Realization of a Laughlin quasiparticle interferometer: Observation of anyonic statistics

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Exchange statistics in 2D

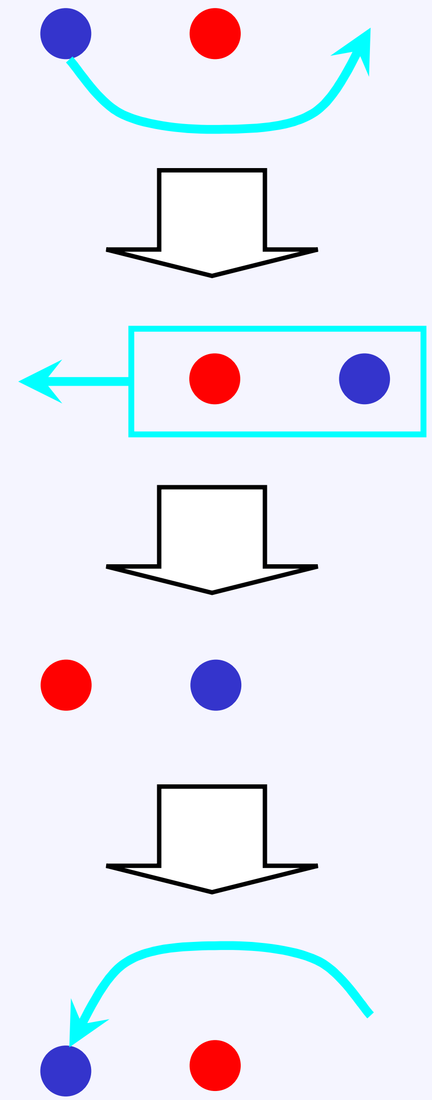
- exchange = half loop + translation
  (exchange)^2 = complete loop
- **in 3D:** loop with particle inside is NOT distinct from loop with no particle inside $\Rightarrow$ $\Theta = j$
- **in 2D:** loop with particle inside IS topologically distinct from loop with no particle inside

$\Rightarrow$ exchange $\Leftrightarrow$ braiding

$\Rightarrow$ NO requirement for Θ to be an integer
e.g., Θ can be any real number

"anyons"

Leinaas, Myrheim 1977; F. Wilczek 1982

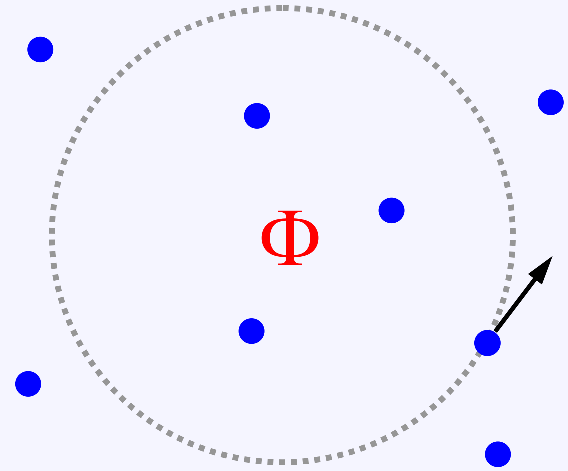


Q: are there such particles in Nature?

A: collective excitations of a many-electron 2D system

Adiabatic transport in magnetic field

- **electrons** $q=e, \quad \Theta_e=1, \quad N_e$



encircling electron acquires Berry phase $\exp(i\gamma)$

$$\gamma(T) = \frac{e}{\hbar}\Phi + 2\pi\Theta_e N_e = 2\pi\left(\frac{\Phi}{h/e} + \Theta_e N_e\right)$$

two contributions: Aharonov-Bohm + statistics

$\Rightarrow$ statistical contribution is NOT observable: $\exp[i2\pi\Theta_e N_e] = +1$

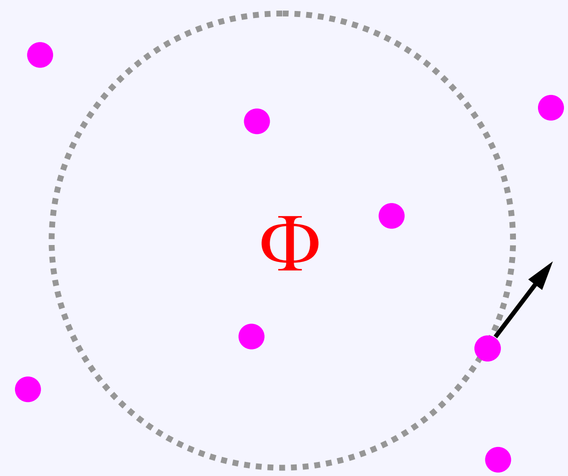
$$\Phi = \oint_C \mathbf{A}\cdot d\mathbf{l}$$

$$\Theta N = i\oint_C d\,\Re \left\langle \Psi(\Re; z_j) \middle| \frac{\partial}{\partial \Re}\Psi(\Re; z_j)\right\rangle$$

Fractional statistics in 2D

- $f = 1/3$ Laughlin quasihole

$$q = e/3, \quad \Theta_{1/3} = 2/3, \quad N_{qh}$$



Ψ of encircling quasihole acquires phase

$$\gamma_m = \frac{q}{\hbar}\Phi + 2\pi\,\Theta_{1/3} N_{qh} = 2\pi m$$

change between *m* and *m*+1

$$\Delta\gamma = \frac{q}{\hbar}\Delta\Phi + 2\pi\,\Theta_{1/3}\Delta N_{qh} = 2\pi$$

- $q = e/3$,

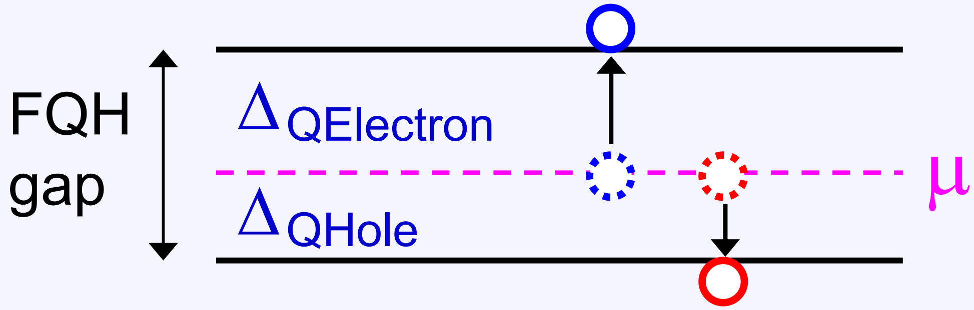
when flux changes by $\Delta\Phi = \Phi_0 = h/e$, Ψ acquires AB phase $\Delta\gamma = 2\pi/3$

$\Rightarrow$ need $\Theta_{1/3} = 2/3$ for single-valued Ψ (QAD period is Φ_0, NOT $3\Phi_0$!)

$$\Delta\gamma = \frac{e}{3\hbar}\Delta\Phi + 2\pi\,(2/3)\times 1 = 2\pi$$

How can one make FQH quasiparticles?

