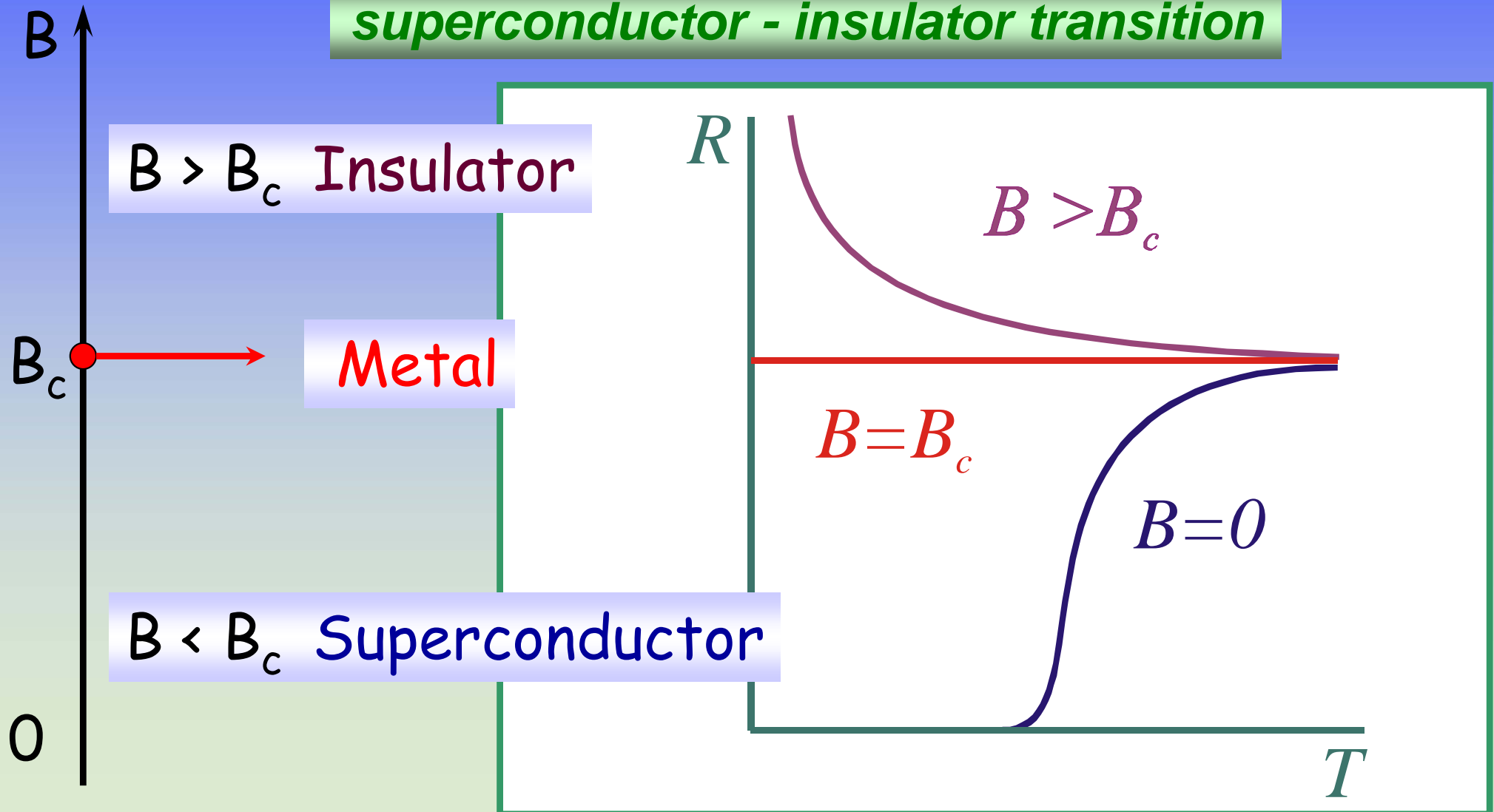
superconductivity  
Cooper pairing  
attractive interaction

# Suppression of Superconductivity by **Magnetic Field**

$T = 0$

**Field-induced**

*superconductor - insulator transition*



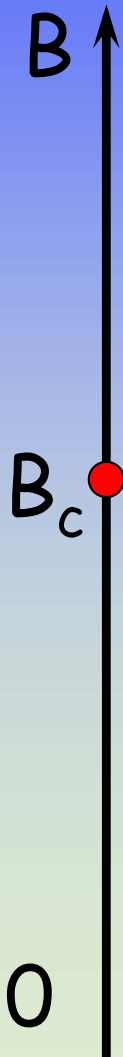
# Field-induced superconductor – insulator transition

$T = 0$

\*\*\* Quantum phase transition \*\*\*

Duality between the dynamics of Cooper pairs and vortices

Matthew P.A. Fisher, PRL **65**, 923 (1990)



$B > B_c$  Insulator:

a condensate of vortices; Cooper pairs are localized

**Metal:**

The resistance has a finite, nonzero value at  $T = 0$ .

This value is *universal* -  $R_c = h / (2e)^2$

$B < B_c$  Superconductor:

a condensate of Cooper pairs; vortices are localized

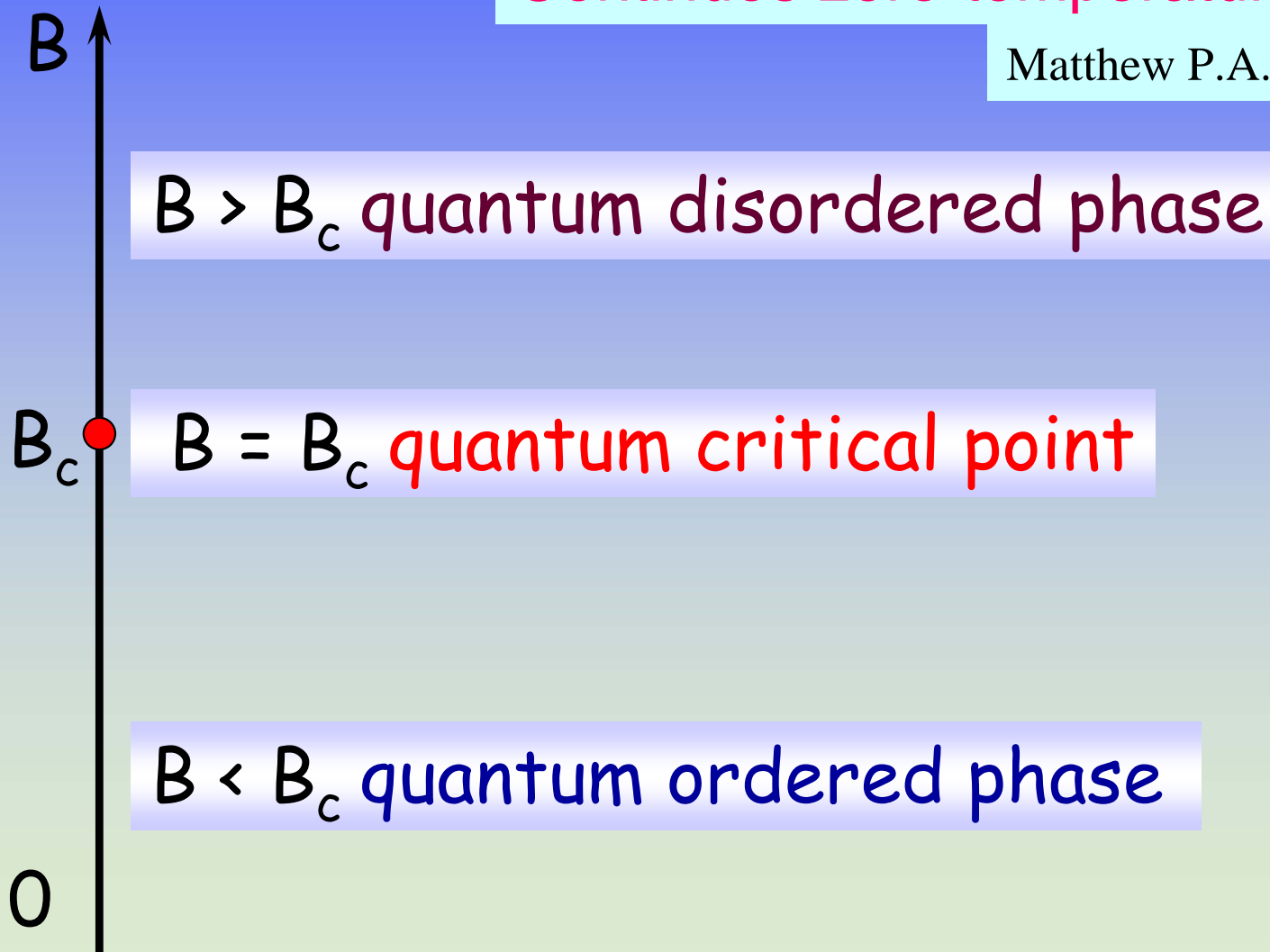
# Field-induced superconductor – insulator transition

$T = 0$

\*\*\* Quantum phase transition \*\*\*

Continuous zero-temperature phase transition

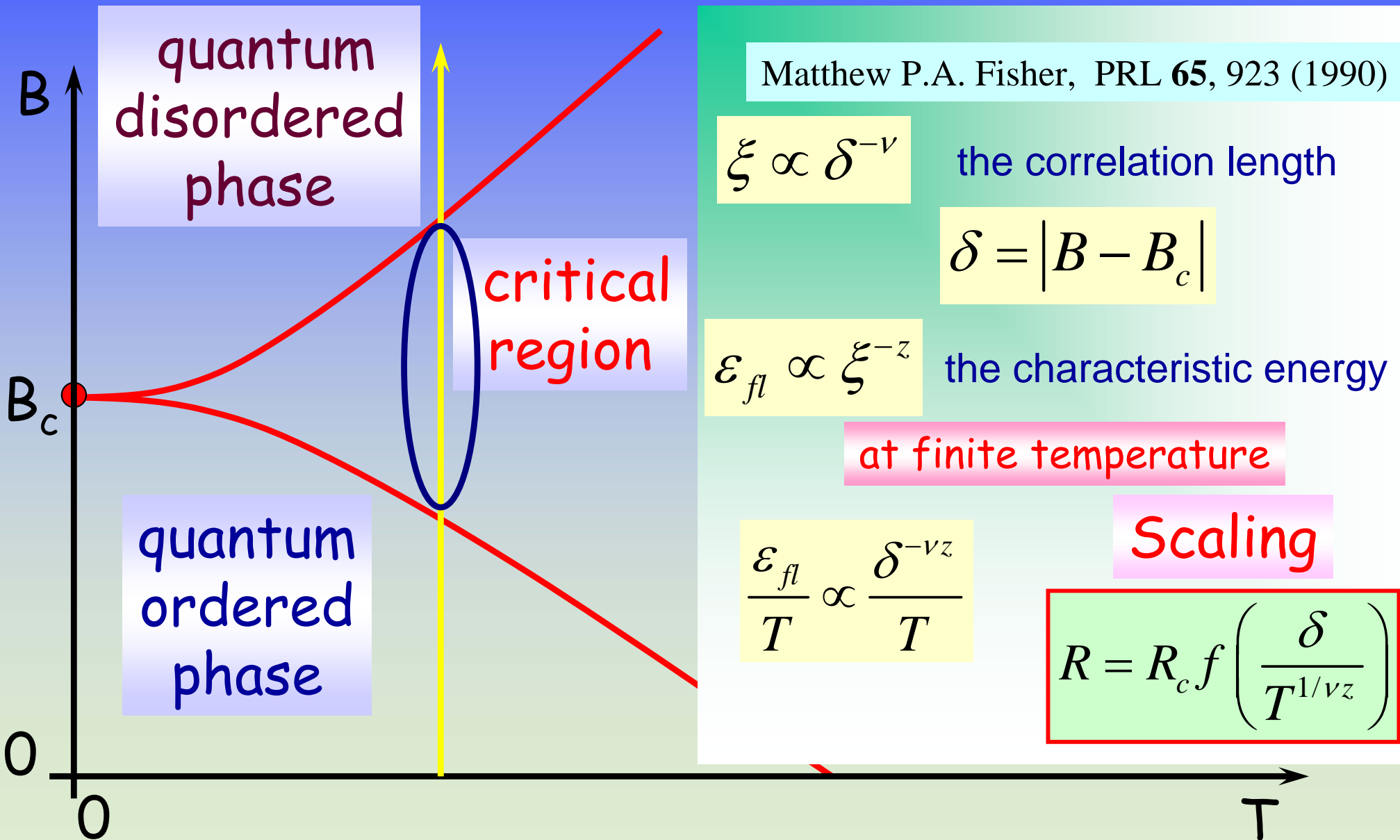
Matthew P.A. Fisher, PRL **65**, 923 (1990)



# Field-induced superconductor – insulator transition

$T > 0$

\*\*\* Quantum phase transition \*\*\*



# Field-induced superconductor – insulator transition

\*\*\* Quantum phase transition \*\*\*

## EXPERIMENT

✓ Fan-shaped curves

$$\begin{aligned} dR/dB > 0 & \text{ at } B < B_c \\ dR/dB < 0 & \text{ at } B > B_c \end{aligned}$$

✓ Scaling

$$R = R_c f(|B - B_c|/T^{1/\nu z})$$

✓ Negative magnetoresistance

at high magnetic fields

(as result of the break up of the localized Cooper pairs)

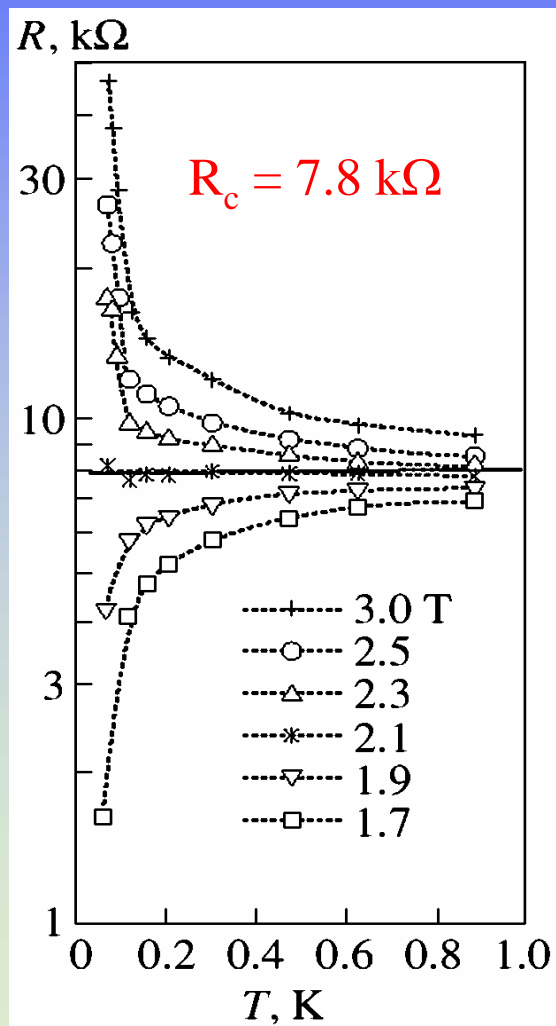
V.F. Gantmakher, et. al. JETP Lett. 68, 363 (1998)



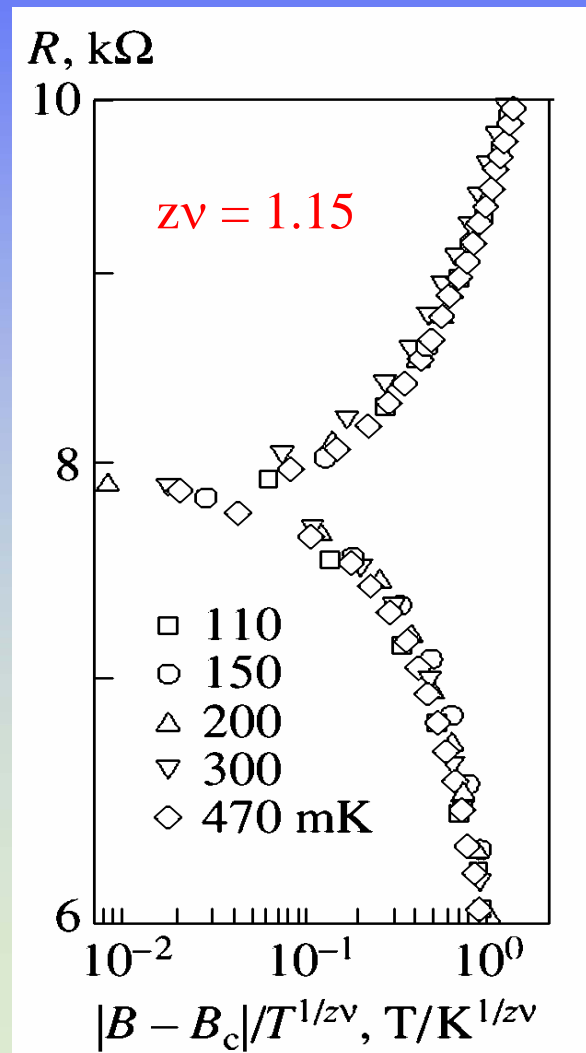
# Field-induced superconductor – insulator transition

V. F. Gantmakher, M. V. Golubkov, V. T. Dolgoplov, A. A. Shashkin, G. E. Tsydynzhapov, *a*-InO<sub>x</sub>  
JETP Lett. **71**, 160 (2000); **71**, 473 (2000)

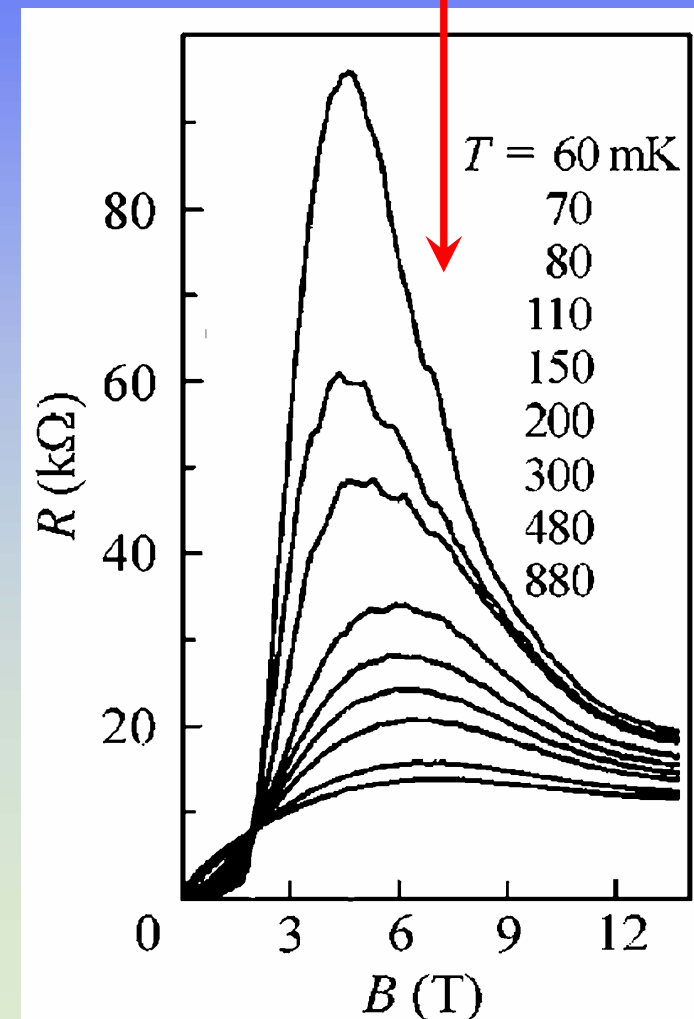
## Fan-shaped curves



## Scaling



## Negative magnetoresistance

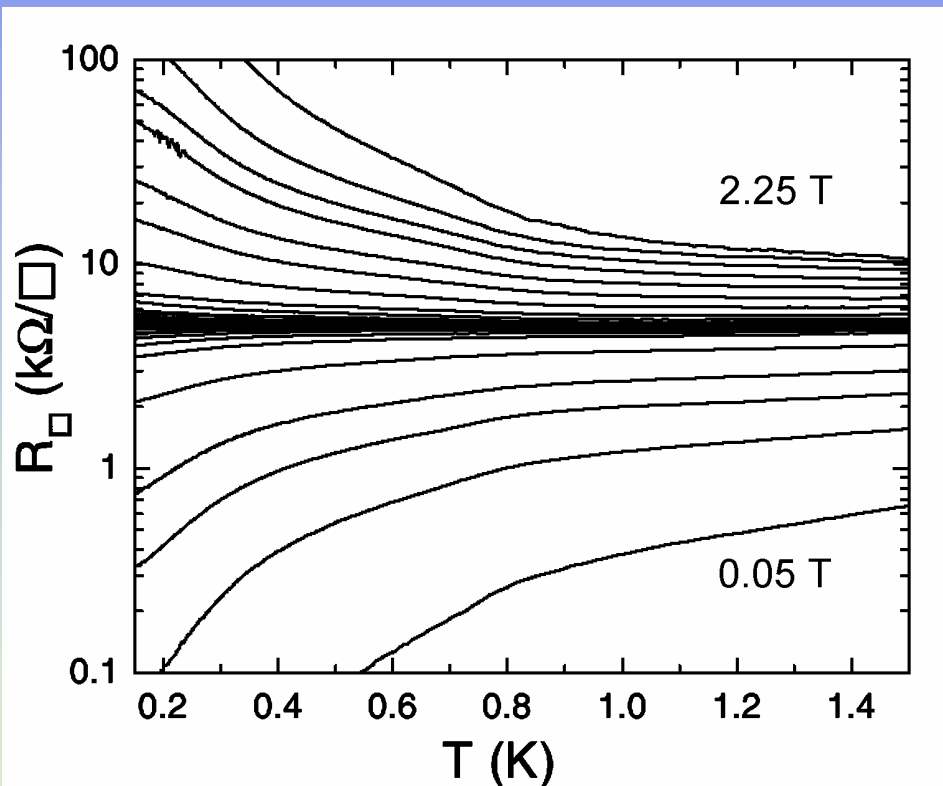


# Field-induced superconductor – insulator transition

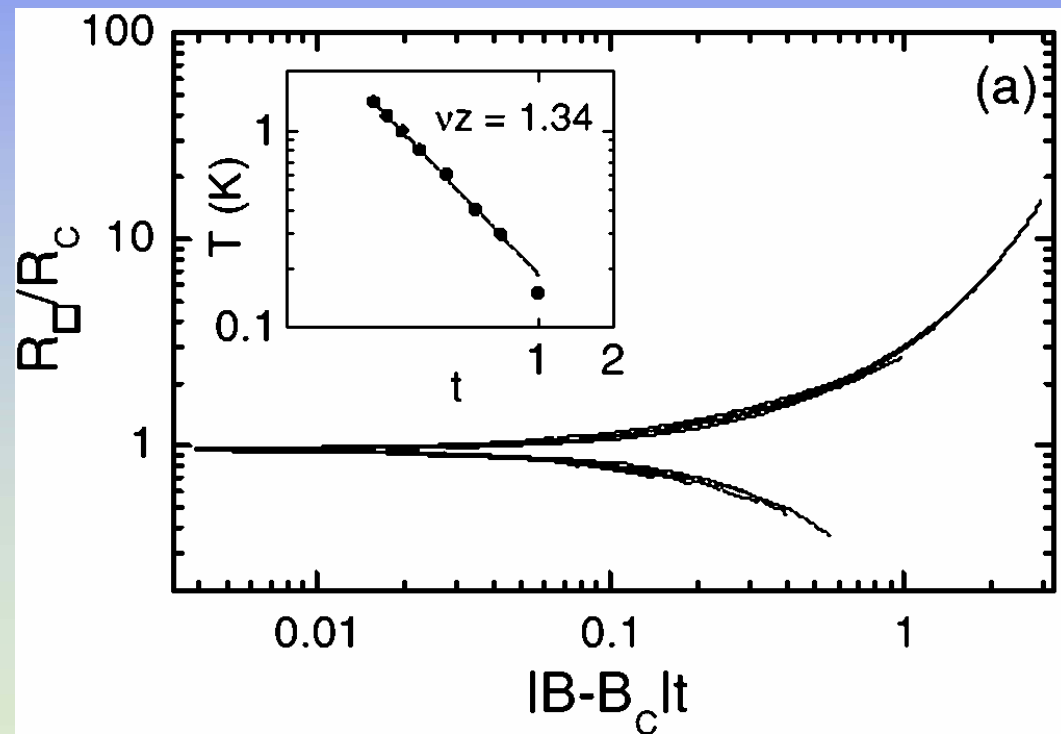
E. Bielejec and Wenhao Wu, PRL **88**, 206802 (2002)

