(hopping=insulator)

metal-insulator transition in disordered two-dimensional electron gas

(no Mottness)

Alexander Finkel'stein

Dept of Condensed Matter Physics
Weizmann Institute of Science
Rehovot, Israel

Alex Punnoose & A.F. ,
Phys. Rev. Lett. **88** (2002)
and
Science **310**, 429 (2005).

Conclusions

The interplay of interactions and disorder fundamentally revises the common belief that 2D electron systems become insulating at low enough temperatures.

Using a large-N approximation scheme (valleys), we obtained a two-parameter scaling theory that exhibits a metal-insulator transition in 2d.

The transition between the metallic and insulating phases is controlled by a finite-resistance unstable fixed point.

The theory is internally consistent: there are no divergences in the interaction amplitudes at finite temperatures.

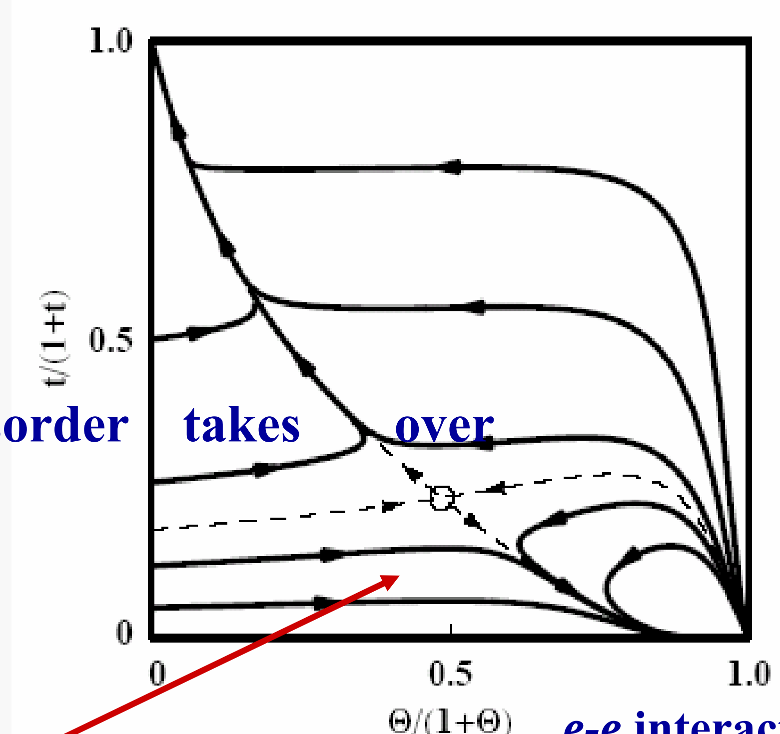
The two-stage route (1984) to the MIT transition (first spins, next charges) has been revised.

The spin-susceptibility close to the transition diverges. The g-factor remains finite => this divergence is not related to any magnetic instability.

Numerically the parameters of the fixed point appeared to be small. This gives arguments in favor that the 2-loop calculations are adequate and sufficient in the large-N limit.

MIT for non-interacting electrons in D>2:
a competition between dimensions and the interference
(ignoring spin-orbit case)

MIT for interacting electrons in d=2:
a competition between disorder and *e-e* interactions



disorder takes over

***e-e* interaction**

metallic phase stabilized by *e-e* interaction

Metal: classical physics

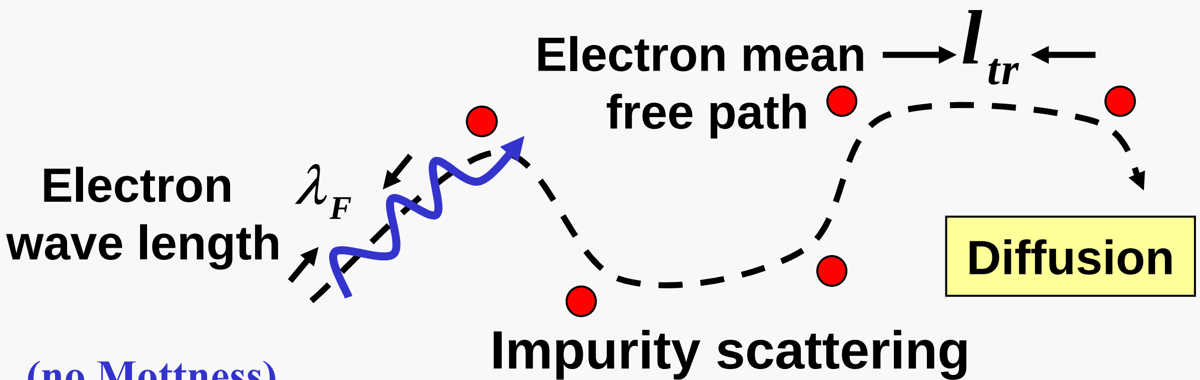
$$\sigma = \frac{ne^2\tau_{tr}}{m}$$

$$l_{tr} >> \lambda_F$$ (electron wave length)

$$\sigma = ne\mu$$

μ is **mobility**

$$\tau_{tr} = l_{tr} / v_F$$



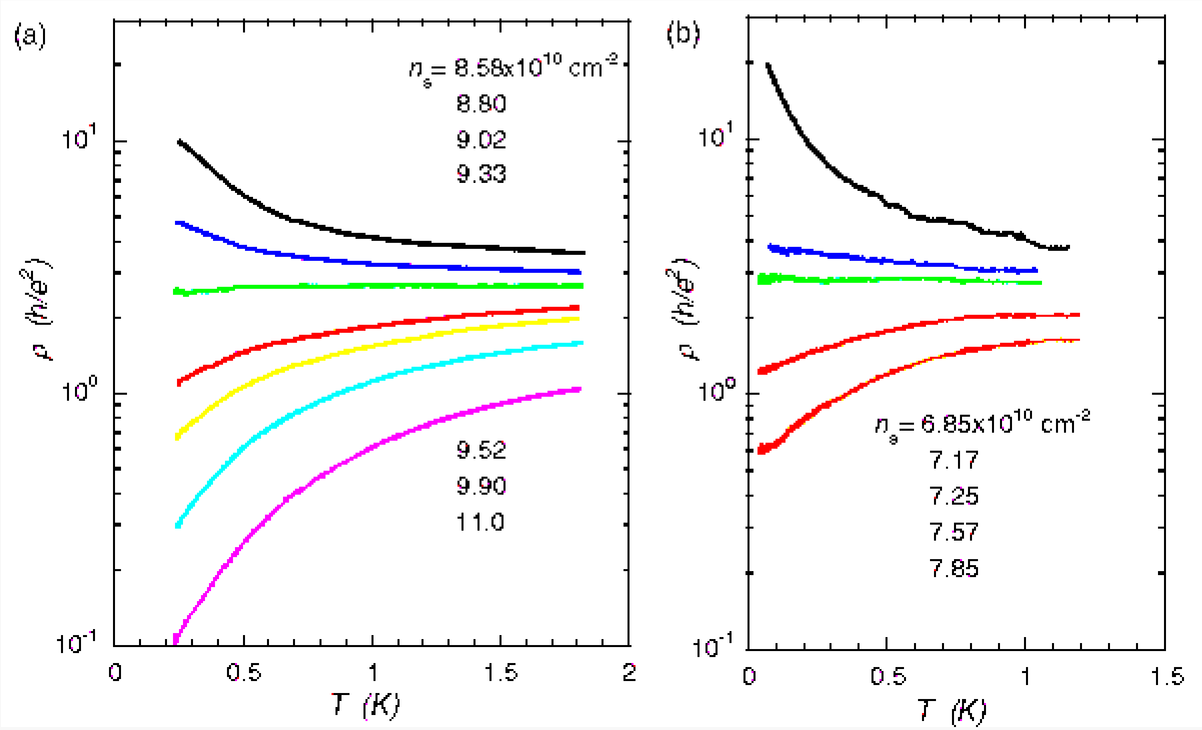
(no Mottness)

Transition: $p_F l_{tr} \sim \hbar$ Ioffe – Regel ; Mott

$$\sigma_{2d} = e^2 \frac{(p_F{}^2 / 2\pi\hbar^2)(l_{tr} / v_F)}{m} \sim e^2 /(2\pi\hbar) \; ; \;\; 2\pi\hbar / e^2 \approx 25 \;\; k\Omega$$

Insulator: area of quantum physics (importance of interference)

Close look at the critical region - C



$$\beta(g_c) = 0\ ??$$

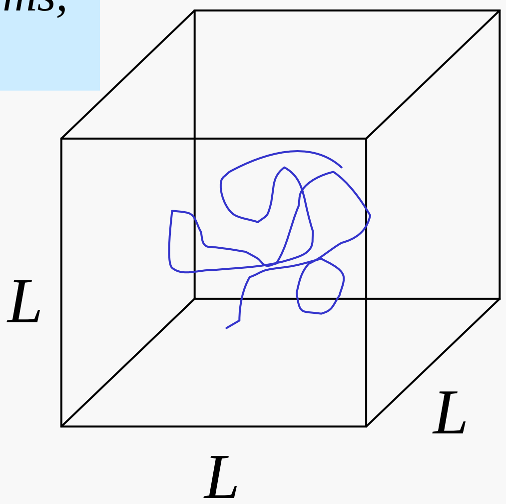
Universal behavior in ultra-clean silicon MOSFETs: different samples from different wafers: (a) V. M. Pudalov, (b) Heemskerk & Klapwijk

- **Resistivities are essentially the same at the separatrices:** $\rho_c \sim \pi h/e^2$
  (critical densities are very different)

S V Kravchenko and M P Sarachik, Rep. Prog. Phys. 67 (2004) 1-44

Scaling ideas in transport : *Thouless (74,77); Abrahams, Anderson, Licciardello, Ramakrishnan (79); Wegner (79).*

Renormalization Group transformation:
increasing of the blocks size from l_{tr} *to* L



dimension $d=2$ is of special importance;
the geometrical factor in the Ohm's law,
$g \sim (L/l_{tr})^{d-2}$, disappears at $d=2$.

g(L)–
dimensionless
conductance of the sample
of the size L in
units of e^2/h .

