

Current-induced phase textures and pairbreaking in multilayers and two-gap superconductors

Alex Gurevich¹ and Valeri Vinokur²

¹University of Wisconsin, Madison, WI

²Argonne National Lab. Argonne, IL

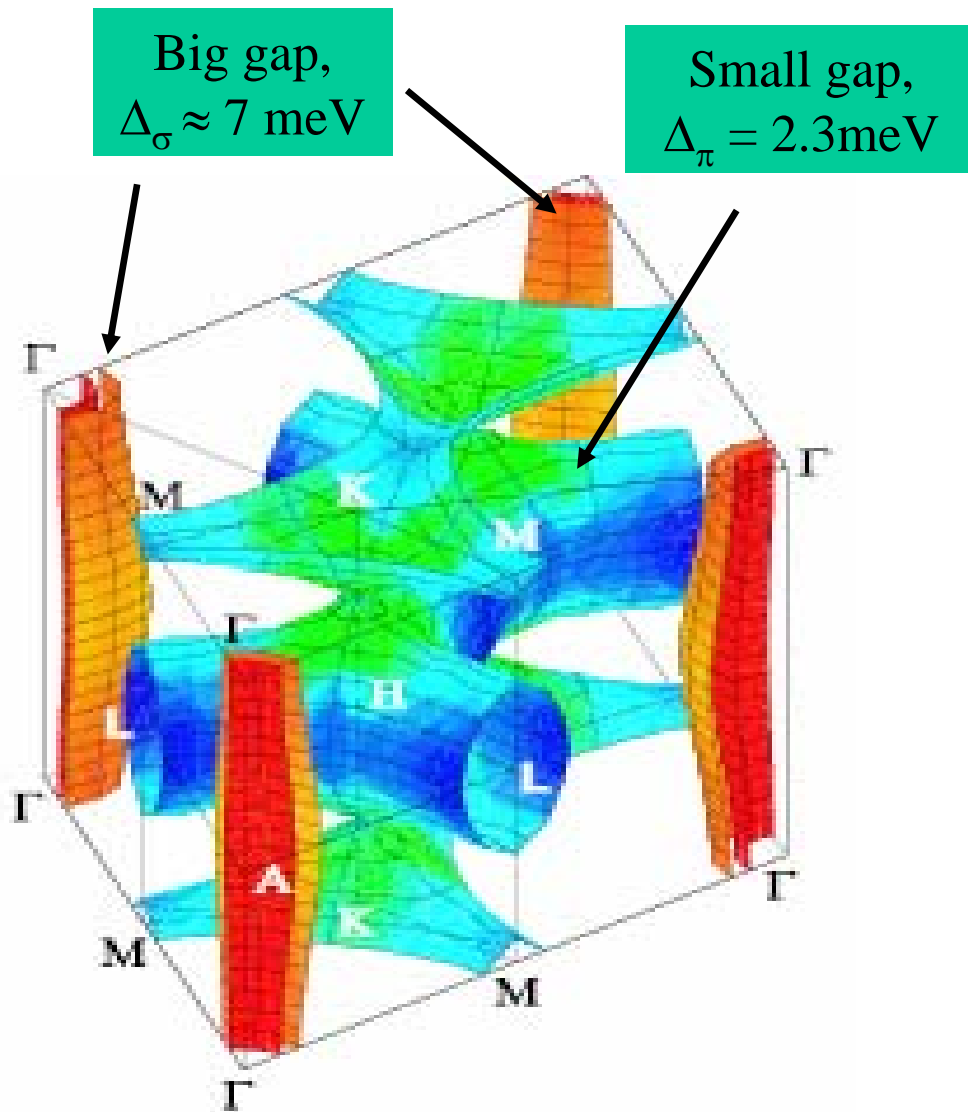
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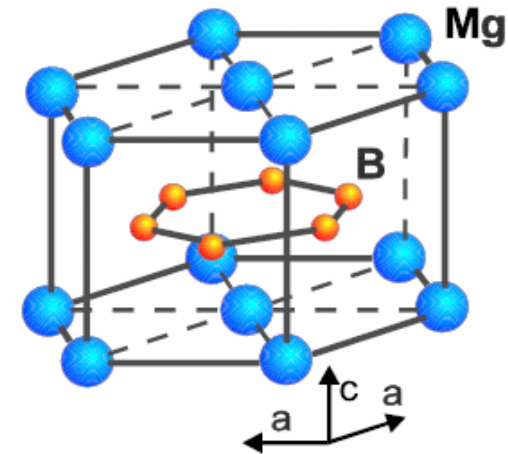
Outline