Be films

## Fan-shaped curves



## Scaling



# Field-induced superconductor – insulator transition



VOLUME 74, NUMBER 15

PHYSICAL REVIEW LETTERS

10 APRIL 1995

## Superconducting-Insulating Transition in Two-Dimensional $a$ -MoGe Thin Films

Ali Yazdani\* and Aharon Kapitulnik

Department of Applied Physics, Stanford University, Stanford, California 94305

$\text{Mo}_{21}\text{Ge}_{79}$ ,  $d = 8$  nm,  $R_c = 1750 \Omega$

### Fan-shaped curves

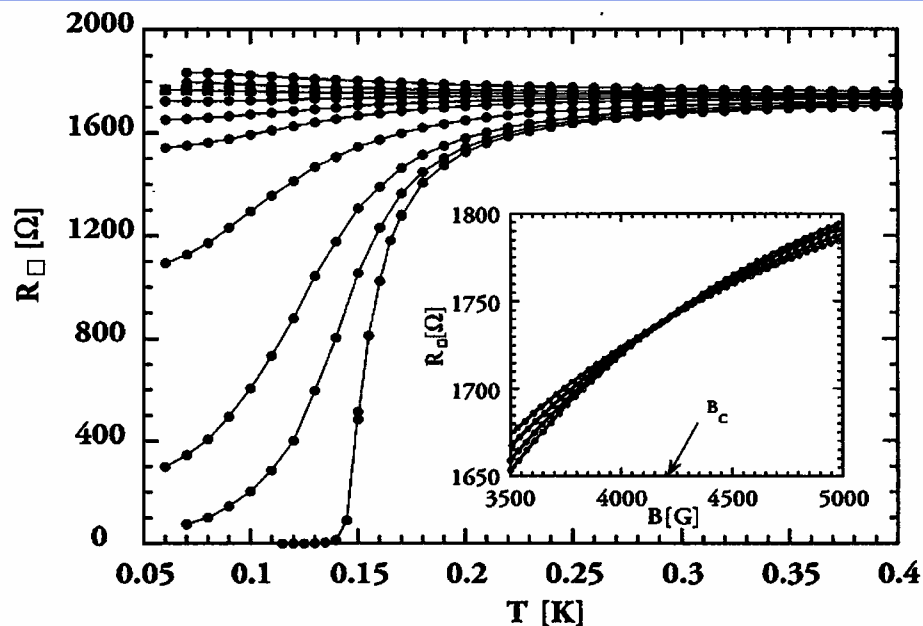


FIG. 1. Zero bias resistance of sample 2 plotted versus temperature at  $B = 0, 0.5, 1.0, 2.0, 3.0, 4.0, 4.4, 4.5, 5.5, 6$  kG. In the inset,  $R_{\square}(B, T, E = 0)$  for the same sample measured versus field, at  $T = 80, 90, 100, 110$  mK.

### Scaling

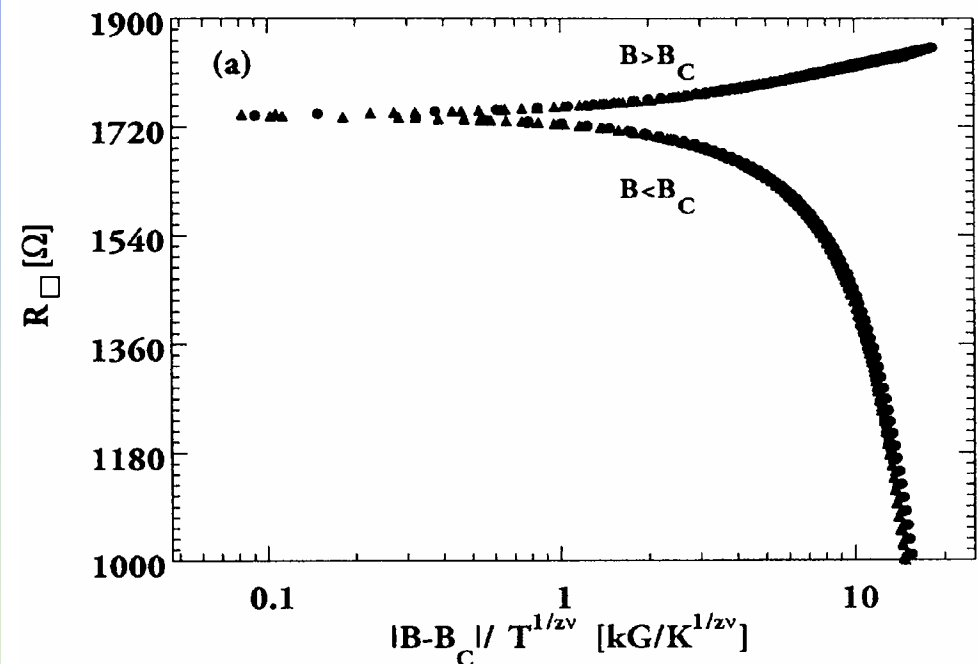


FIG. 3. Top: Scaling of  $R_{\square}(B, T, E = 0)$  for sample 2 measured at  $T = 80, 90, 100, 110$  mK ( $B_c = 4.19$  kG,  $\nu_z = 1.36$ ).

# Field-induced superconductor – insulator transition



PHYSICAL REVIEW B

VOLUME 58, NUMBER 5

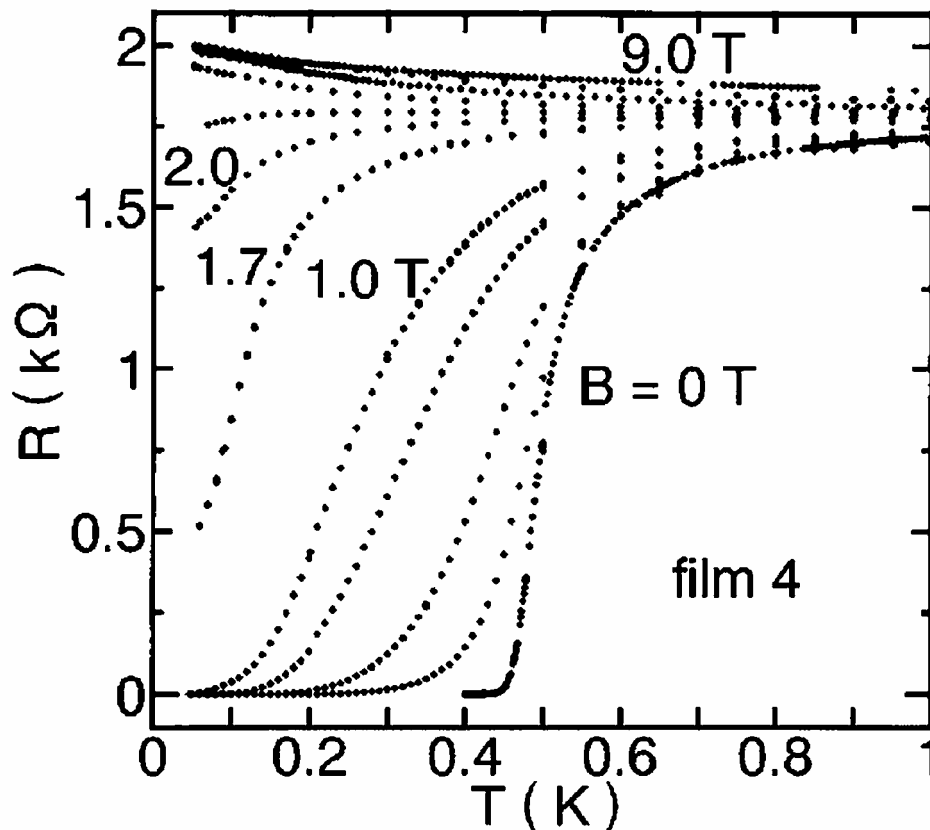
1 AUGUST 1998-1

## Anomalous magnetoresistance near the superconductor-insulator transition in ultrathin films of $a\text{-Mo}_x\text{Si}_{1-x}$

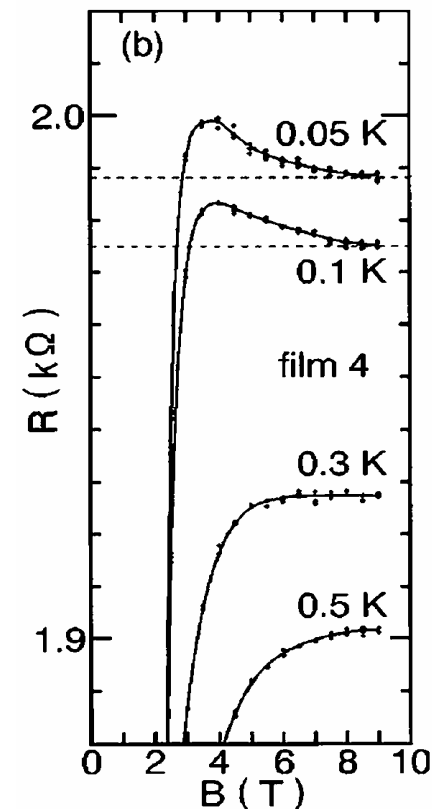
S. Okuma, T. Terashima, and N. Kokubo

*Research Center for Very Low Temperature System, Tokyo Institute of Technology, 2-12-1, Ohokayama, Meguro-ku,  
Tokyo 152-8551, Japan*

### Fan-shaped curves



### Negative magnetoresistance

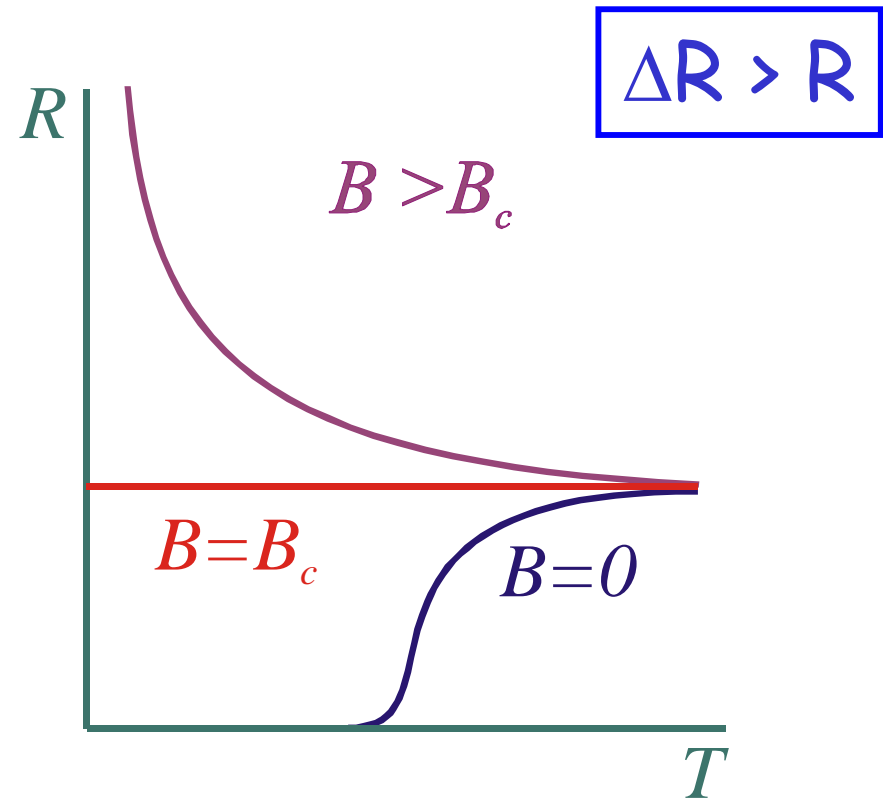
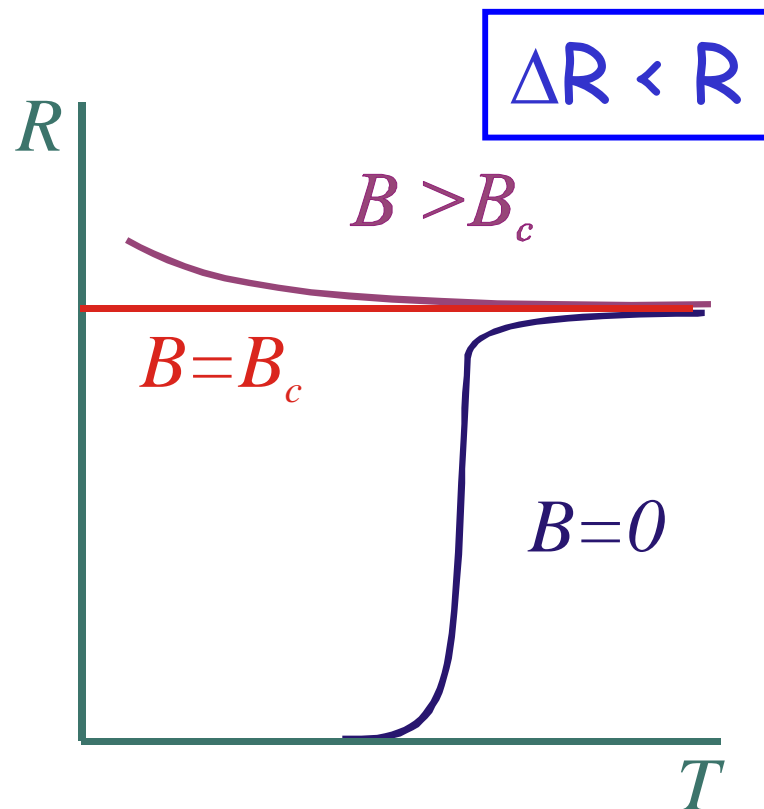


# Suppression of Superconductivity by **Magnetic Field**

**Field-induced...**

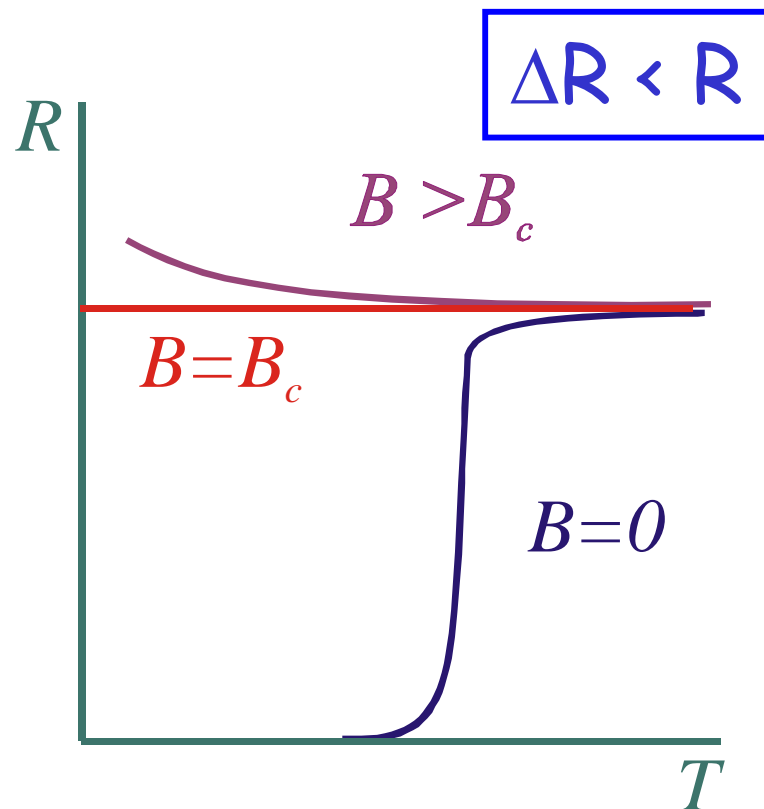


*superconductor –  
insulator  
transition*



# Suppression of Superconductivity by **Magnetic Field**

## Field-induced...



This reminds us of the behavior of a disordered metal with quantum corrections to the conductivity rather than that of an insulator.

# Quantum corrections to conductivity

one-particle interference

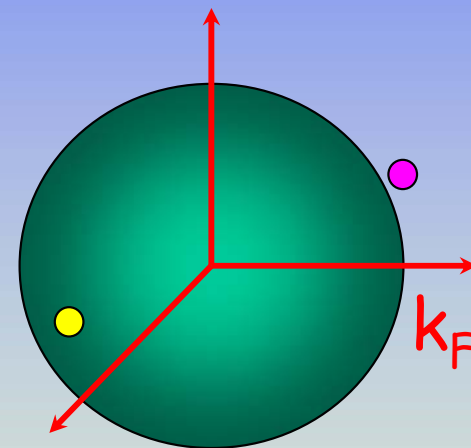
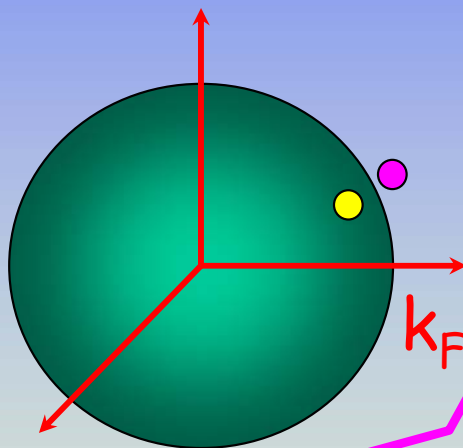
two-particle interference

(e-e interaction)

~~Weak  
Localization~~

Diffusion Channel

Cooper Channel



Aronov -  
Altshuler

Aslamasov -  
Larkin

Density-Of-  
States

Maki -  
Thompson

Superconducting fluctuations

# Quantum corrections to conductivity

Aronov -  
Altshuler

$$\delta_{AA} \sigma(T)$$

$$\propto \ln(T)$$

Aslamasov -  
Larkin

Density-Of-  
States

Maki -  
Thompson

Superconducting fluctuations

at  $T \ll T_{c0}$  in magnetic field ???

V.M. Galitski and A.I. Larkin, PRB **63**, 174506 (2001)

