V.F.Gantmakher

General theory of quantum phase transitions applied to transport phenomena

1. **Metal - insulator transitions, 3D and 2D**
2. **Superconductor - insulator transitions**

Argonne 2005

S.Sachdev, Quantum Phase Transitions (Cambridge University Press, Cambridge 2000)

The partition function $Z = \sum_i e^{-\varepsilon_i / T}$ determines probability p_i for state ε_i to exist

$$p_i = \frac{e^{-\varepsilon_i / T}}{Z}$$

and probability P_i for the system to be in the point q of the configurational space

$$P_i(q) = \frac{1}{Z} \sum_i \varphi_i^*(q) \varphi_i(q) e^{-\varepsilon_i / T} .$$

Since $\int P(q) dq = 1$, the partition function is

$$Z = \sum_i \int \varphi_i^*(q) \varphi_i(q) e^{-\varepsilon_i / T} dq$$

$$\tilde{t} = i \frac{\eta}{T}$$

Compare with time-evolution operator $\hat{S} = \exp(-\frac{i}{\eta} \hat{H} t)$ in quantum mechanics

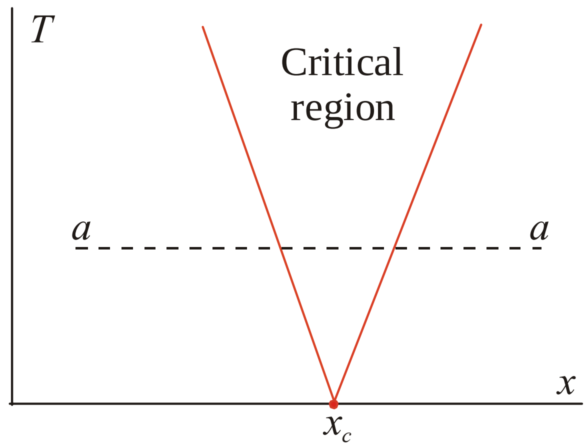
and its diagonal matrix elements $\int \varphi_i^*(q) \varphi_i(q) e^{-i\varepsilon_i t / \eta} dq$

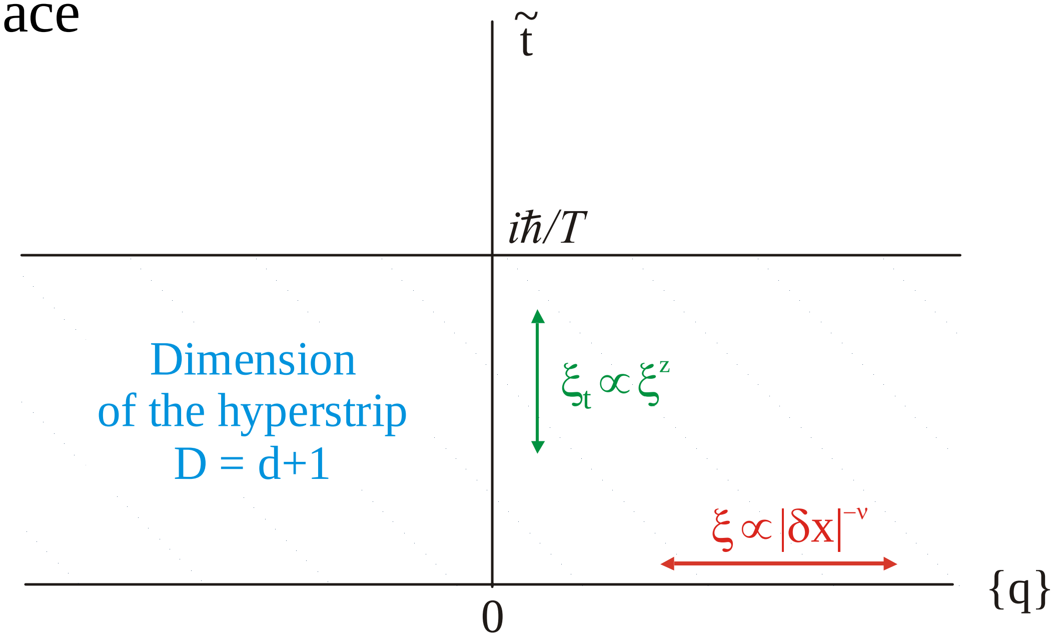
The mapping

of the quantum phase transition in *d*-space

to the classical phase transition in

$D = d + 1$ – dimensional strip



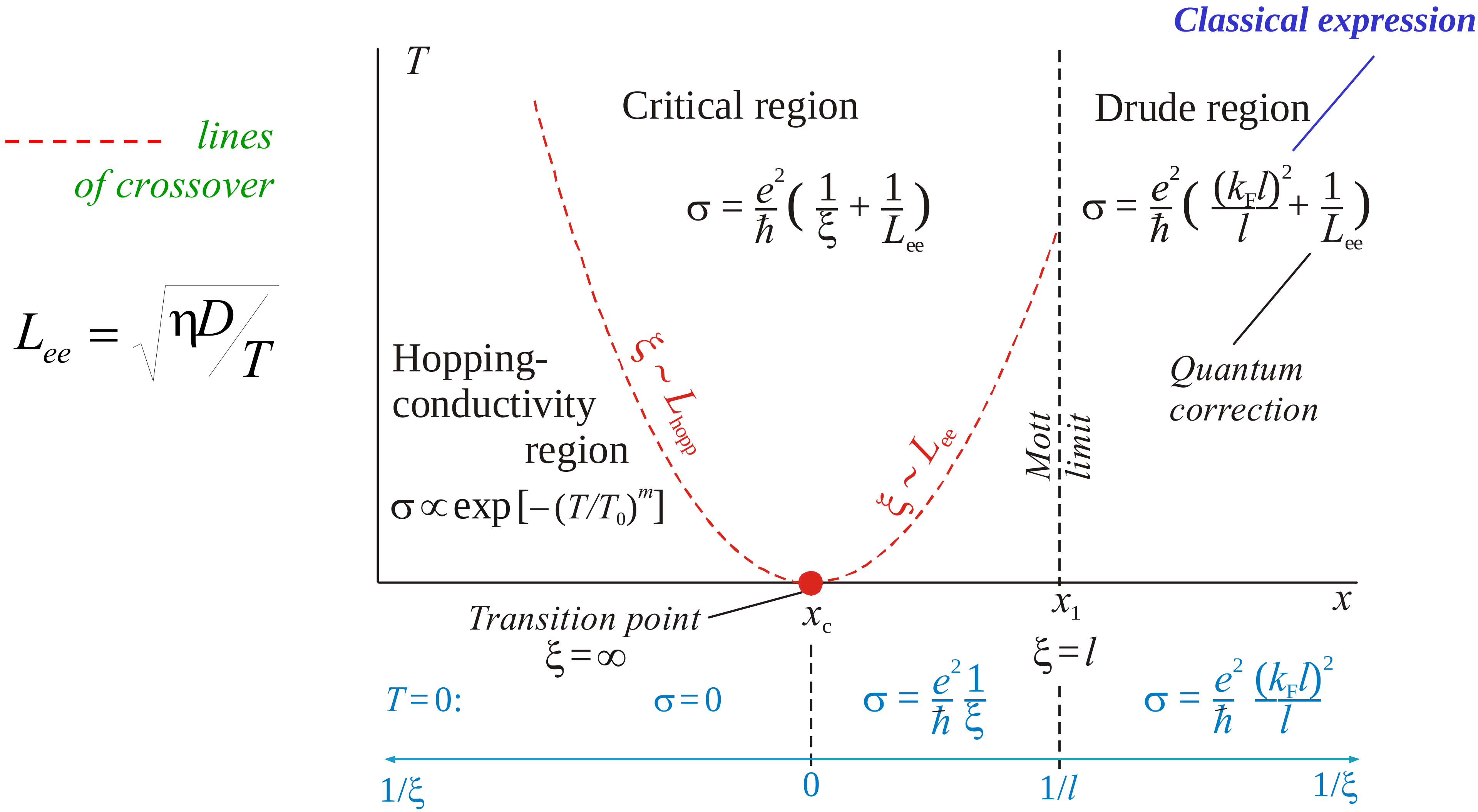


Two lengths: ξ and $L_\varphi \propto \xi_t^{1/z} \propto (\eta / T)^{1/z}$

Scaling expression for conductance $\sigma \propto \xi^{2-d} f\left(\frac{L_\varphi}{\xi}\right)$ ***contains***

