Radiation from a single Josephson S-I-S junction

L.N. Bulaevskii, LANL
A.E. Koshelev, ANL

Content

Introduction

- Josephson prediction of radiation from tunneling junction, 1962.

- Experimental observation by Dmitrenko et al. and Langenberger et al.,1965.

- Radiation power from 0.16 cm X 0.025 cm Sn-SnO-Sn junction in magnetic field 1.9 G was 10^{-12} W, while power fed into junction was 3×10^{-7} W.

- Langenberger et al. gave only “electric engineer” estimate for radiation power in terms of impedances.

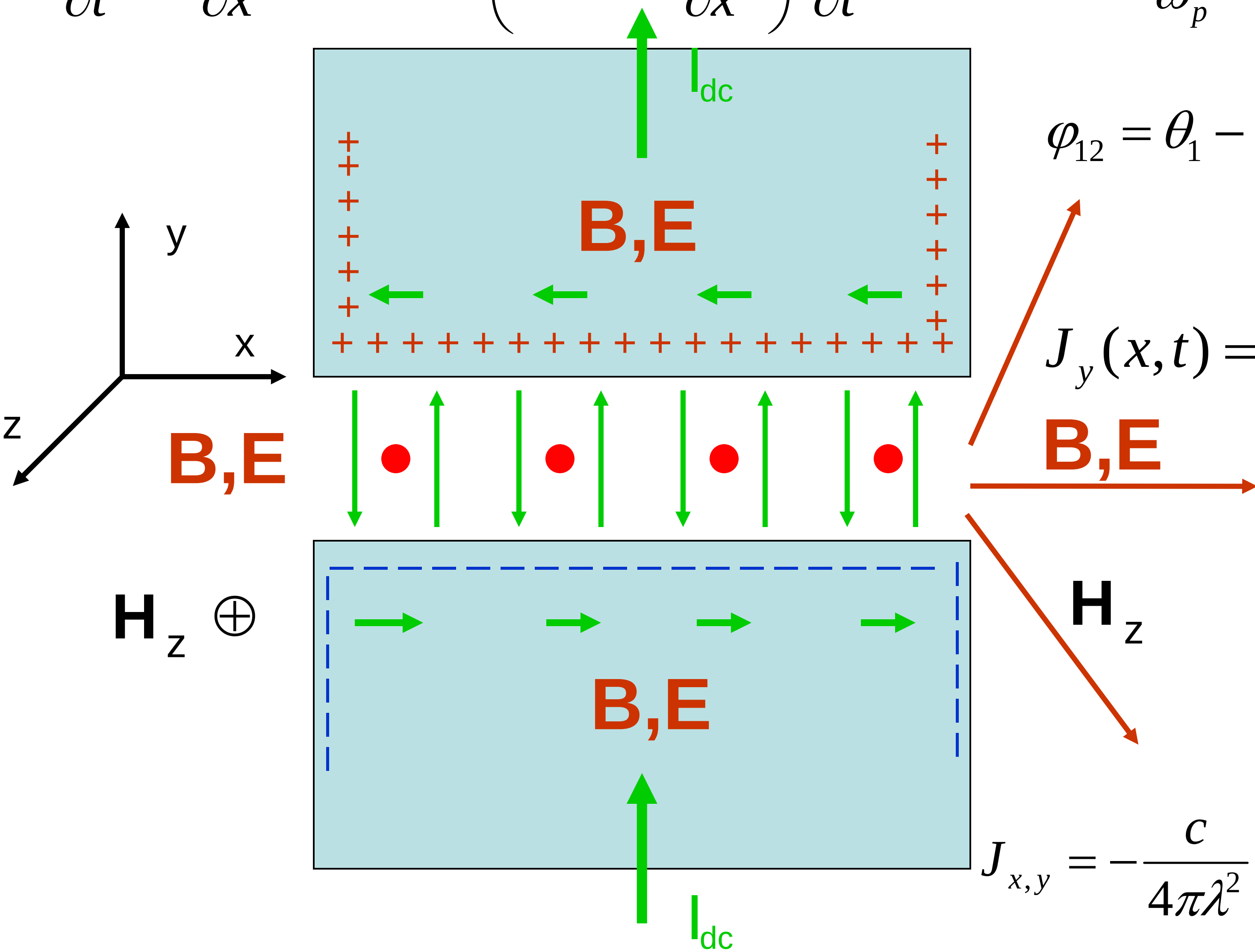
- Proposal to obtain THz radiation from intrinsic junctions in layered superconductors, Latyshev and Matsuda, 2001 and Tachiki et al., 2005.