An analytical expression for the fluctuation conductivity in the region close to the transition line at low temperatures

$$T_{c0} \tau \ll 1,$$

$$ds = \frac{2e^2}{3p^2 h} \frac{\epsilon}{\epsilon} \ln \frac{r}{h} - \frac{3}{2r} + y(r) + 4[ry'(r) - 1] \frac{\gamma}{\epsilon}$$

$$t = T / T_{c0} \ll 1,$$

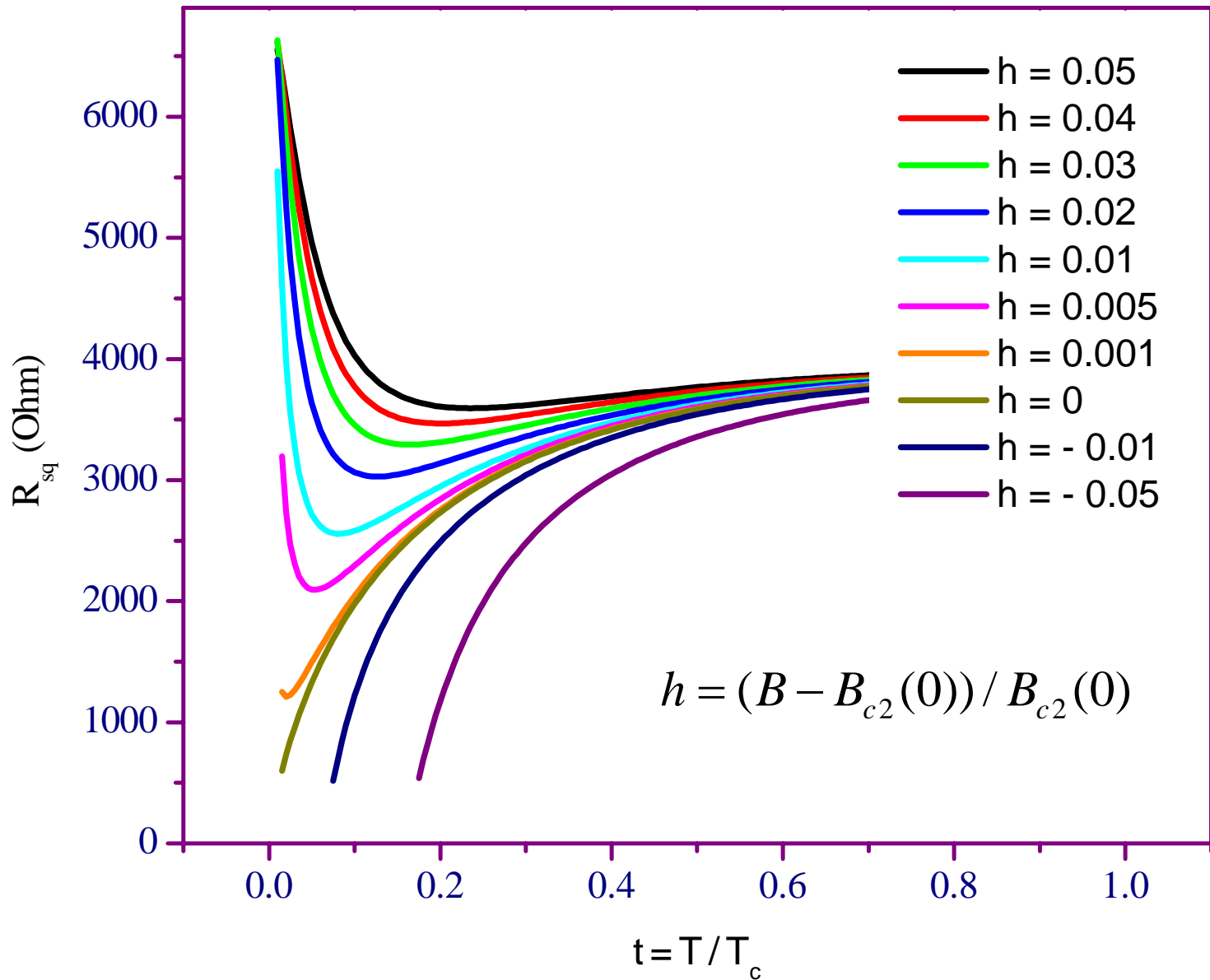
$$h = (B - B_{c2}(T)) / B_{c2}(0) \ll 1$$

$$r = (1/2g)h/t$$

$$\gamma = 1.781$$



# Superconducting fluctuations at low temperature



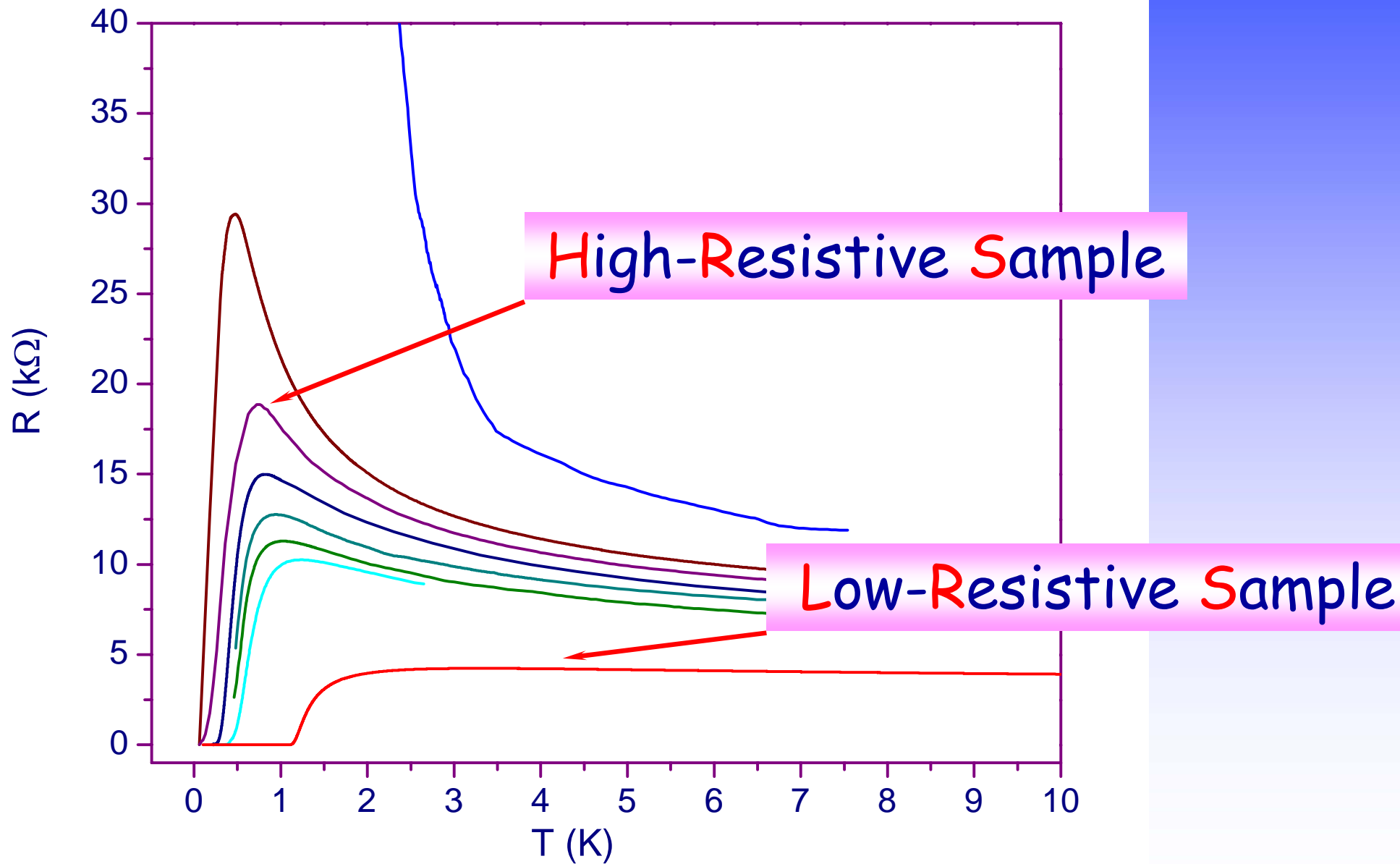
# Experiment

## TiN films

- ✓ TiN films were formed by atomic layer chemical vapor deposition onto a Si/SiO<sub>2</sub> substrate.
- ✓ The films consist of a dense packing of the crystallinities.
- ✓ The average size of the crystallinities is ~ 30 nm.