two scaling variables : ξ ***and*** $u = L_\varphi / \xi$

(x, T) – diagram, which follows from AALR-1979



The descriptoin includes **two** *lenghts:* ξ *and* L_{ee} *,*

both diverge in the transition point

Function σ(T) in the critical region

In metal region at $T \neq 0$

$$\sigma = \sigma_{03} + \frac{e^2}{\eta}\frac{1}{L_T} \qquad L_T = \sqrt{D\eta / T},$$

In the vicinity of the transition at $T = 0$

$$\sigma = \frac{e^2}{\eta}\frac{1}{\xi}$$

Interpolation

$$\sigma = \frac{e^2}{\eta}\left(\frac{1}{\xi} + \frac{1}{L_T}\right)$$

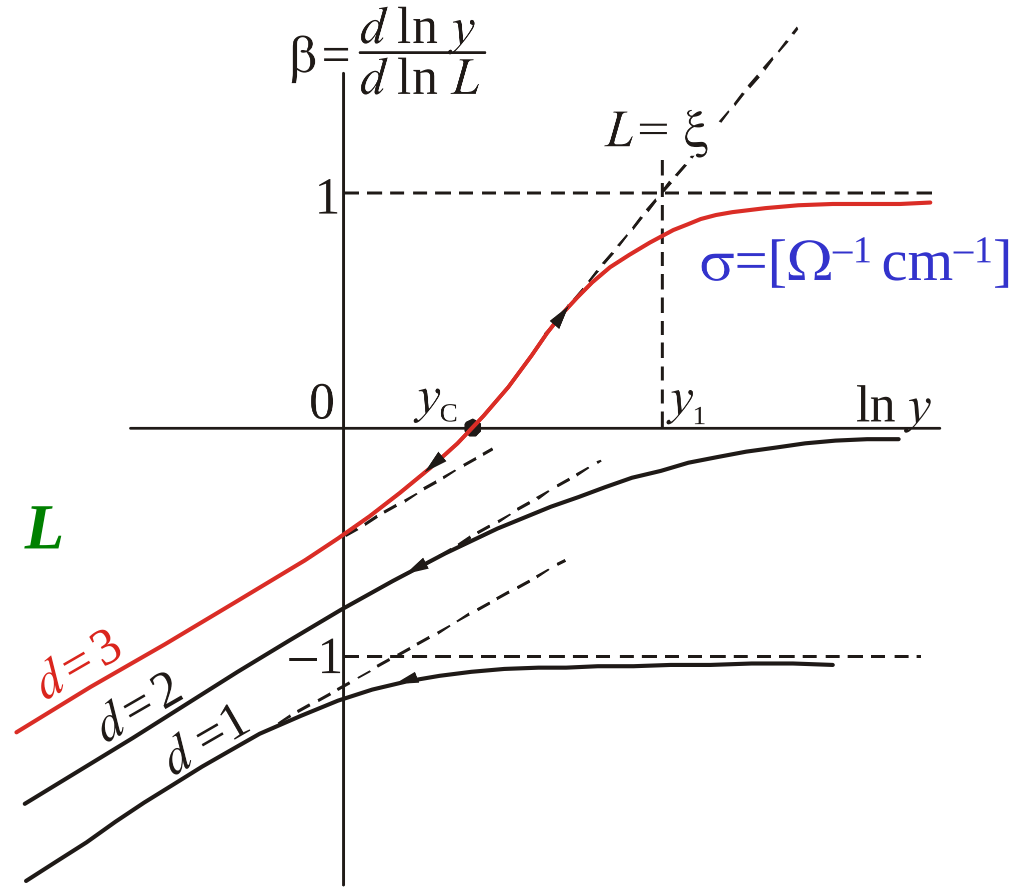
$$\begin{cases} \sigma = \dfrac{e^2}{\eta}\left(\dfrac{1}{\xi} + \dfrac{1}{L_T}\right) \\ \sigma = e^2 g_F D \end{cases} \xrightarrow{\xi \to \infty} \sigma = \frac{e^2}{\eta}\left(\frac{1}{\xi} + (T g_F)^{1/3}\right)$$

The Einstein relation

E.Abrahams, P.W.Anderson, D.C.Licciardello, and T.W.Ramakrishnan,
Phys.Rev.Lett. **42**, 673 (1979)

(AALR-1979)

y(L) is conductance of a cube of size L



With *d=3*,

1. at y_c the metal-insulator transition takes place;

2. the value of the three-dimensional conductivity of a rather large sample may be infinitesimal;

3. the metal--insulator transition is continuous.

Metal - insulator transition

	AALR	QPT
Theoretical scheme	AALR	QPT
Driving factor	*Disorder*	*Interaction*
Features in common	*Two lengths, ξ and L_φ, in the vicinity of the transition* *Similar shape of the critical region*	
Scaling expression **3D**	$\sigma \propto \frac{1}{\xi} + \frac{1}{L_\varphi}$	$\sigma \propto \frac{1}{\xi} f\left(\frac{L_\varphi}{\xi}\right) \qquad f(x) = 1 + {}^{1}\!/_{x}$

2D noninteracting electrons

Larkin and Khmelnitskii, 1982

k_F^2

$$\sigma = \sigma_{02} - \frac{e^2}{\eta} \ln \frac{L_T}{l}$$

$$\sigma_{02} = \frac{ne^2 l}{\eta k_F} \approx \frac{e^2}{\eta}(k_F l)$$

Estimate of crossover temperature:

$\sigma \sim 0$

$$k_F l = \ln L_T / l \qquad L_T \equiv \xi = l \exp(k_F l)$$

Inserting $L_T = \sqrt{D\eta / T}$ ***, we get*** $T_\xi = D\eta / \xi^2$

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Estimate of crossover temperature:

$\sigma = 0$

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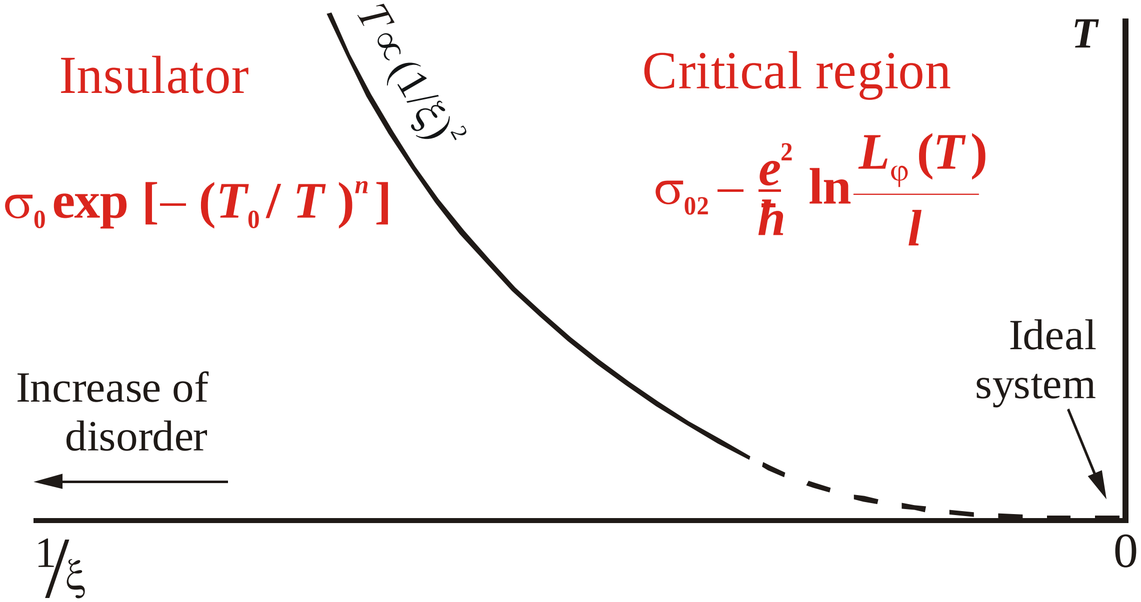

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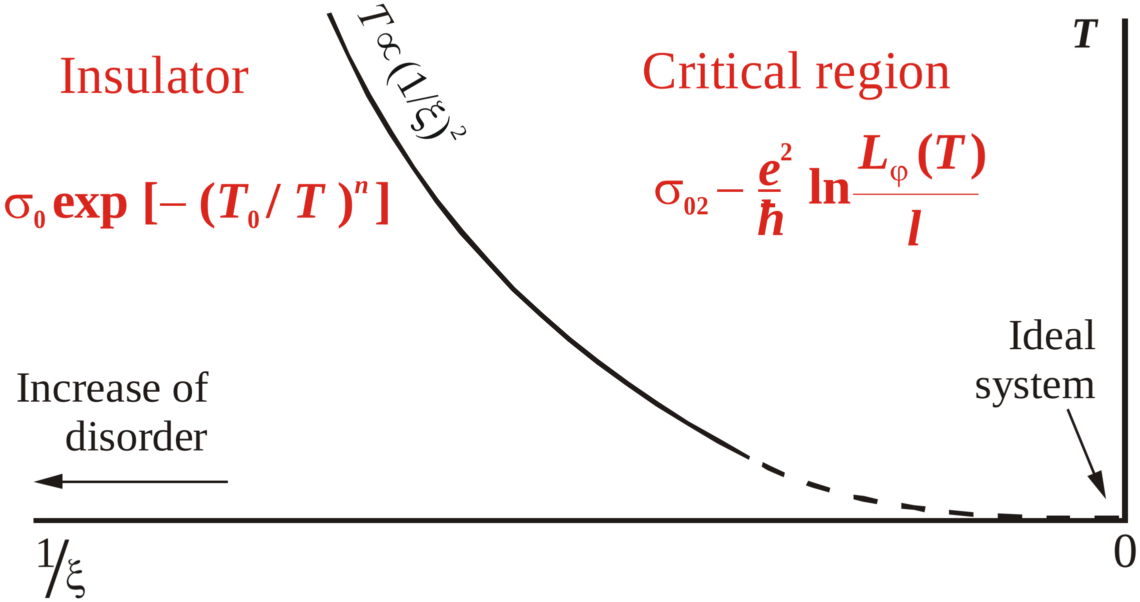

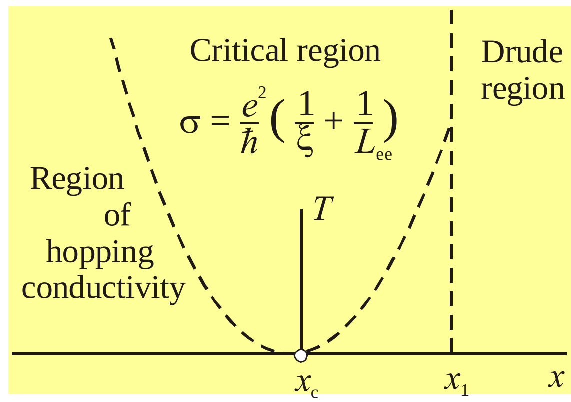

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(k_F^2 → n)

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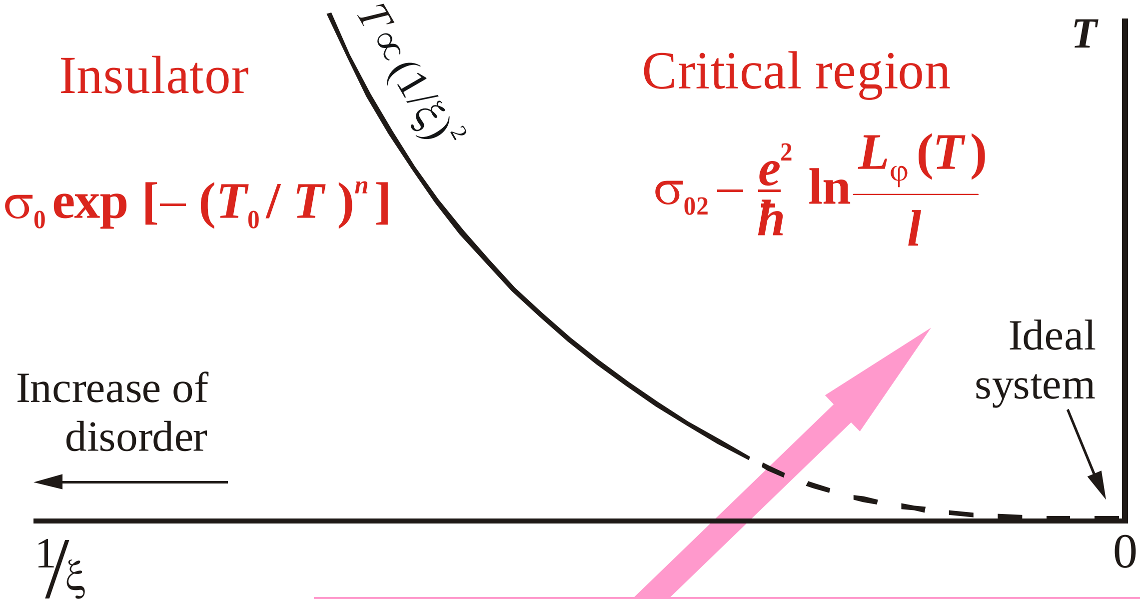

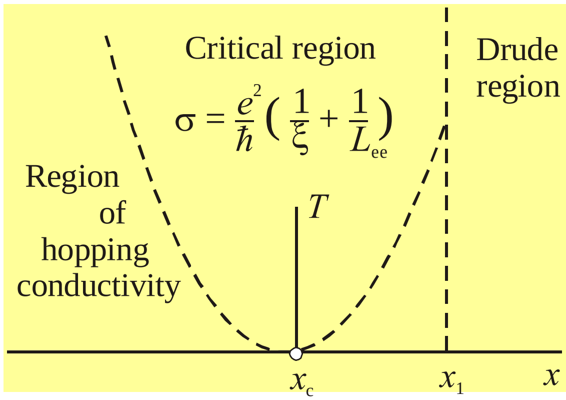

$$k_F l - \ln\frac{L_T}{l} = k_F l - \ln\frac{L_T}{\xi\exp(-k_F l)} = \ln\frac{L_T}{\xi}$$

