

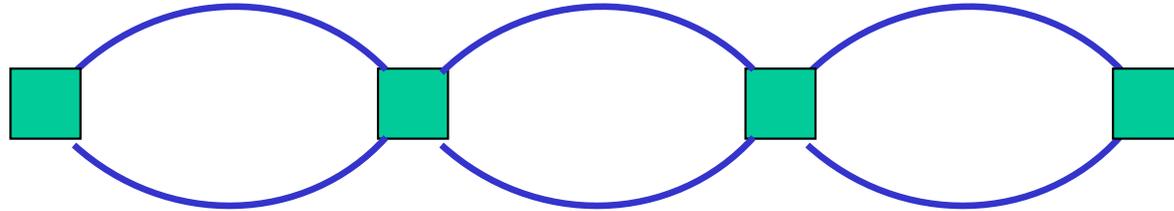
Bosonization and renormalization group for a clean Fermi gas with a repulsion in arbitrary dimensions.

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Contents

1. New method of bosonization in $d > 1$ including spin excitations.
2. New logarithmic contributions due to spin excitations, renormalization group.
3. Non-Fermi liquid behavior of a Fermi liquid, a possibility of a phase transition.

The main assumption of the Fermi liquid theory:



■ = Constant (weakly depends on temperature)

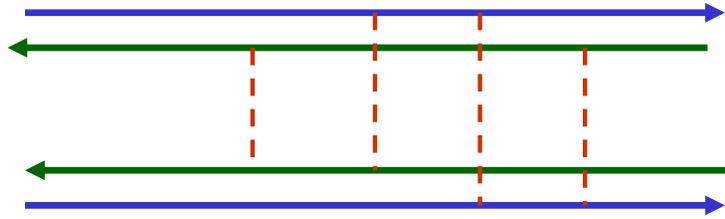
Not always true: e.g. for an attraction one has a Cooper instability.

What about repulsion?

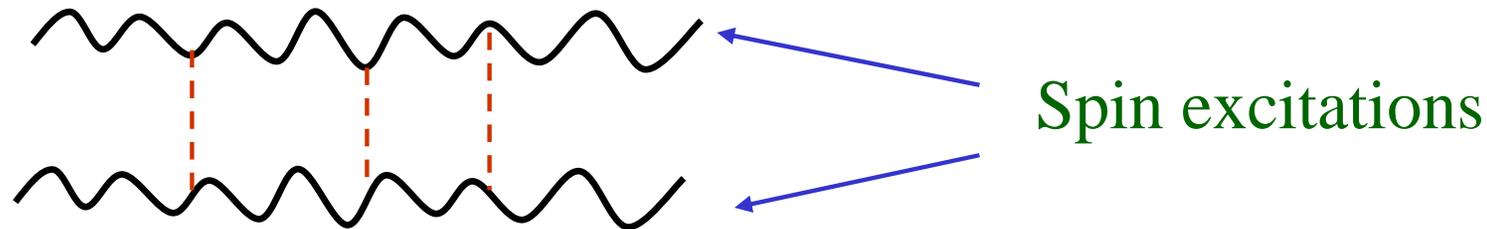
Nothing in the weak coupling limit (common knowledge).

In this work: there are non-trivial correlations for repulsion in the weak coupling limit and even an instability!

Everything comes not from two electron correlations but from four electron ones! → A reason why nobody (?) has noticed these instabilities before.



Equivalent representation



Reduction from electrons to collective excitations: Bosonization

The scheme of the method.

1. Singling out slow varying $k < k_c \leq p_F$ pairs in the interaction.

$$\mathcal{L}_{int} \rightarrow \tilde{\mathcal{L}}_{int} = \frac{1}{2} \sum_{\sigma, \sigma'} \int dP_1 dP_2 dK \quad (2.10)$$

$$\{V_2 \chi_{\sigma}^*(P_1) \chi_{\sigma}(P_1 + K) \chi_{\sigma'}^*(P_2) \chi_{\sigma'}(P_2 - K) - V_1 (\mathbf{p}_1 - \mathbf{p}_2) \chi_{\sigma}^*(P_1) \chi_{\sigma'}(P_1 + K) \chi_{\sigma'}^*(P_2) \chi_{\sigma}(P_2 - K)\}$$

No pairs of the type $\chi\chi$, $\chi^*\chi^*$ but they are not necessary because only $V(2p_F)$ and $V(0)$ play a role.