# Experiment

# Temperature dependence

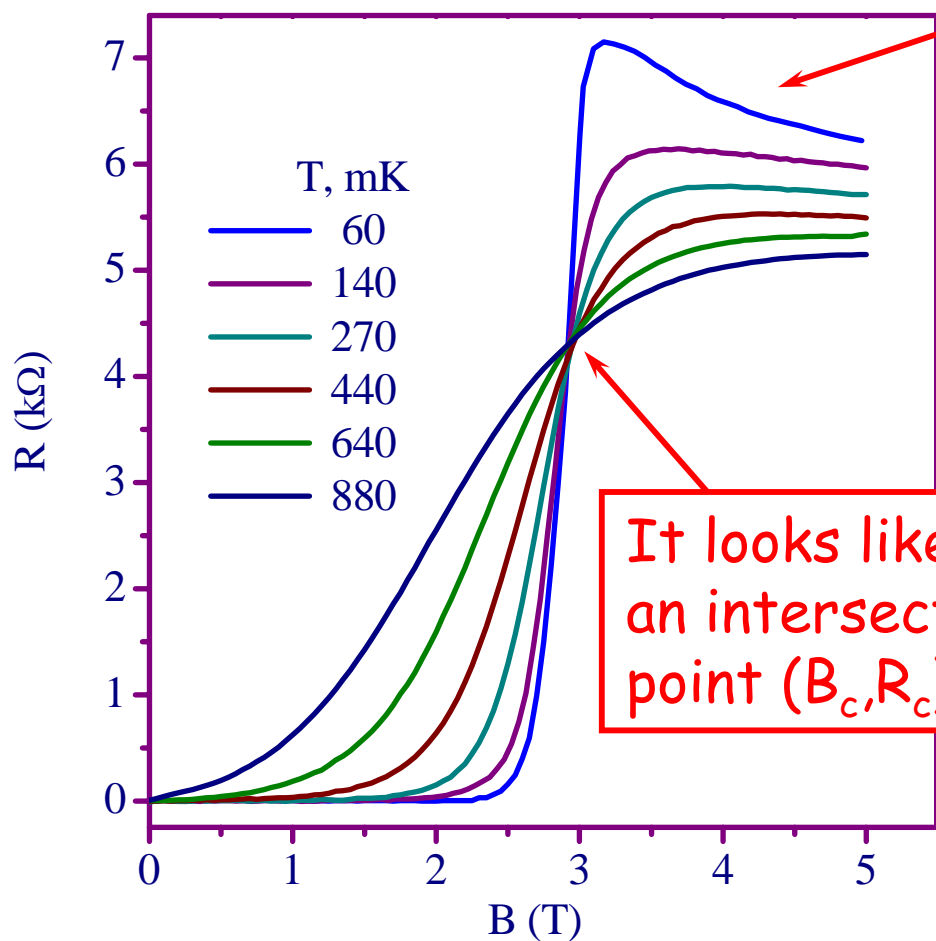


# Experiment

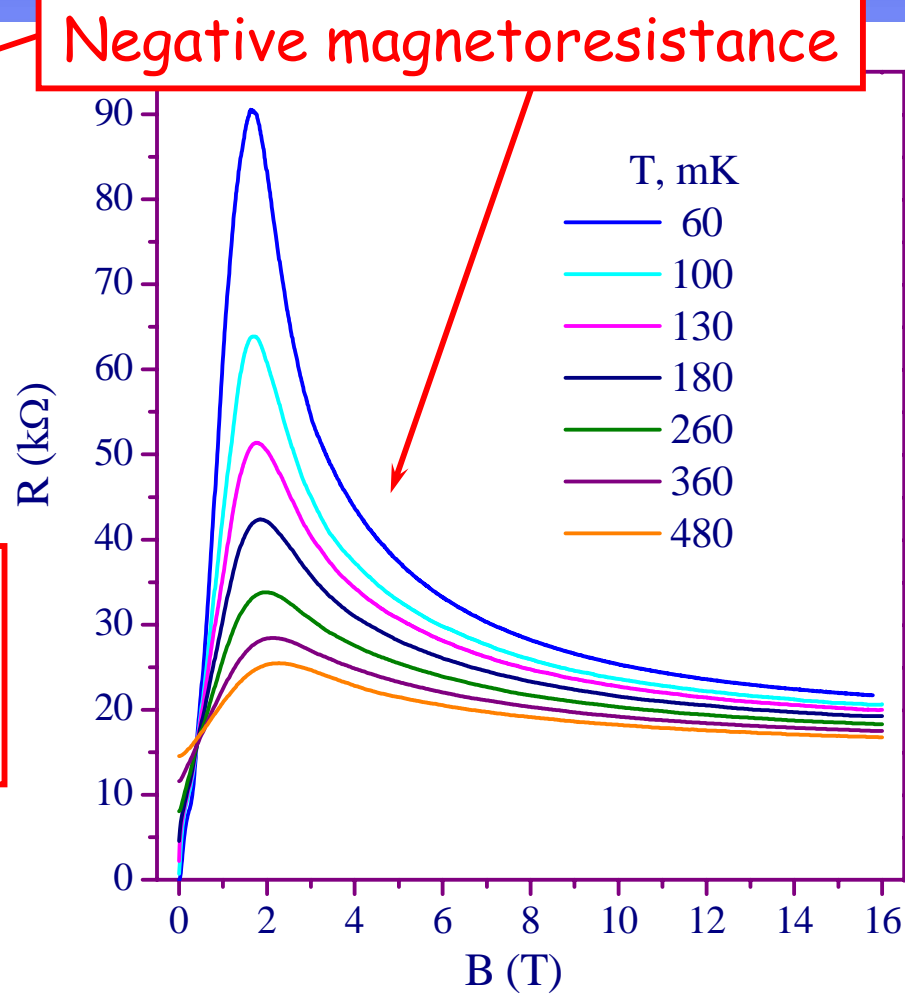
# TiN films

## Magnetic-field dependence

### Low-Resistive Sample



### High-Resistive Sample

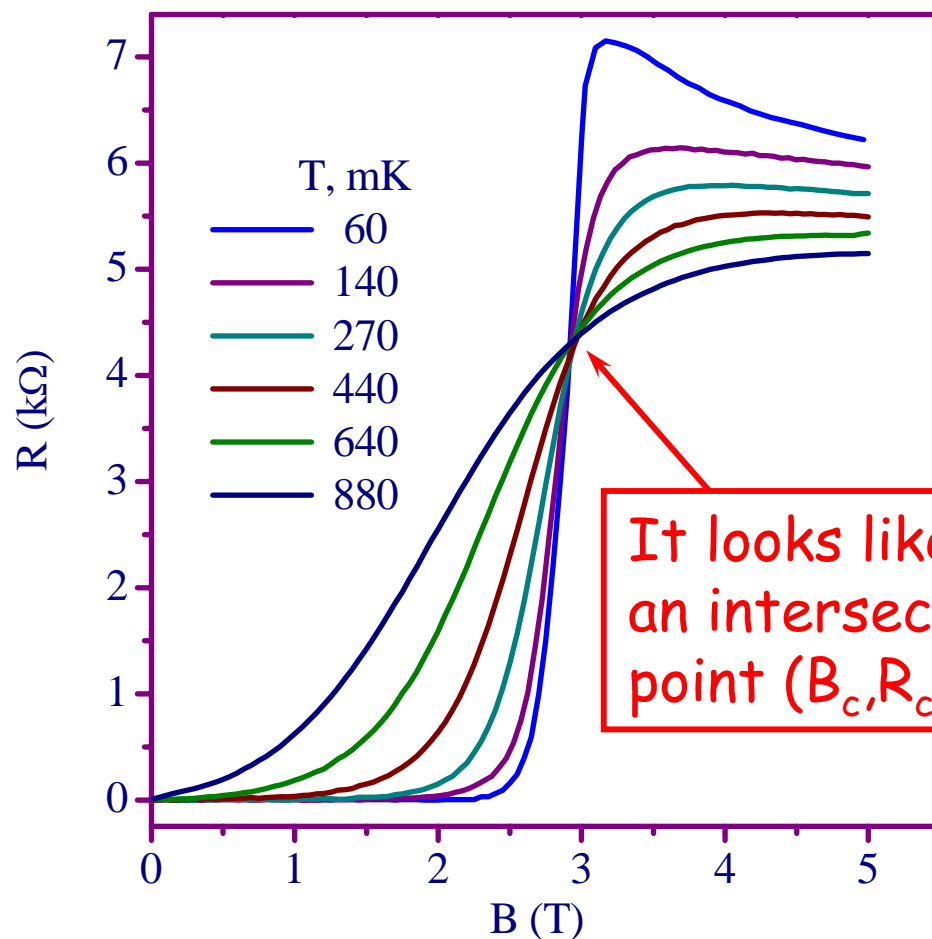


# Experiment

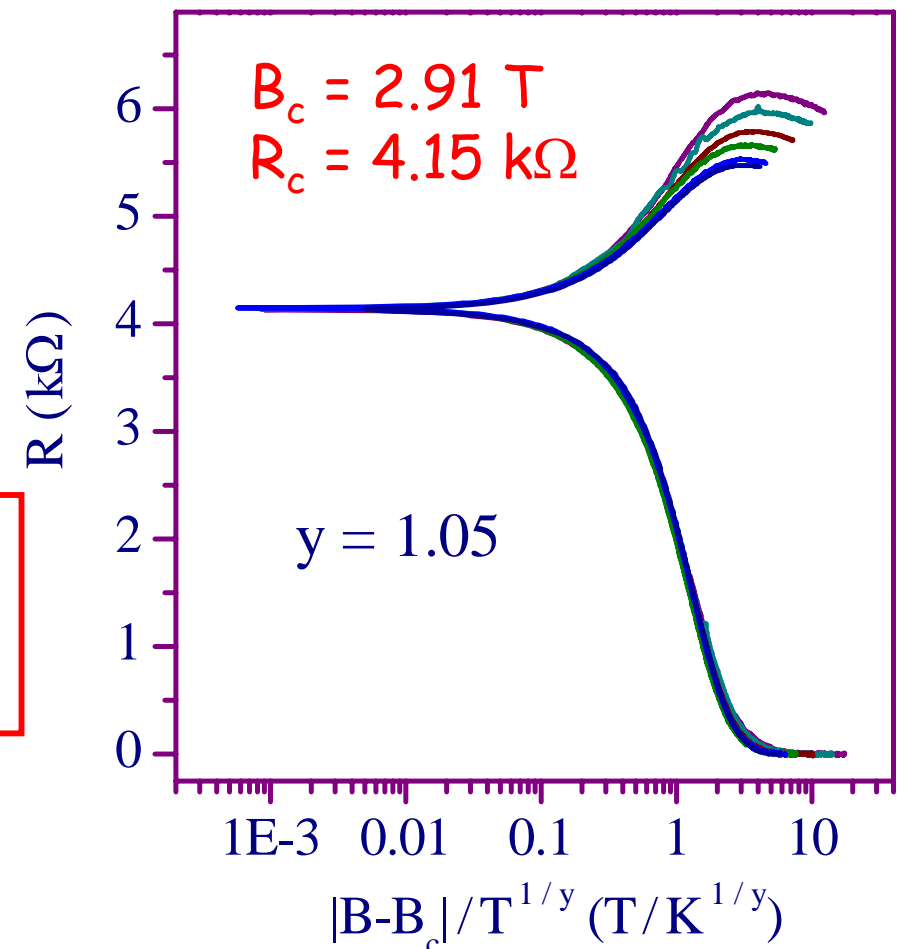
# TiN films

## Magnetic-field dependence

### Low-Resistive Sample



### Scaling

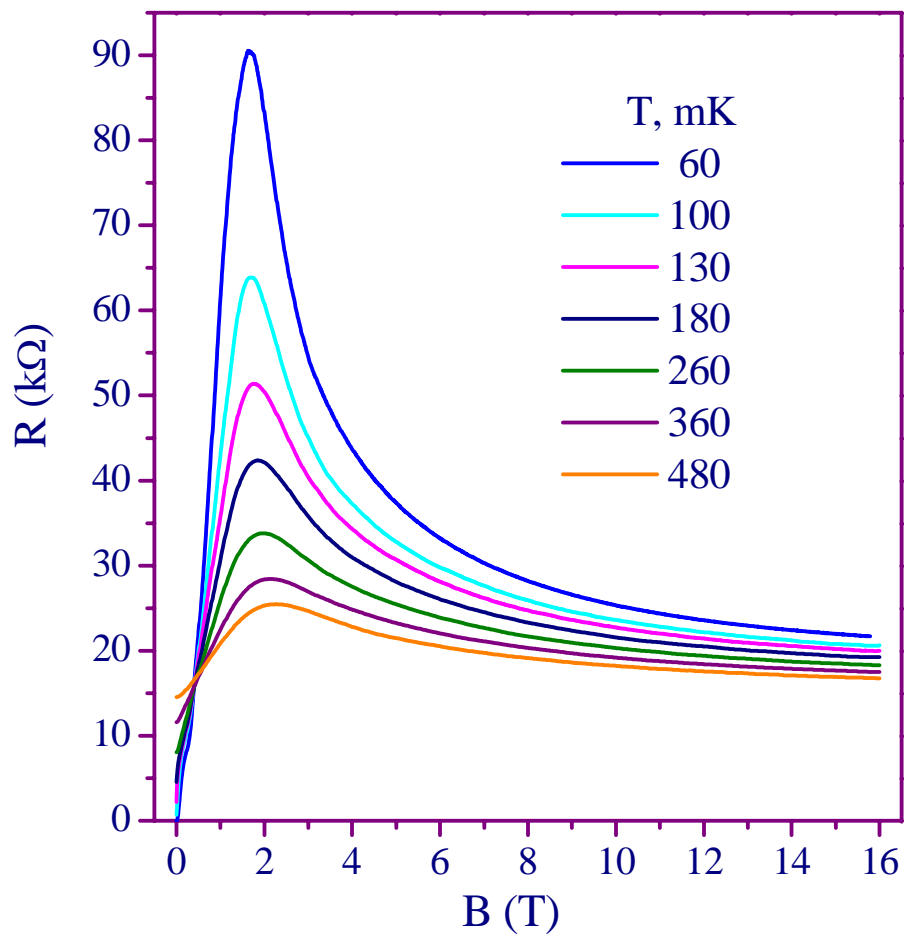


# Experiment

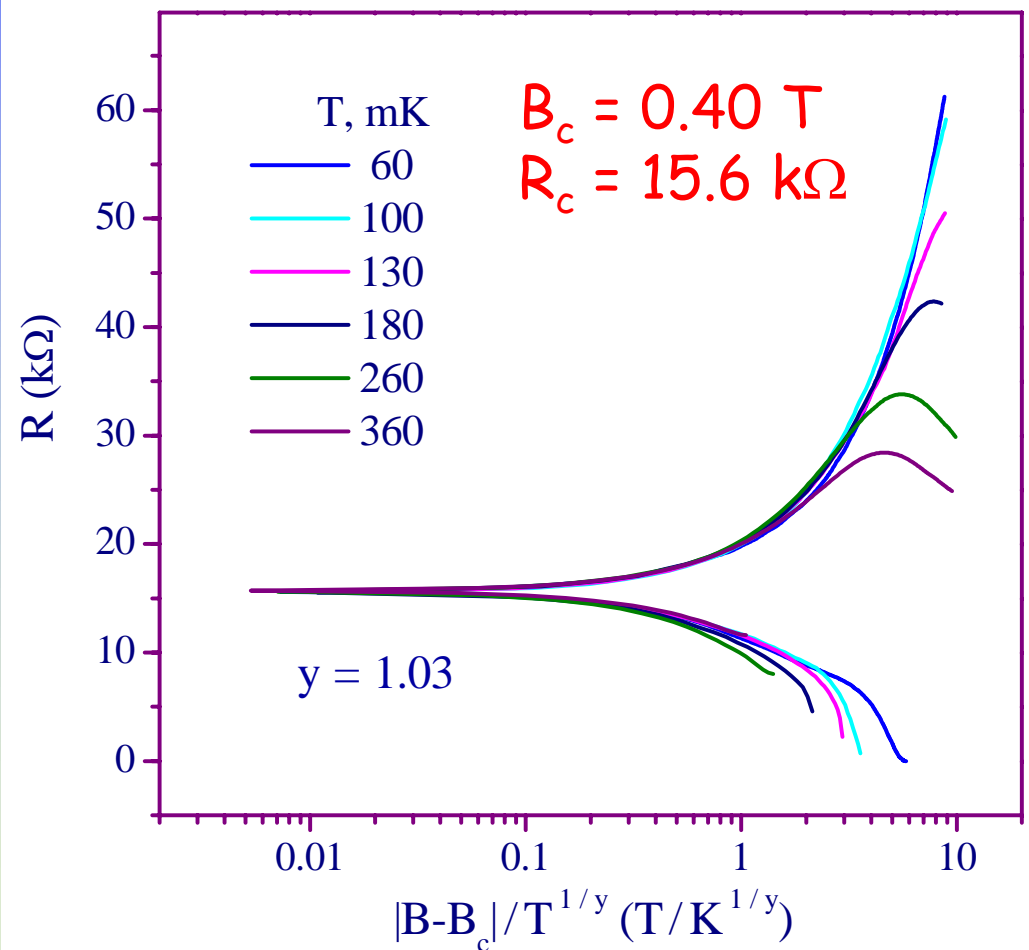
# TiN films

## Magnetic-field dependence

### High-Resistive Sample



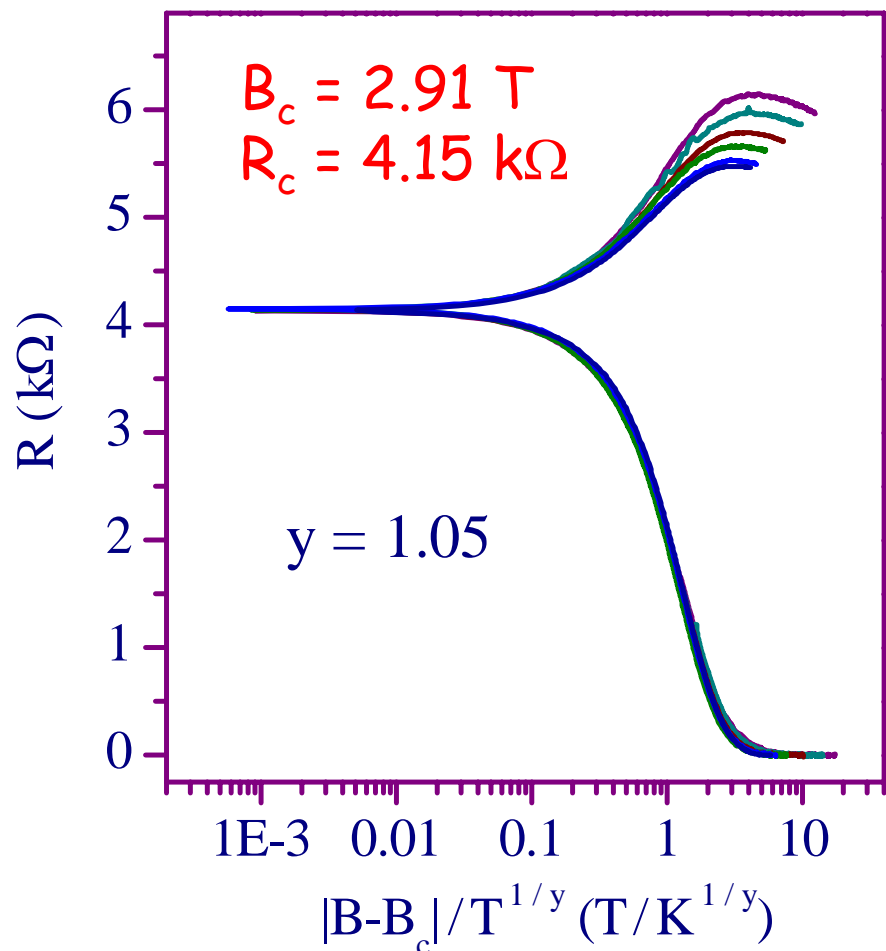
### Scaling



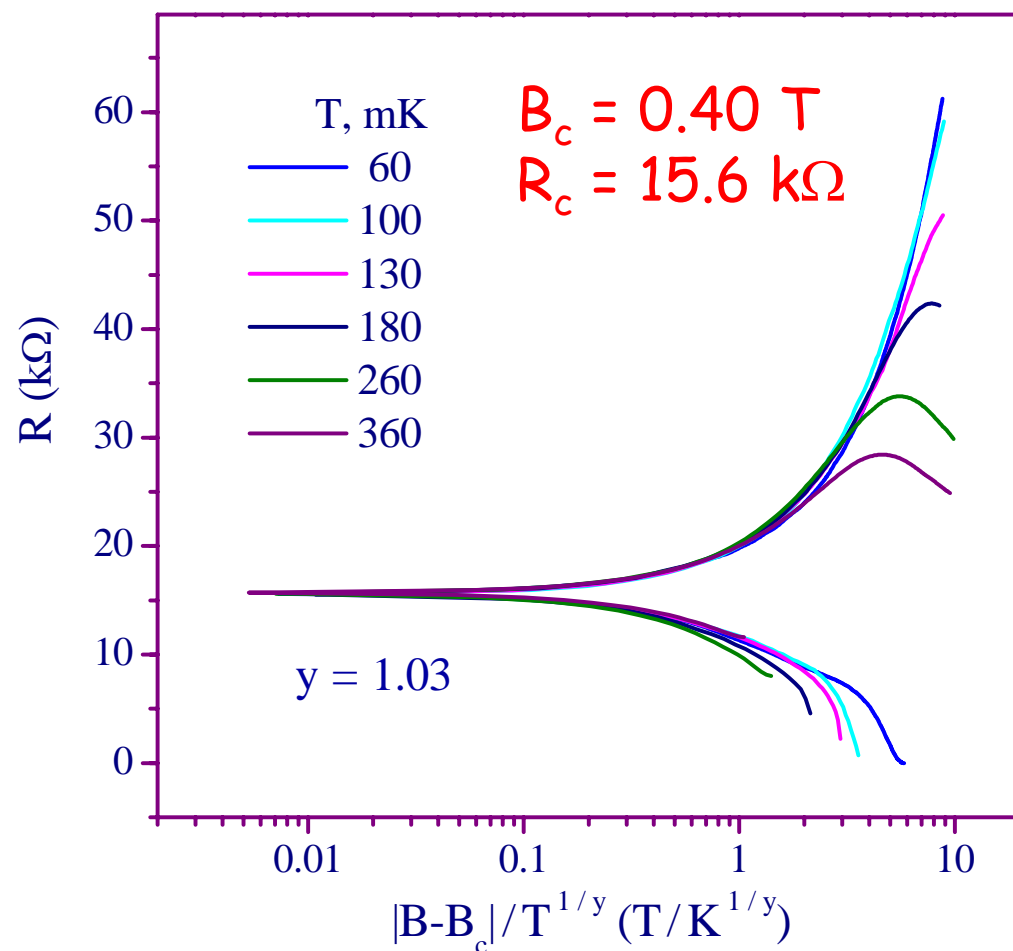
# Scaling

# Magnetic-field dependence

## Low-Resistive Sample



## High-Resistive Sample



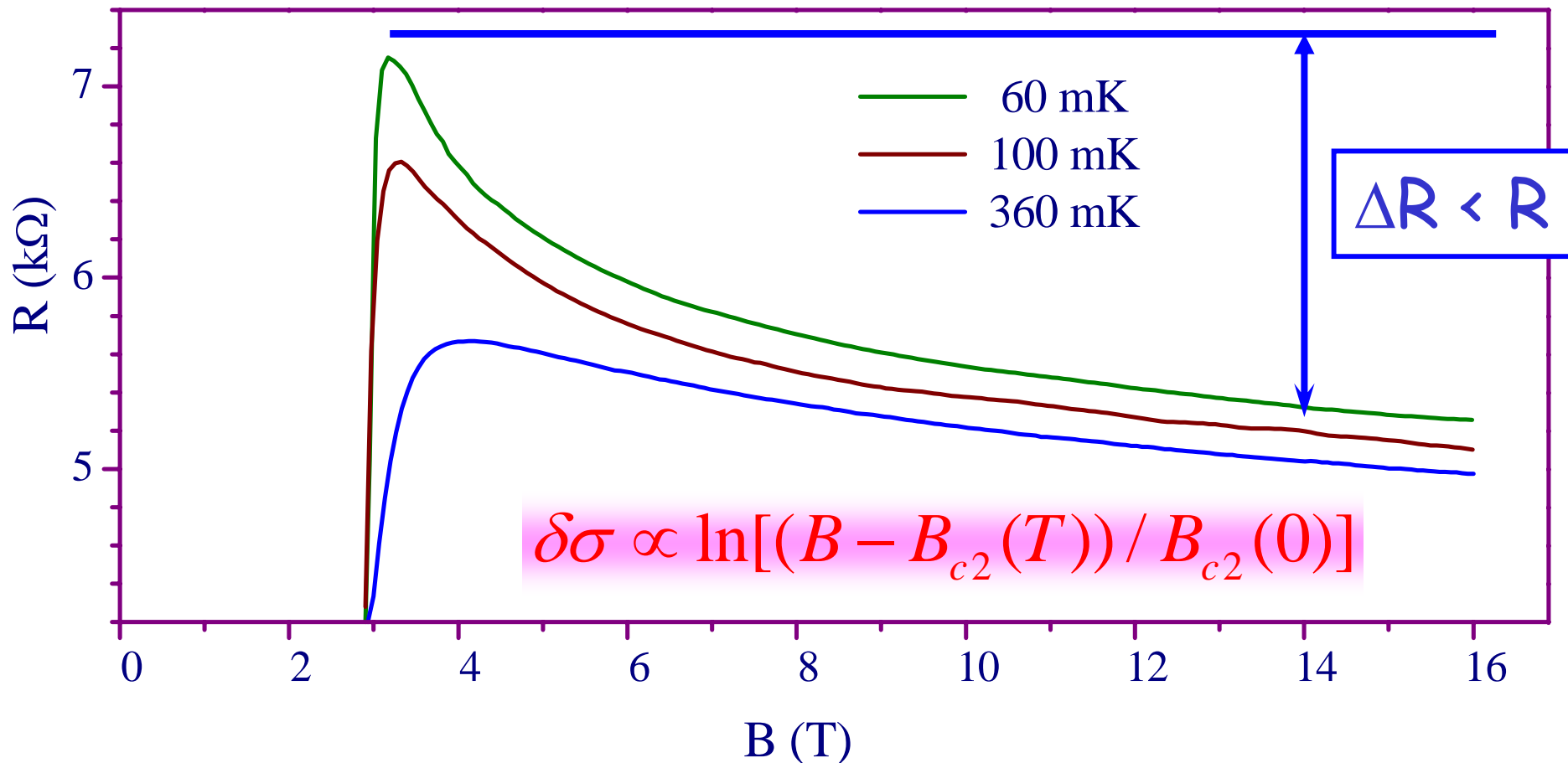
$y (= \nu z) \approx 1$  for both samples !

# Magnetic-field dependence

Low-Resistive Sample

Negative magnetoresistance

comparison with Galitski - Larkin calculations of the quantum corrections

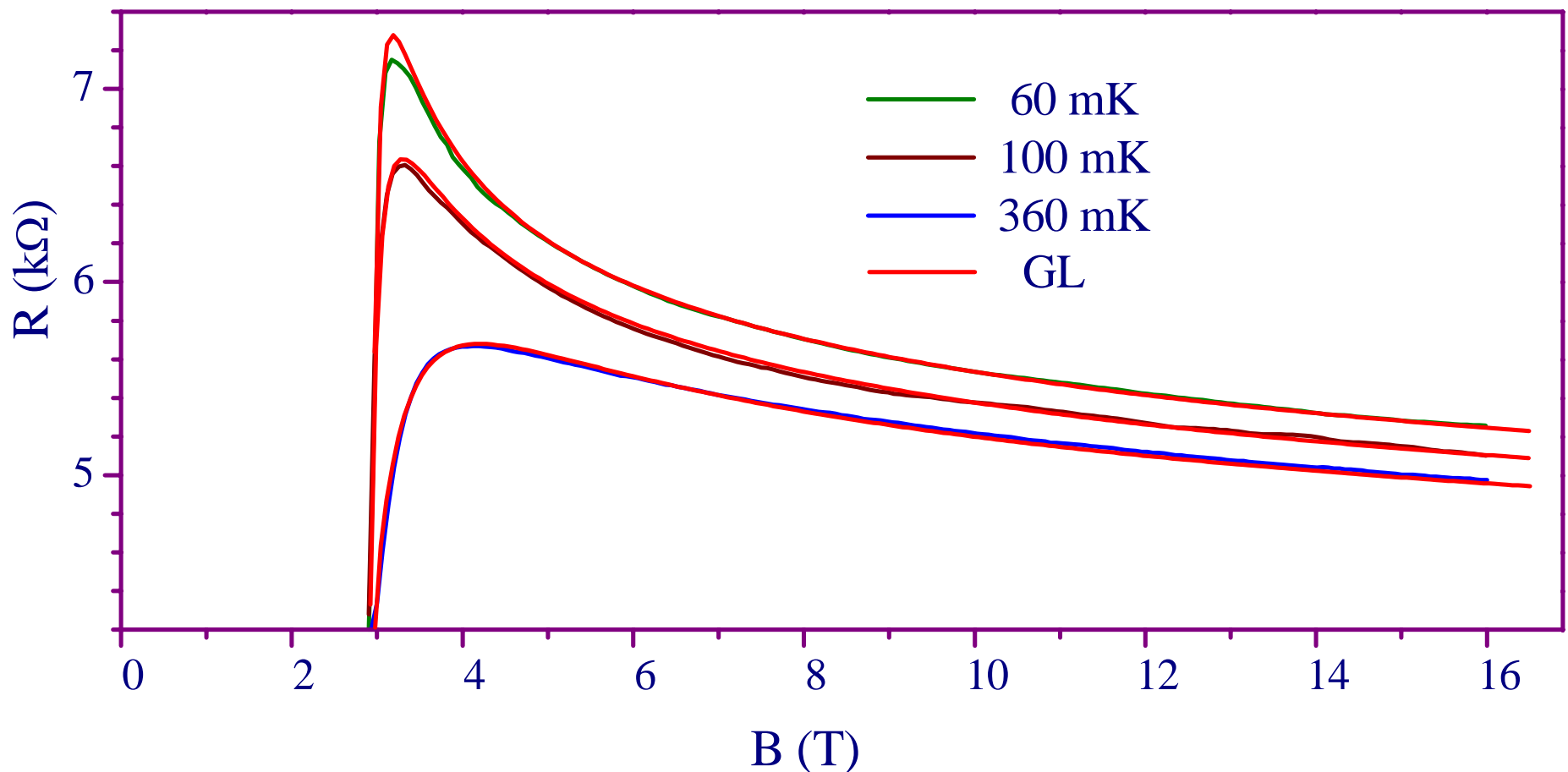




comparison with Galitski - Larkin calculations  
of the quantum corrections

$$T_c = 2 \text{ K}, B_{c2}(0) = 2.8 \text{ T}$$

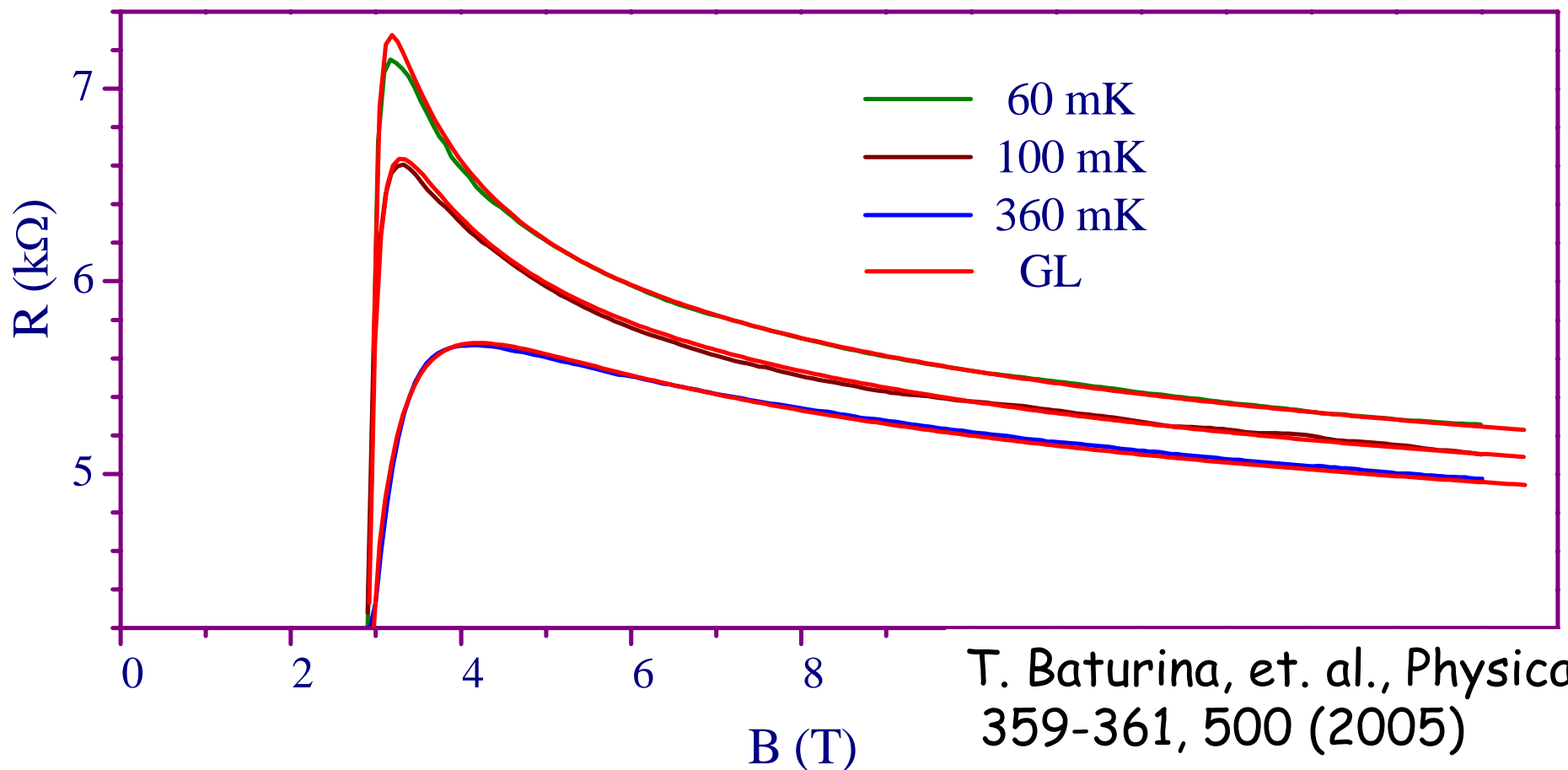
$$ds = \frac{2e^2}{3p^2h} \frac{\hat{e}}{\hat{e}} \ln \frac{r}{h} - \frac{3}{2r} + y(r) + 4[ry'(r) - 1]$$



comparison with Galitski - Larkin calculations  
of the quantum corrections

$$T_c = 2 \text{ K}, B_{c2}(0) = 2.8 \text{ T}$$

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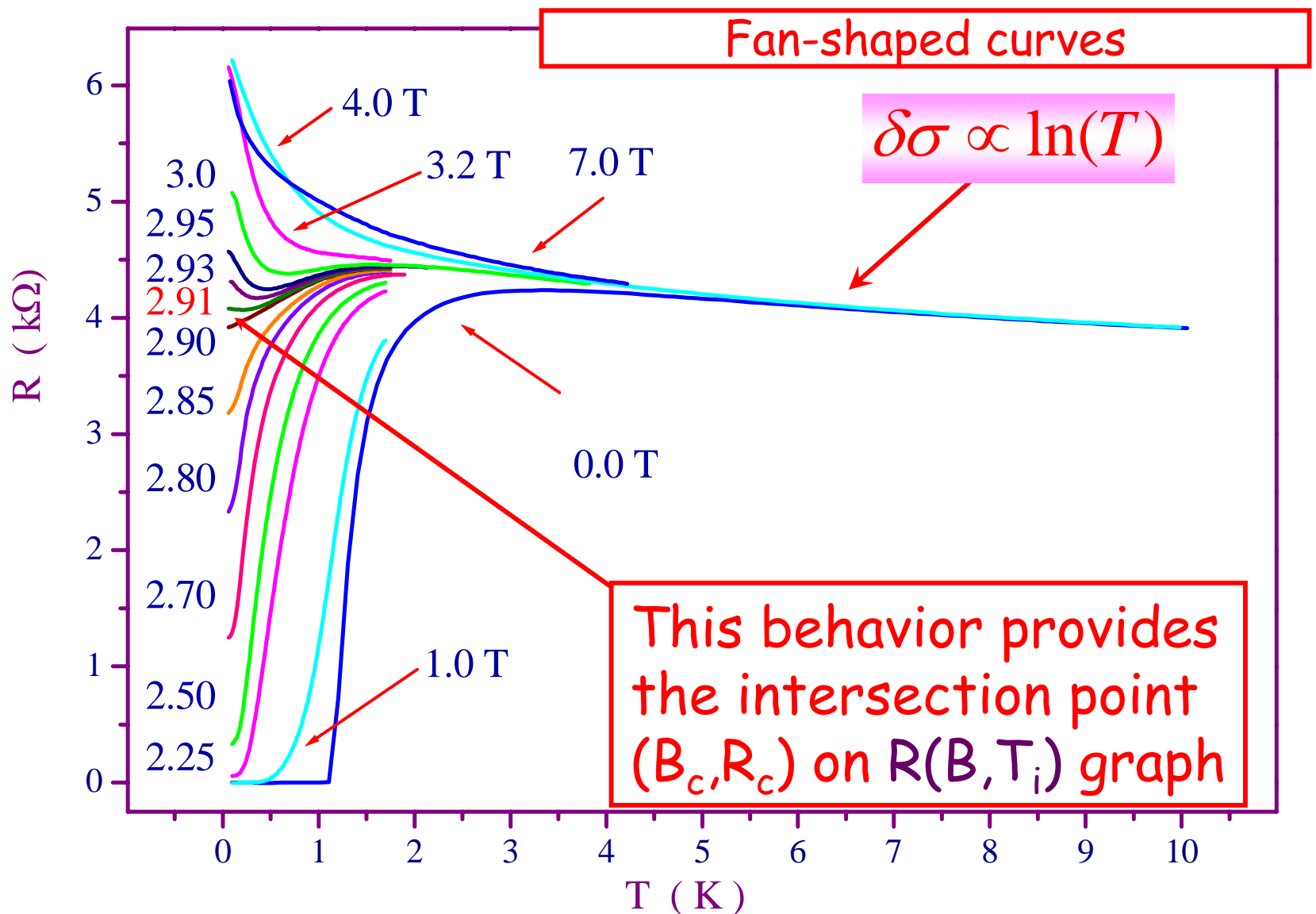