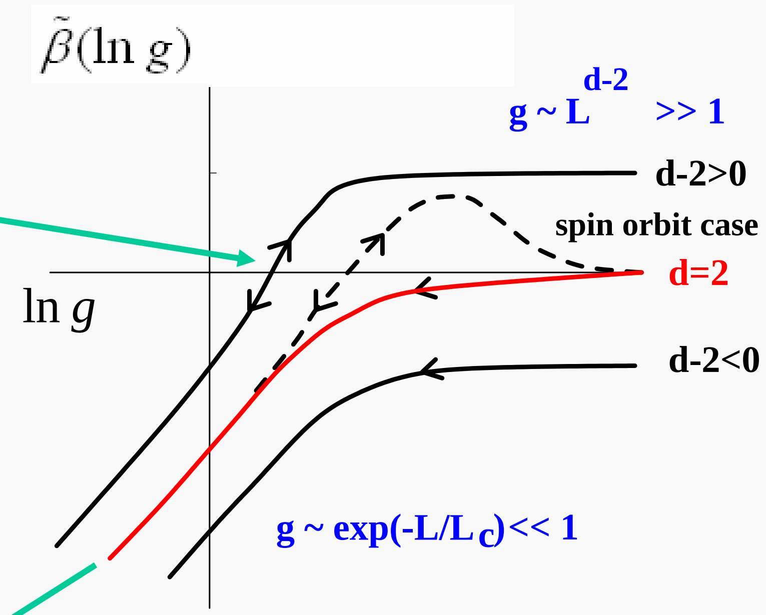
$$\boxed{\frac{dg}{d\xi} = \beta(g)\;;\;\; \xi = \ln\;(L/l_{tr}).}$$

this equation has a very specific form: $\beta(g;\cancel{L/l_{tr}})$ does not depend explicitly on L

M-I transition: $\beta(g = g_{crit}) = 0.$

Abrahams, Anderson, Licciardello, Ramakrishnan (79)

$$\frac{d\ln g}{d\xi} = \tilde{\beta}(\ln g)\ ;\ \xi = \ln(L/a).$$



M-I transition:

a competition between the dimension and the interference (ignoring spin-orbit case)

scale is controlled by the temperature

2D electron systems eventually become localized

$$\ln(1/T\tau) > 1/\rho$$

the conductance g is assumed to be the only relevant scaling parameter

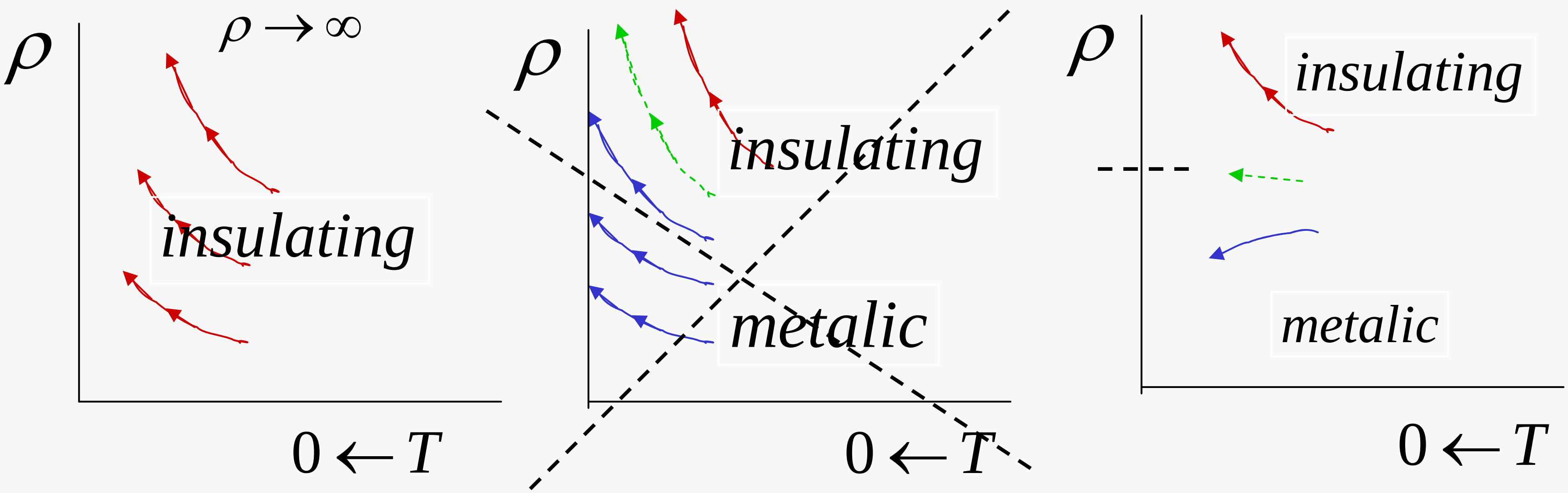
a fundamental difference between experimentalists and theorists:

ρ **versus** σ

behavior of the resistance in 2D

(scale is controlled by the temperature)

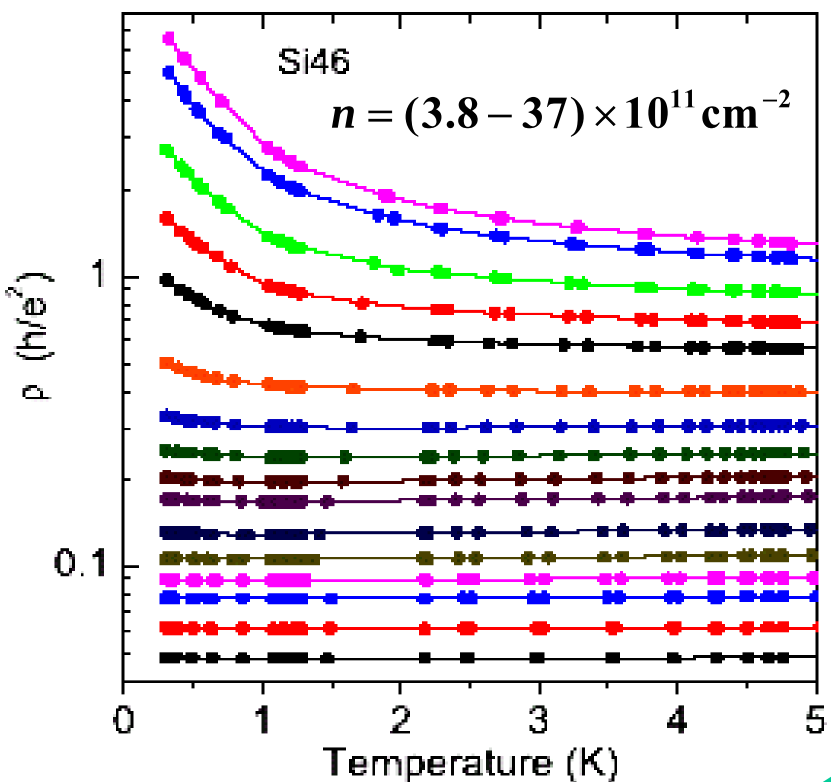
no M - I Transition **M - I Transition**



Si-MOSFET

low mobility μ=1,500cm²/Vs

(data provided by V. M. Pudalov)

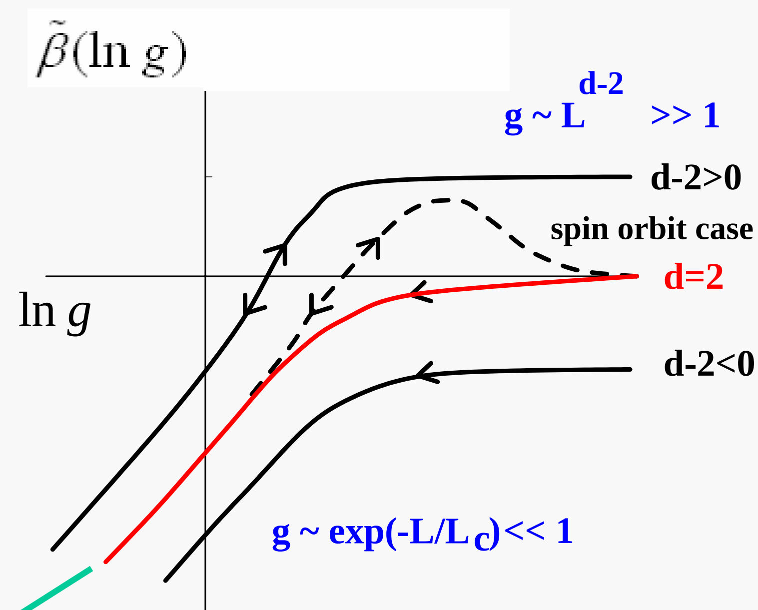


2D electron states eventually become localized

$$\ln(1/T\tau) \geq 1/\rho$$

Abrahams, Anderson, Licciardello, Ramakrishnan (79)