- large 2D electron system
  (include donors = neutral)

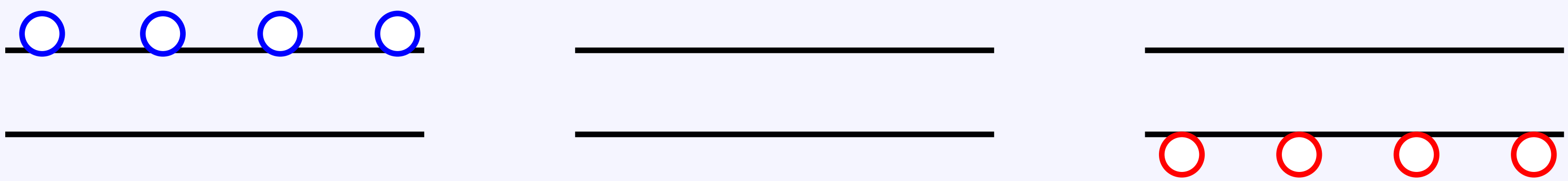


FQH condensate at f

quantum number: $f = \dfrac{\sigma_{XY}}{e^2/h}$

variable: $\nu = \dfrac{hn}{eB}$

A: change B, electron density n is fixed $\Rightarrow$ ν changes; remains neutral

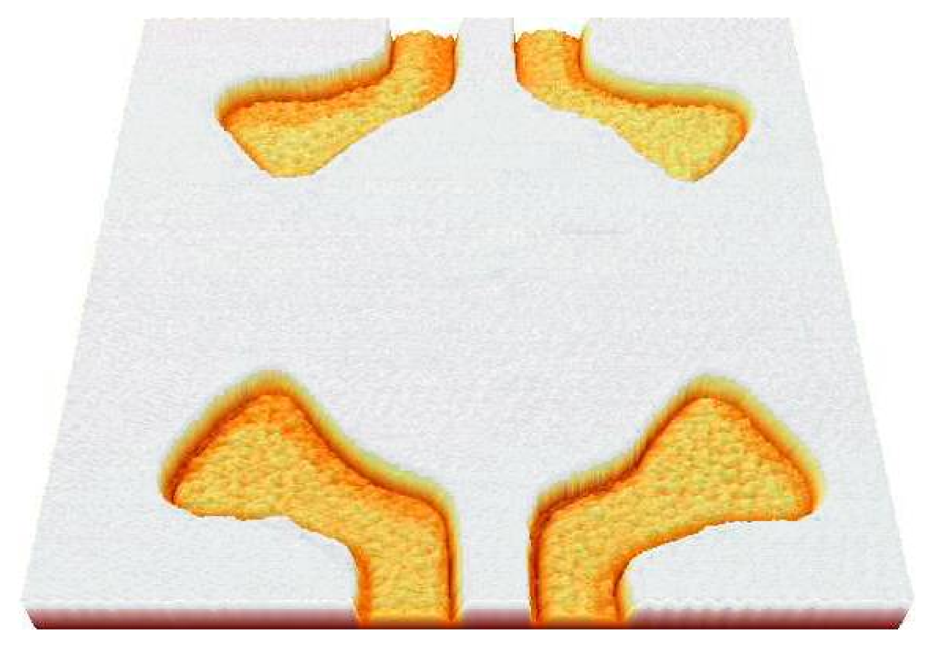


$$B < B_f \qquad \nu = \frac{B_f}{B} f > f$$

$$B = B_f \qquad \text{"exact filling"} \qquad \nu = f$$

$$B > B_f \qquad \nu = \frac{B_f}{B} f < f$$

Aharonov-Bohm interferometer: Samples



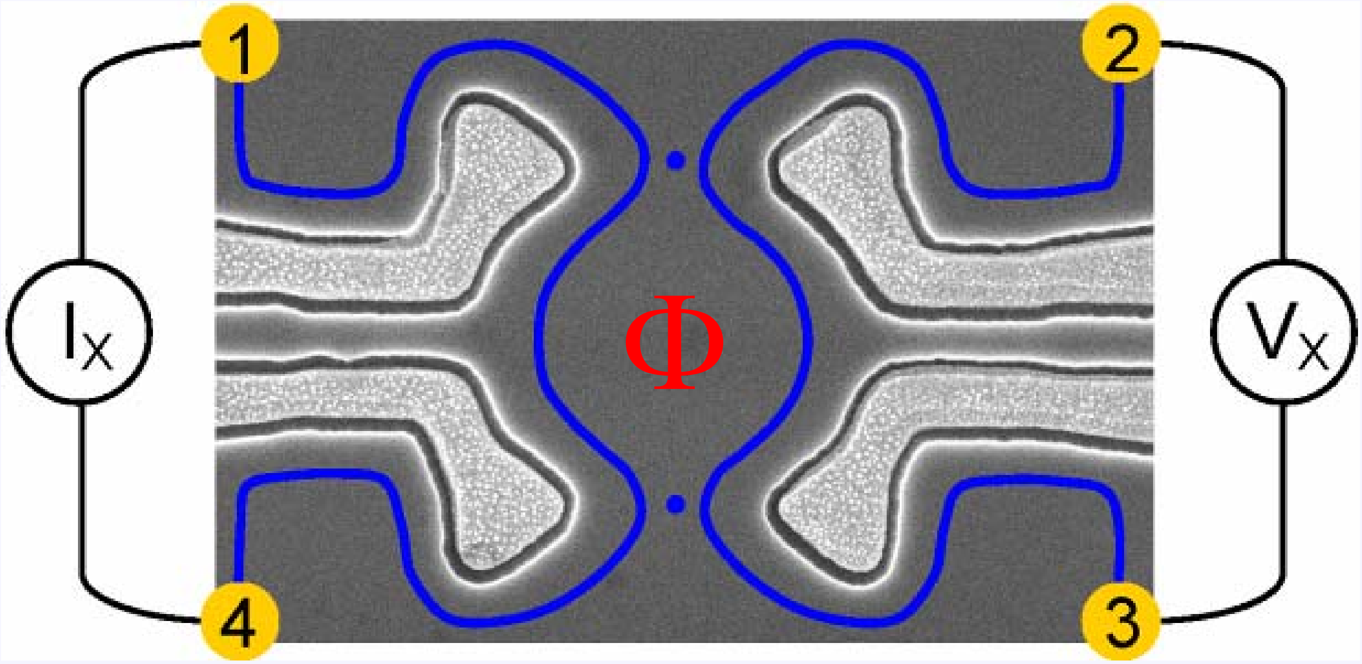
2D electrons ≈300 nm below surface in these low *n*, high μ GaAs/AlGaAs heterojunctions suitable for FQHE