Example of a Cooper logarithm with these vertices.

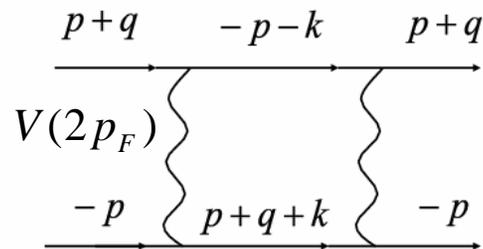


FIG. 2: Superconducting loop. Logarithm comes from small momenta $k < k_c$.

2. Decoupling of the slow pairs by the Hubbard-Stratonovich transformation \longrightarrow electron motion in a slow field $\Phi \longrightarrow$ writing equations for quasiclassical Green functions Φ is a 2x2 spin matrix.

$$\left(\frac{\partial}{\partial \tau} + \frac{\partial}{\partial \tau'} - iv_F \mathbf{n} \nabla \right) g_{\mathbf{n}}^{\Phi}(\mathbf{R}, \tau, \tau') + (g_{\mathbf{n}}^{\Phi}(\mathbf{R}; \tau, \tau') \Phi_{\mathbf{n}}(\mathbf{R}, \tau') - \Phi_{\mathbf{n}}(\mathbf{R}, \tau) g_{\mathbf{n}}^{\Phi}(\mathbf{R}; \tau, \tau')) = 0 \quad (2.37)$$

$$g_{\mathbf{n}}^{\Phi}(\mathbf{R}, \tau, \tau') = i \int_{-\infty}^{\infty} G_{\mathbf{p}}^{\Phi}(\mathbf{R}, \tau, \tau') \frac{d\xi}{\pi}$$

The solution

$$g_{\mathbf{n}}^{\Phi}(\mathbf{R}, \tau, \tau') = T_{\mathbf{n}}(\mathbf{R}, \tau) g_0(\tau - \tau') T_{\mathbf{n}}^{-1}(\mathbf{R}, \tau')$$



Generalization of the Schwinger Ansatz

Equations for

$$M_{\mathbf{n}}(x) = \frac{\partial T_{\mathbf{n}}(x)}{\partial \tau} T_{\mathbf{n}}^{-1}(x)$$

Another representation:

$$M_{\mathbf{n}}(x) = \rho_{\mathbf{n}}(x) + \mathbf{S}_{\mathbf{n}}(x) \sigma$$

$S_n(r, \tau)$ Spin

$\rho_n(r, \tau)$ Charge

Equations for the charge and spin excitations: starting point for the calculations.

$$\left(-\frac{\partial}{\partial \tau} + i v_F \mathbf{n} \nabla_{\mathbf{R}}\right) \rho_{\mathbf{n}}(x) = -i \frac{\partial \varphi_{\mathbf{n}}(x)}{\partial \tau}$$

$$\left(-\frac{\partial}{\partial \tau} + i v_F \mathbf{n} \nabla_{\mathbf{R}}\right) \mathbf{S}_{\mathbf{n}}(x) + 2i [\mathbf{h}_{\mathbf{n}}(x) \times \mathbf{S}_{\mathbf{n}}(x)] = -\frac{\partial \mathbf{h}_{\mathbf{n}}(x)}{\partial \tau}$$

↔ No interaction leading to infrared divergences.

Effective interaction leading to divergent contributions at $T=0$ (logarithmic in any dimensions)

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{-i\omega + v_F k} \frac{1}{i\omega + v_F k} \propto \int \frac{d^{d-1} k_{\perp}}{(2\pi)^{d-1}} \ln \frac{\varepsilon_F}{T}$$



Non-trivial effective model for spin excitations.