$$\frac{d\ln g}{d\xi} = \tilde{\beta}(\ln g)\ ;\ \xi = \ln(L/a).$$

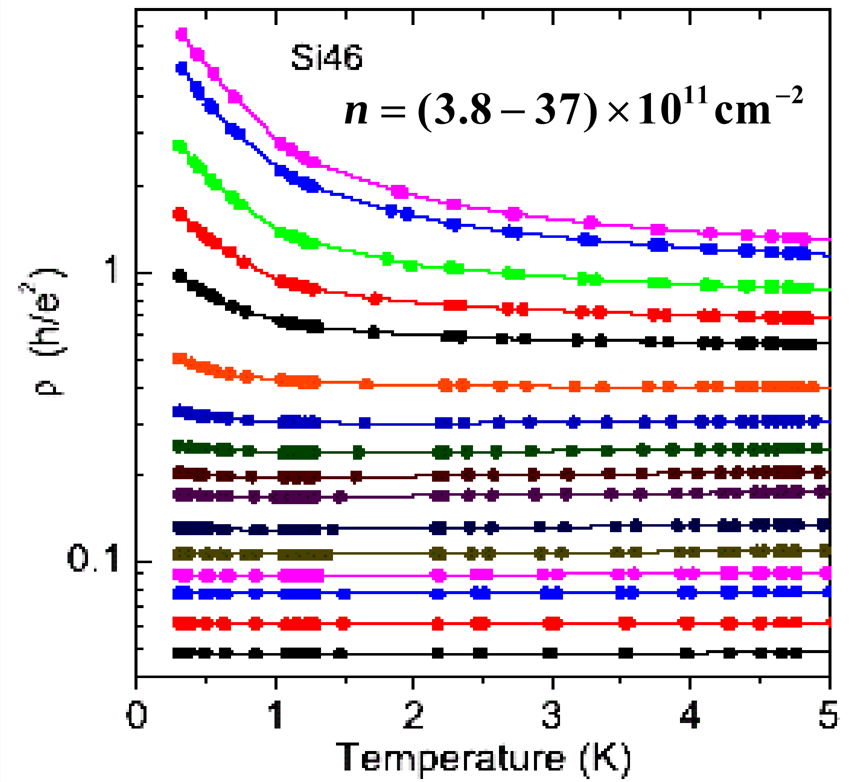


the conductance g is assumed to be the only relevant scaling parameter

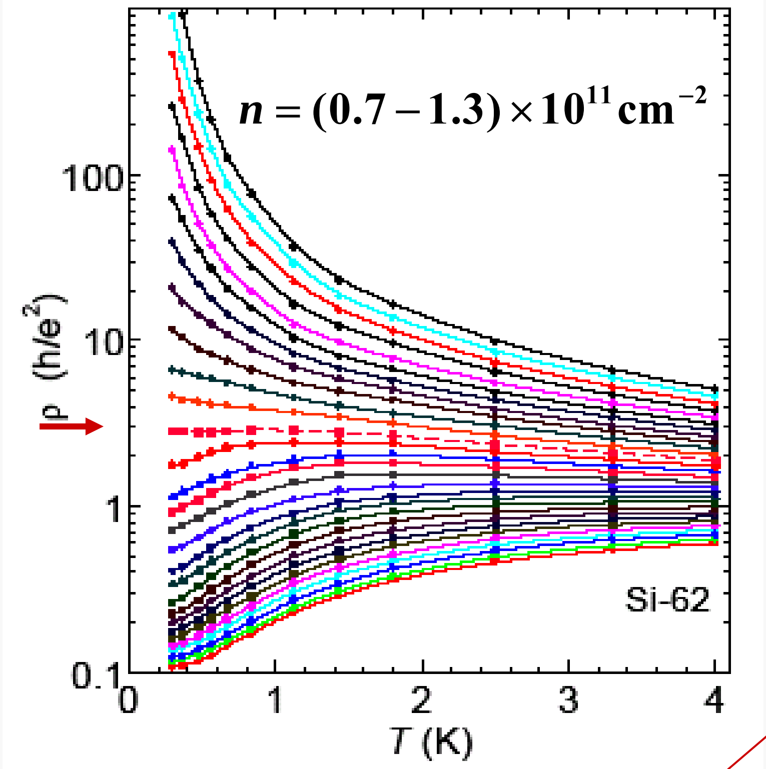
Transport experiments in 2d (Si-MOSFET)

S. Kravchenko, et al., (1994)

low mobility μ=1,500 cm²/Vs



high mobility μ=39,000 cm²/Vs



2D electron systems eventually become insulating

$$\ln(1/T\tau) \geq 1/\rho$$

$$n_c \sim 10^{11}\,cm^{-2} \qquad \rho_c \sim \frac{\pi h}{e^2}$$

the M-I transition?

what is specific for high mobility samples?

at the metal-insulator transition: $l_{tr} \sim \lambda \quad \Rightarrow \quad E_F \sim 1/\tau_{tr}$

$$\mu^{-1} = \frac{m}{e}\tau_{tr}{}^{-1}$$

the higher mobility, the **lower** density can be reached **remaining** in the **metallic** phase

$$E_F \sim n$$

MOSFETs: $n = 10^{11} cm^{-2}$

$$E_F = \frac{\pi\hbar^2 n}{2m} \approx 6K$$

$$r_s \equiv \frac{E_{ee}}{E_F} \simeq 10$$

$$E_{ee} = \frac{e^2}{\varepsilon}(\pi n)^{1/2} \approx 60K$$

Low-Density Spin Susceptibility and Effective Mass of Mobile Electrons in Si Inversion Layers

V. M. Pudalov,[1,2] M. E. Gershenson,[1] H. Kojima,[1] N. Butch,[1] E. M. Dizhur,[3] G. Brunthaler,[4] A. Prinz,[4] and G. Bauer[4]

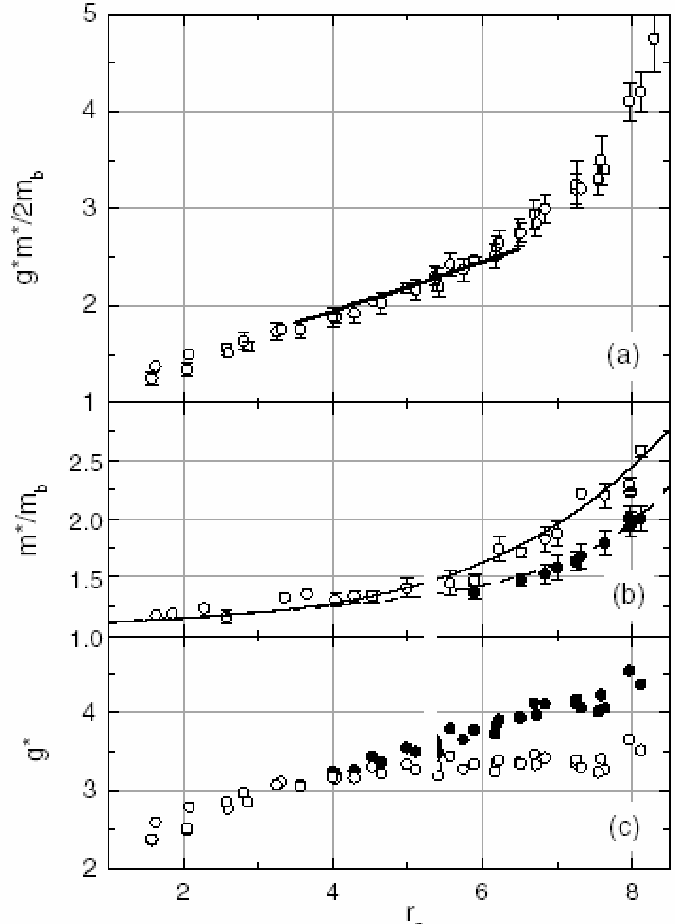
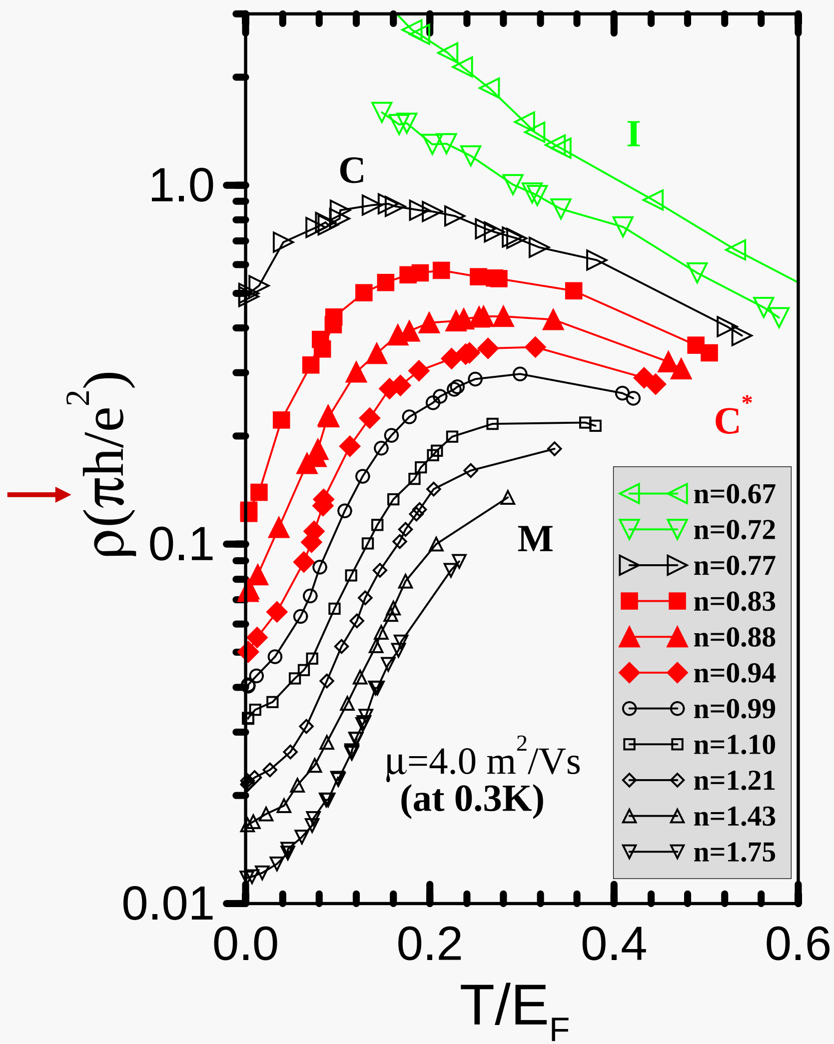


FIG. 4. Parameters g^*m^*, m^*, and g^* for different samples as a function of r_s (dots). The solid line in (a) shows the data by Okamoto *et al.* [10]. The solid and open dots in (b) and (c) correspond to two different methods of finding m^* (see the text). The solid line and the dashed line in (b) are polynomial fits for the two dependences $m^*(r_s)$. The values of g^* shown in (c) were obtained by dividing the g^*m^* data by the smooth approximations of the experimental dependences $m^*(r_s)$ shown in (b).