T. Baturina, et. al., Physica B  
359-361, 500 (2005)

# Temperature dependence

Low-Resistive Sample

$R(T, B_i)$



# LRS

## Temperature dependence

comparison with Galitski - Larkin calculations of the quantum corrections

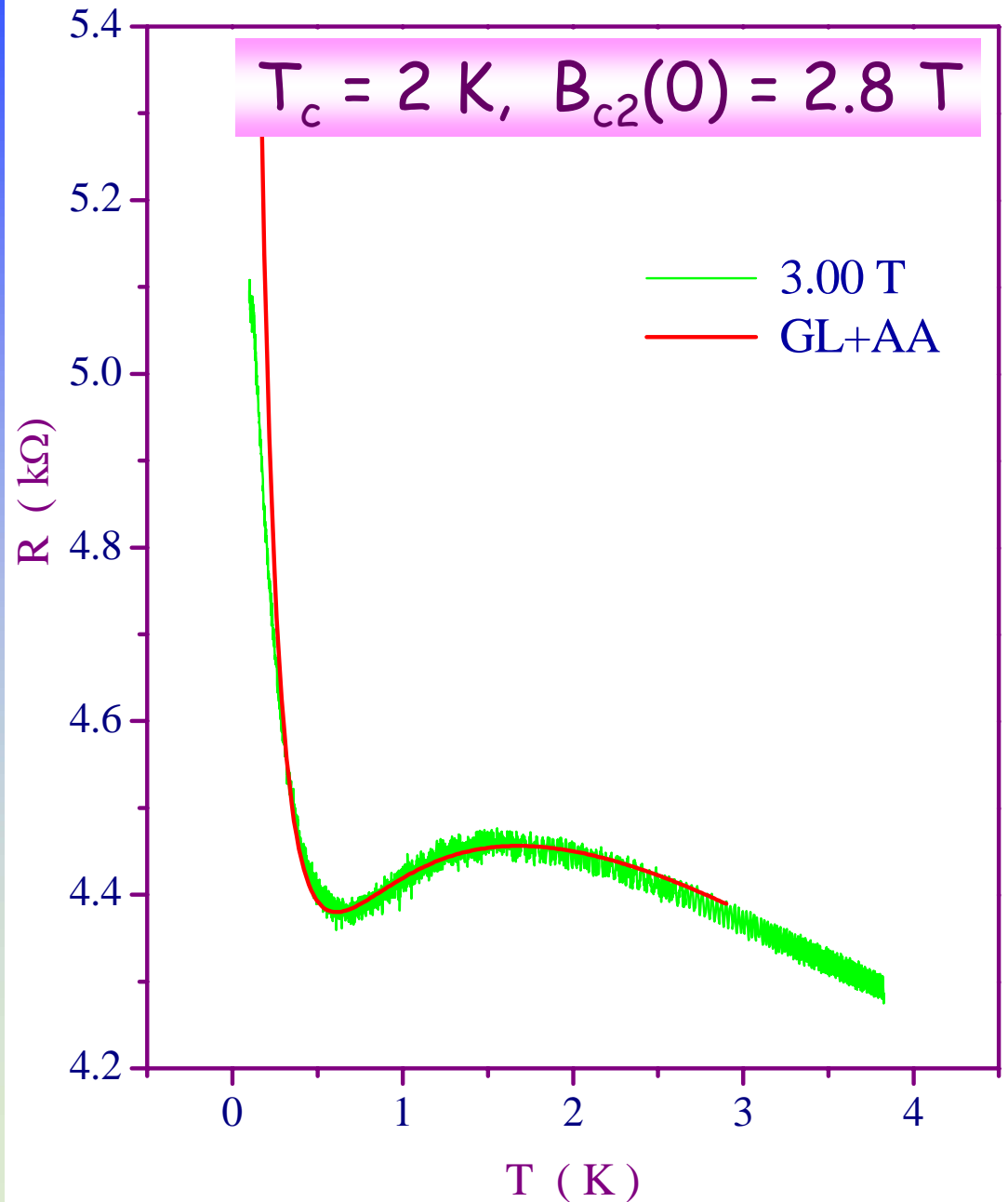
Cooper Channel

+

Aronov - Altshuler

$$\delta\sigma_{AA} \propto \ln(T)$$

Diffusion Channel



# LRS

## Temperature dependence

comparison with Galitski - Larkin calculations of the quantum corrections

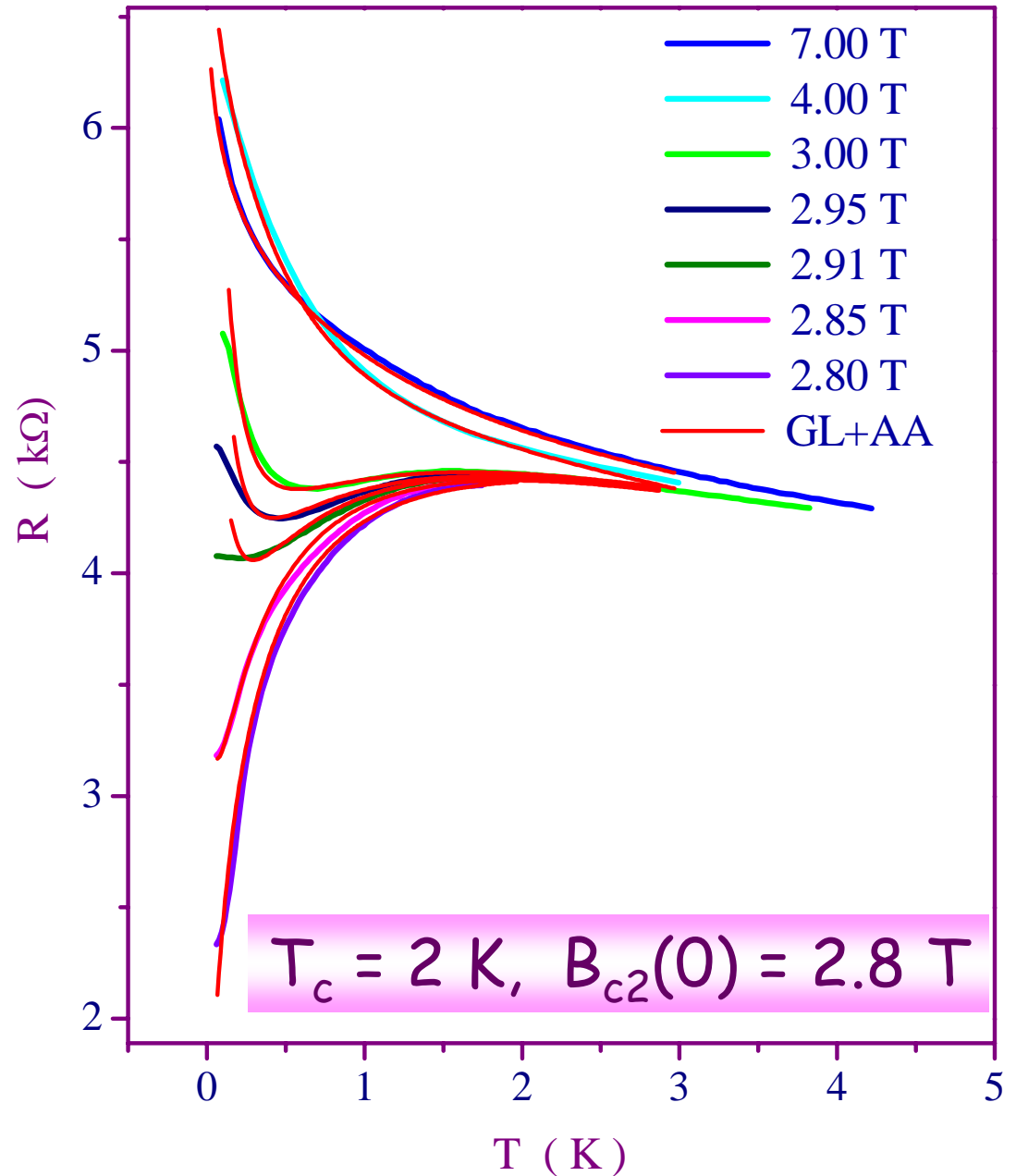
Cooper Channel

+

Aronov - Altshuler

$$\delta\sigma_{AA} \propto \ln(T)$$

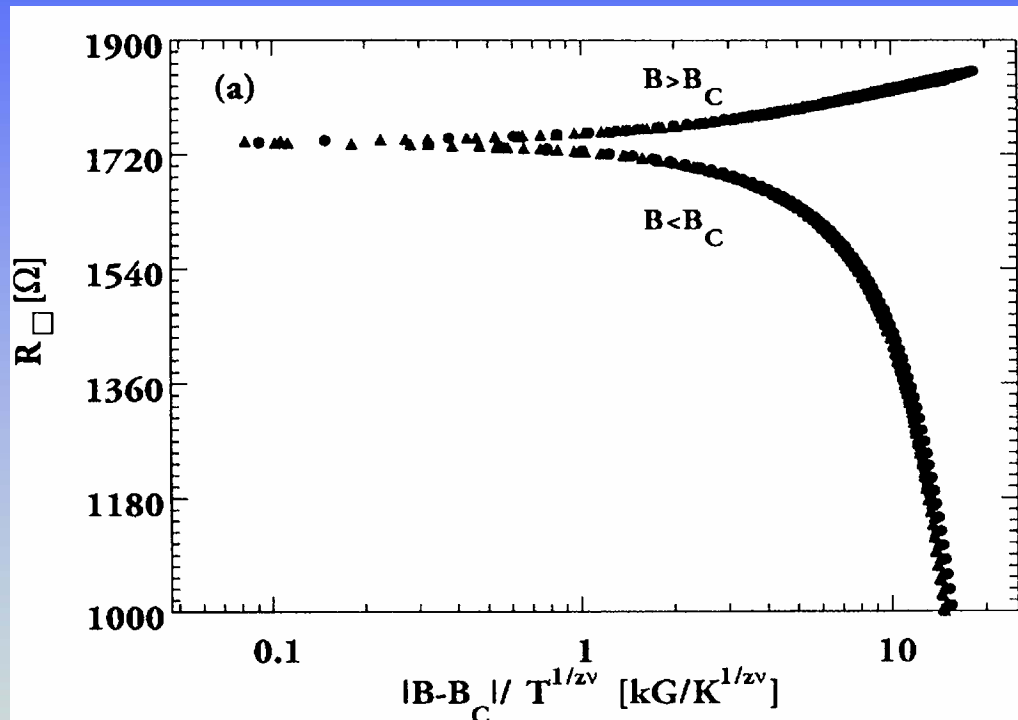
Diffusion Channel



# Superconducting fluctuations at low temperature

## Scaling

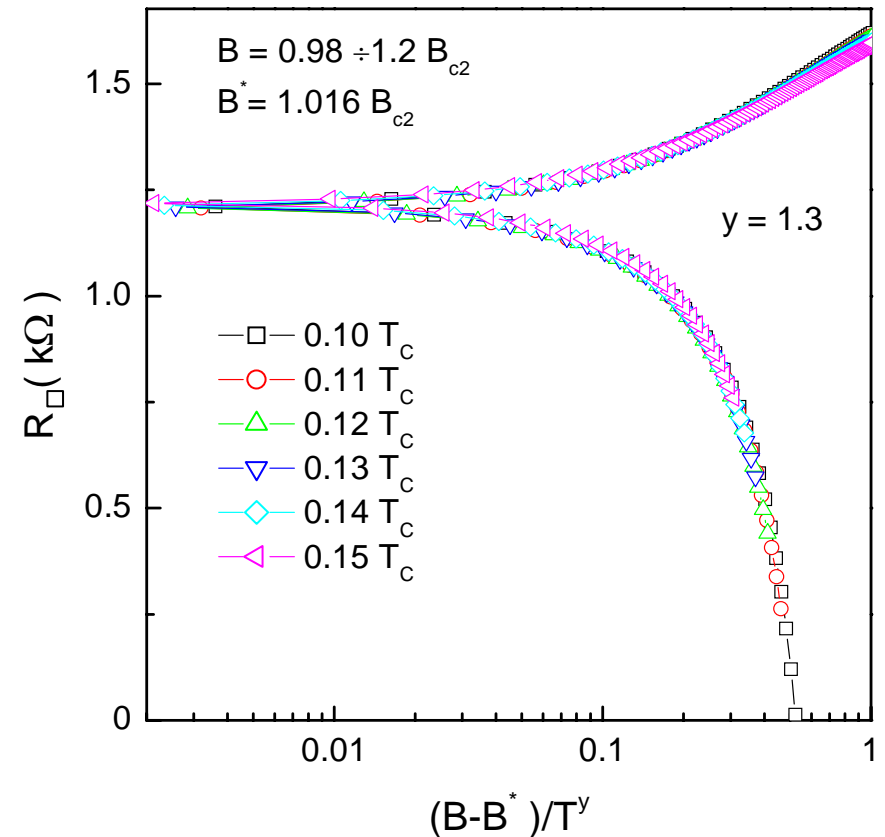
A. Yazdani and A. Kapitulnik (1995)



$$T_C = 0.15 \text{ K} \quad B_C = 4.19 \text{ kG}$$

$$T = 0.08 \div 0.11 \text{ K} \quad B - B_C < 1 \text{ kG}$$

$$z\nu = 1.36$$



Theoretical expression does not contain scaling properties, but in restricted region of values  $T$  and  $B$  scaling presentation can be done

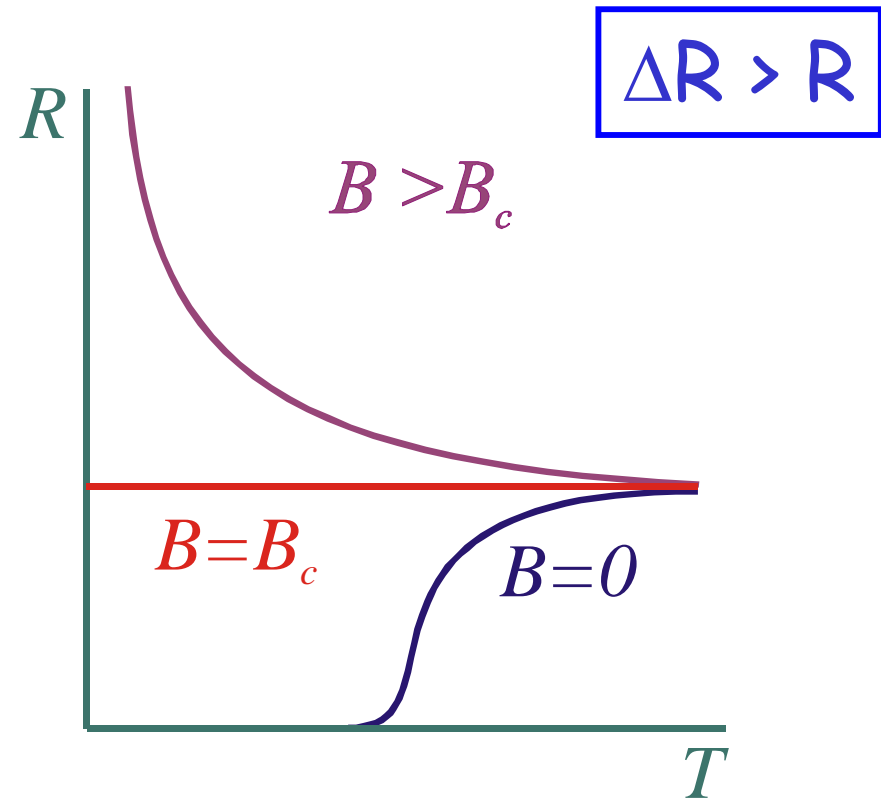
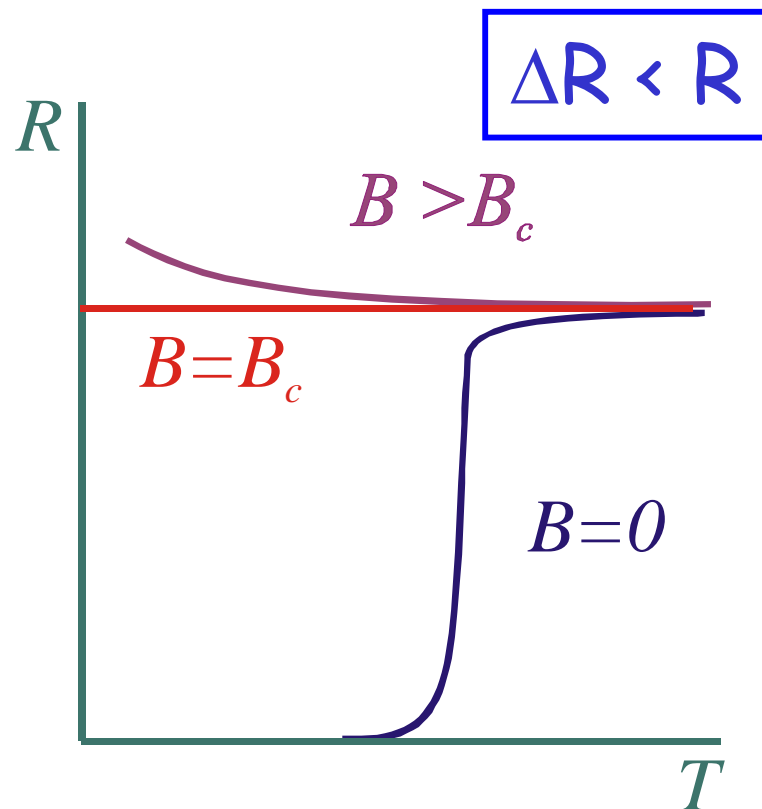
[V.F. Gantmakher, S.N. Ermolov, G.E. Tsydynzhapov, A.F. Zhukov, T.I. Baturina, JETP Lett. 77, 424 (2003)]

# Suppression of Superconductivity by **Magnetic Field**

## Field-induced...

*superconductor –  
metal  
transition*

*superconductor –  
insulator  
transition*



# Suppression of Superconductivity by **Magnetic Field**

## Conclusion (first step)

*superconductor –  
metal  
transition*

V.M. Galitski and A.I. Larkin,  
PRB **63**, 174506 (2001)

*superconductor –  
insulator  
transition*

Matthew P.A. Fisher,  
PRL **65**, 923 (1990)

### Common features

- ✓ Fan-shaped structure of  $R(T, B_i)$  curves
- ✓ Negative magnetoresistance in high fields
- ✓ Scaling

**All main features** can be explained in the frames of the theory of **the quantum corrections** to the conductivity in disordered metals !



**Let's take a close look**

**(second step)**





N. Hadacek, M. Sanquer, and J.-C. Villegier,  
Double reentrant superconductor-insulator transition in TiN films,  
PRB 69, 024505 (2004)