- Better understanding of the mechanism of radiation in terms of phase difference is needed to derive radiation.

Sine-Gordon equation for the phase difference

$$\frac{\partial^2 \varphi}{\partial t^2} - \frac{\partial^2 \varphi}{\partial x^2} + \sin\varphi + \left(\gamma_T - \gamma_S \frac{\partial^2}{\partial x^2}\right)\frac{\partial \varphi}{\partial t} = 0, \qquad \gamma_T = \frac{\sigma_T}{\omega_p}, \qquad \gamma_S = \frac{2\pi\sigma_q\omega_p\lambda^2}{c^2}$$

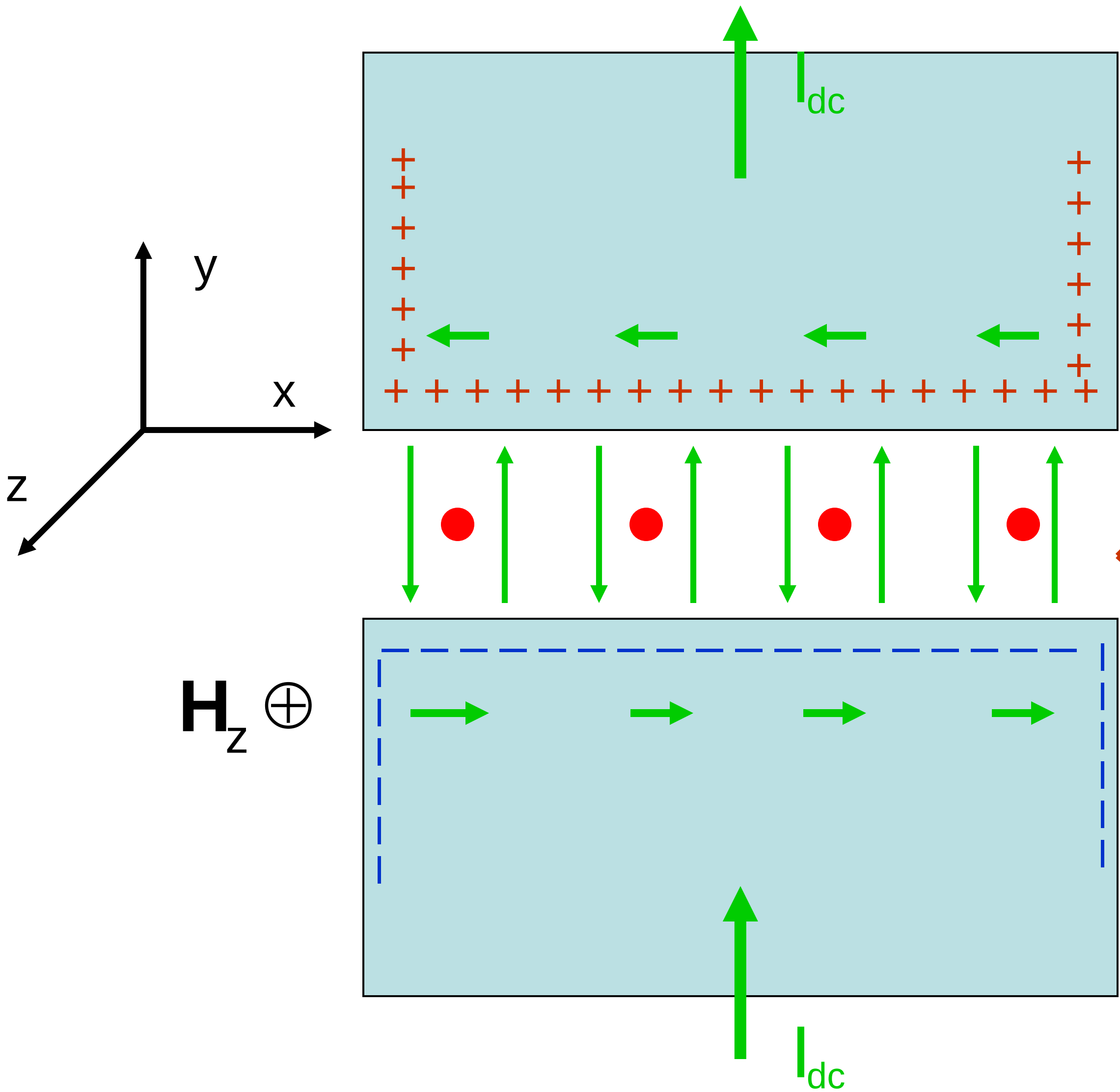
$$\varphi_{12} = \theta_1 - \theta_2 + \frac{2\pi}{\Phi_0}\int_1^2 A_y dy$$