ρ (T) in a high mobility sample



Pudalov, et al., ('98)

I - insulating region

C - narrow critical region containing the separator

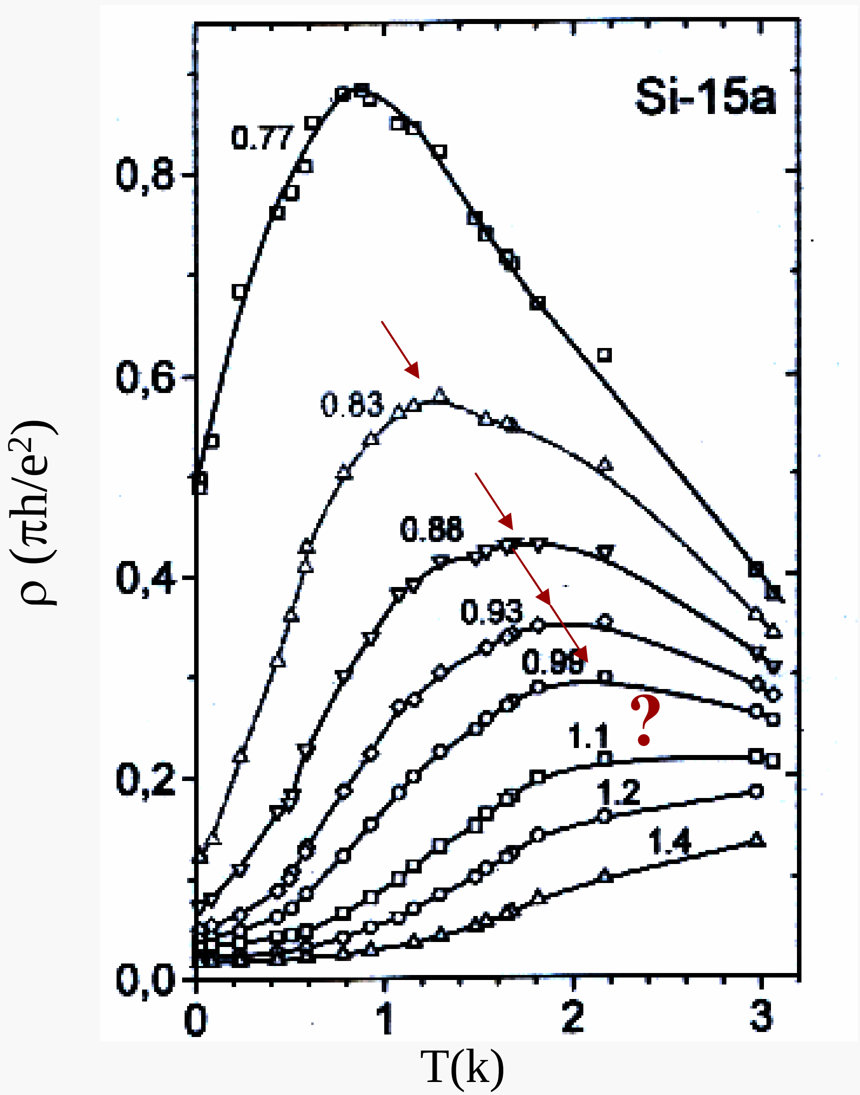
the regions of my interest :
strongly interacting electrons in the diffusive regime

C* - non-monotonic region: $\rho \le h/e^2$

$$T < \hbar/\tau \le E_F$$

M - region with no clear maximum

Closer look at the region - C*



(data provided by V. M. Pudalov)

diffusive electrons not too far from the M-I transition.

$$\rho \le h/e^2$$

$$T < \hbar/\tau \le E_F$$

A universal picture with single parameter scaling?

$$\frac{d\rho}{d\xi} = \beta_{''\infty''}(\rho)$$

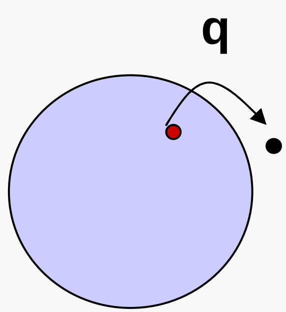
there are **non monotonic** curves near the M-I transition.

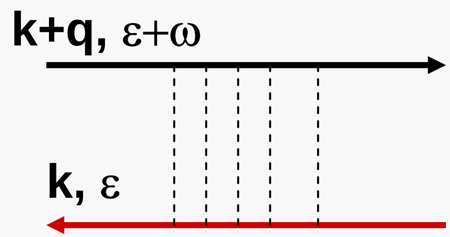
Low energy modes in a diffusive system

(non-linear σ–model with *e-e* interactions)

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)$$

particle-hole excitations

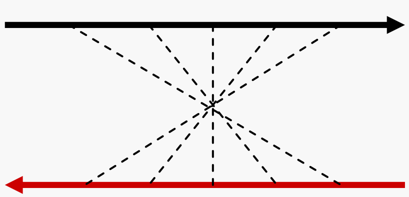


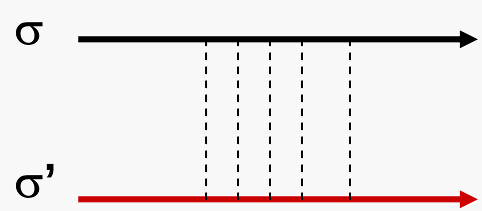


$$\approx \frac{1}{Dq^2 - i\,z\,\omega}$$

disorder average propagator
(diffuson)

diffusive pole





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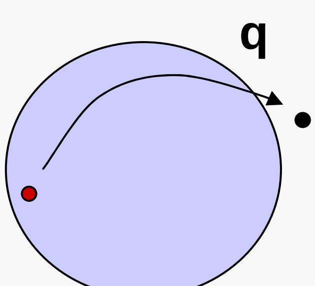


particle-particle propagator
(cooperon)

spin-1/2
- 1 singlet
- 3 triplet modes

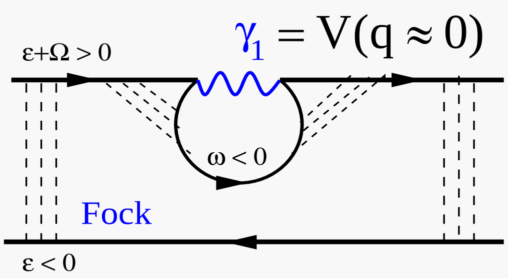
electron-electron interaction in disordered 2D gas

$(\varepsilon_F \tau)^{-1} << 1$

$T\tau << 1$

the slow diffusion of the electrons results in non-analytic corrections to the diffusion coefficient (conductivity) and the *e-e* interaction amplitudes

Altshuler, Aronov('80)



singlet

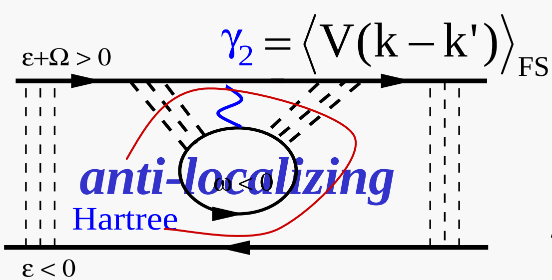
$$\gamma_s = \gamma_1 - \frac{1}{2}\gamma_2$$

$$\delta\sigma(T) = -\frac{e^2}{2\pi^2}\left(\gamma_s + 3\gamma_t\right) \ln(1/T\tau)$$

two contributions of opposite signs (a la Hartree-Fock)

notice factor 3 here

$$\boxed{\delta\sigma(T) = -\frac{e^2}{2\pi^2}\left(1 - 3\frac{\gamma_2}{2}\right) \ln(1/T\tau)}$$



triplet

$$\gamma_t = -\frac{1}{2}\gamma_2$$

(a prototype of the one-loop corrections)

$$\delta\sigma(T) = -\frac{e^2}{2\pi^2}\left(1 - 3\frac{\gamma_2}{2}\right)\ln(1/T\tau)$$

In conventional conductors
γ_2 is small and the net effect of interactions is localizing

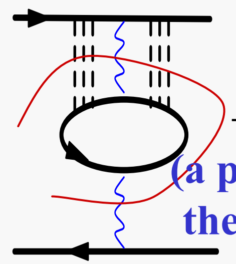
can γ_2 win?

Can the trend change and become anti-localizing?

warning:
this "Hartree-antilocalization" should not be confused with the Spin-Orbit effect.

Can γ_2 become large as a result of the renormalizations?

The slow diffusion of the electrons results in non-analytic corrections to the ***e-e*** interaction amplitudes

$\delta\gamma_2 =$  $+ \cdots\cdots$

(a prototype of the one-loop corrections)

$$\propto \rho \;\ln(1/T\tau)$$

γ_2 in the presence of disorder increases at low temperatures.

A. Finkel'stein (83)

Physical meaning of γ_2 :

$$\gamma_t = -\frac{1}{2}\gamma_2$$

$$1+\gamma_2 = \frac{1}{1+F_0^{\sigma}}$$

$1+\gamma_2$ describes enhancement of the spin susceptibility (Stoner-factor enhancement);

for repulsive electrons $F_0^{\sigma} < 0$.

$$D_s = D/(1+\gamma_2) \qquad \sigma / e^2 = 2\nu D$$

spin-diffusion coefficient

"Can γ_2 become large as a result of the renormalizations? "

is equivalent to

" Does the gas of interacting disordered electrons approach a magnetic instability? "

Two parameter scaling

A. Finkel'stein (83)

finite ρ increases γ_2

while, γ_2 reduces ρ

the interplay of disorder and interaction is captured by a set of two coupled Renormalization Group equations for ρ and γ_2:

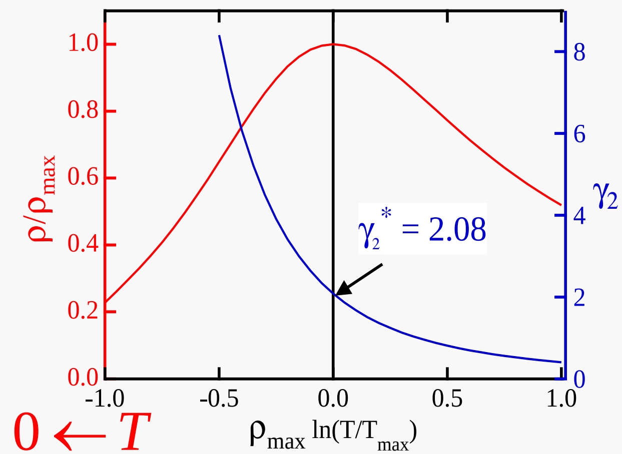
to all orders in γ_2

$$\frac{d\rho}{d\xi} = \rho^2 \left[\overset{\textit{cooperon}}{1} + \overset{\textit{singlet}}{1} - 3\overbrace{\left(\frac{1+\gamma_2}{\gamma_2}\ln(1+\gamma_2) - 1\right)}^{\textit{triplet}}\right]$$