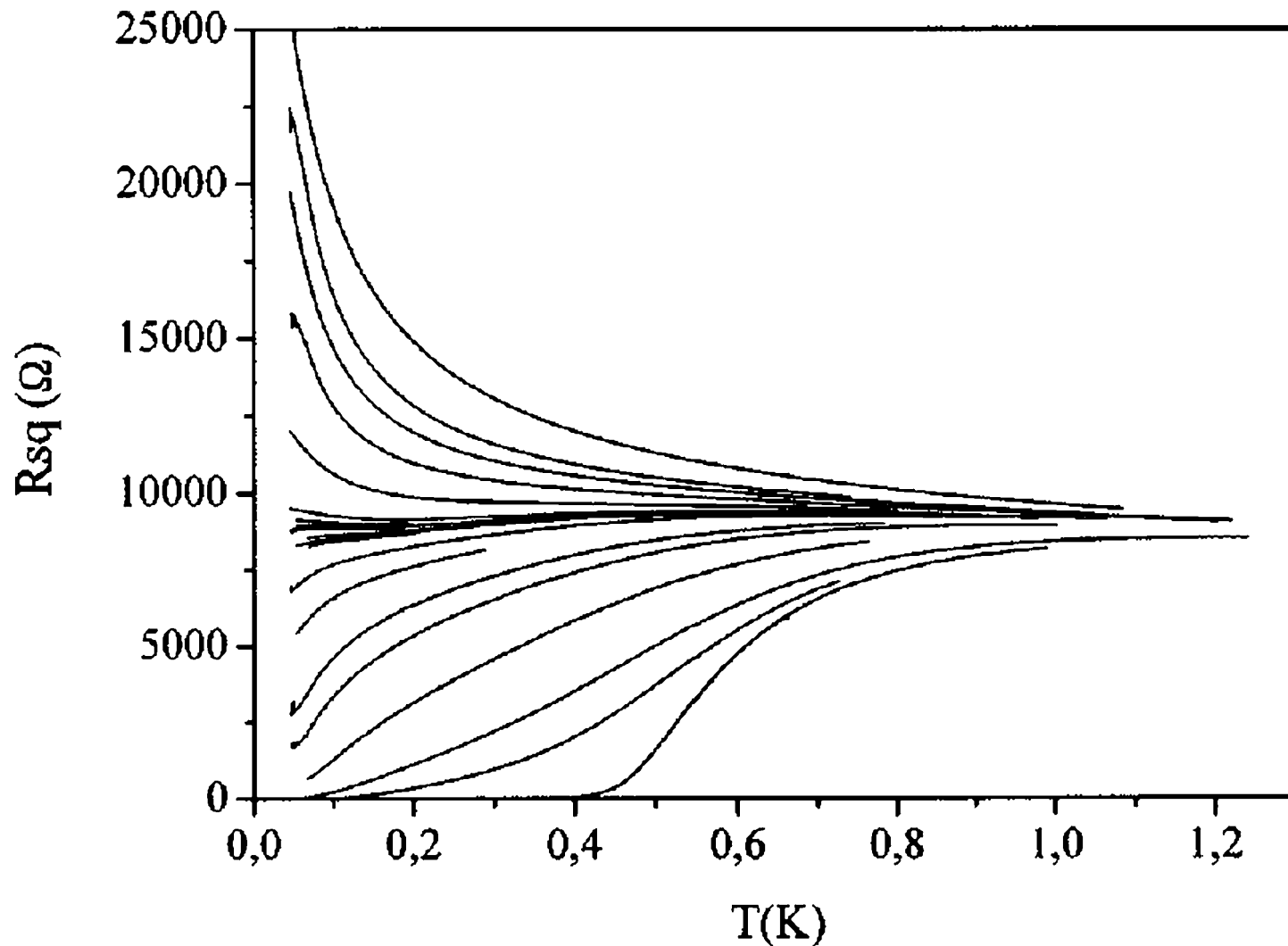
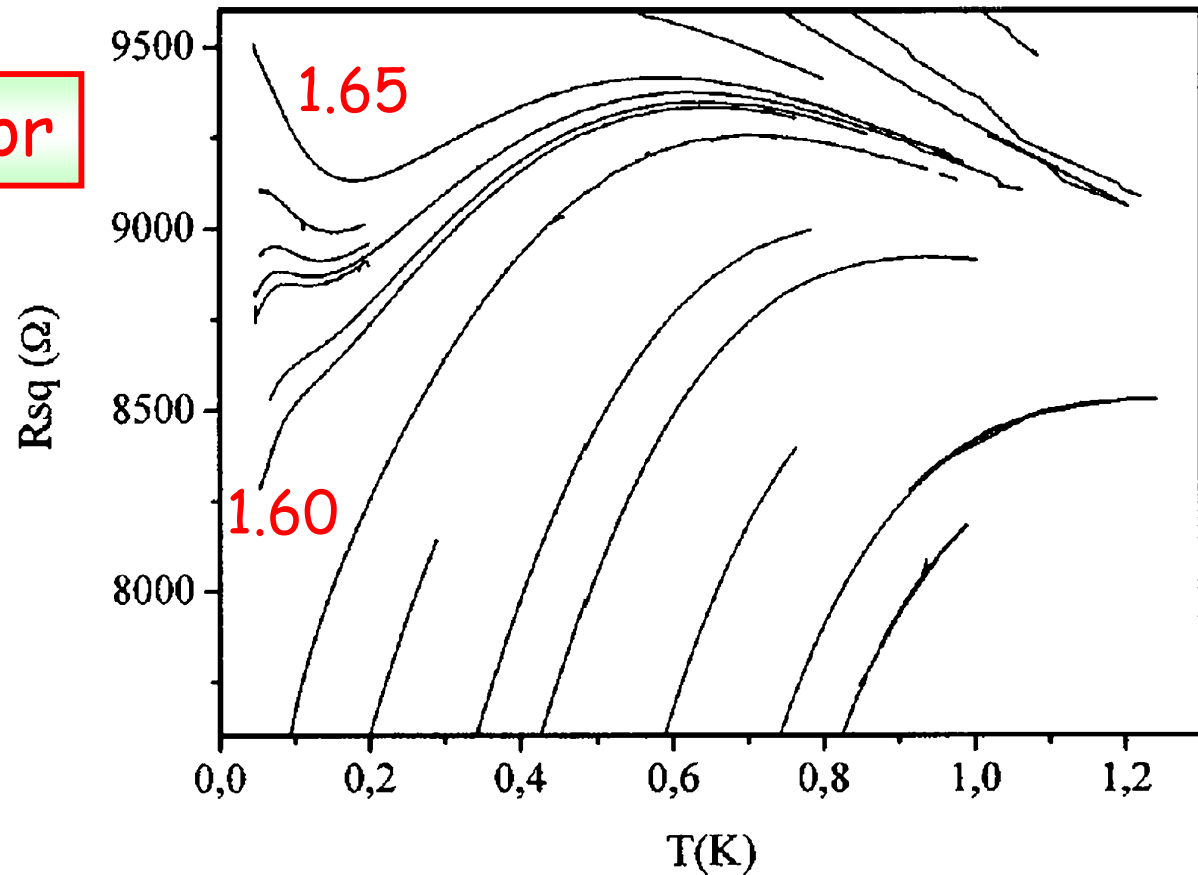
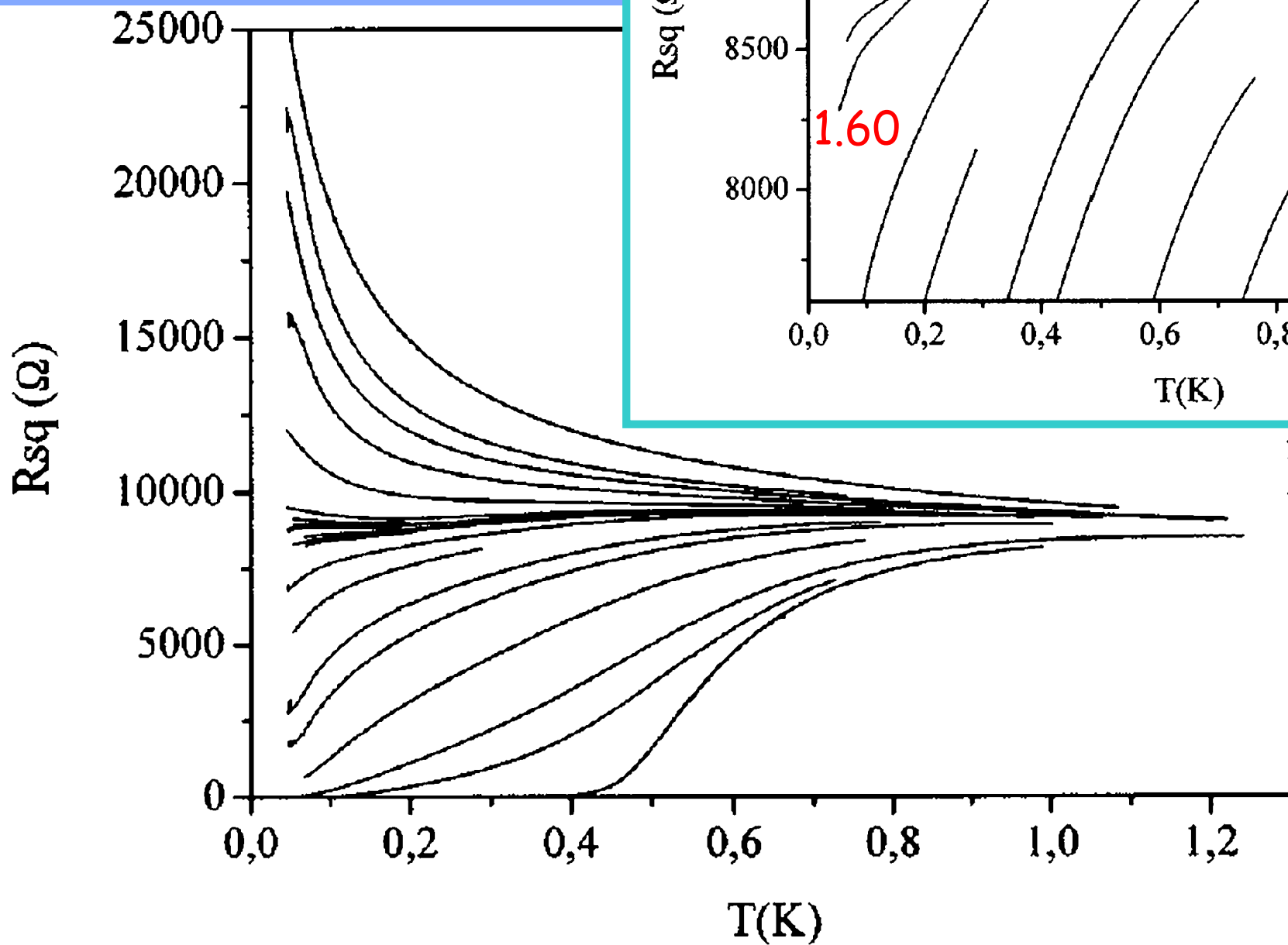
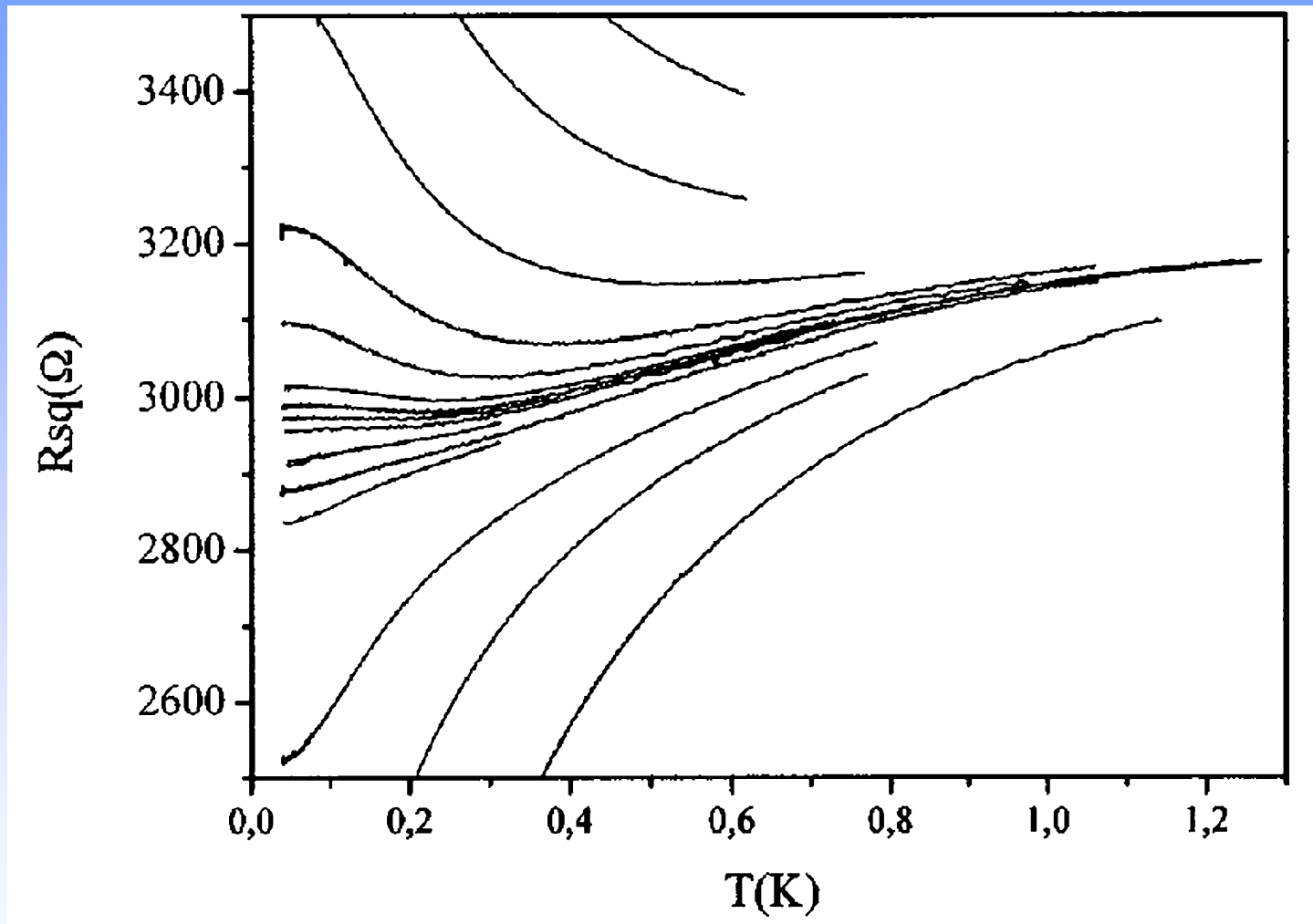


FIG. 4. Resistance per square versus temperature for various perpendicular magnetic fields. Top: TiN NH63 sample. From bottom to top the magnetic field is 0, 0.3, 0.6, 1.0, 1.3, 1.4, 1.5, 1.57, 1.60, 1.615, 1.625, 1.635, 1.64, 1.645, 1.65, 1.7, 1.8, 1.9, 2.0, 2.4 T.

double reentrant behavior

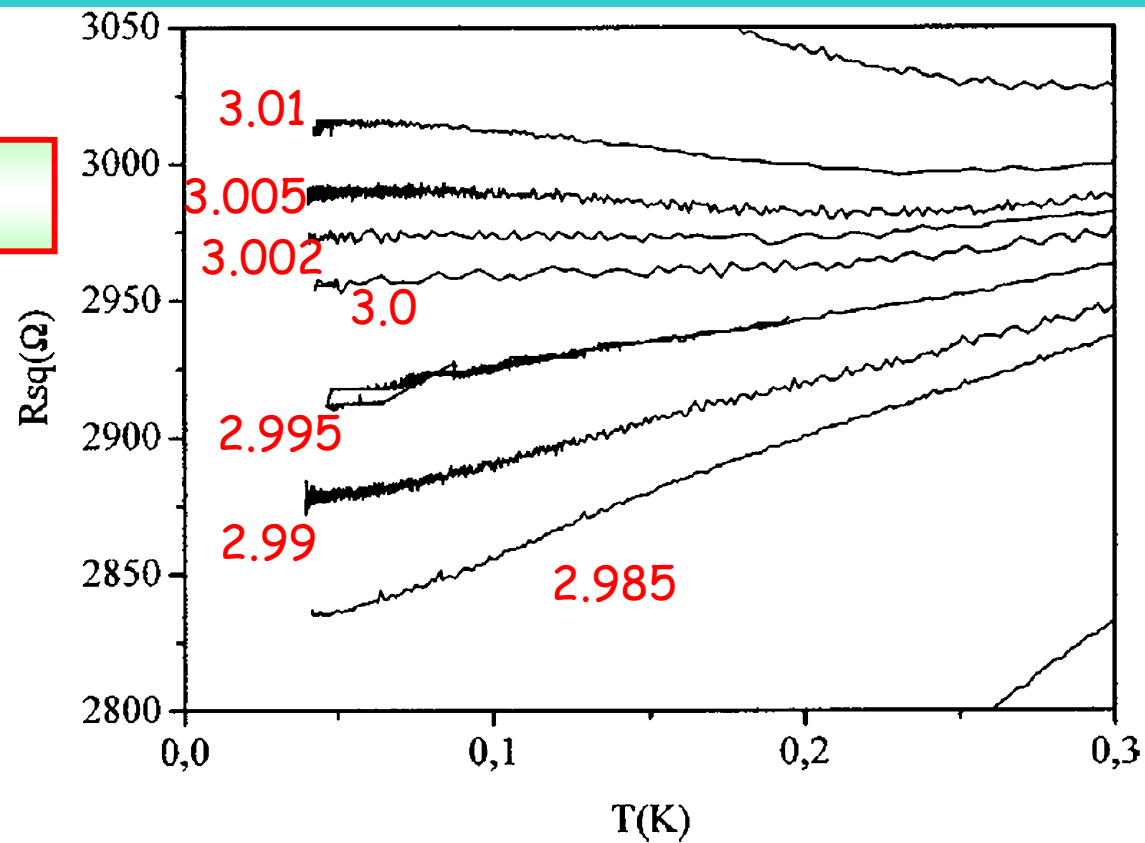
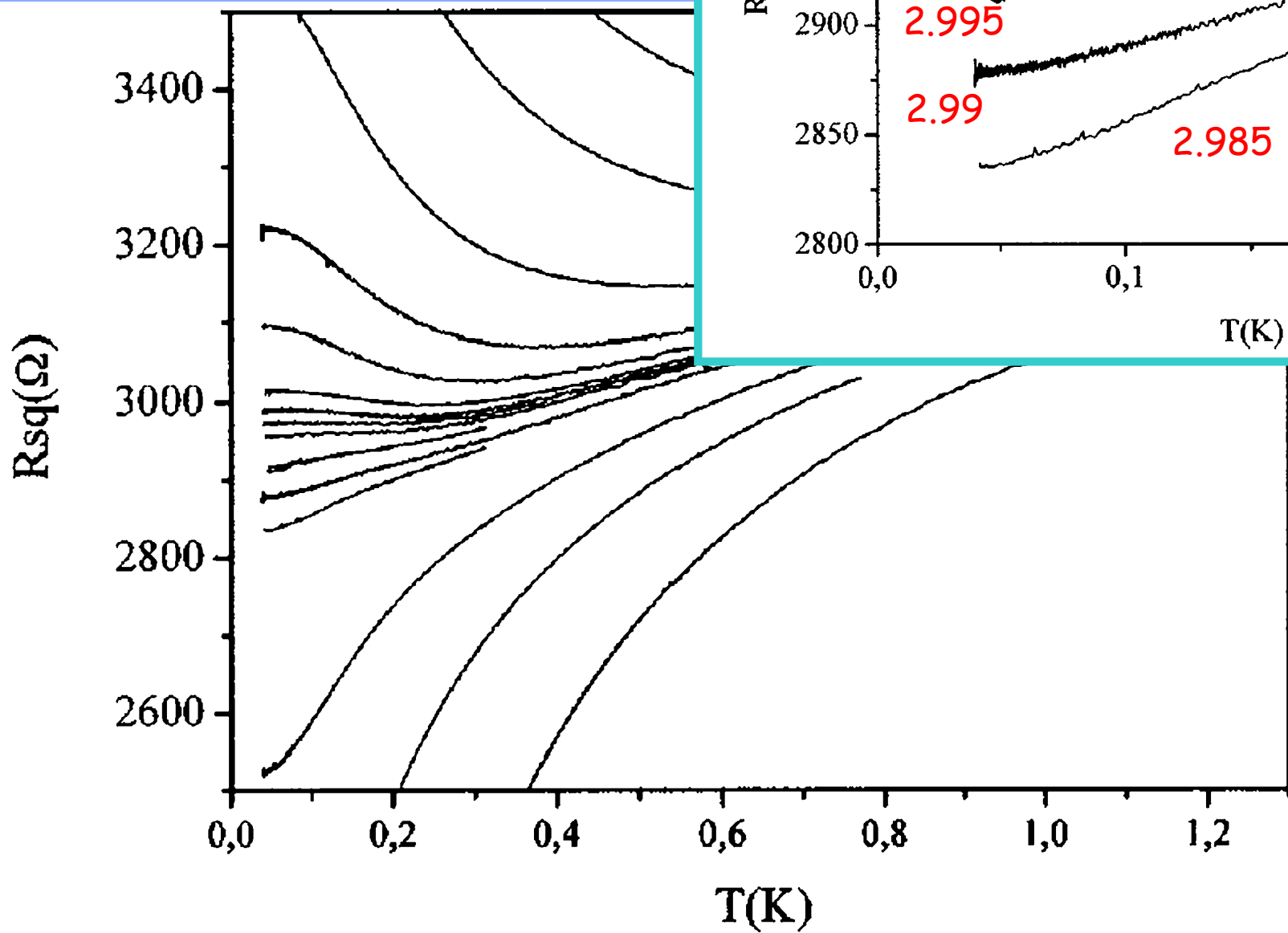


N. Hadacek, M. Sanquer, and J.-C. Villegier,  
Double reentrant superconductor-insulator transition in **TiN** films,  
PRB 69, 024505 (2004)



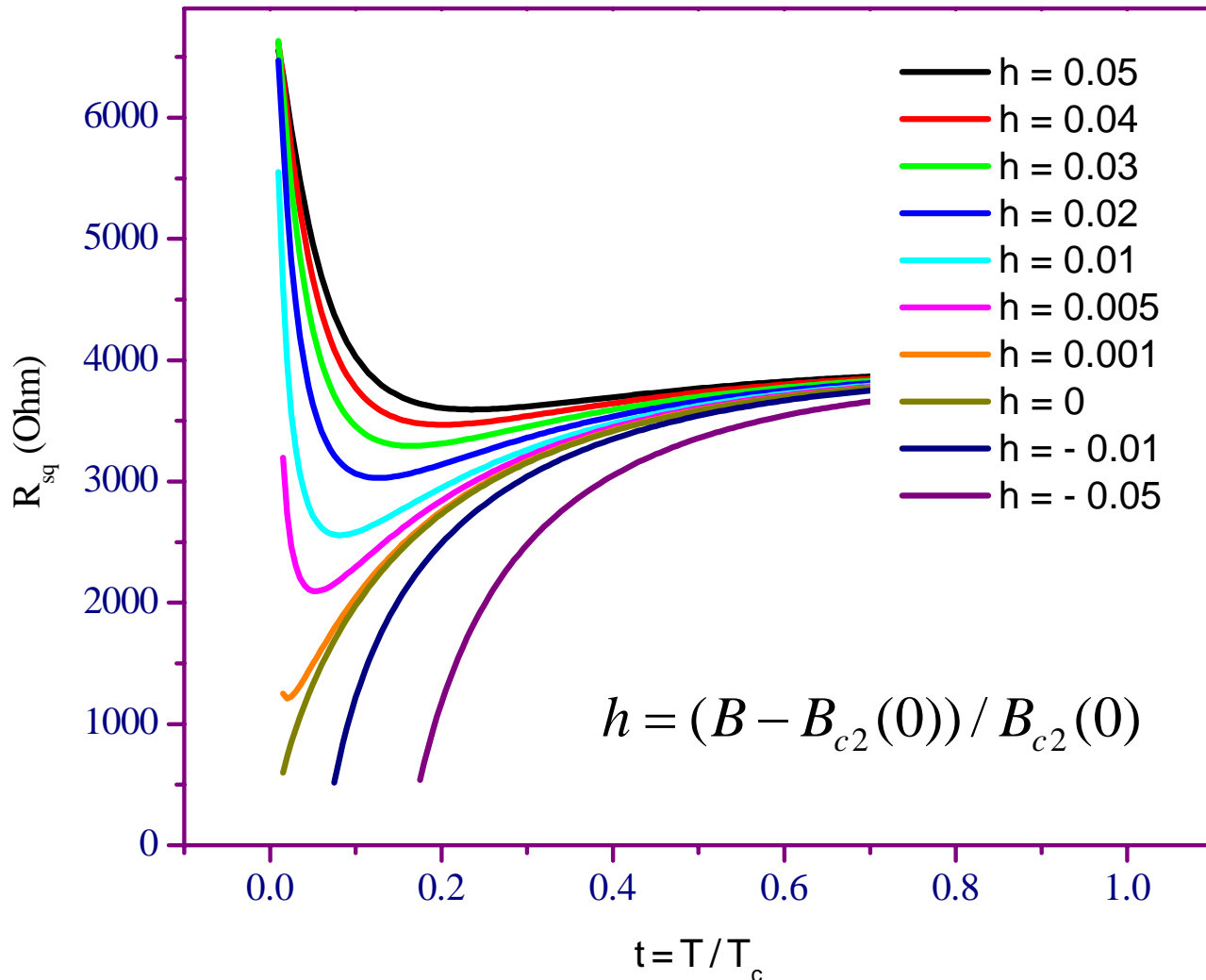
Bottom: TiN NH57 sample. From bottom to top the magnetic field is 2.8, 2.9, 2.95, 2.985, 2.99, 2.995, 3.0, 3.002, 3.005, 3.01, 3.02, 3.1, 3.25, 3.5 T.