lithographic island R≈1,050 nm
etched 150 nm, Au/Ti in trenches

$$R_{XX} = V_X / I_X$$

$$G_T \approx R_{XX} / R_{XY}^2$$

A-B flux Φ



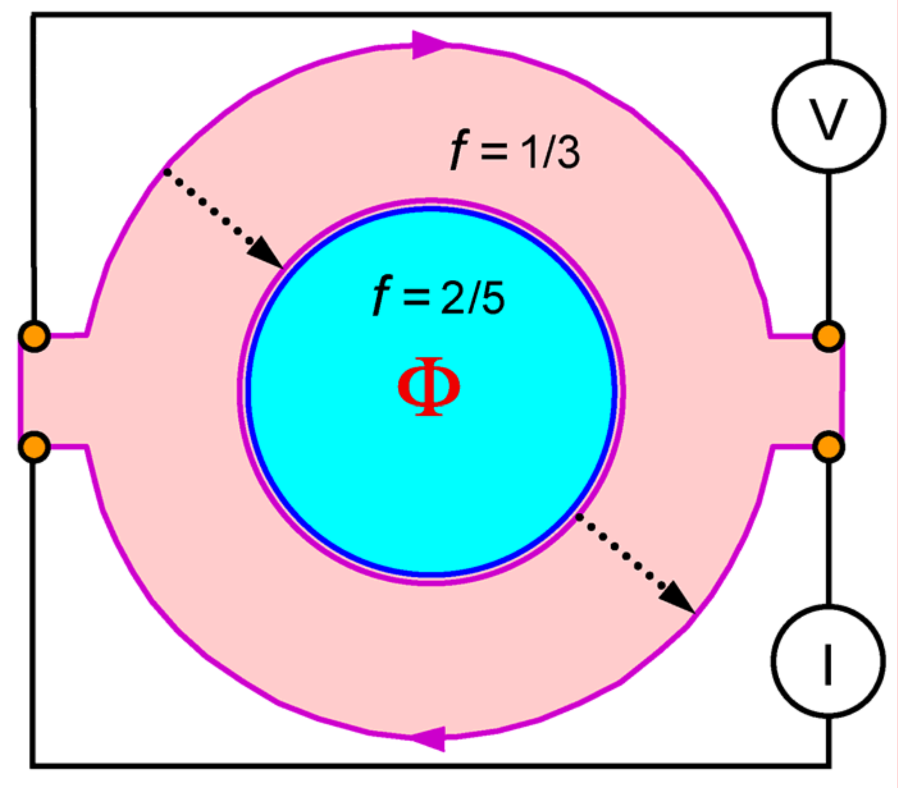
Laughlin quasiparticle interferometer: Idea

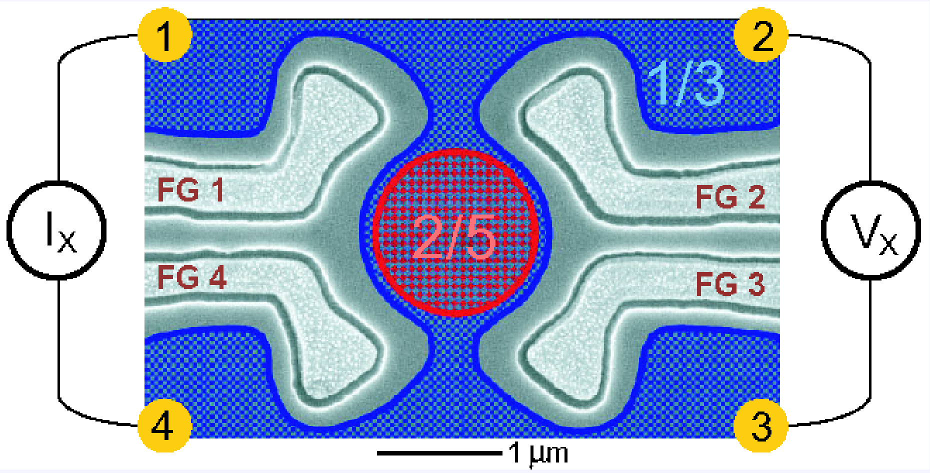
- an $e/3$ QP encircles the $f = 2/5$ island

⇒ if $e/3$ QP path is quantum-coherent, expect to see an interference pattern, e. g., Aharonov-Bohm vs. Φ through *inner* island

- $\delta\Phi = h/2e$ creates one e/5 QP in the $f = 2/5$ island

⇒ expect to observe effects of fractional statistical phase (neither bosons nor fermions produce an observable statistical effect)