$$\frac{d\gamma_2}{d\xi} = \rho\,\frac{(1+\gamma_2)^2}{2}$$

$$\xi = \ln(1/T\tau)\ ,\quad T\tau \ll 1$$

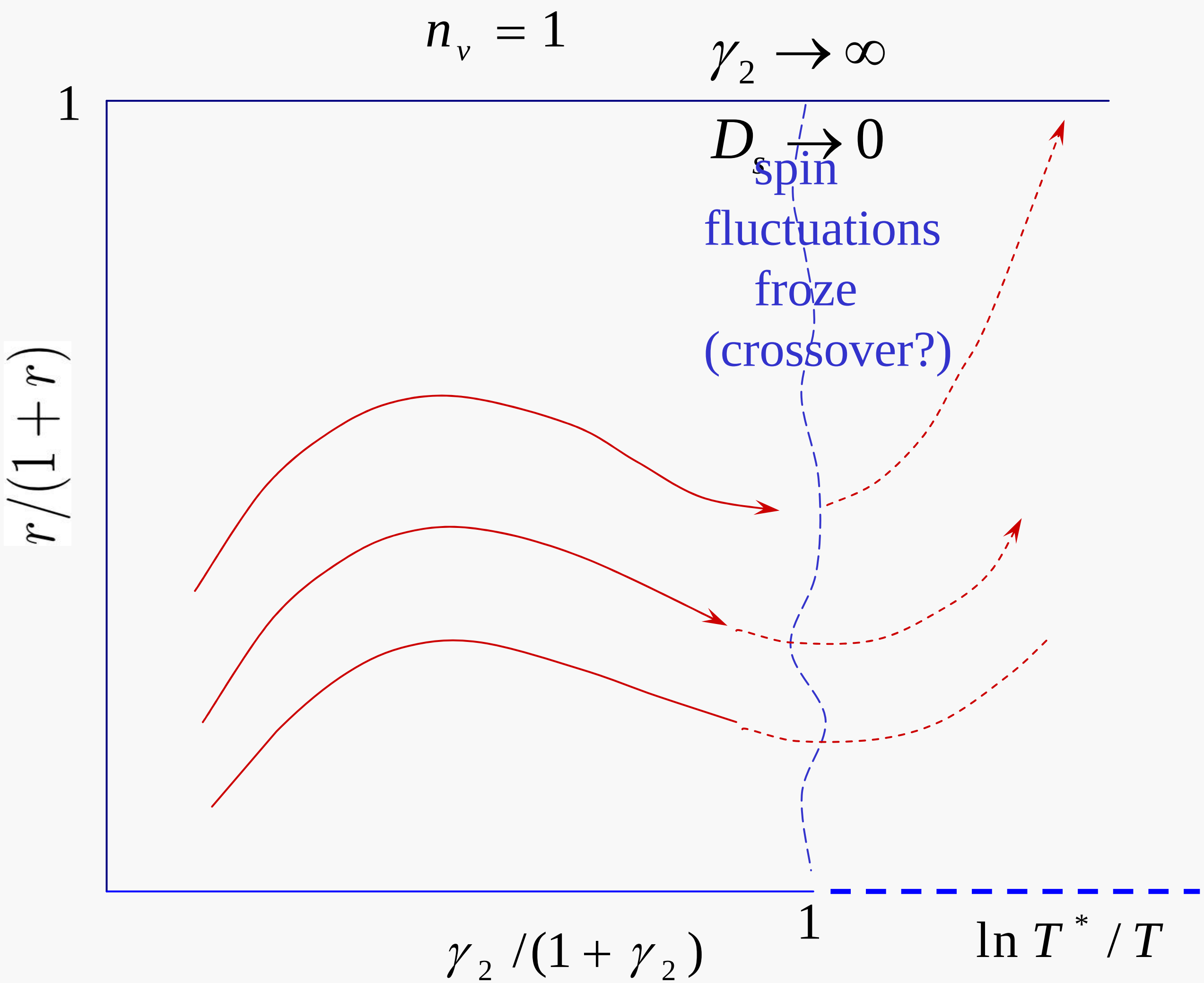
Solution of the RG-equations with γ_2 diverging at $T^* \approx T_{\max}\, e^{-1}$



after rescaling the solutions are described by a *single* curve

the case of Coulomb interaction only; no free-electron limit in this equations.

developing of the insulating state occurs in 2 stages: first spins, only afterwards the localization of charges (1984).



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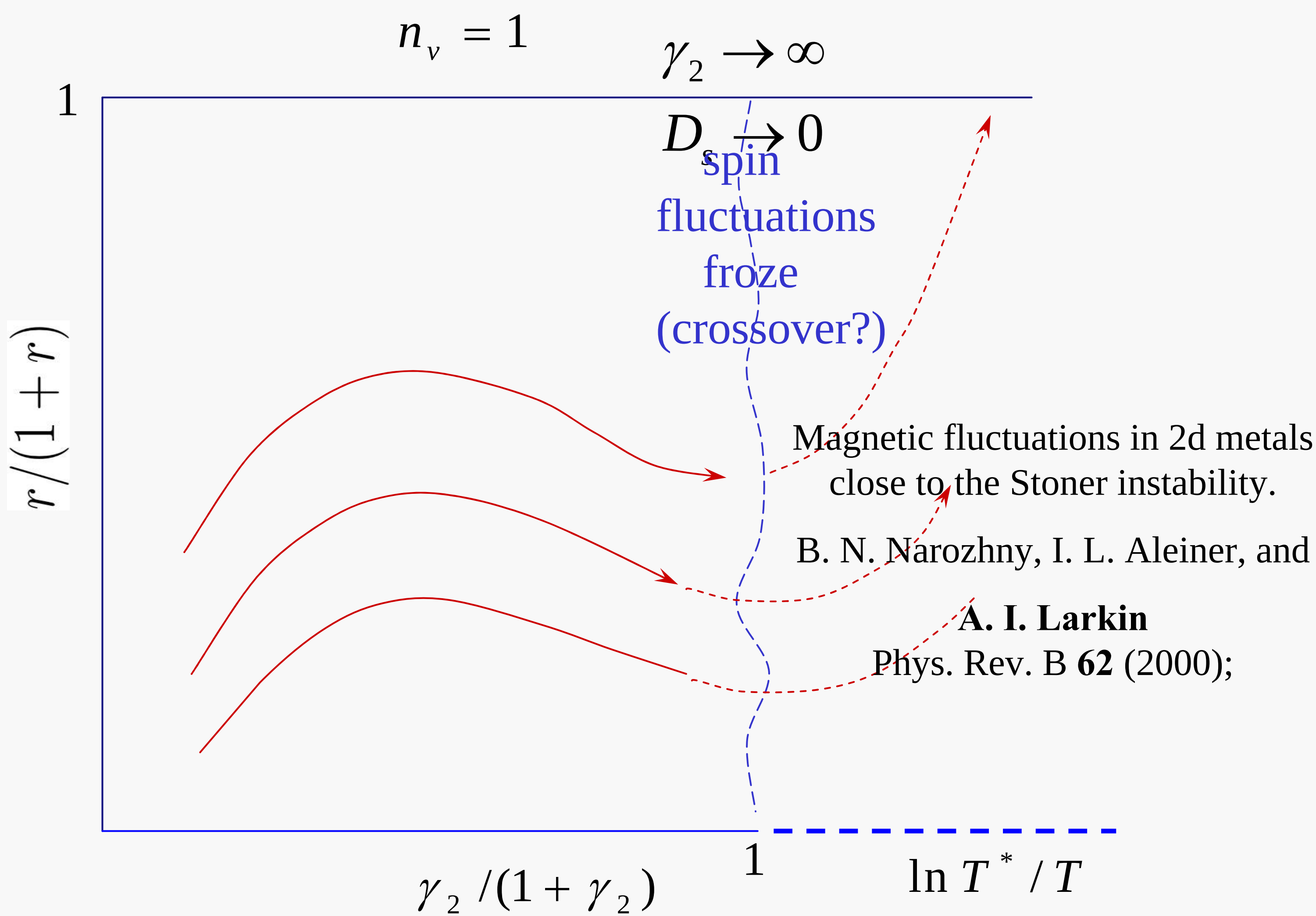
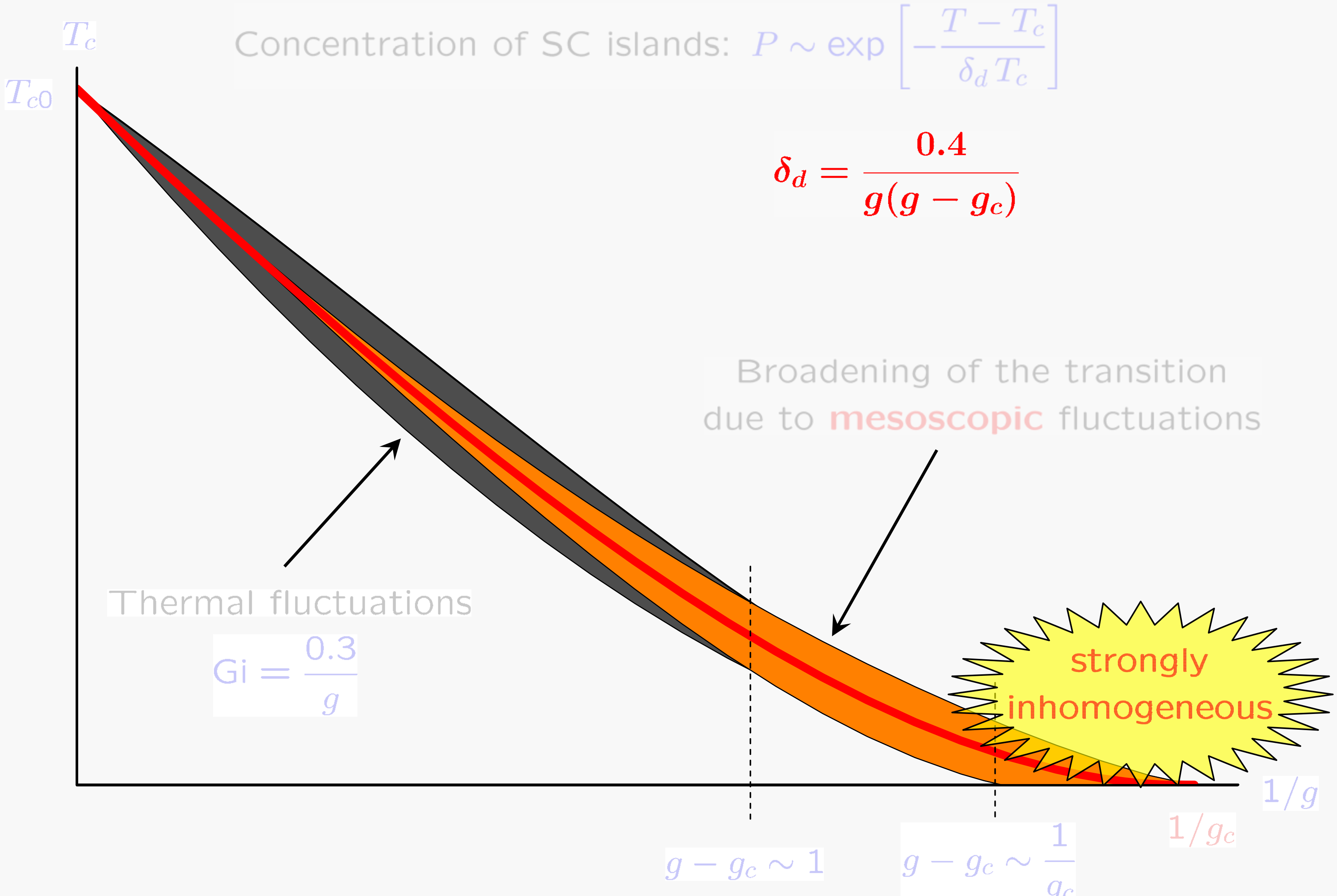
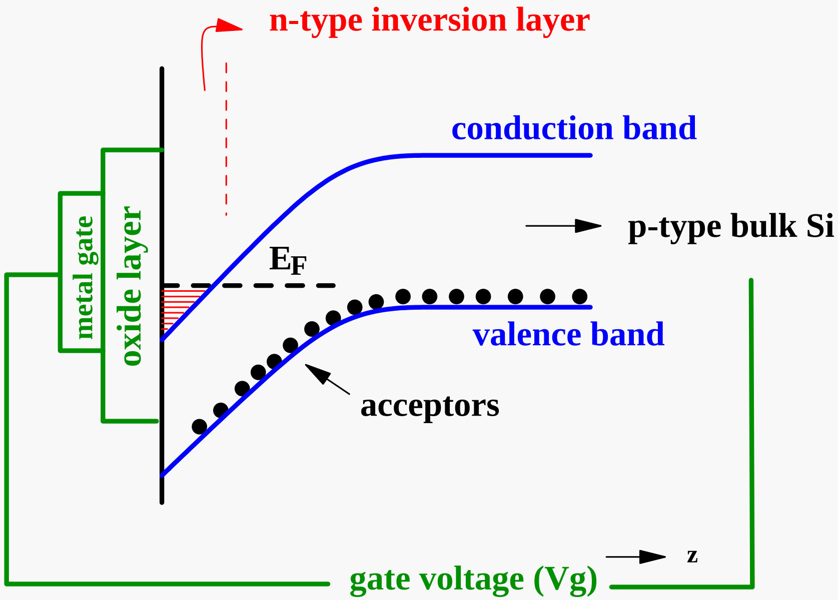