$$J_y(x,t) = J_c \sin\varphi + \sigma_T E_y$$

$$J_{x,y} = -\frac{c}{4\pi\lambda^2}\left(\frac{\Phi_0}{2\pi}\nabla_{x,y}\theta - A_{x,y}\right)$$

Relation between phase difference and ac fields



$$\frac{\partial \varphi}{\partial x} = \frac{2\pi}{\Phi_0} \int_{-\infty}^{+\infty} dy B_z \approx \frac{4\pi\lambda}{\Phi_0} B_z$$

$$\frac{\partial \varphi}{\partial t} = -\frac{2\pi c}{\Phi_0} \int_{-\infty}^{+\infty} dy E_y \approx \frac{2\pi c d}{\Phi_0} E_y$$

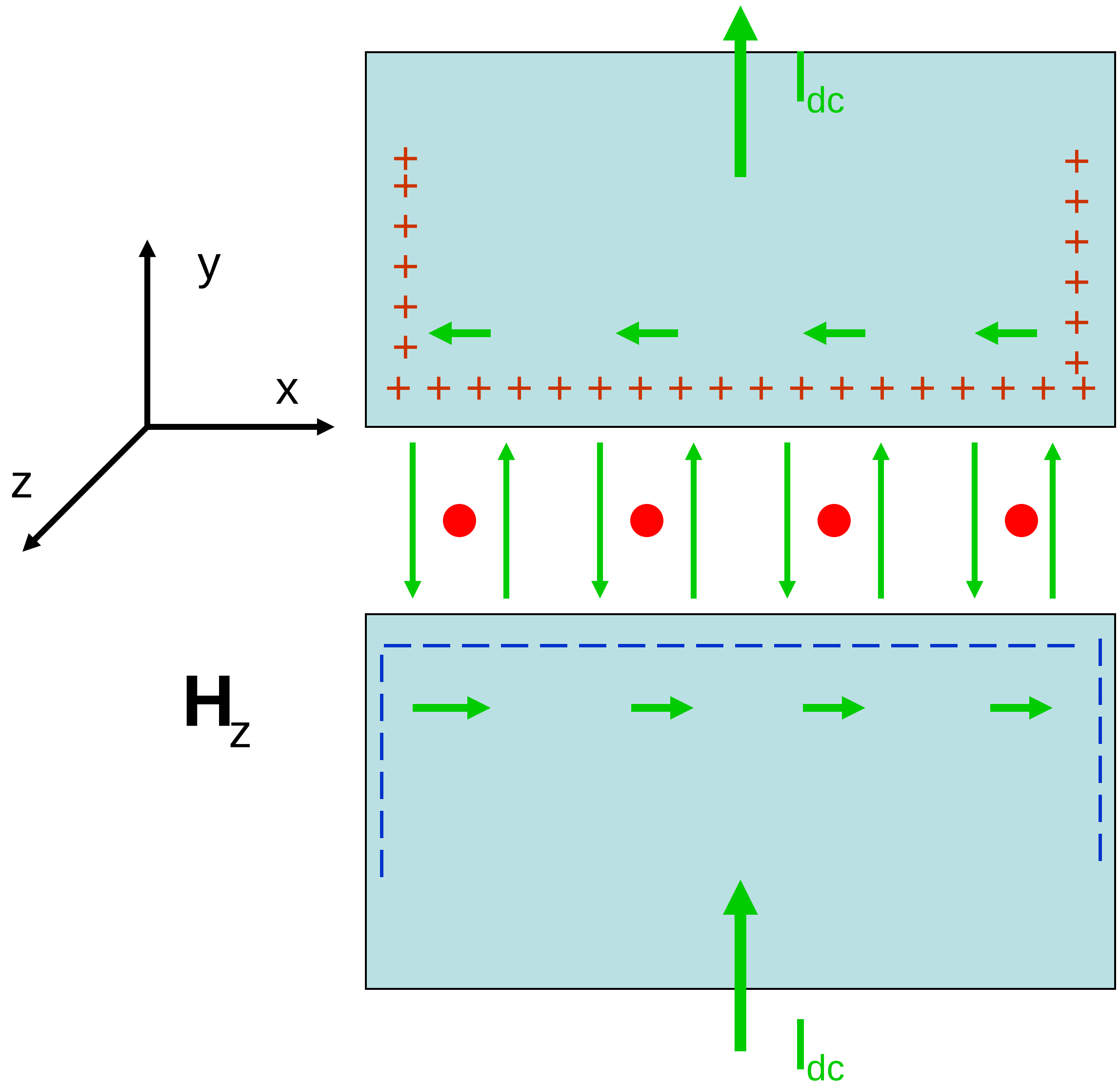
$$P_x(\omega) = \frac{c}{8\pi} B_z(\omega) E_y(\omega)$$

Standard boundary condition

$$\frac{\partial \varphi}{\partial x} = \frac{4\pi\lambda H_z}{\Phi_0}$$

excludes radiation.

The boundary conditions



At boundary superconductor-outer space at $\omega = 0$

$$B_z = H_z$$

At junction edge for $\omega \neq 0$

$$\frac{\partial \varphi}{\partial t} = -\frac{2\pi c}{\Phi_0} \int_{-\infty}^{+\infty} dy E_y \approx \frac{2\pi c d}{\Phi_0} E_y$$

$$\frac{\partial \varphi}{\partial x} = \frac{2\pi}{\Phi_0} \int_{-\infty}^{+\infty} dy B_z \approx \frac{4\pi\lambda}{\Phi_0} B_z$$

General relation between $B_z(\omega)$ and $E_y(\omega)$ follows from the Maxwell equations in the outer space.

Boundary condition for 1D junction

In the outer space $k^2 = k_x^2 + k_y^2 = k_0^2 = \omega^2 / c^2$, no incoming waves.