Logarithmic in any dimensions → RG treatment

Effective field theory for the spin excitations. (Due to necessity of averaging over \mathbf{h} , use of supervectors).

The operator $\hat{L}_u = -\frac{\partial}{\partial \tau} + iv_F (\mathbf{n} \nabla_{\mathbf{r}}) + 2iu\hat{h}$ is not hermitian \rightarrow
 doubling the size of the supervectors ψ

Effective Lagrangian

$$Z = \int \exp(-L[\psi]) D\psi$$

Z -Partition function, L -Lagrangian

$$L[\psi] = L_0[\psi] + L_2[\psi] + L_3[\psi] + L_4[\psi]$$

$$L_0[\psi] = -2iv \int \overline{\psi_\alpha(X)} H_0 \psi_\alpha(X) dX$$

$$H_0 = \begin{pmatrix} -iv_F (\mathbf{m} \nabla) \tau_3 \Sigma_3 & -\partial/\partial \tau \\ -\partial/\partial \tau & -iv_F (\mathbf{m} \nabla) \tau_3 \Sigma_3 \end{pmatrix}$$

$L_0[\psi]$ -Lagrangian of free excitations

Interaction terms

$$L_4[\psi] = -8\nu \varepsilon_{\alpha\beta\gamma} \varepsilon_{\alpha\beta_1\gamma_1} \int \left(\overline{\psi_\beta(X)} \tau_3 \psi_\gamma(X) u \right) \\ \times \tilde{\Gamma} \left(\overline{u \psi_{\beta_1}(X)} \tau_3 \psi_{\gamma_1}(X) \right) dX \quad (4)$$

$$\varepsilon_{\alpha\beta\gamma} = -\varepsilon_{\alpha\gamma\beta} = -\varepsilon_{\beta\alpha\gamma} = 1$$

$$L_3[\psi] = -8\nu \sqrt{2} i \varepsilon_{\alpha\beta\gamma} \int \left(\overline{\psi_\beta(X)} \tau_3 \psi_\gamma(X) u \right) \\ \times \tilde{\Gamma} \left(\overline{\bar{F}_0 \partial_x(u)} \tau_3 \psi_\gamma(X) \right) dX$$

$$L_2[\psi] = 4i\nu \int \left(\overline{\psi_\alpha(X)} \tau_3 \overleftarrow{\partial}_x(u) F_0 \right) \\ \times \tilde{\Gamma} \left(\overline{\bar{F}_0 \partial_x(u)} \tau_3 \psi_\gamma(X) \right) dX$$

L_0, L_4 are supersymmetric

L_2, L_3 violate the supersymmetry \Rightarrow contribution to thermodynamics

Next step: renormalization group treatment.

Results of the solutions of the RG equations.

Quartic interaction: $\gamma_1(\xi)$ -forward scattering, $\gamma_3(\xi)$
backward scattering

$$\gamma_3(\xi) = \frac{1}{\xi_b^* + \xi},$$

$$\gamma_1(\xi) = \frac{1}{\xi_f^* - \xi}$$

ξ -Logarithmic variable

Cubic interaction:

$$\beta_3^+(\xi) = \frac{1}{\xi_b^* + \xi}, \quad \beta_3^-(\xi) = \frac{\xi_b^*}{(\xi_b^* + \xi)^2}$$

$$\beta_1^+(\xi) = \frac{1}{\xi_f^* - \xi}, \quad \beta_1^-(\xi) = \frac{\xi_f^*}{(\xi_f^* - \xi)^2}$$