- **Systems with two different weakly coupled current-carrying parallel channels:**
 - **Multilayer structures**
 - **2 – gap superconductors (π and σ bands in MgB_2)**
- **How does current destroy superconductivity?**
 - **Current-induced decoupling of channels and formation of equilibrium phase textures**
 - **Current sharing and suppression of current pairbreaking in the weaker channel**
 - **Global pairbreaking**
- **Manifestations**
 - **Current - controlled 4 terminal devices**
 - **Current - induced interband phase textures in MgB_2**

Two-gap superconductivity in MgB₂



Liu, Mazin and Kortus (2002);
Choi et al, (2002)



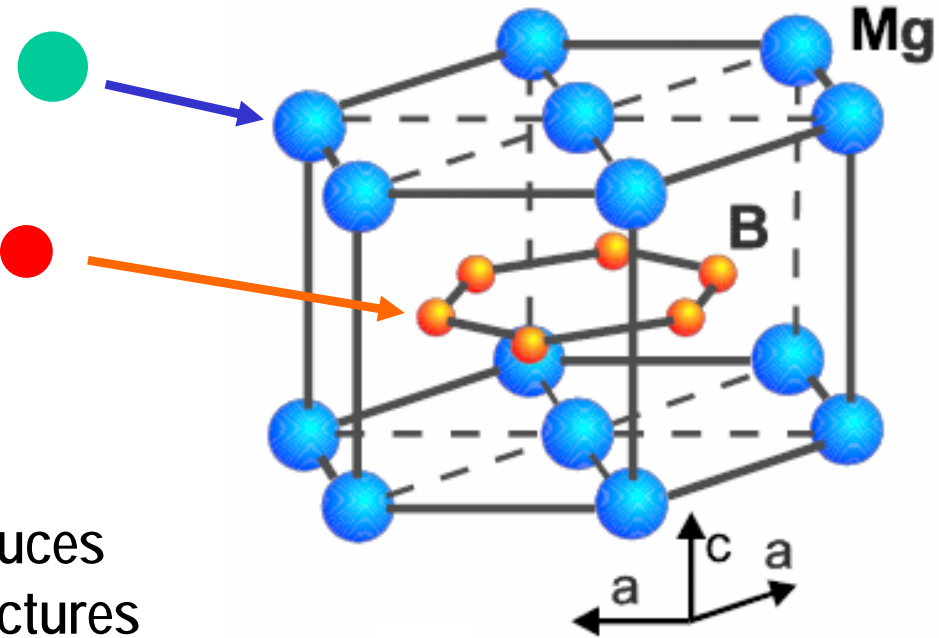
- 2D big gap for in-plane σ -orbitals s and 3D small gap for out-of-plane π -orbitals
- Weak interband coupling due to orthogonal p_z and p_{xy} orbitals of B

High $T_c = 40\text{K}$, but
low H_{c2} of MgB₂ single crystals:

$$H_{c2}^{\perp}(0) \approx 3.5 \text{ T}$$
$$H_{c2}^{\parallel}(0) \approx 13 \text{ T}$$

Tunable impurity scattering

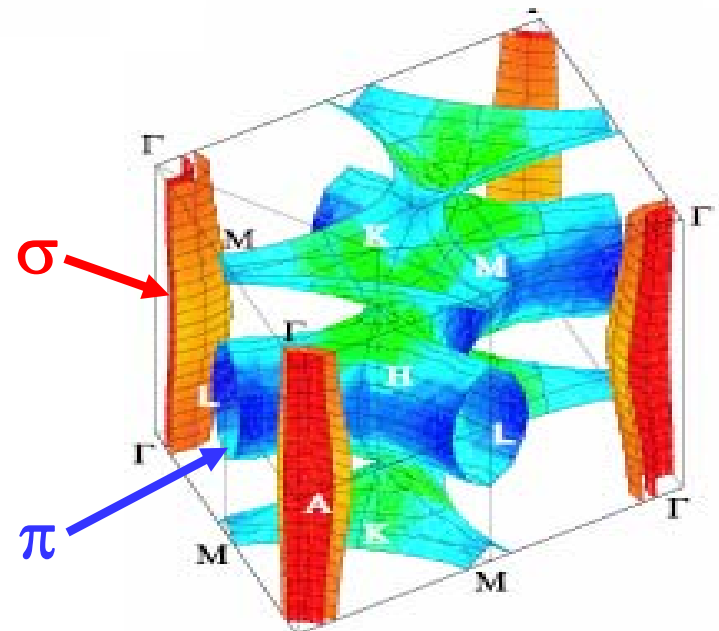
- Mg - substitution: 3D intraband π scattering
- B - substitution: 2D intraband σ scattering
- Weak interband scattering
- Selective atomic substitution produces quenched impurity or vacancy structures