2D interacting electrons

Larkin and Khmelnitskii, 1982

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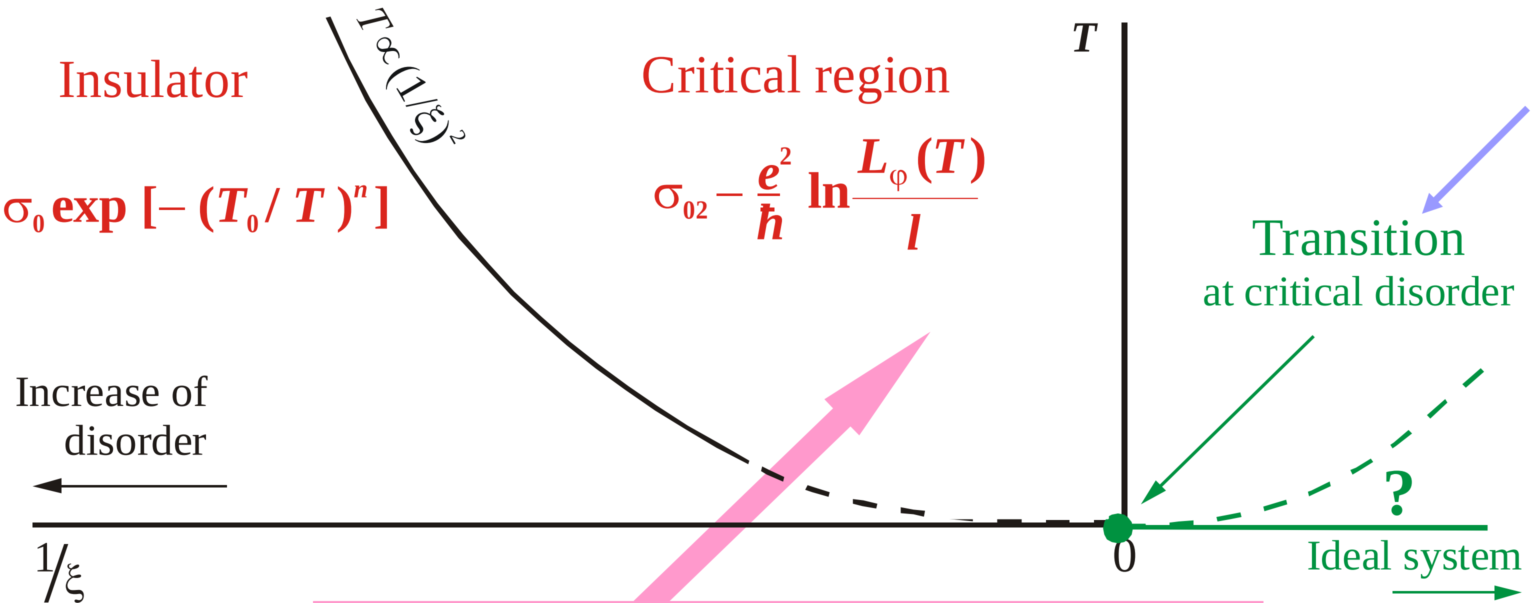
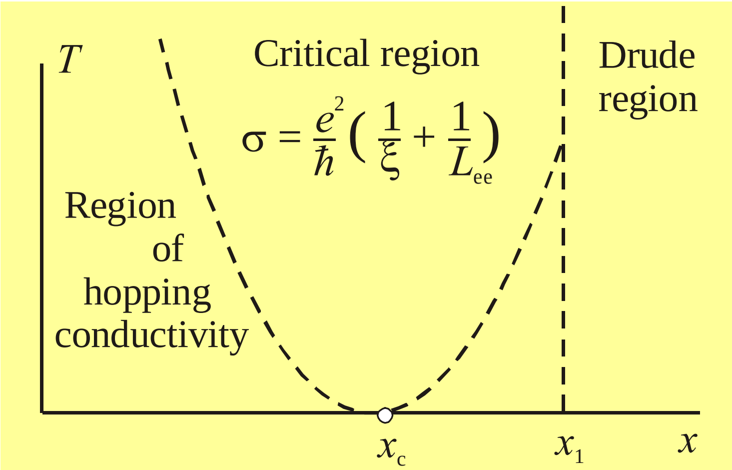
*A. Punnoose, A.Finkelstein, Science, **310**, 289 (2005)*



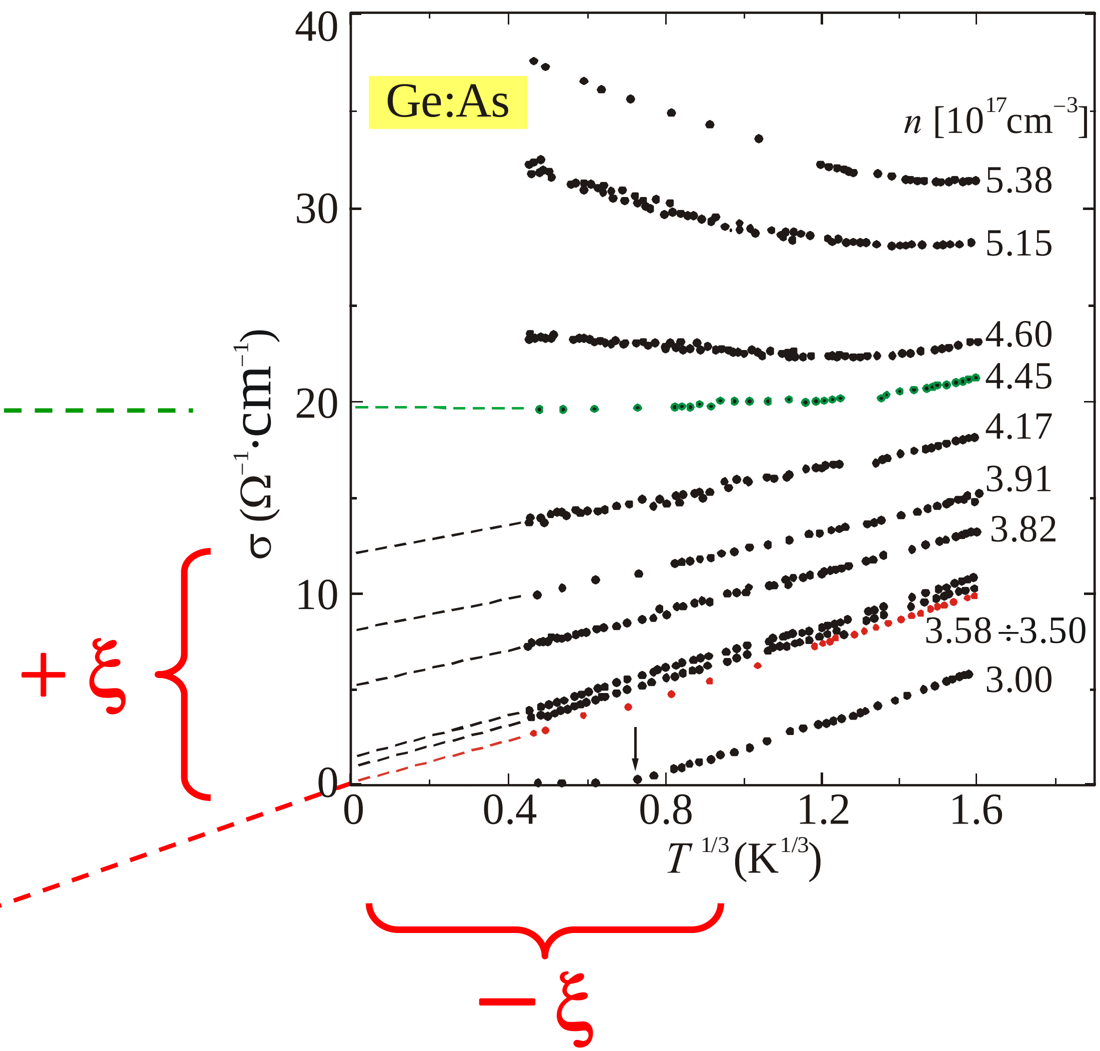
$$k_F l - \ln\frac{L_T}{l} = k_F l - \ln\frac{L_T}{\xi\exp(-k_F l)} = \ln\frac{L_T}{\xi}$$