Relation between fields at the boundary leads – outer space:

$$B_{0z}(\omega, y) = \frac{|k_0|}{2\pi} \int_{-|k_0|}^{+|k_0|} dk_y \frac{E_{0y}(\omega, k_y)}{\sqrt{k_0^2 - k_y^2}} e^{ik_y y} - \frac{ik_0}{2\pi} \int_{k_y^2 > k_0^2} dk_y \frac{E_{0y}(\omega, k_y)}{\sqrt{k_y^2 - k_0^2}} e^{ik_y y},$$

Integrate both sides and use

$$\left(\frac{\partial \varphi}{\partial x}\right)_\omega = \frac{2\pi}{\Phi_0} \int_{-\infty}^{+\infty} dy B_z(\omega, y), \qquad \left(\frac{\partial \varphi}{\partial t}\right)_\omega = \frac{2\pi c}{\Phi_0} \int_{-\infty}^{+\infty} dy E_y(\omega, y).$$

boundary conditions at right and left sides

$$\left(\frac{\partial \varphi}{\partial x}\right)_\omega = \mu \frac{1}{c} \left(\frac{\partial \varphi}{\partial t}\right)_\omega . \qquad k_0 w >> 1.$$

Condition $B_z = H_z$ at the boundary leads – outer space

$$\left(\frac{\partial \varphi}{\partial x}\right)_{\omega=0} = \frac{4\pi\lambda H_z}{\Phi_0} \quad \text{at} \quad H_z << \frac{\Phi_0}{2\pi\lambda^2}$$

inside junction at distances bigger than λ from the edge.

Boundary condition for small circular 2D junction

For junction with the radius $R, \quad k_0 R << 1$

$$\left(\frac{\partial\varphi}{\partial r}\right)_{\omega,r=R} = -\frac{1}{\omega R \ln^2[2e^{-C}/(|k_0|R)]}\left|\left(\frac{\partial\varphi}{\partial t}\right)_{\omega,r=R}\right|^2 .$$

Radiation and dissipation power

$$\frac{W_{rad}}{w} = \frac{\Phi_0^2 \omega_p^2 \omega}{64\pi^3 c^2} \left| \left(\frac{\partial \varphi}{\partial (\omega_p t)} \right)_{\omega, x=0} \right|^2 \left(1 - \frac{1}{2} k_0^2 \lambda^2 \right)$$

1D junction:

$$W_{rad} = \frac{\Phi_0^2 \omega_p^2 (1 - k_0^2 \lambda^2 / 2)}{32\pi^2 c \ln^2 [2e^{-C} / (|k_0| R)]} \left| \left(\frac{\partial \varphi}{\partial (\omega_p t)} \right)_{\omega, r=R} \right|^2$$

2D circular junction with radius R

$$\frac{W_{dis}}{w} = \frac{\Phi_0^2 \omega^2 L_x}{4\pi^2 c^2} \left[\frac{\sigma_T}{d} + \frac{\sigma_q \lambda}{\lambda_J} \left\langle \left| \left(\frac{\partial \varphi}{\partial x} \right)_{\omega} \right|^2 \right\rangle_x \right]$$

1D junction

I-V characteristic $IV = W_{dis}(\omega) + W_{rad}(\omega), \quad \omega = 2eV/\hbar$