Calibration with electrons ⇒ outer A-B ring r

$$2\Delta B_2 = 2.85 \text{ mT}$$

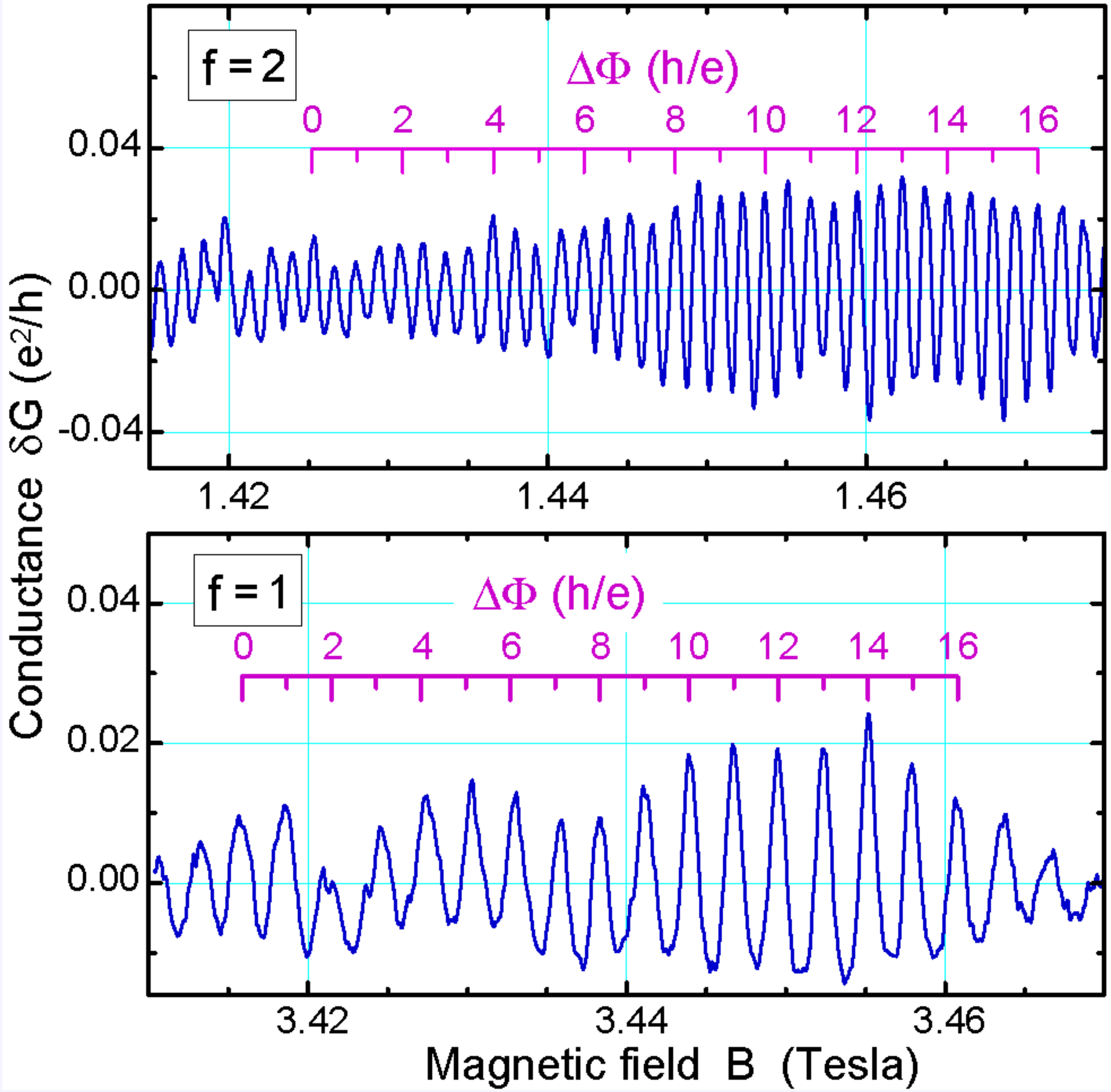
$$\Delta B_1 = 2.81 \text{ mT}$$

$$\Downarrow$$

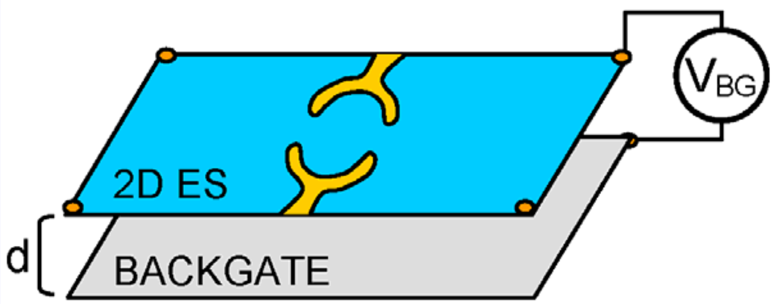
$$\pi r^2 \Delta B_1 = h/e;$$

$$r = \sqrt{h/\pi e \Delta B_1}$$

$$\approx 685 \text{ nm}$$



Interference of electrons in the outer ring *vs.* backgate



small perturbation:

$$\delta n / n \approx 0.0017$$

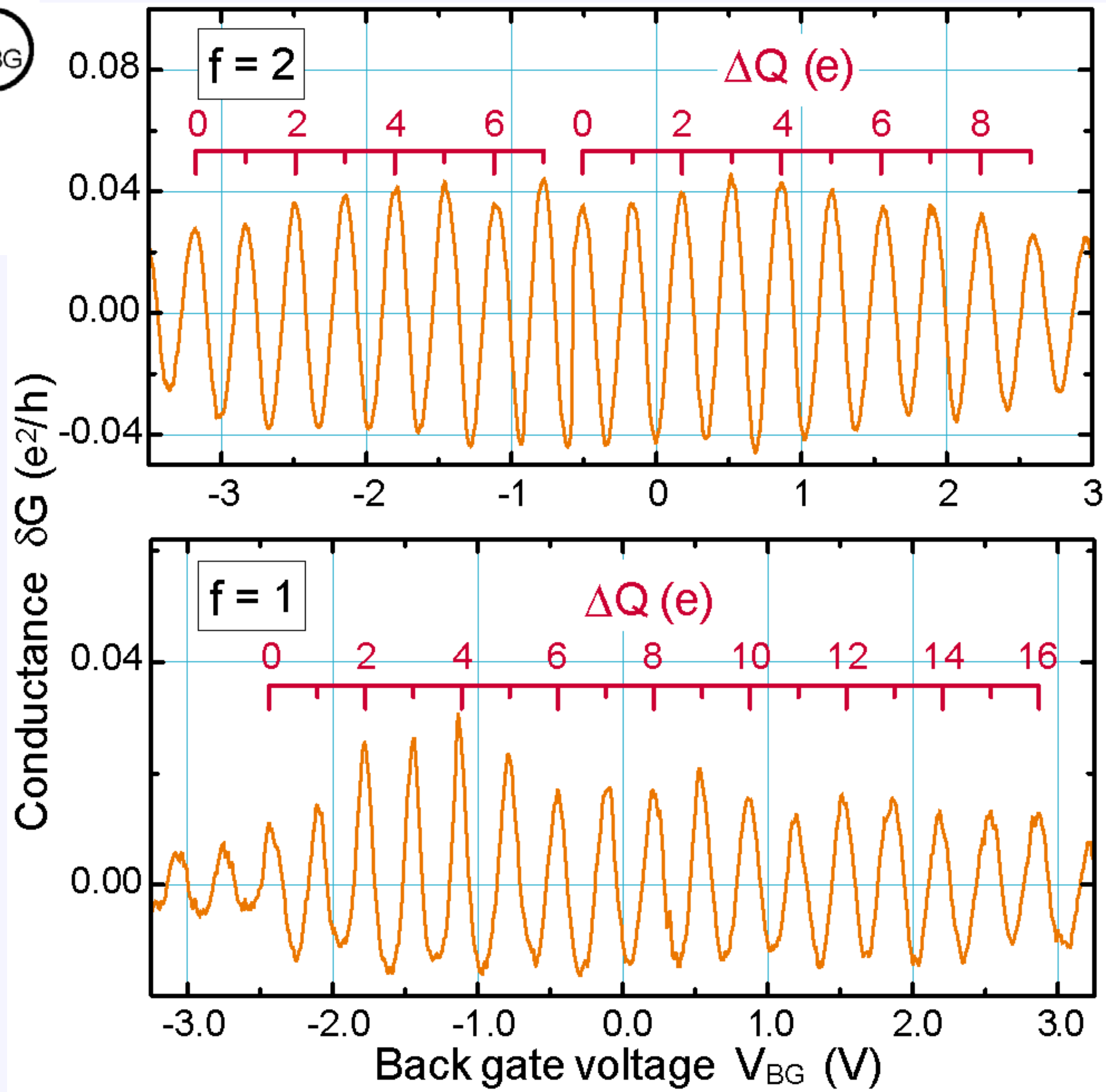
upon 1 V

need to calibrate

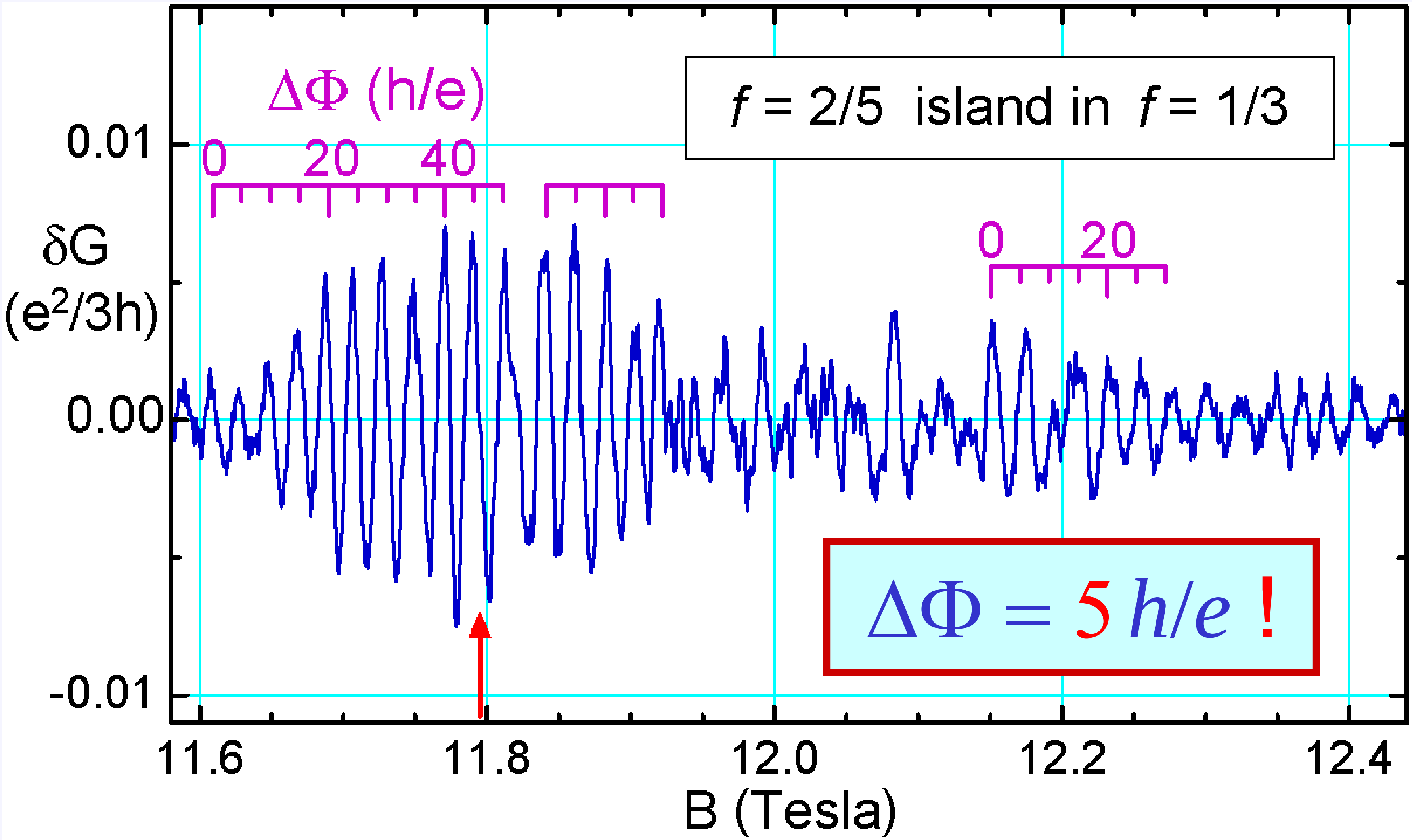
$$\Delta Q = e, \ \Delta V_{BG}$$

$$\Downarrow$$

$$\frac{\delta Q}{\delta V_{BG}}$$



Observation of an Aharonov-Bohm "superperiod"



Aharonov-Bohm interference of e/3 Laughlin quasiparticles circling the island of the f = 2/5 FQH fluid

Observation of Aharonov-Bohm "superperiod"

Aharonov-Bohm superperiod of $\Delta\Phi > h/e$
has never been reported before

Derivation of the Byers-Yang theorem uses a singular gauge transformation at the center of the AB ring, where electrons are excluded

Present interferometer geometry has no electron vacuum within the AB path $\Rightarrow$ BY theorem is not applicable (no "violation" of BY theorem)

N. Byers and C.N. Yang, PRL 1961; C.N. Yang, RMP 1962

LQP interferometer: Flux and charge periods

AB "superperiod"

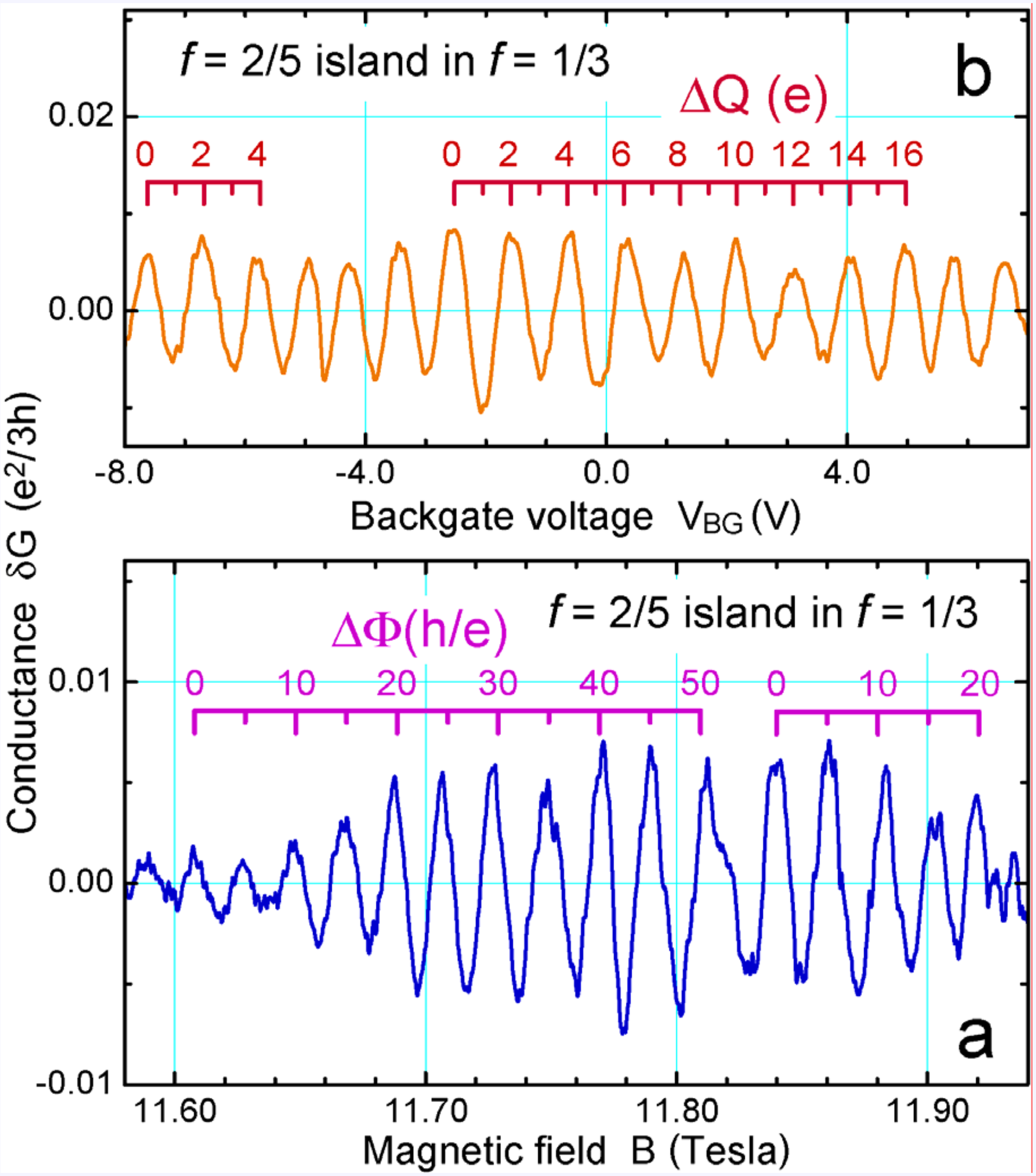
$$\Delta\Phi = 5\, h/e$$

$\Delta\Phi = 5h/e$

$\Downarrow$

creation of ten $e/5$ LQPs in the island

backgate voltage period of $\Delta Q = 10(e/5) = 2e$



Fractional statistics of Laughlin quasiparticles

- $f = 1/3$ quasiparticles: $q = e/3$ $f = 2/5$ quasiparticles: $q = e/5$
- Berry phase period must be 2π: $\Delta\gamma = 2\pi$