the saturation behavior

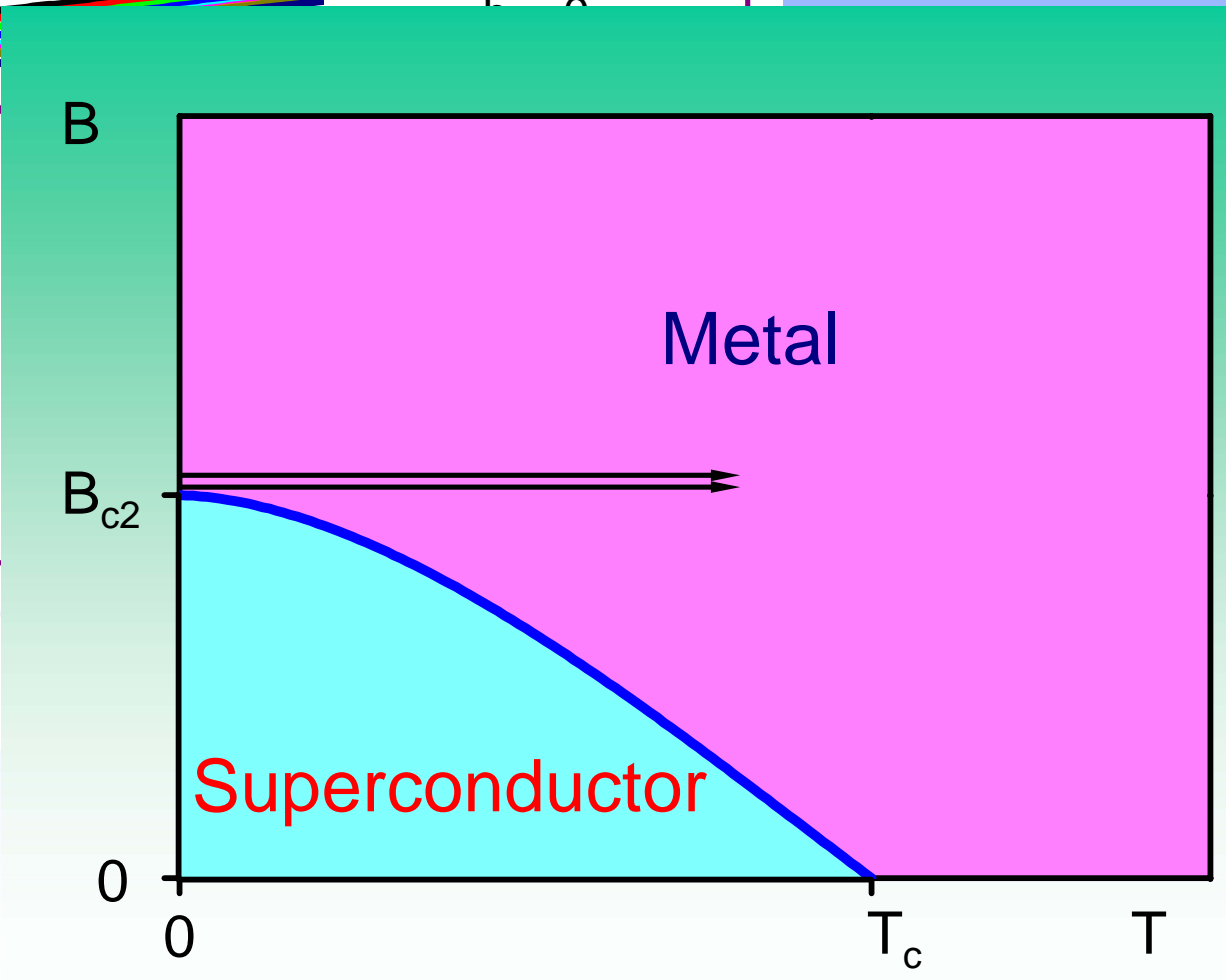
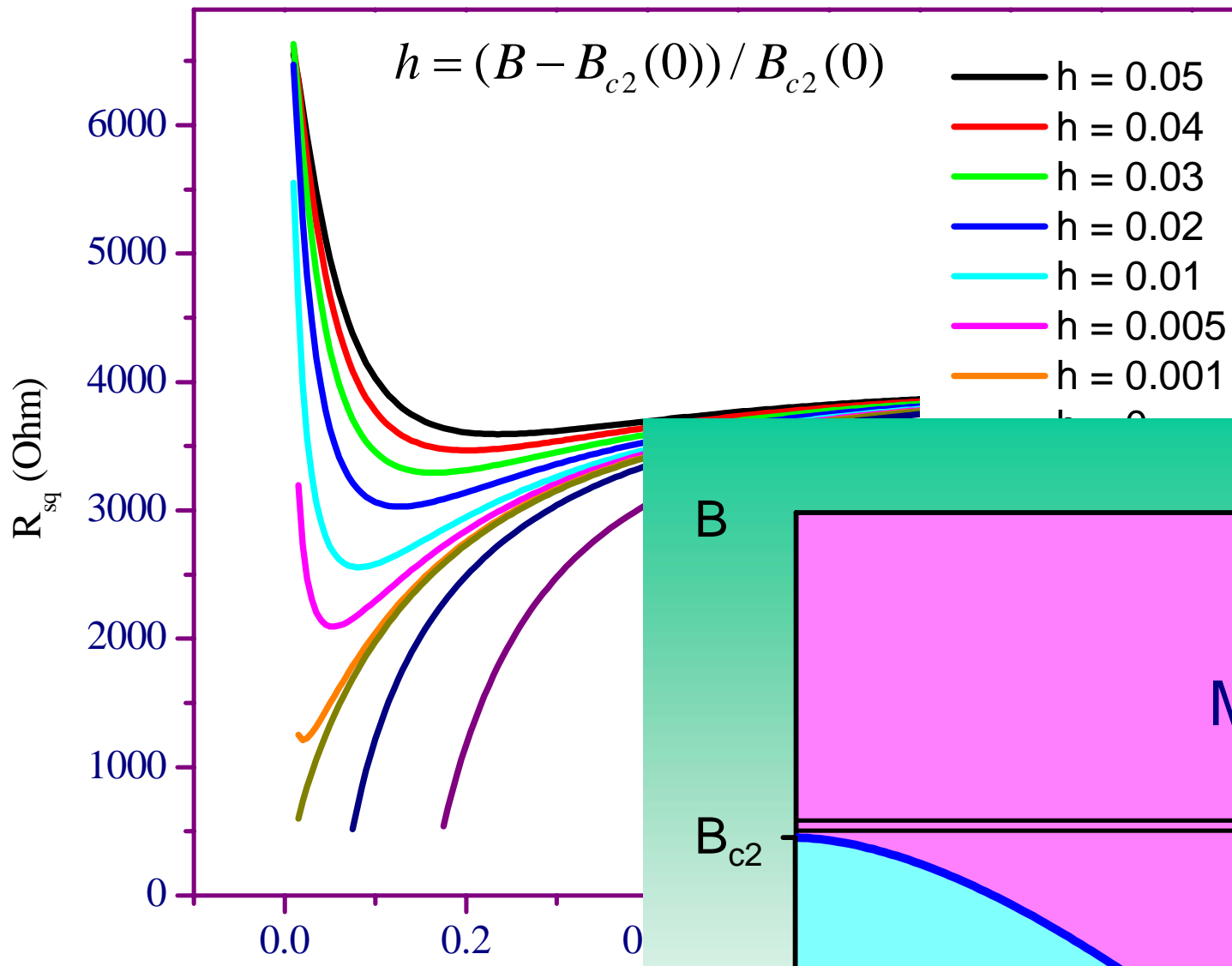


# Superconducting fluctuations at low temperature

small change of  $B$  leads to a drastic change of the shape of  $R(T)$  curves







Temperature dependence of the superconducting critical field,  $B_{c2}$   
 E. Helfand and N.R. Werthamer, PRL 13, 686 (1964); PR 147, 288 (1964);  
 E. Helfand, N.R. Werthamer, and C. Hohenberg, PR 147, 295 (1964)

# Superconducting fluctuations at low temperature

What about the saturation and double reentrant behavior?..

Practically, always there is

inevitable dispersion of  $B_{c2}(0)$  along the film...

fragmentation on two phases:  
in the very region of the magnetic fields close to  $B_{c2}(0)$  the film most likely constitutes the superconducting islands embedded in the normal metal

The more resistive film, the larger fluctuations in  $T_c$  and  $B_{c2}(0)$

# How we can include this effect in our calculations?

First way

fragmentation on two phases:

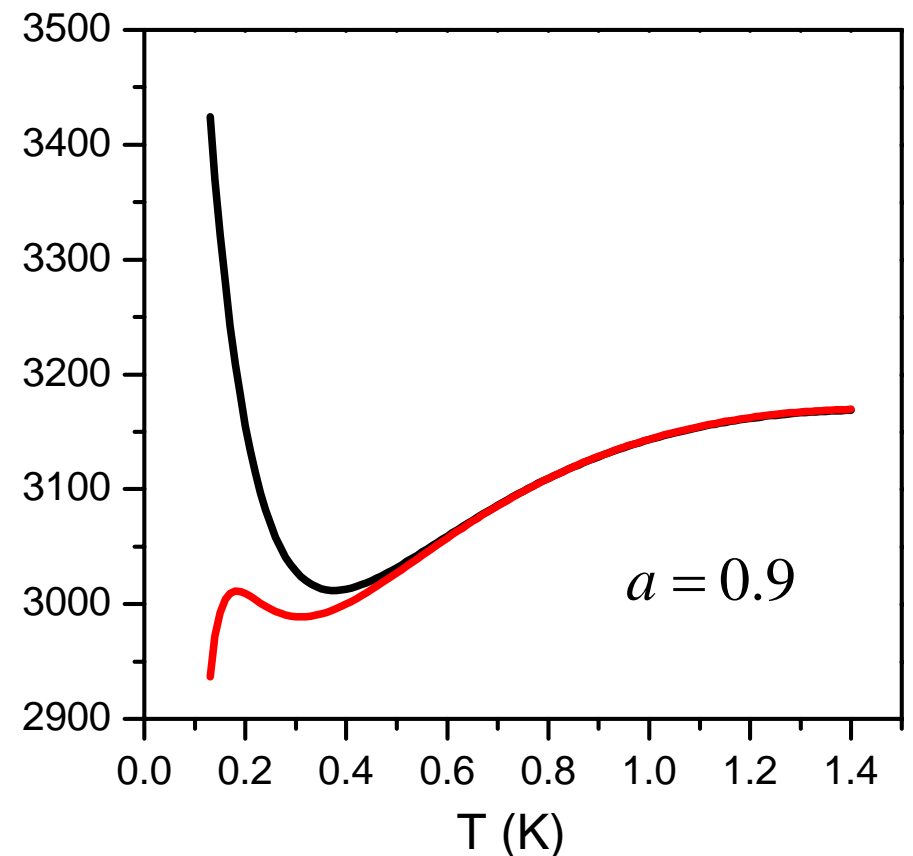
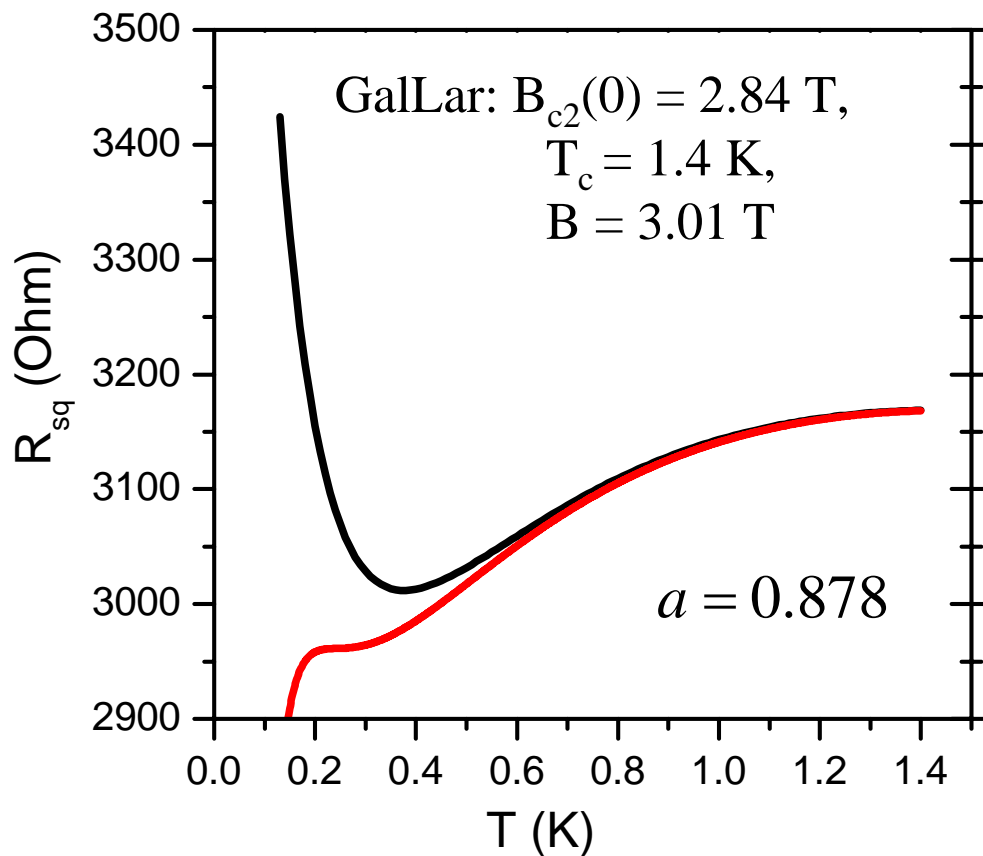
$$\Delta G = a \cdot \Delta G_1 + (1-a) \cdot \Delta G_2$$

$$\Delta G_1: B_{c2}(0) = 2.81 \text{ T}, T_c = 1.4 \text{ K}$$

$$\Delta G_2: B_{c2}(0) = 3.03 \text{ T}, T_c = 1.5 \text{ K}$$

the saturation behavior

double reentrant behavior

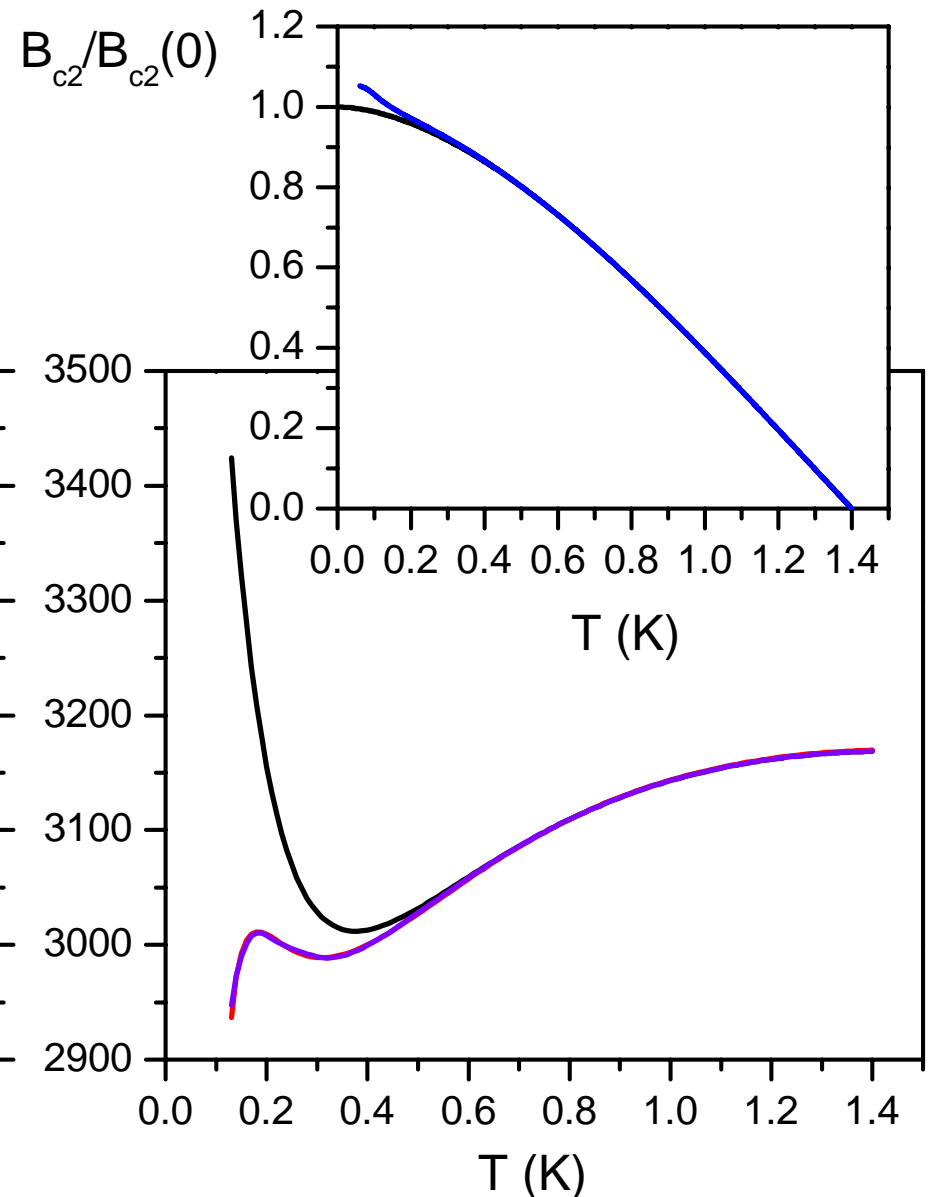
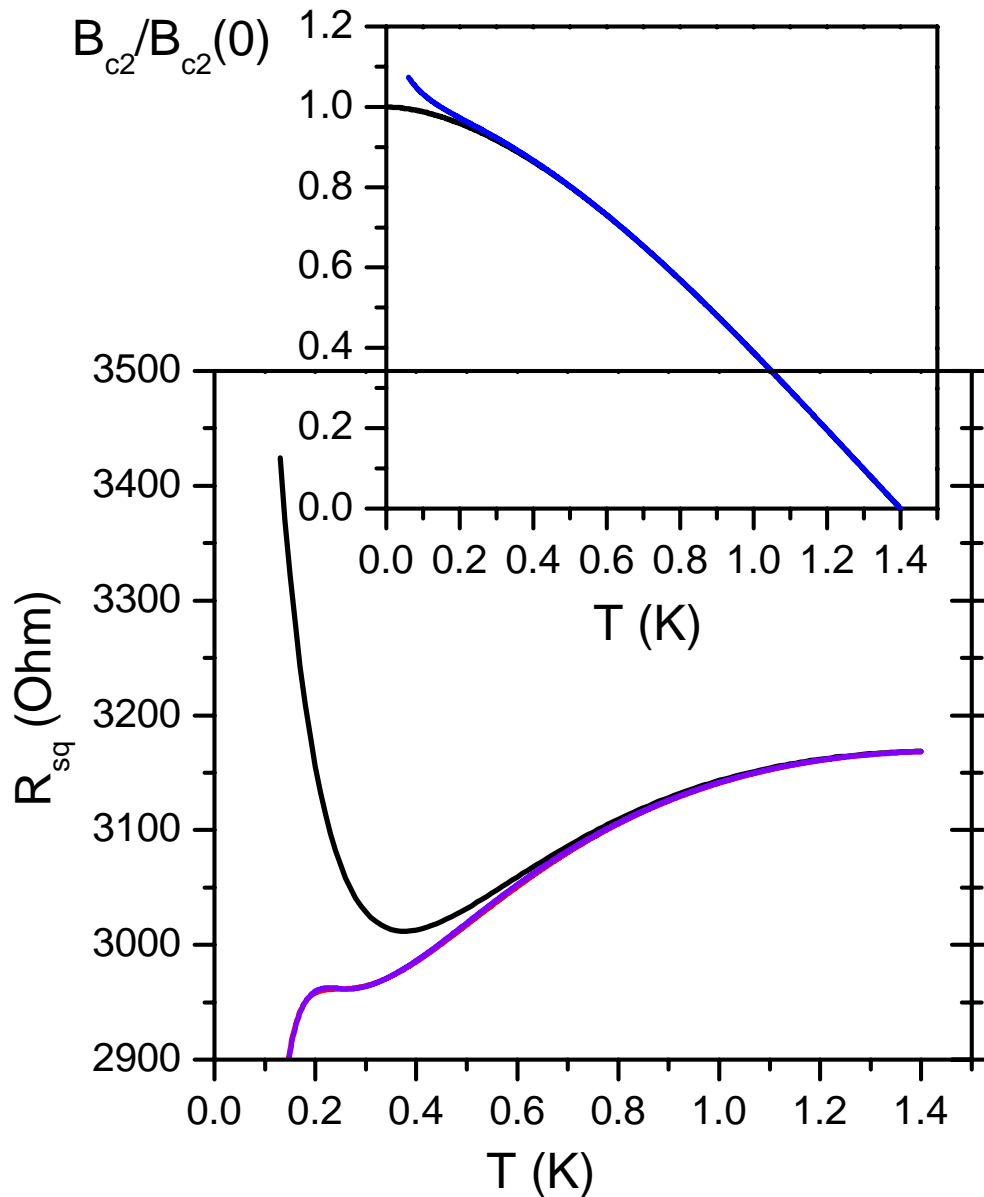


Second way

modified  $B_{c2}(T)$

the saturation behavior

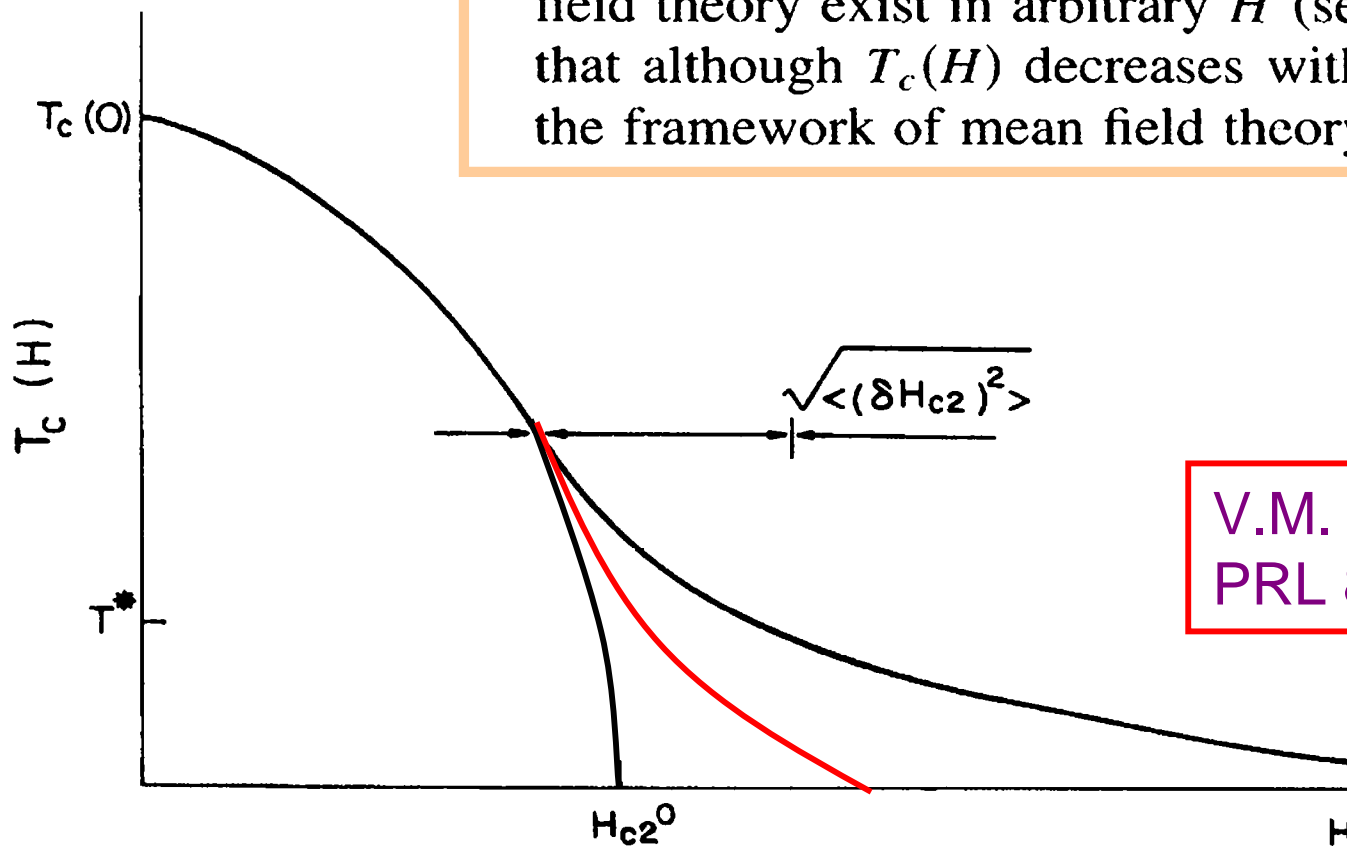
double reentrant behavior



# Mesoscopic effects in Disordered Superconductors near $H_{c2}$

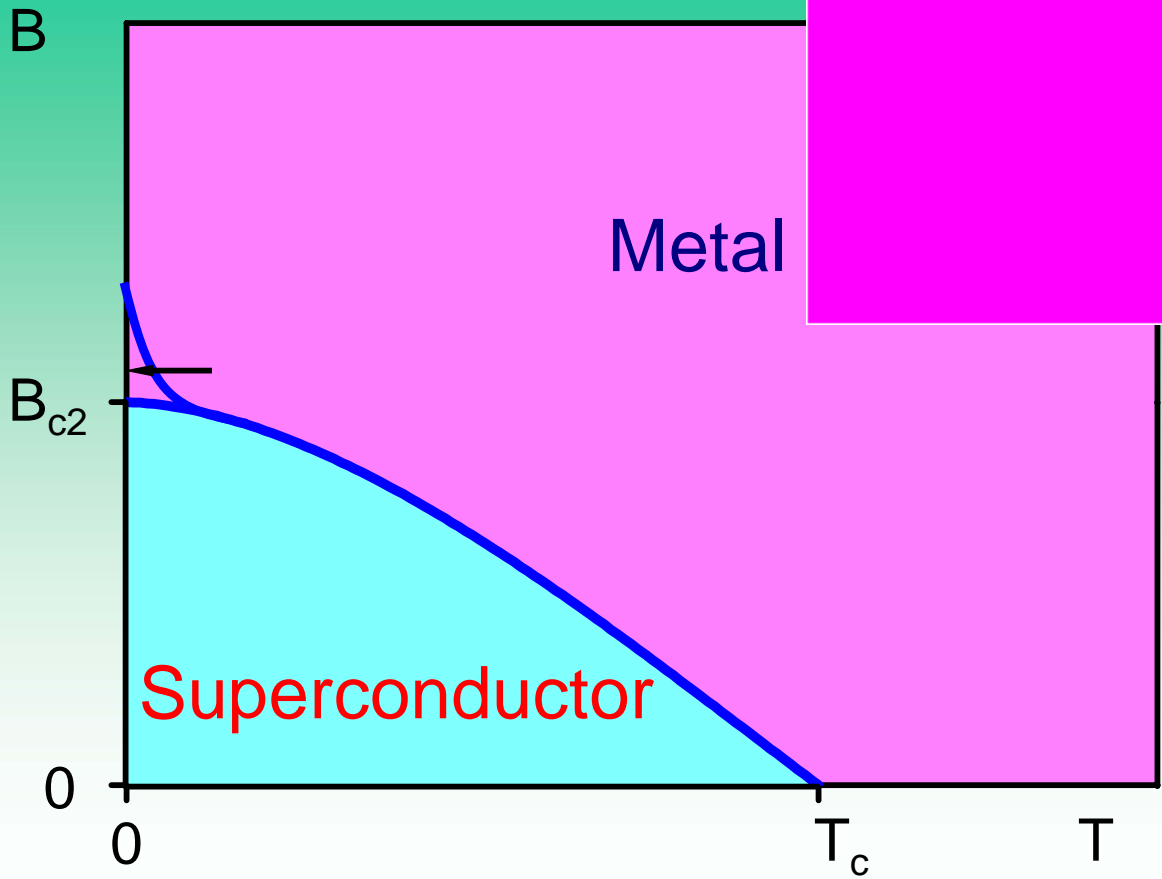
B. Spivak and Fei Zhou, PRL 74, 2800 (1995)