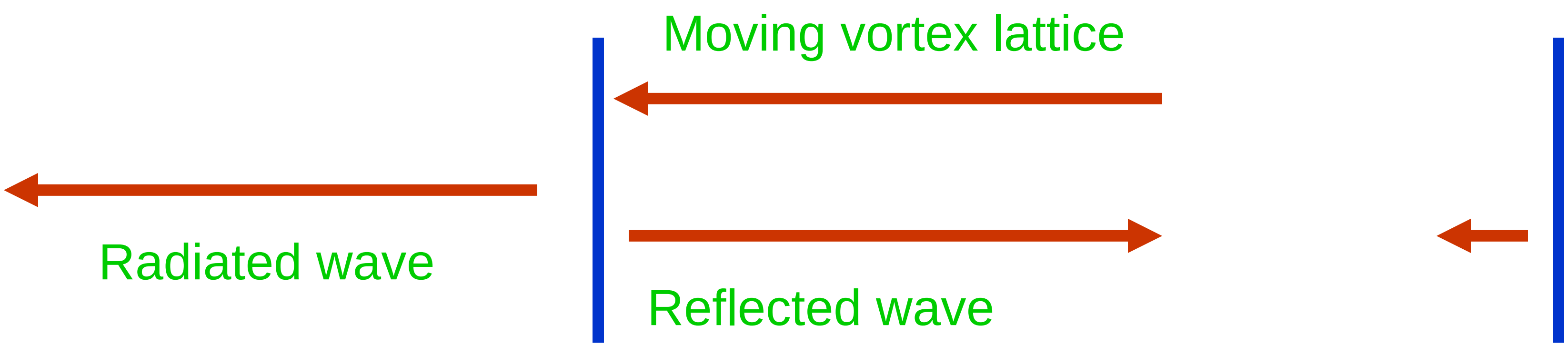
To derive radiation power and I-V characteristics we need to find $\varphi(x,t)$ by solving sine–Gordon equation with bound. conditions.

Results in the high field limit

$$\varphi(x,t) = \omega t - hx + \Theta(x,t), \quad \Theta(x,t) << 1, \qquad h = \frac{2\pi\lambda H_z}{\Phi_0},$$

$$\Theta(x,t) = -\frac{\sin(\omega t - hx)}{h^2\lambda_J^2} + \mathrm{Re}[a_1 e^{-\kappa x + i\omega(t - x/c_s)} + a_2 e^{\kappa x + i\omega(t + x/c_s)}].$$

a_1, a_2 are defined by boundary conditions.



Reflections lead to formation of almost standing waves (Fiske resonances broadened by dissipation and radiation).

Results in the high field limit, 1D

At resonance, $\omega_n = \pi n \lambda_J / L_x$,

$$\frac{W_{rad}}{w} \approx \frac{\Phi_0^2 \omega_p^4}{64\pi^3 c^2 \omega_n} \frac{2[1-\cos(hL_x)]}{(h\lambda_J)^2(\kappa L_x + 2\xi)^2}, \qquad \kappa = \gamma\omega_p / (2\omega\lambda_J) << 1,$$

$$\gamma = \gamma_T + \gamma_S h^2 \lambda_J^2, \qquad \xi = \sqrt{d/(2\lambda\varepsilon)}.$$

$$\frac{I}{w} \approx \frac{\sigma_T L_x V}{d} + \frac{\Phi_0 \omega_p^2}{8\pi (hL_x)^2 \omega} \sqrt{\frac{\varepsilon}{2d\lambda}} \frac{1-\cos(hL_x)}{\kappa L_x + 2\xi}.$$

Approach is valid if $\gamma h L_x >> 1$ or $\xi h L_x >> 1$.

Drops of I at $V > V_{res}$ lead to steps in I-V characteristics.

Junction of Langenberger et al. was in nonlinear regime.

Weaker non-resonance radiation is present at H=0.

Nonlinear regime

For rough estimate we can take

$$\frac{\partial \varphi}{\partial(\omega_p t)} \approx 1.$$

At $\omega \approx \omega_p = 2\pi \times 10^{10}$ Hz we get

$W_{rad} \approx 0.2 \times 10^{-12}$ w.

Numerical calculations are needed to find field and size dependence of the radiation power.

Conclusions

- Boundary conditions in the presence of radiation are formulated.
- In a similar way boundary conditions for intrinsic junctions in layered crystals and for stack of usual junctions opened into free space are derived.
- They account for radiation and describe also the electromagnetic coupling of junctions via open space.
- For small junction in addition to dipole radiation also radiation due to electromagnetic modes inside junction is present.
- It does not scale as R^4 for small R and so it is bigger for small junctions. It may contribute to decoherence in junctions used as a qubits.
- Quantization for junctions in the presence of radiation is an intriguing problem due to dynamic boundary conditions.