⇒ an $-e/3$ encircling $\Delta N = 10$ of $f = 2/5$ LQPs:

$$\Delta\gamma = \frac{-e/3}{\hbar}\Delta\Phi + 2\pi\Theta^{1/3}_{2/5}\Delta N_{2/5} = 2\pi\left(-\frac{5}{3} + 10\,\Theta^{1/3}_{2/5}\right) = 2\pi$$

relative statistics $\Theta^{1/3}_{2/5} = \frac{1}{10}\left(1 + \frac{5}{3}\right) = \frac{4}{15}$

* Inputs: q's (from prior antidot experiments), but NOT Θ's

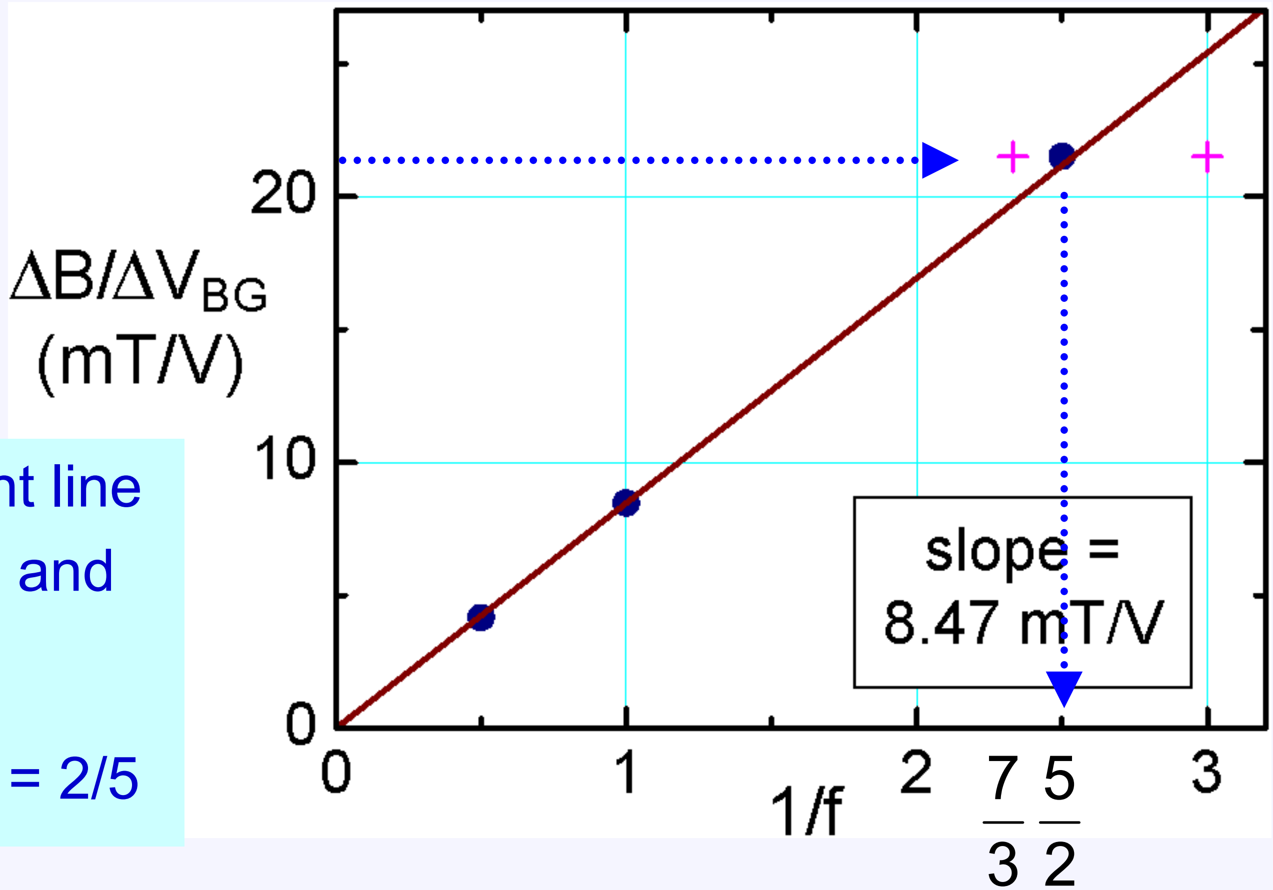
† Integer statistics would allow addition of one LQP per period, anyonic statistics forces period of ten LQPs!

‡ Net island charging (neglecting statistics) is not energetically possible, leads to huge charging energy ~1,000 K for the tenth AB period

Q: How do we know the island filling?

A: The ratio of oscillations periods is independent of edge ring area S

$$\frac{S\,\Delta B}{S\,\Delta V_{BG}} \propto \frac{N_\Phi}{N_e} \equiv \frac{1}{f}$$