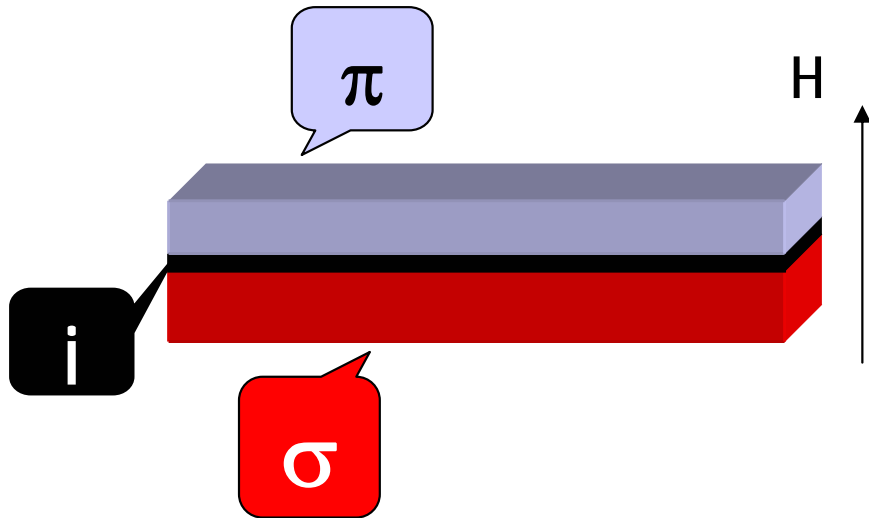
Anisotropic intraband diffusivities:

$$D_{\sigma}^{(c)} \ll D_{\sigma}^{(ab)}, \quad D_{\pi}^{(c)} \approx D_{\pi}^{(ab)}$$

D_{σ}/D_{π} is a variable material parameter

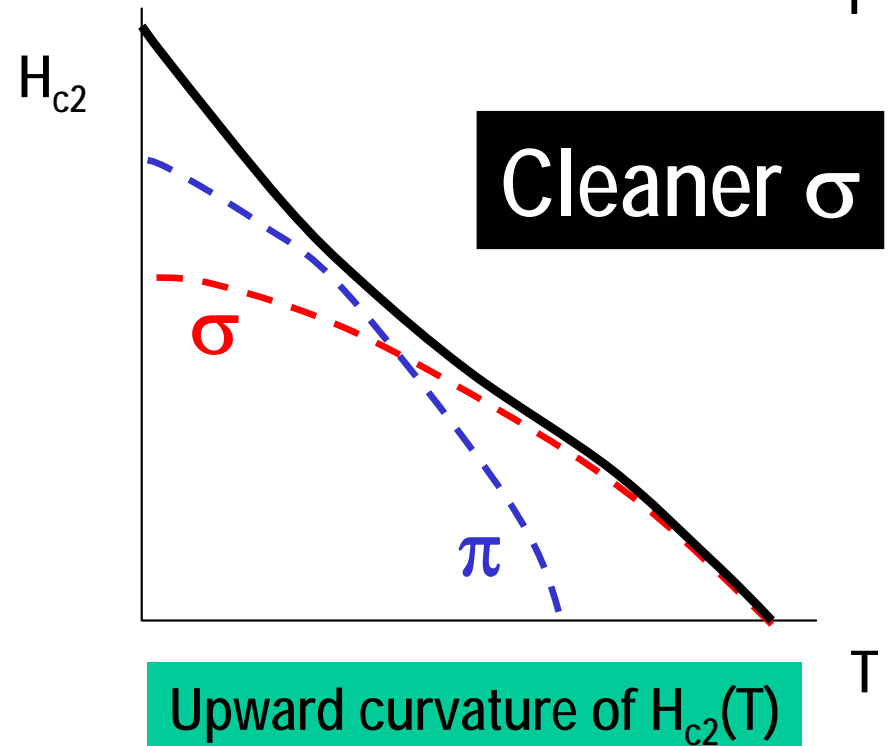
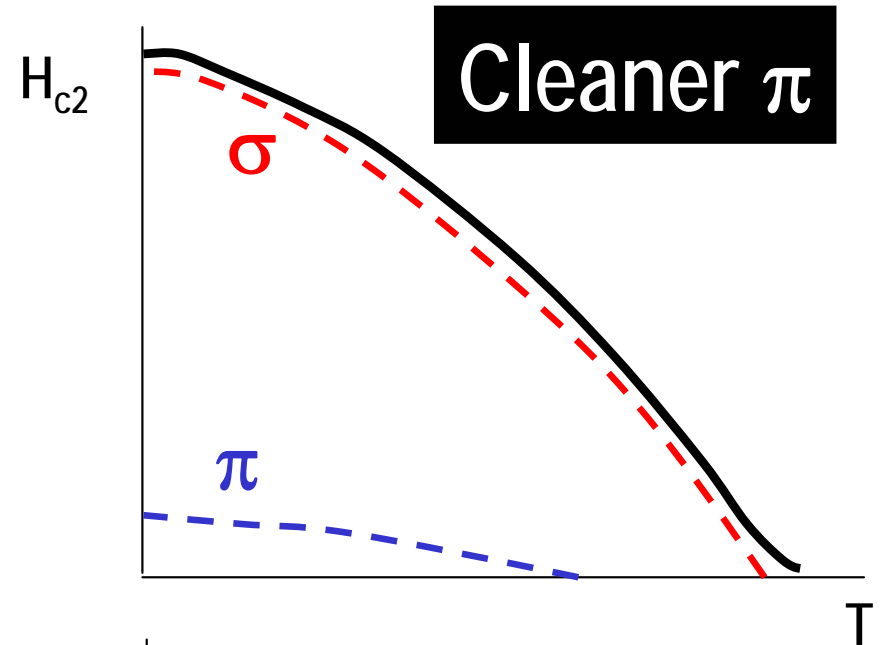