Quadratic interaction:

$$\Delta_1(\xi) = \text{const} = \Delta(0)$$

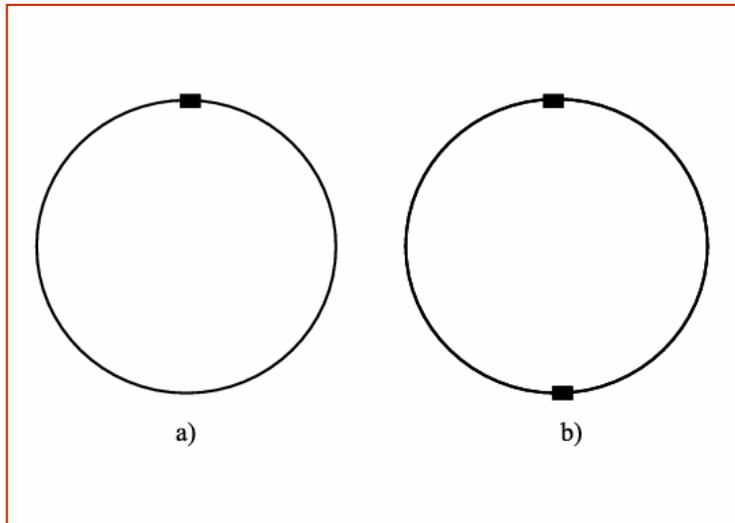
$$\Delta_3^{-+}(\xi) = \frac{2\xi_b^{*2}}{(\xi_b^* + \xi)^3} - \frac{\xi_b^*}{(\xi_b^* + \xi)^2}$$

$$\Delta_3^{+-}(\xi) = \frac{2}{\xi_b^* + \xi} - \frac{1}{\xi_b^*},$$

$$\Delta_3^{++}(\xi) = \Delta_3^{--}(\xi) = \frac{\xi_b^*}{(\xi_b^* + \xi)^2}$$

Results for the specific heat δC

Only the backscattering amplitudes $\Delta_3(\xi)$ contribute to thermodynamics in the one loop approximation.



Thermodynamic potential in the first and second order in the effective Δ
 A non-trivial contribution comes from **b)** only.

$$\delta c_{d=2} = -\frac{3\zeta(3)}{\pi} \left(\frac{T}{\varepsilon_F}\right)^2 (\Gamma_{cb}^2 + 6Y(0))$$

Charge excitations

$$\delta c_{d=3} = -\frac{3\pi^4}{10} \left(\frac{T}{\varepsilon_F}\right)^3 \left(\Gamma_{cb}^2 + 6 \int_{T/\varepsilon_0}^1 \frac{Y(\theta)}{\theta} d\theta \right)$$

Spin excitations

The function Y is not universal and depends on the cutoff.

$$Y(\theta) = \frac{2\Gamma_b^2}{X^2} \int_0^X \frac{\ln(1+t)}{(1+t)^2} \ln\left(\frac{X}{t}\right) dt$$

$$X = -\alpha_d \Gamma_b \ln[\max\{\theta, T/\varepsilon_0\}]$$

$$Y(\theta) = \Gamma_b^2 \left(\frac{1}{2} - \frac{10}{9} X \right) \quad \text{for } X \leq 1$$

$$Y(\theta) \simeq \frac{2\Gamma_b^2 \ln X}{X^2} \quad \text{for } X \leq 1$$

$$Y_{X=0} = \Gamma_b^2 / 2 \quad \longrightarrow \quad \text{Agreement with:}$$

Chubukov, Maslov, Gangadharaiah, Glazman (2005) for 2d,

Chubukov, Maslov, Millis (2005) for 3d

(Direct diagrammatic expansion)

What about $\gamma_1(\xi) = \frac{1}{\xi_f^* - \xi}$?

Does this instability mean anything?

Yes (preliminary).

Formation of an order parameter $Q(u)$:

$$Q_{\alpha\beta}(u) = \langle \psi_\alpha \bar{\psi}_\beta \rangle \quad 0 < u < 1$$

$$Q_{\alpha\beta} = -Q_{\beta\alpha} \quad \longrightarrow \quad Q_\gamma = \varepsilon_{\alpha\beta\gamma} Q_{\alpha\beta}$$

Change of the specific heat at low temperatures

$$\delta C \propto -\frac{T}{\ln(\varepsilon_F / T)} \quad \longleftrightarrow \quad \text{Universal}$$

Conclusions.

There can be a non-trivial life and even a phase transition in the Fermi gas with a repulsion due to spin excitations.

All these effects should become very pronounced near quantum critical points like normal metal-magnetic states transitions (may be, in the high T_c cuprates(?))