I.Shlimak, M.Kaveh, R.Ussyshkin, et al., Phys.Rev.Lett. **77**, 1103 (1996);
J.Phys.:Cond.Matt. **9**, 9873 (1997)

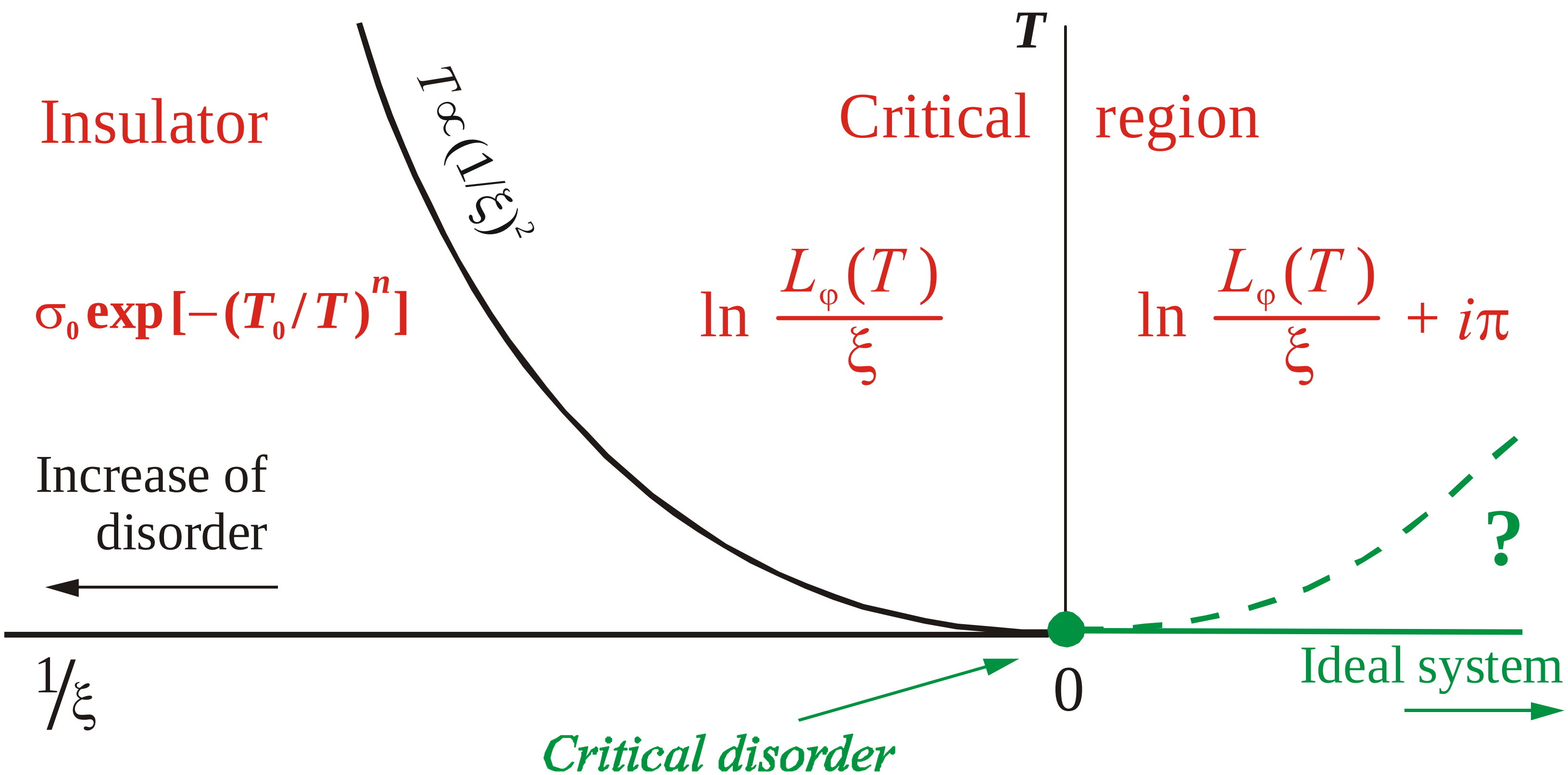


Critical region

$$\sigma = \frac{e^2}{\eta}\left(\pm \frac{1}{\xi} + (g_F T)^{1/3}\right)$$



2D interacting electrons
(hypothetical phase diagram)



H. Fröhlich,
Proc. R. Soc.
221, *296 (1954)*

Metal - insulator transition

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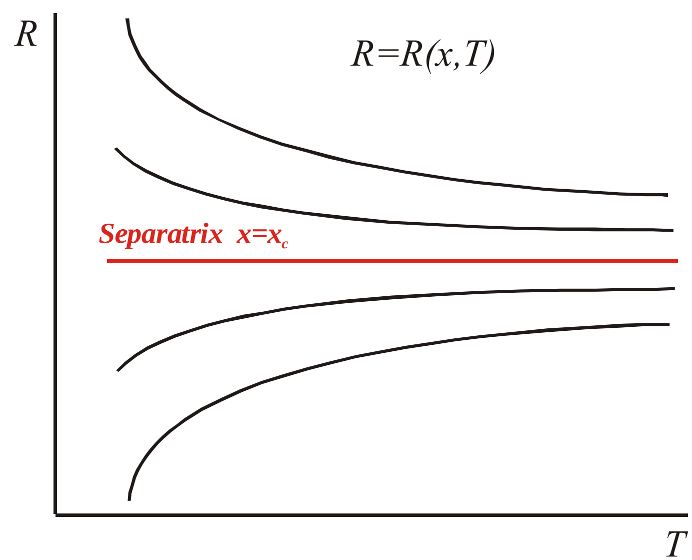
Scaling expression **2D**	—	$\sigma \propto \ln\left(\frac{L_\varphi}{\xi}\right)$

2D superconductor-insulator transition
(theoretical aspect)