Superconductivity in disordered thin films: Giant mesoscopic fluctuations.

Skvortsov and Feigel'man PRL, 95 (2005)



2DEG in Si-MOSFETs: two valleys

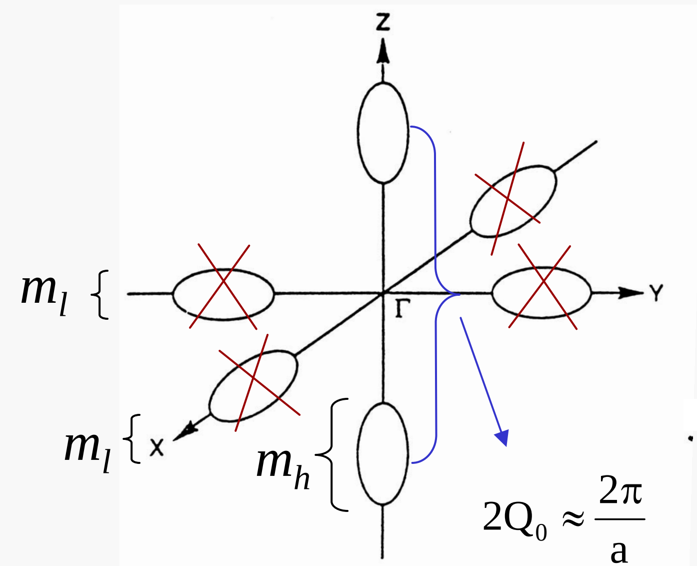


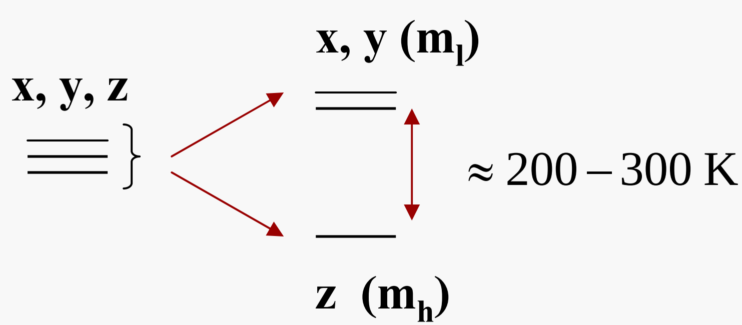
Energy $\left(\frac{k_z^2}{2m_z}\right)$ is quantized in the z-direction

$$k_z \ll Q_0\ !$$

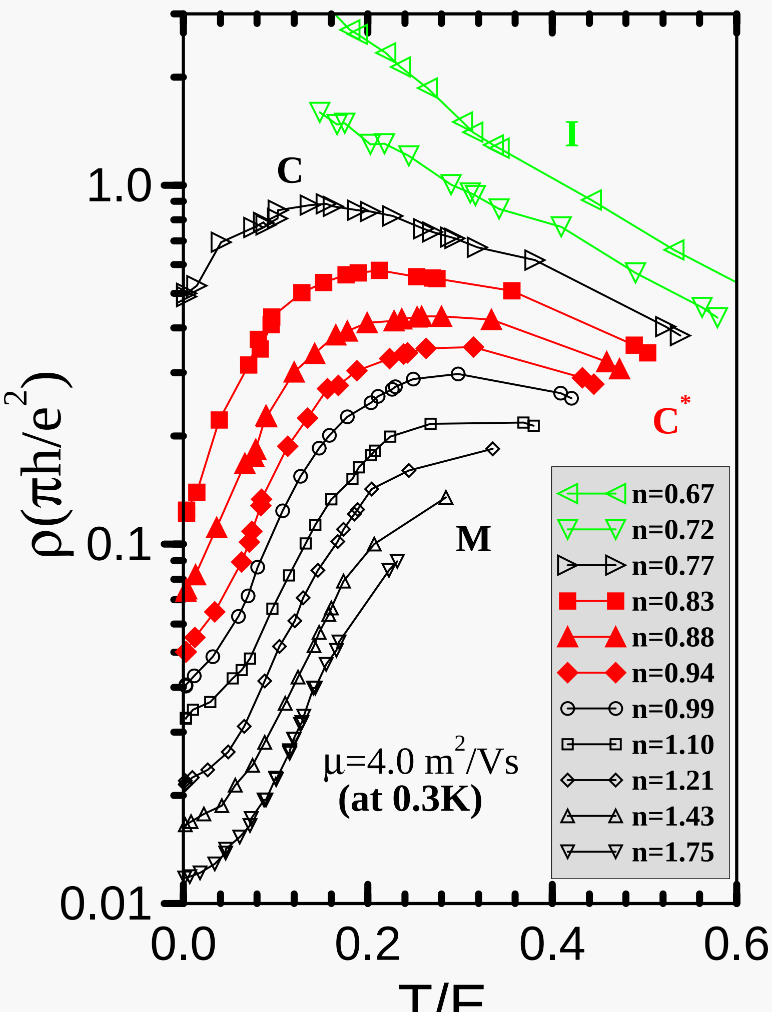
No intervalley scattering, but still electrons in different valleys are coupled by e-e interaction.

Lifting of the valley degeneracy in a (001) layer

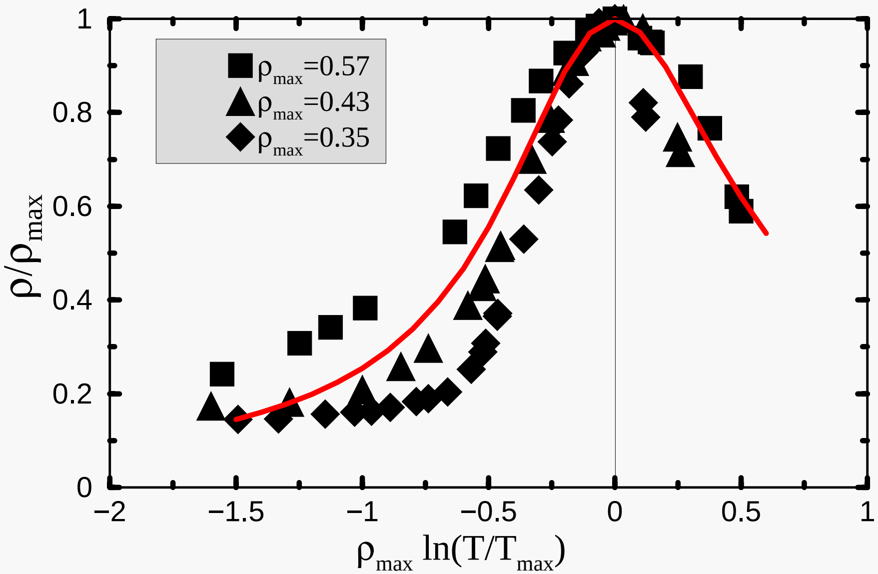




Analysis of the region C* in a high-mobility sample with RG for two valleys



Pudalov, et al., ('98)



Data from the region C* in a high-mobility sample

- the drop of five times in $\rho(T)$ and its saturation has been captured in the correct temperature interval
- **no adjustable parameters** are used

A. Punnoose and AF, PRL (2002)

- RG equations successfully describes the region **C***

$$\rho(T) = \rho_{max} R(\eta) \quad and \quad \eta = \rho_{max} \ln(T_{max}/T)\,.$$

(i) $\rho(T)$ is non-monotonic; data collapses on a single curve

(ii) effect of an in-plane magnetic field at $(g\,\mu_B H_{||} / kT) \sim 1$

- the theory is consistent down to $T^* \approx T_{\max}\, e^{-30000}$ $(\gamma_2 \to \infty)$

$(T^* \approx T_{\max}\, e^{-1})$

for one valley case

$$\ln(\ln 1/T^*) \sim (2n_v)^2$$

- in the limit $n_v \to \infty$ the theory is internally consistent including T=>0

- **one-loop result:** for 2-valleys, since T* is practically zero, the existence of the ***Metal to Insulator*** transition in 2D is (logically) unavoidable, providing that the localization at strong disorder is undisputable.