Ratios fall on straight line forced through (0,0) and the f = 1 data point

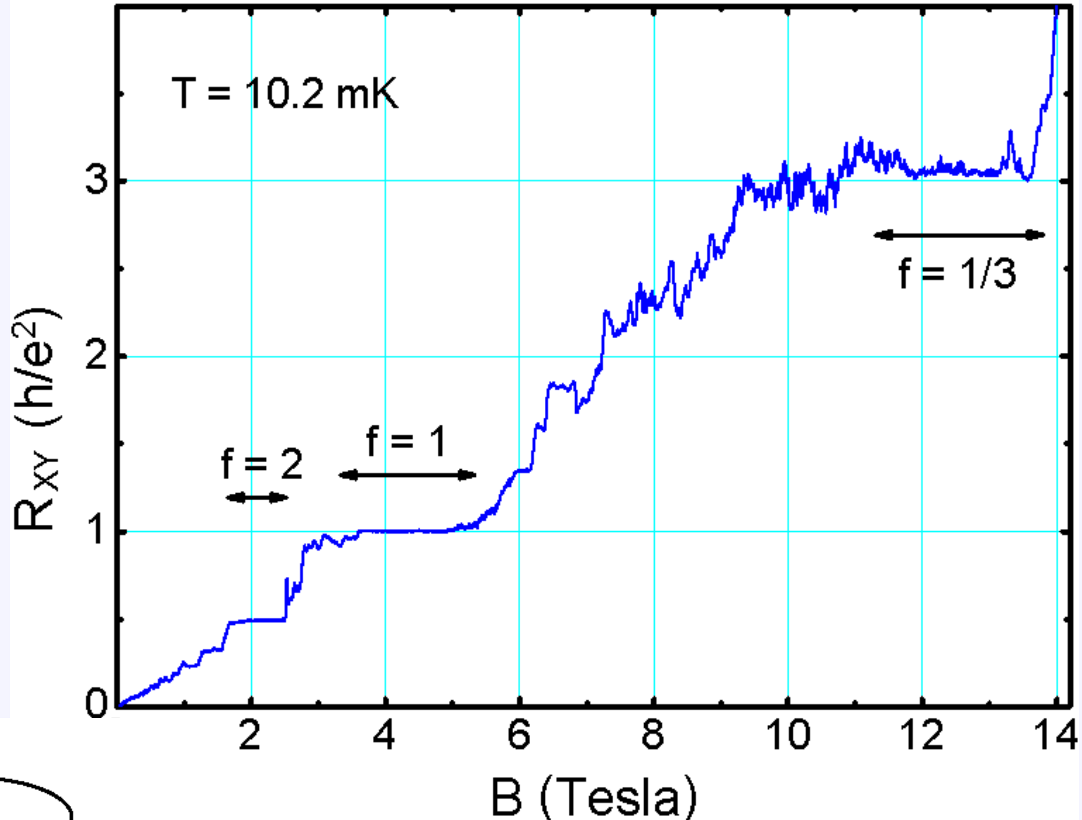
⇒ island filling is f = 2/5

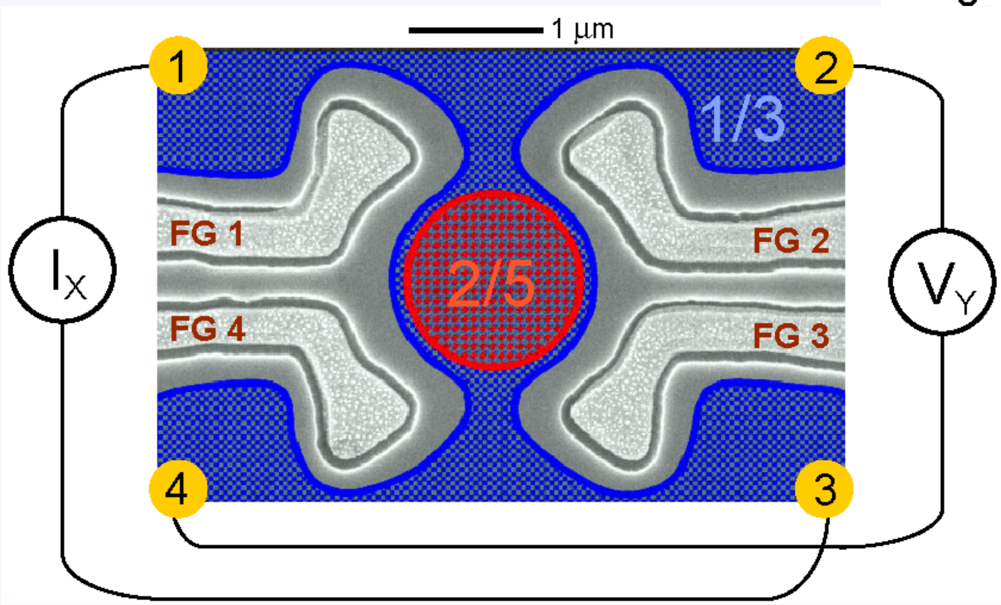
⇒ no edge depletion model is used to establish island filling

Q: How do we know the $f_C = 1/3$ FQH fluid surrounds the island?

A: Quantized plateau
$R_{XY} = 3h/e^2$ at 12.3 T
($f_I = 2/5$) confirms
conduction through
uninterrupted $f_C = 1/3$

C = constriction
I = island

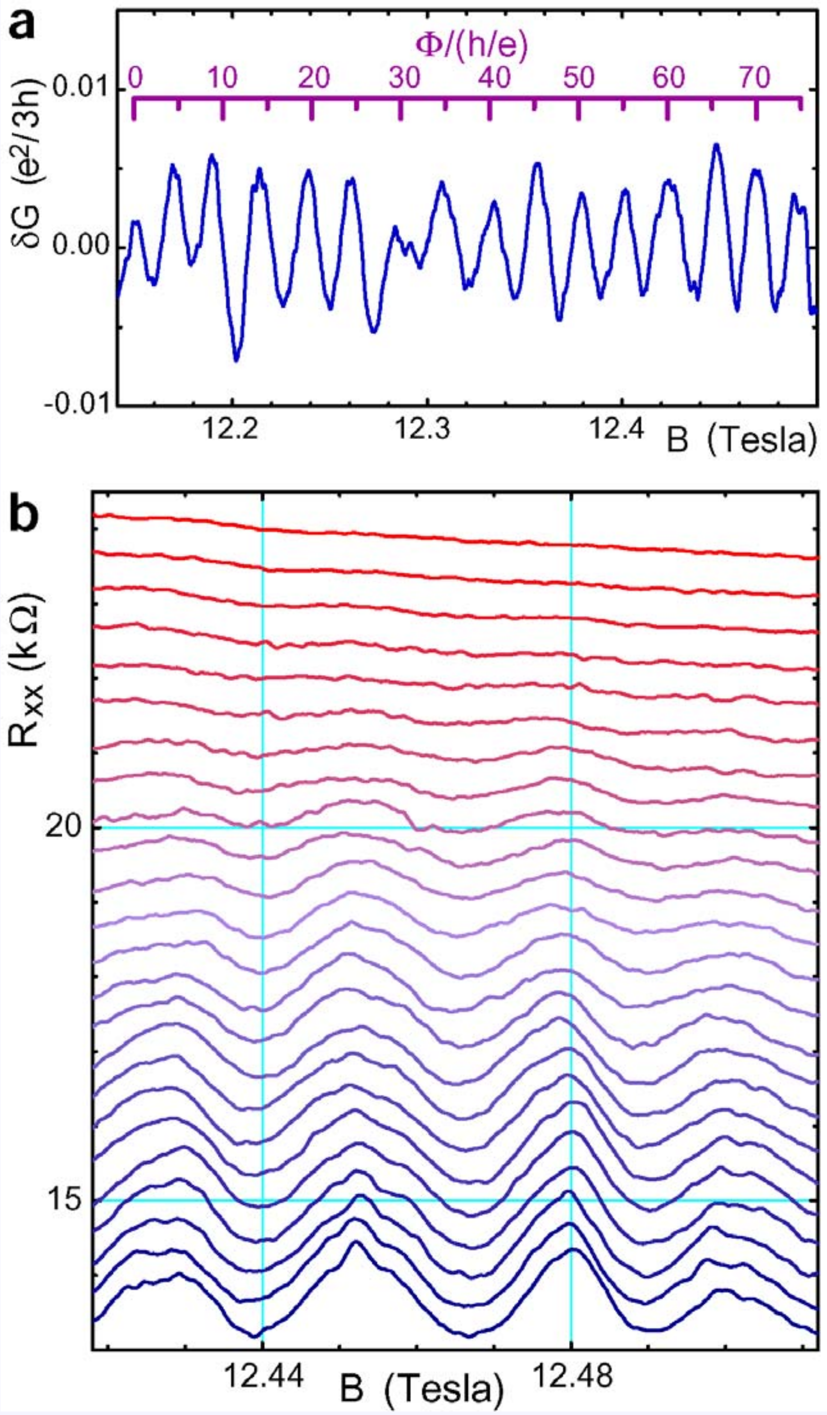




A-B oscillations vs. T

interference of $e/3$ quasiparticles circling $f = 2/5$ island

$10.2 \leq T \leq 141$ mK



Thermal dephasing of conductance amplitude

theory:

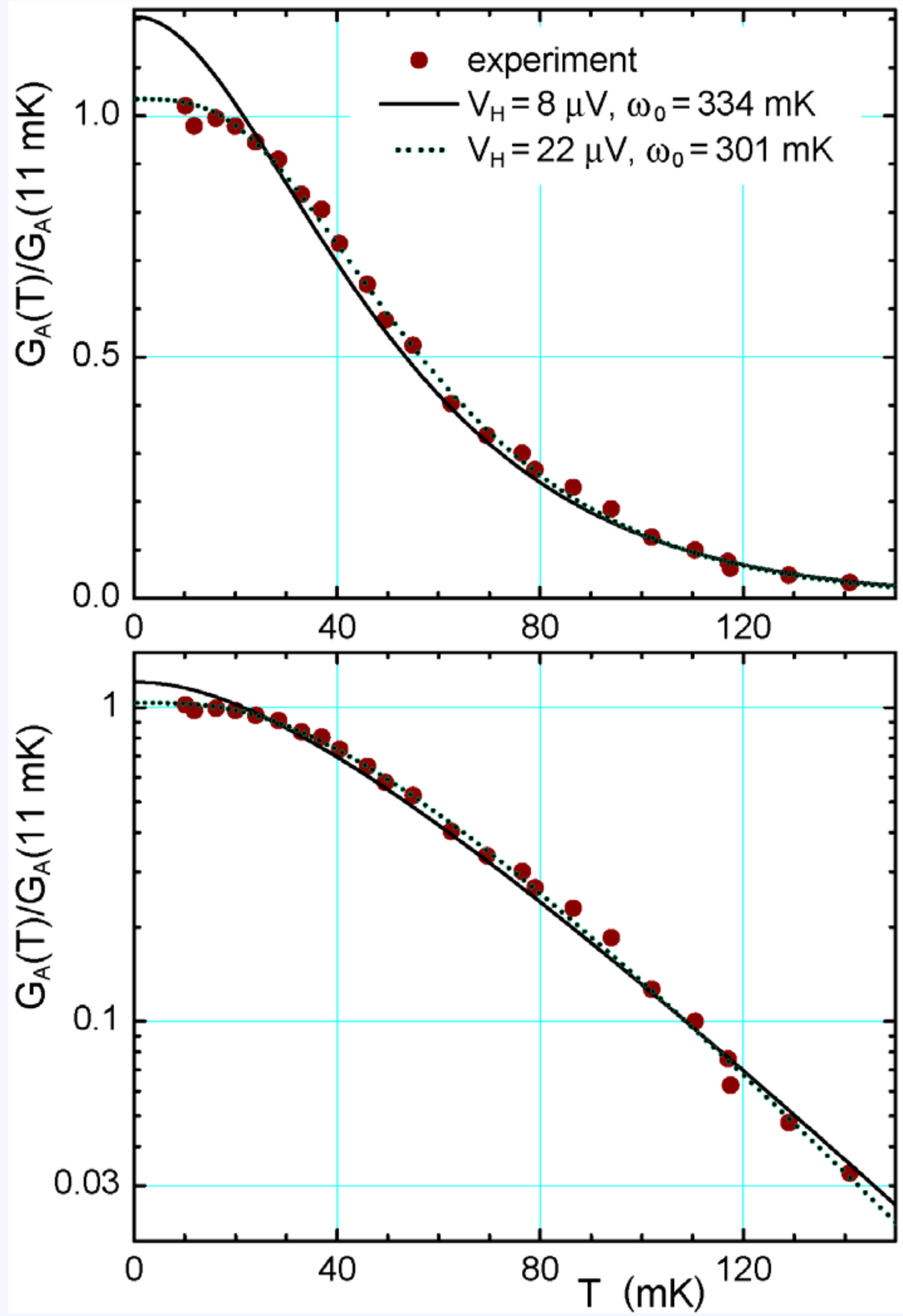
g = 1/3 chiral Luttinger liquid (χLL) two-point interferometer

C. Chamon *et al.*, PRB 1997

- "oscillation frequency" $\omega_0 = \dfrac{4\pi u}{C}$
- finite bias → Hall voltage V_H
- 8 μV ≈ in the V_H = 0 limit
- high-T: $G \propto \exp(T/T_0)$

NOT activated:

$$G \propto \exp(T_0/T)$$

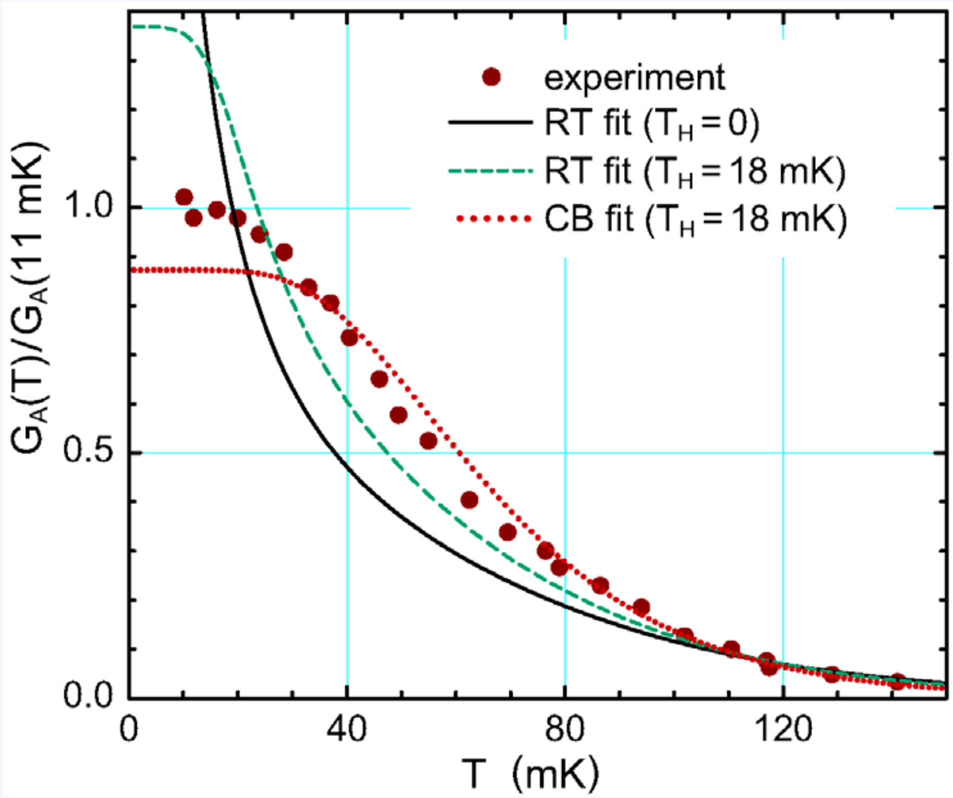


T-dependence is different from RT and CB

Fermi liquid theory:

RT – single particle resonant tunneling

CB – "orthodox" Coulomb blockade



T_H – "electron heating temperature"

T_H = 18 mK for a small quantum antidot

Maasilta & Goldman, PRB 1997

(Larger) interferometer T_H is expected to be $\leq$ 18 mK

Direct observation of anyonic statistics

- no fit to a detailed model is necessary:

  an f = 1/3 LQP encircling one f = 2/5 LQP and

  single-valuedness of wave function requires

  relative statistics to be fractional

$$\Theta^{1/3}_{2/5} = \frac{4}{15}$$

- direct: experiment closely models

  definition of anyonic brading statistics in 2D

- the only input: LQP charge $e/3$

  has been measured directly in quantum antidots

- thermal decoherence fits well g = 1/3 χLL theory;

  does not fit RT and CB T-dependencies

Thanks for attention!