Scaling variable in **2D** is

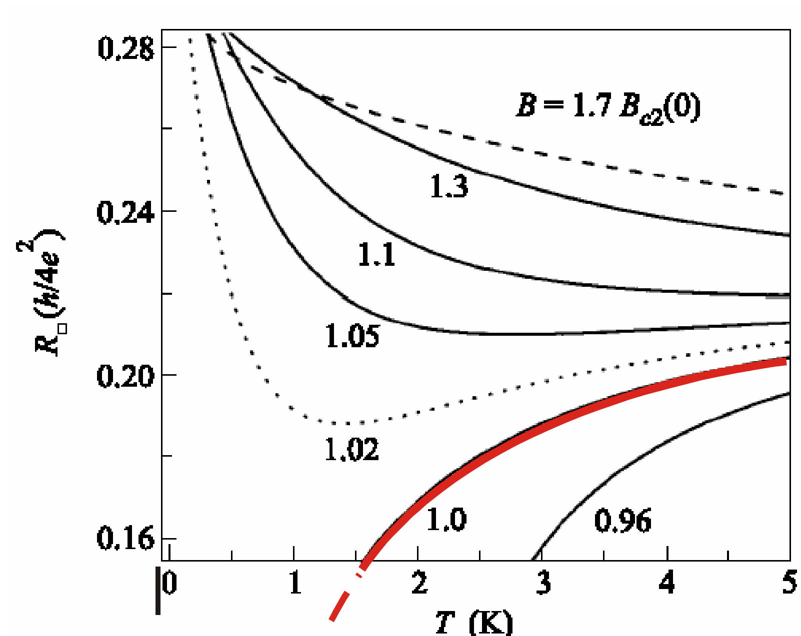
$$\frac{L_\varphi}{\xi} \rightarrow \frac{\Delta x}{T^{1/z\nu}} \equiv u$$ and $$R = R_c f(u)$$

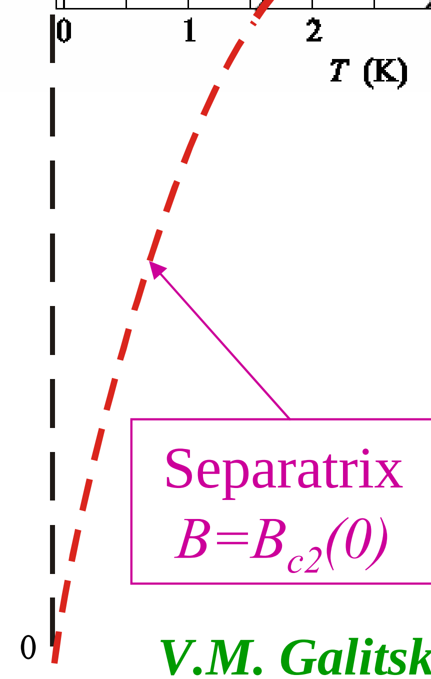
Along the separatrix ***u***= const=0 and

$R=R_c f(0)=R_c$

S.L.Sondhi, S.M.Girvin, J.P.Carini, D.Shahar,

Rev. Mod. Phys. **69,** *315 (1997)*





Rigid calculations of the superconducting fluctuations

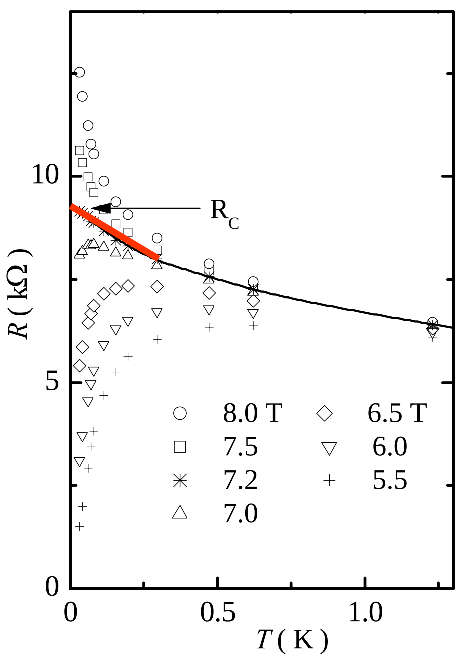
at $T<<T_{c0}$
in magnetic field

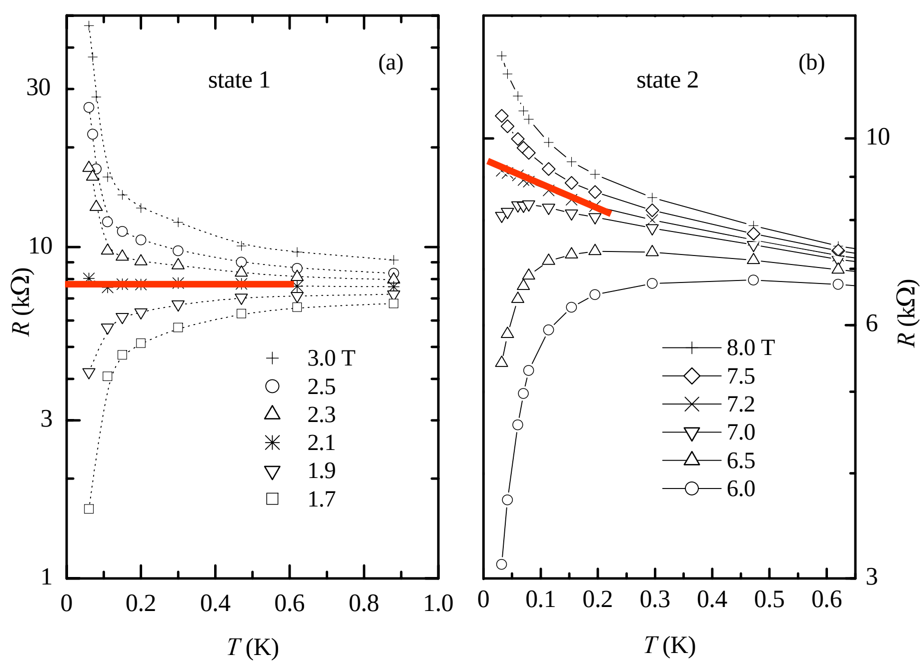
V.M. Galitski and A.I. Larkin,

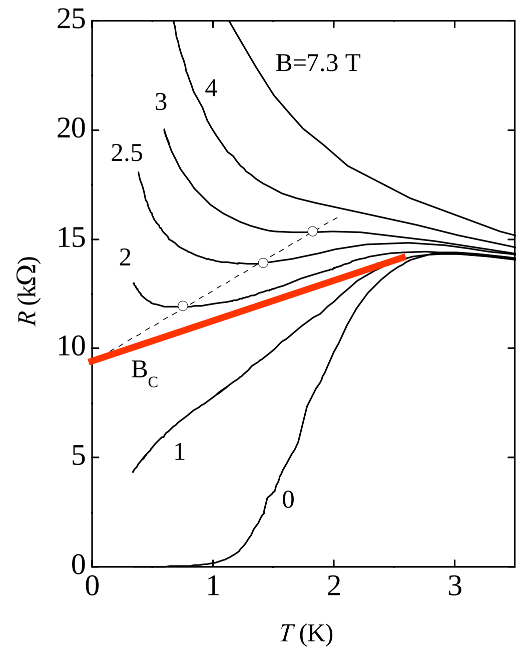
Phys.Rev. B **63,** *174506 (2001)*

2D superconductor-insulator transition
(experimental aspect)