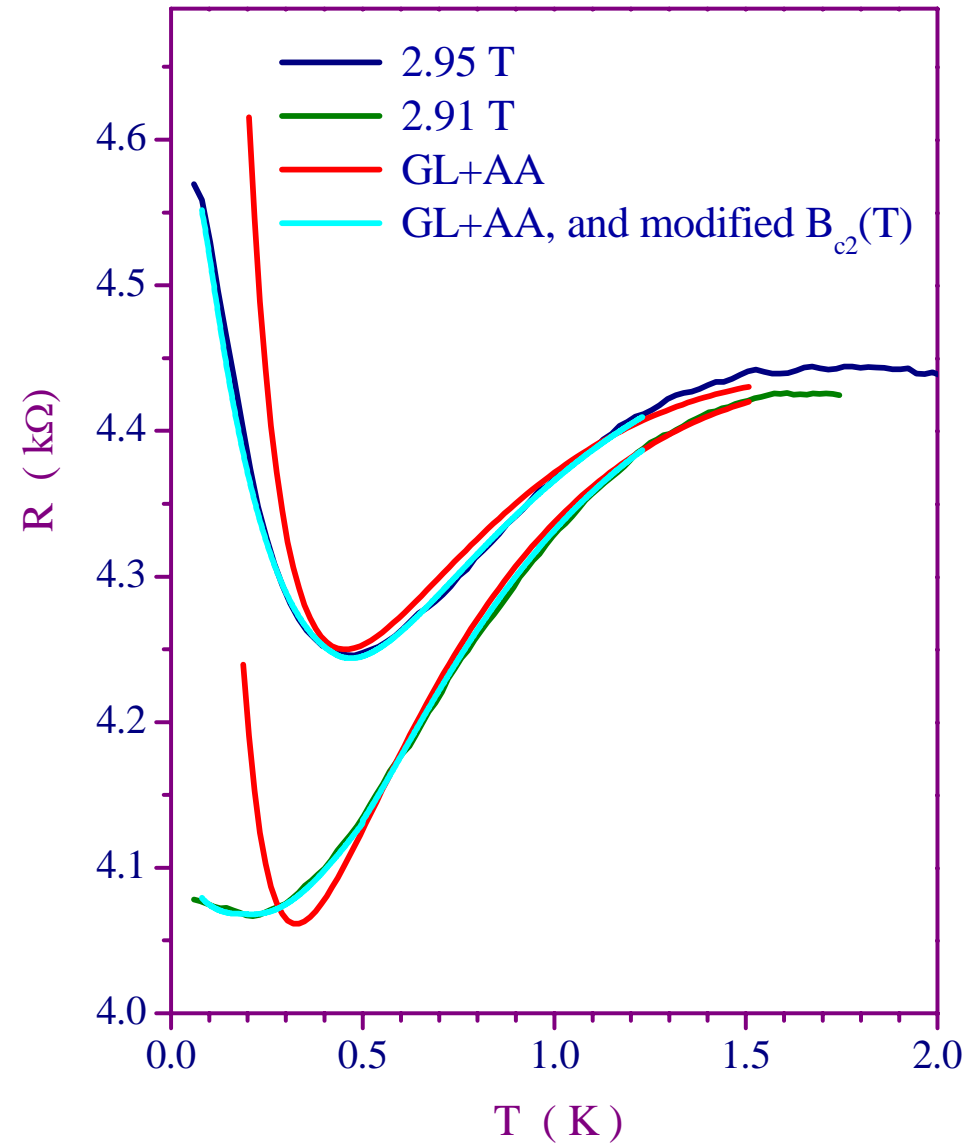
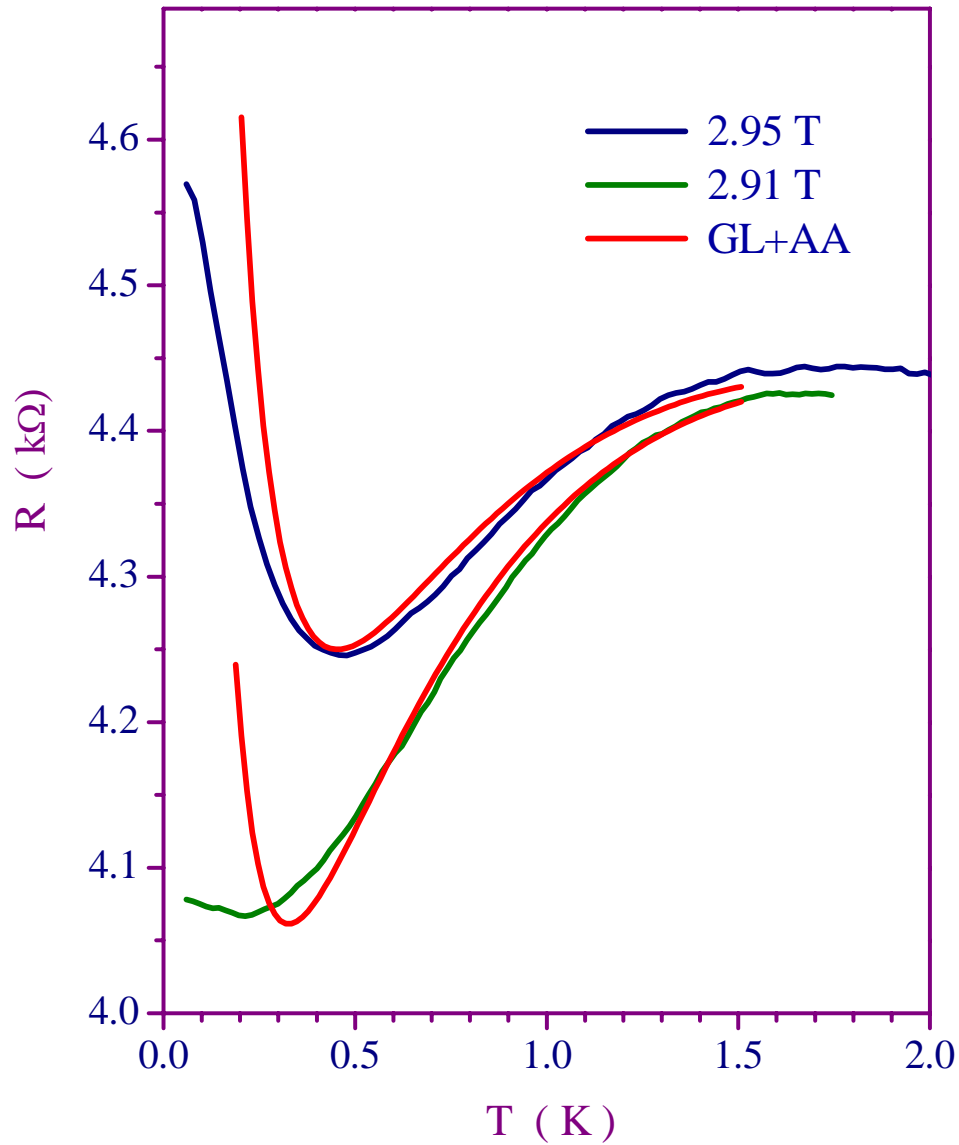
In the case of bulk samples at  $T = 0$ , due to the existence of the mesoscopic fluctuations, superconducting solutions of mean field theory exist in arbitrary  $H$  (see Fig. 2). This means that although  $T_c(H)$  decreases with  $H$  it is never zero in the framework of mean field theory.

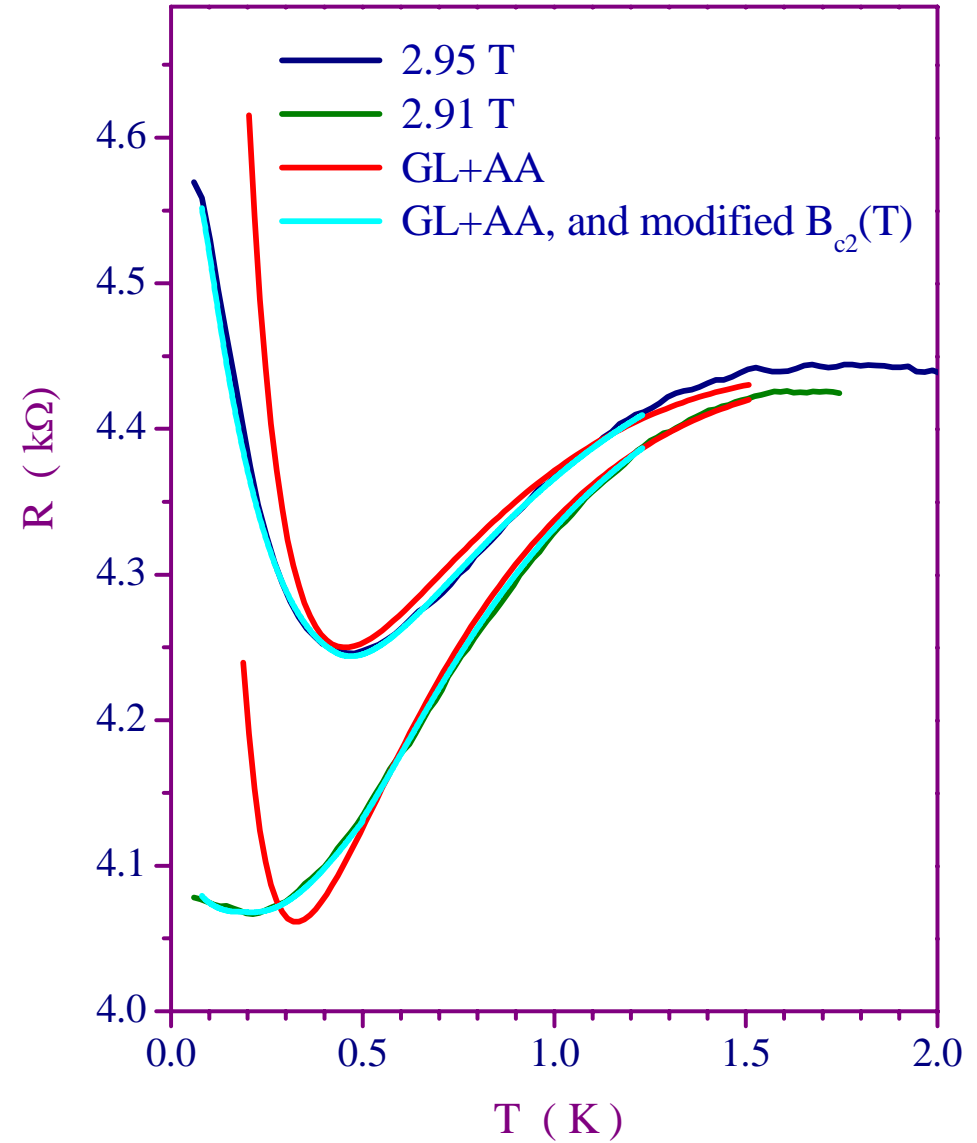
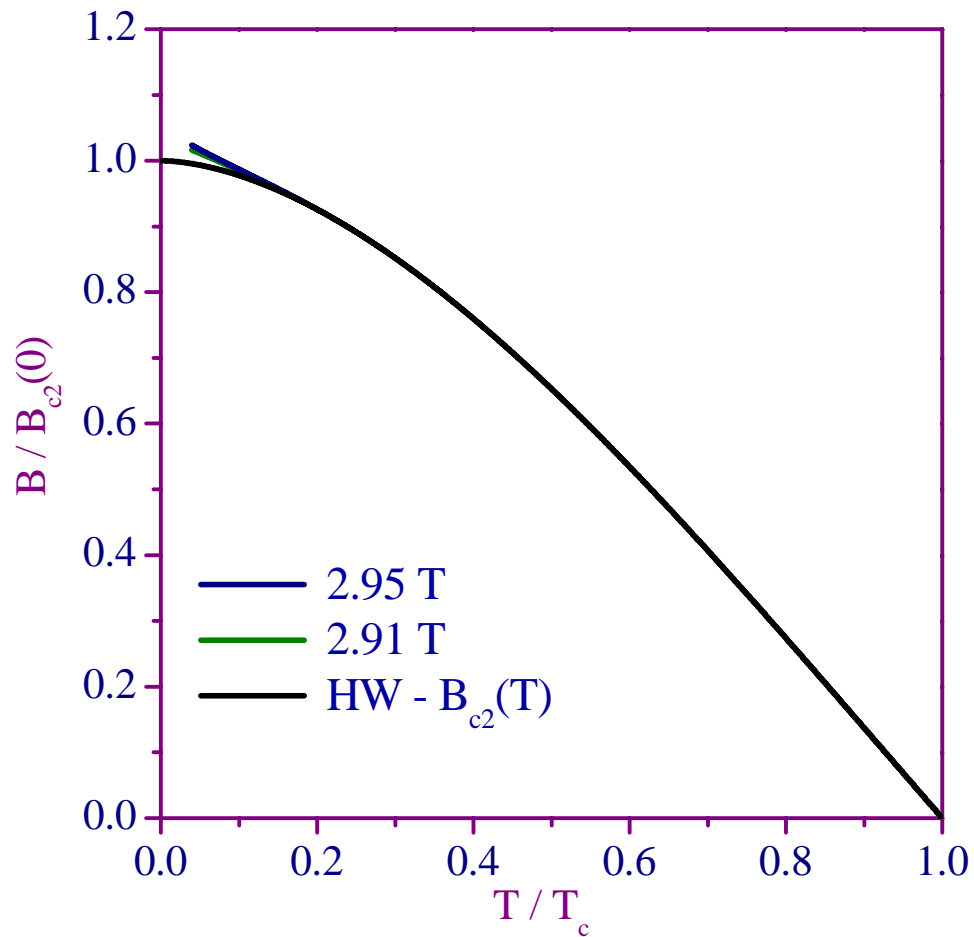


V.M. Galitski and A.I. Larkin,  
PRL 87, 087001 (2001)

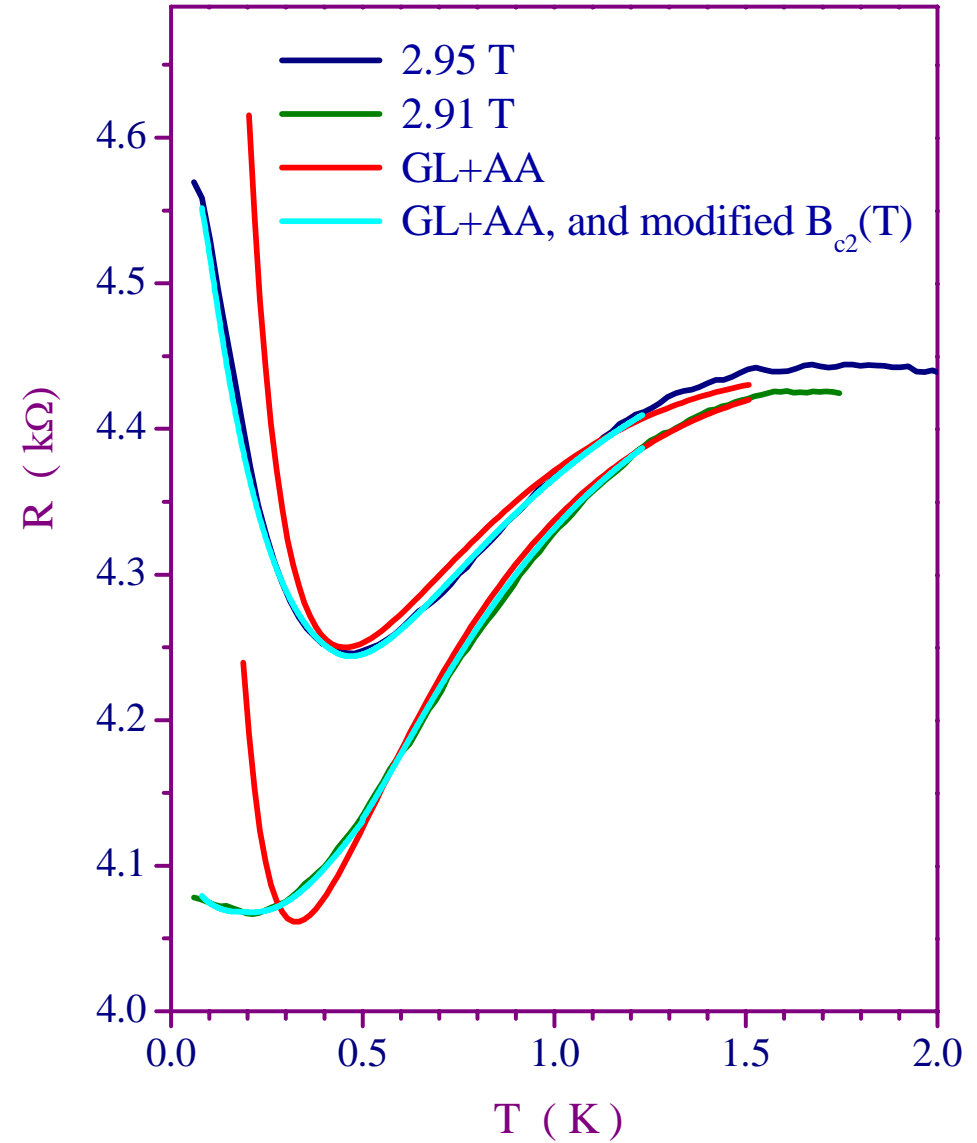
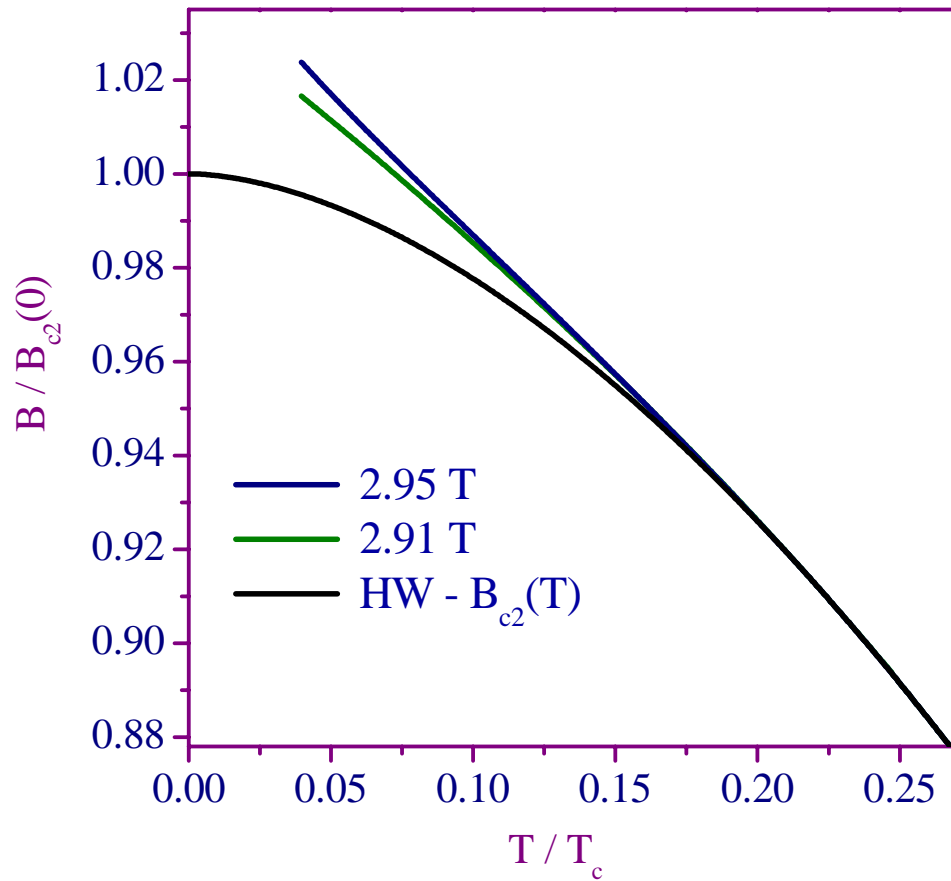
FIG. 2. Qualitative picture of  $T_c(H)$  in bulk samples.











# Suppression of Superconductivity by **Magnetic Field**

## Conclusion

*superconductor –  
metal  
transition*



*superconductor –  
insulator  
transition*

Strong quantum or mesoscopic fluctuations cause the fragmentation of uniformly disordered film on two phases, namely, the formation of local superconducting islands surrounded by a normal metal.

**A uniformly disordered film may appear as a granular one in terms of its superconducting properties**