Valleys: the large-n_v limit

A combined effect of the two spin projections and n_v valleys (flavors) enhances the screening and makes the bare value of γ_2 to scale as $1/(2n_v)$

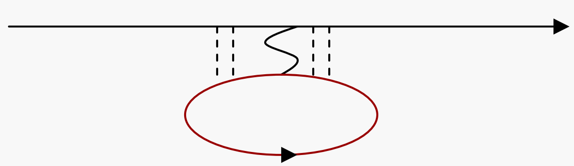
Natural variables: $t = n_v \rho$ and $\theta_2 = 2n_v\gamma_2$

resistance per species

interaction has to be compensated for enhanced screening by the loop summation over spin and valleys; each loop involves a sum over the spin and valley indices

The limit $n_v \to \infty$ is taken keeping t and θ_2 finite

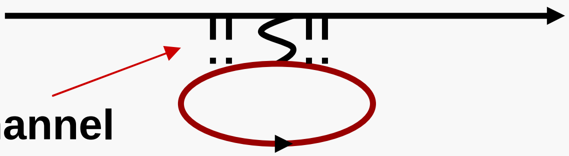
$$\text{loop} \Rightarrow \gamma_2 \sum_{spin+valleys} \int d^2q \Rightarrow 2n_v t\gamma_2 = t\theta_2$$



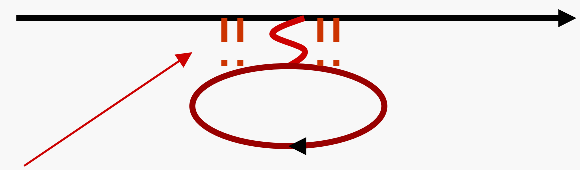
not more than one interaction per loop in the large n_v limit

not more than one interaction per loop

in the large n_v limit

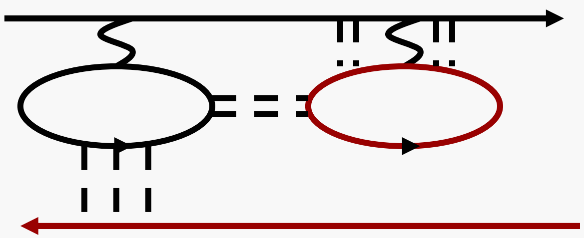


Diffuson channel

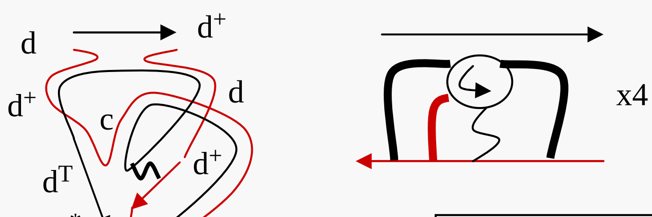


Interaction in the Cooperon channel

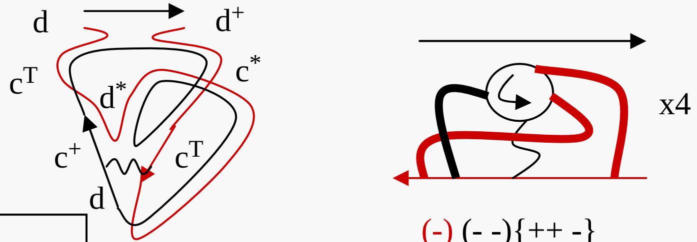
Two loops - t^2 contributions



$$t^2(2n_v\gamma)^2 = t^2\theta^2$$

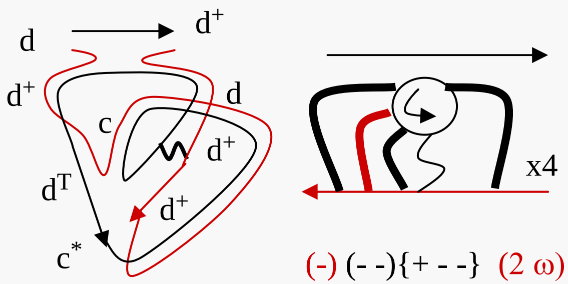


(-) (- -){++ -}

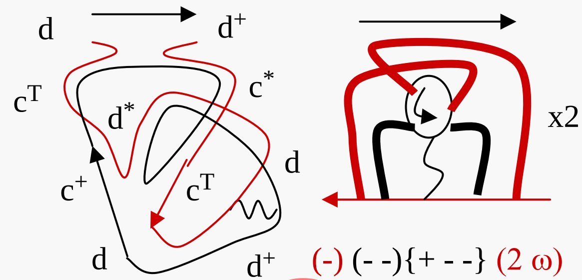


(-) (- -){++ -}

$= + 8\ \alpha\ (\gamma_2 + \gamma_c)\ D(p,\omega)\ D^2(q,\omega)\ k^2$



(-) (- -){+ - -} (2 ω)



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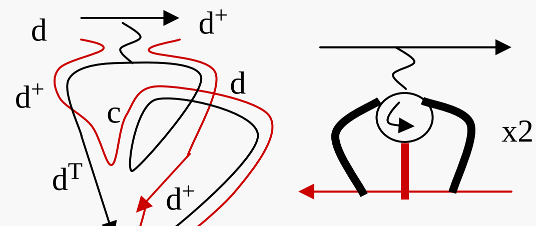
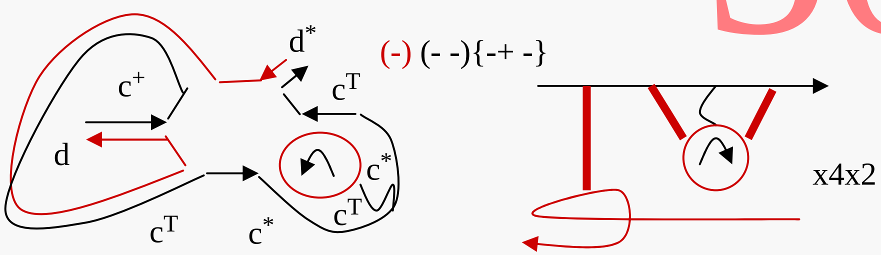


(-) (- +){+ + -} (2 ω)

50

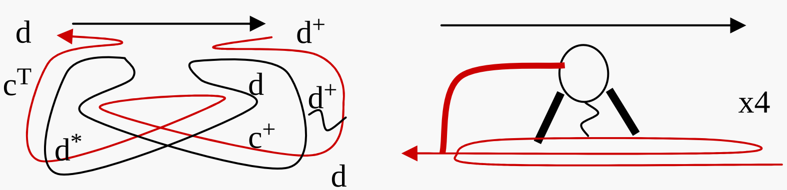
$- 4\ \alpha\ (\gamma_2 + \gamma_c)\ (2\omega)\ D^3(p,\omega)\ D(q,\omega)\ k^2$	$- 2\ \alpha\ (\gamma_2 + \gamma_c)\ (2\omega)\ D(p,\omega)\ D^2(q,\omega)\ k^2$	$- 2\ \alpha\ (\gamma_2 + \gamma_c)\ (2\omega)\ D^2(k+p,\omega)\ D(q,\omega)$



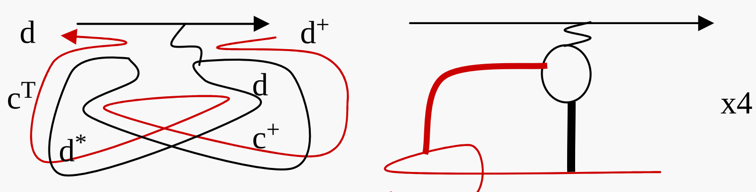
(-) (- -){-+ -}

x2
note the extra factor

$= - 8\ \alpha\ (\gamma_2 + \gamma_c)\ D(p)\ D^2(q,\omega)\ k^2$



(-) (- -){+ + -}



(-) (- +) {+ - -}

$= 4\ \alpha\ (\gamma_2 + \gamma_c)\ D^2(p,\omega)\ D(q,\omega)\ k^2$	$= 4\ \alpha\ (\gamma_2 + \gamma_c)\ D(k+p,\omega)\ D(q,\omega)$

RG equations: β - functions to order t^2 in the large N limit

$$\theta = \gamma_2 + \alpha\,\gamma_c\,; \qquad \alpha = 1, 0$$

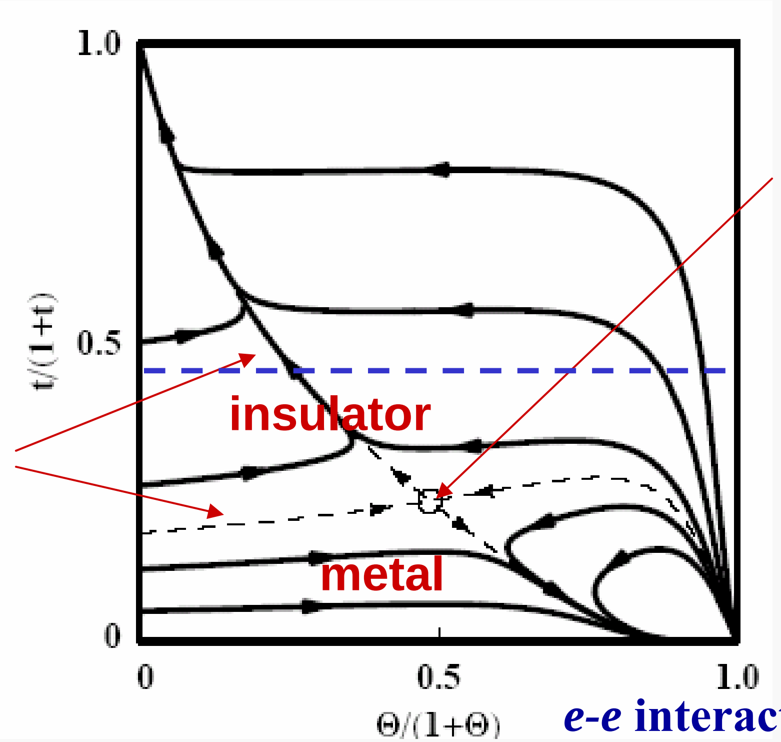
$$\frac{d\ln t}{d\xi} = (\alpha - \Theta)t + (1 - \alpha - \alpha\Theta + c_t\Theta^2)t^2$$

$$\frac{d\Theta}{d\xi} = (1 + \alpha + \alpha\Theta)t - \left(4(1+\alpha)\Theta + 2\alpha\Theta^2 + 4c_\theta\Theta^3\right)t^2$$

$$c_t = \frac{1}{2}\left(5 - \frac{\pi^2}{3}\right) \approx 0.8,$$

$$c_\theta = \frac{1}{2}\left(1 - \frac{\pi^2}{6}\right) \approx 0.08$$

$t \to \infty,\ \Theta \to 0$



separatrices

unstable MIT fixed point

$t \to 0.3,\ \Theta \to 0.9 \quad (\alpha = 1)$

when the cooperon channel is included.