Superconductor-insulator transition in In-O



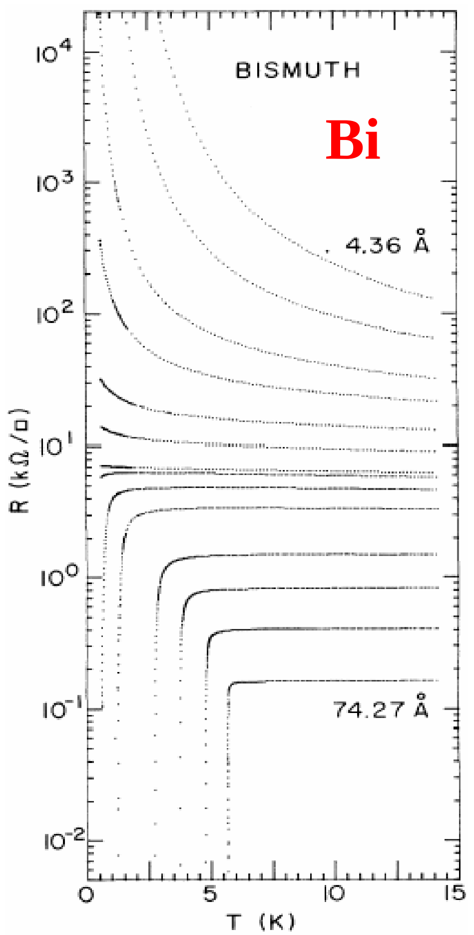




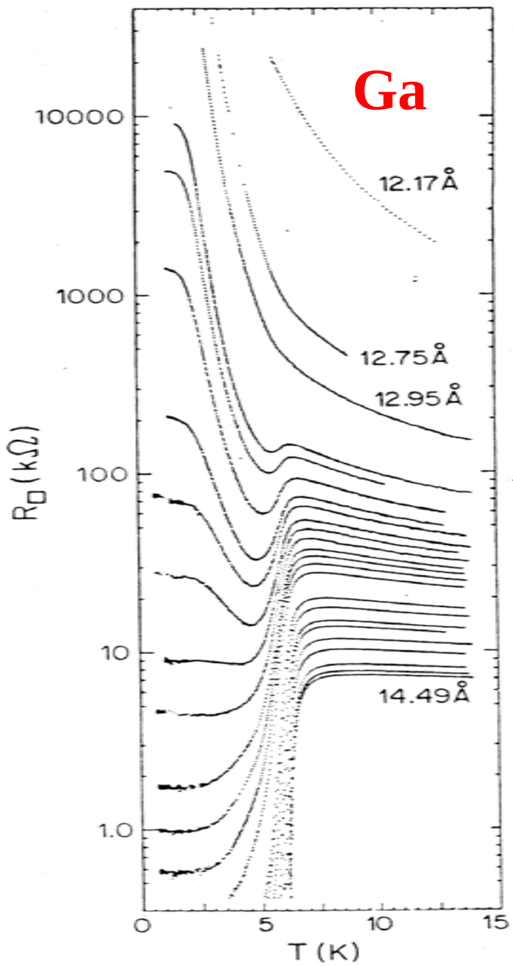
The separatix may have non-zero slope

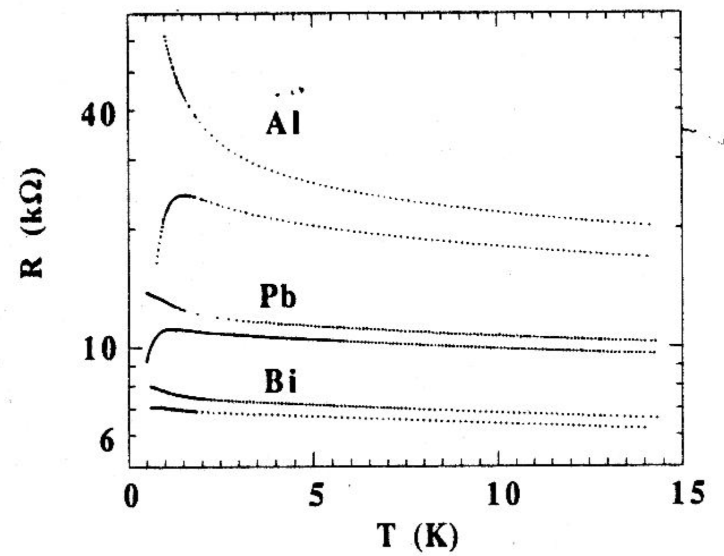
Superconductor-insulator transition in ultrathin films

$d < d_c$: Insulator



$d > d_c$: Superconductor





D.B.Haviland, A.M.Goldman, et al.

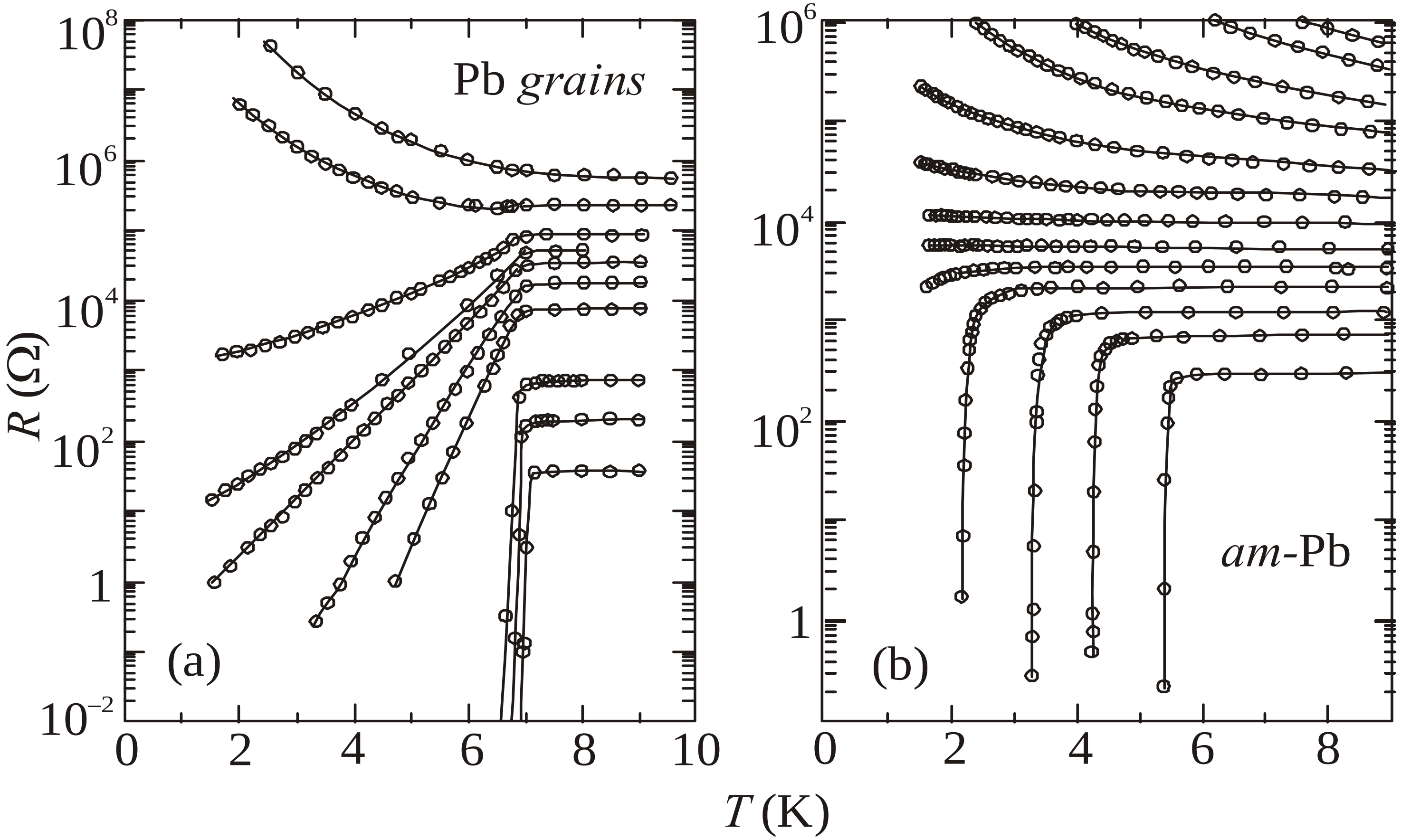
PRL, 1989

PRL, 1986

PRB, 1993

2D superconductor-insulator transition
(experimental aspect)

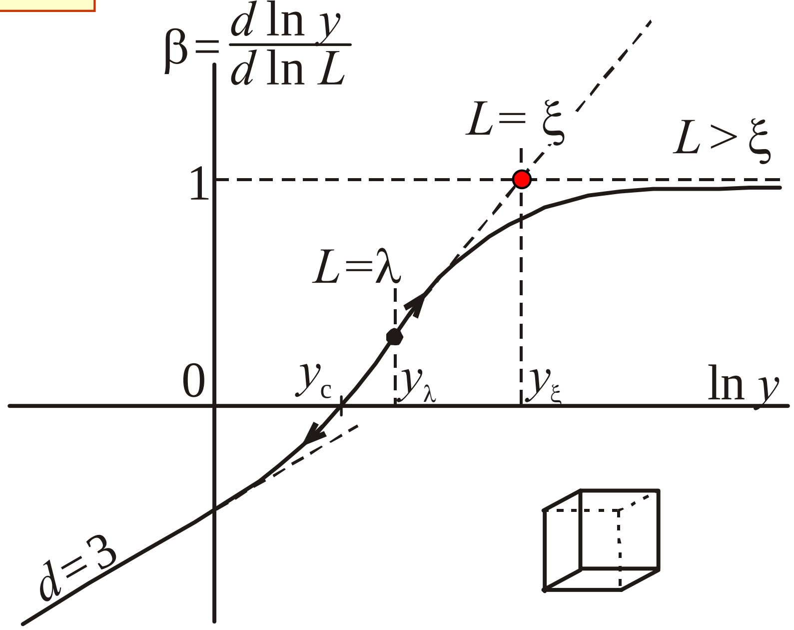
A. Frydman,
Physica C
391, *189 (2003)*



Granular system **Homogeneously disordered system**

Level spacing $\delta\varepsilon = (g_F a^3)^{-1} < \Delta_{sc}$ $\delta\varepsilon = (g_F a^3)^{-1} > \Delta_{sc}$

d = 3



Equation of the straight line

$$\frac{d\ln y}{d\ln L} = s\ln {y}/{y_c}$$

$u=\ln y,\quad x=\ln L$

$$\frac{du}{dx} = s(u-u_c)$$

$$u-u_c = Ue^{sx}$$

Solution

$$\ln {y}/{y_c} = \left(\frac{L}{\lambda}\right)^s \ln {y_\lambda}/{y_c}$$

In the point • $\beta=1$ *and* $\ln y_\xi/y_c = 1/s$.

Hence

$$\ln y_\xi = \ln y_c + {1}/{s} = \text{const} \;\Rightarrow\; y_\xi = A \;\Rightarrow\; Y_\xi = A\frac{e^2}{\eta} \;\Rightarrow\; \sigma = \left(A\frac{e^2}{\eta}\right)\frac{1}{\xi}$$

Quantity ξ *may be written with the help of* λ

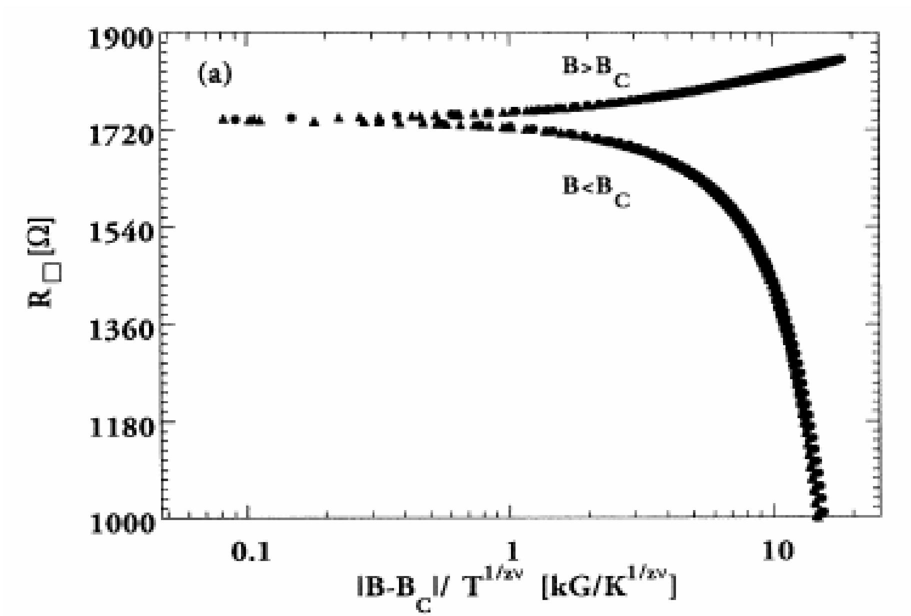
$$\xi \approx \lambda\left(s\ln\frac{y_\lambda}{y_c}\right)^{-1/s},$$

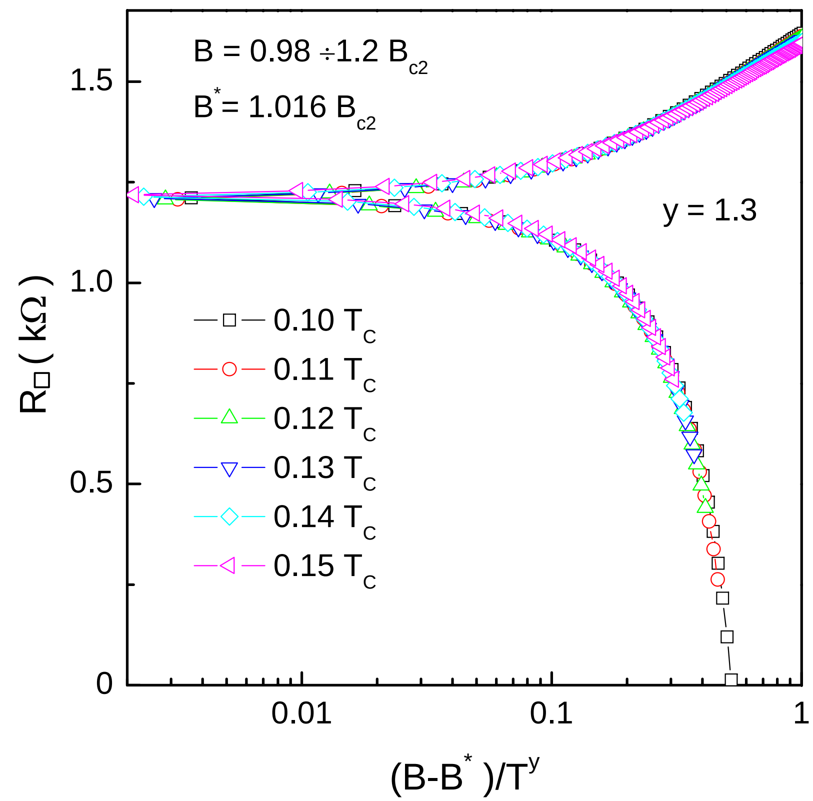
i.e. $\xi\to\infty$ **as** $y_\lambda \to y_c$

Conductance σ may be infinitesimally small

Scaling

A.Yazdani and A.Kapitulnik (1995)





T_C= 0.15 K B_C= 4.19kG

T = 0.08 ÷0.11 K B-B_C < 1 kG

z ν = 1.36

Theoretical expression does not contain scaling properties, but in restricted temperature interval scaling presentation looks credible