***e-e* interaction**

non-linear σ– model with *e-e* interactions

A. Finkel'stein (83)

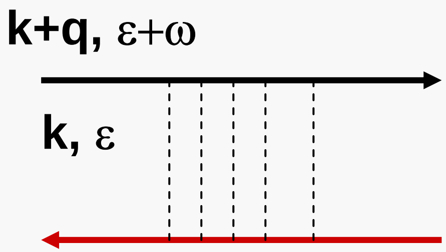
$$S[Q]=\frac{\pi\nu}{4}\int \mathrm{T}r\left[D\left(\vec{\nabla}Q\right)^2-4z\mathrm{Tr}\left(\hat{\varepsilon}Q\right)+Q\hat{\Gamma}Q\right]$$

$$Q_{nm}^{\alpha\beta\ ij} \qquad TrQ=0 \qquad Q^2=I$$

one more RG-equation: $\boldsymbol{\frac{d\ln z}{d\xi}=\varsigma(t,\Theta)}$

The parameter z describes
the renormalization of the DOS of the diffusion modes

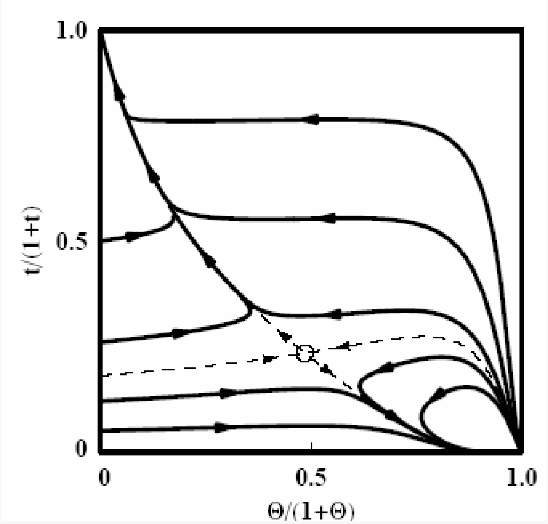
Physical consequence of the existence of the fixed point: thermodynamics at MIT have a critical behavior



$$\approx \frac{1}{Dq^2 - i\,z\,\omega}$$

$$\boxed{\frac{d\ln z}{d\xi} = \varsigma(t,\Theta)}$$

$$\boxed{z = 1/T^{\varsigma_c}}$$



At the transition:

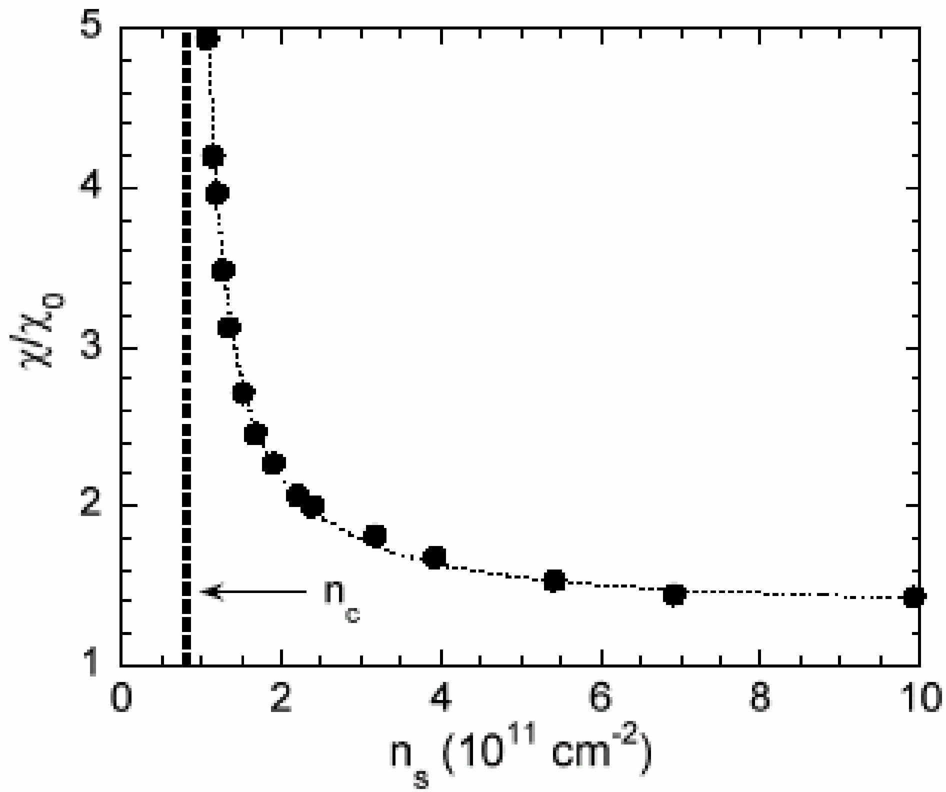
$$\varsigma_c = \varsigma(t_c, \Theta_c) \approx 1/4$$

The parameter z corresponds to the DOS of the diffusion modes

Specific heat: $C_v \sim (z\nu)T \sim T^{1-\varsigma_c}$

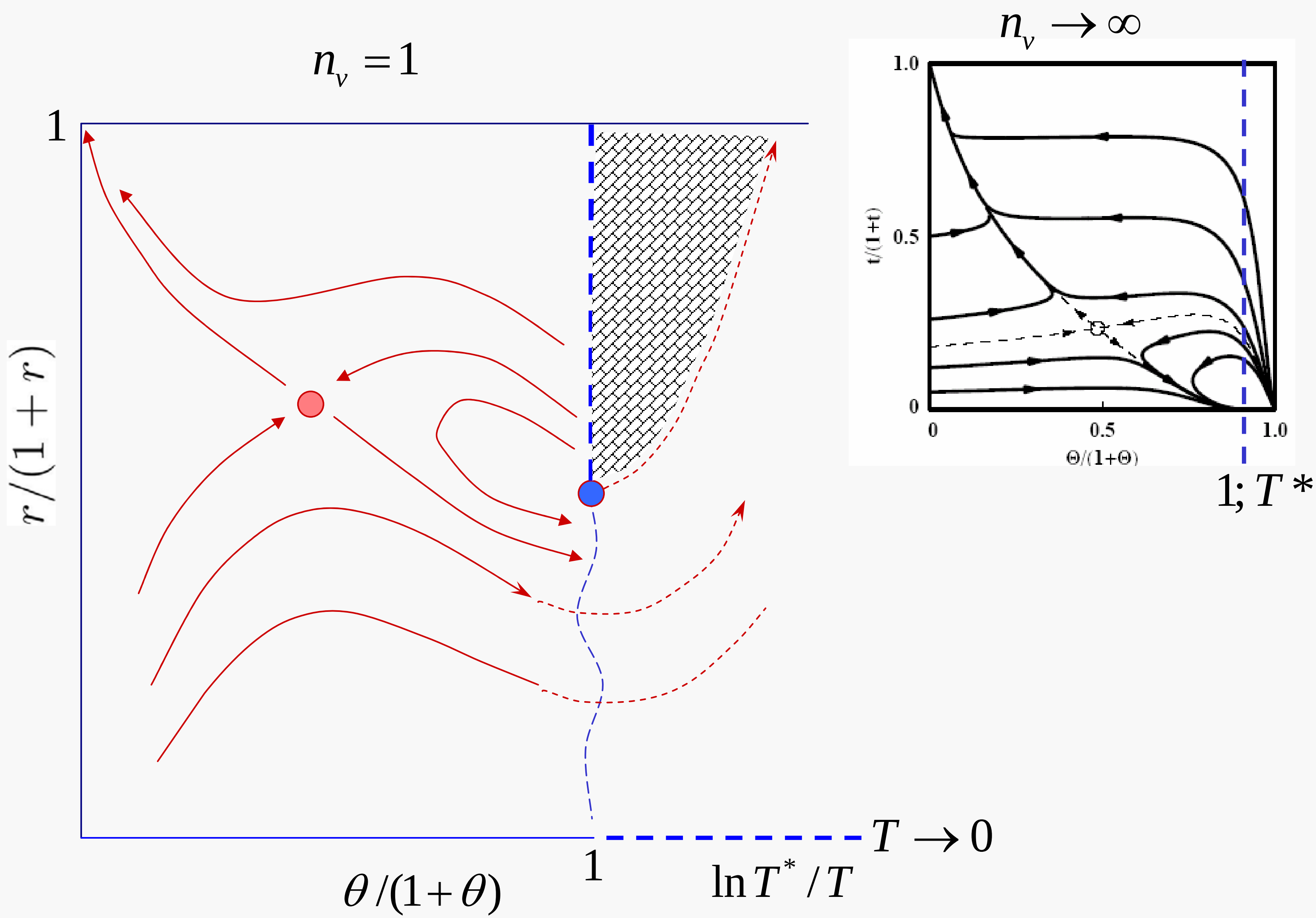
Spin susceptibility: $\chi/\chi_0 = z(1+\gamma_2) \sim 1/T^{\varsigma_c}$

Pauli spin susceptibility



*Prus et al., PRB **67,** 205407 (2003)*

A. A. Shashkin et al., (2004-2005)


$n_v = 1$
$n_v \to \infty$
1
$r/(1+r)$
$\theta/(1+\theta)$
1
$\ln T^*/T$
$T \to 0$
1.0
0.5
0
t/(1+t)
0
0.5
1.0
Θ/(1+Θ)
1; T^*

Conclusions

The interplay of interactions and disorder fundamentally revises the common belief that 2D electron systems become insulating at low enough temperatures.

Using a large-N approximation scheme (valleys), we obtained a two-parameter scaling theory that exhibits a metal-insulator transition in 2d.

The transition between the metallic and insulating phases is controlled by a finite-resistance unstable fixed point.

The theory is internally consistent: there are no divergences in the interaction amplitudes at finite temperatures.

The two-stage route (1984) to the MIT transition (first spins, next charges) has been revised.

The spin-susceptibility close to the transition diverges. The g-factor remains finite => this divergence is not related to any magnetic instability.

Numerically the parameters of the fixed point appeared to be small. This gives arguments in favor that the 2-loop calculations are adequate and sufficient in the large-N limit.

Thanks to Alex Punnoose

for fruitful collaborations