Toy model for MgB₂

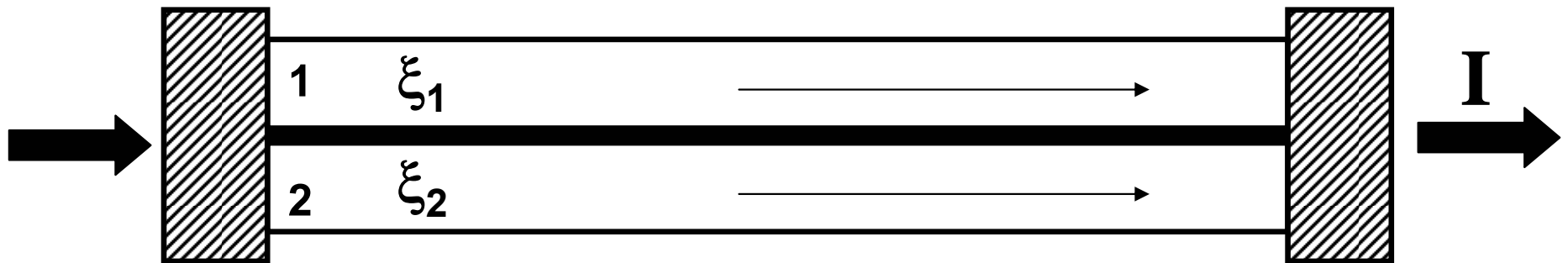


- Two weakly coupled thin films (σ and π)
- $T_c^{(\sigma)} > T_c^{(\pi)}$
- Weak Josephson coupling analogous to interband coupling

More details on H_{c2} in multilayers in:
S. Takahashi and M. Tachiki, PRB 33, 4620 (1986)
For MgB₂ see: AG, PRB 67, 184515 (2003)



Phase locked current state



Same phases $\chi_1 = \chi_2$ to minimize the Josephson energy,

$$W_J = (\hbar J_c / 2e) [1 - \cos(\chi_1 - \chi_2)]$$

Current-carrying state: $\Psi_1 = \Delta_1 \exp(i\chi_1)$, $\Psi_2 = \Delta_2 \exp(i\chi_2)$,

$$\nabla \chi_1 = \nabla \chi_2 = \mathbf{q}$$

$$\mathbf{Q} = \nabla \chi + 2\pi \mathbf{A} / \phi_0$$

$$Q = -\frac{8\pi^2}{c\phi_0} \left(\frac{\Lambda_1^2}{d_1} + \frac{\Lambda_2^2}{d_2} \right) I$$

What happens at higher currents?

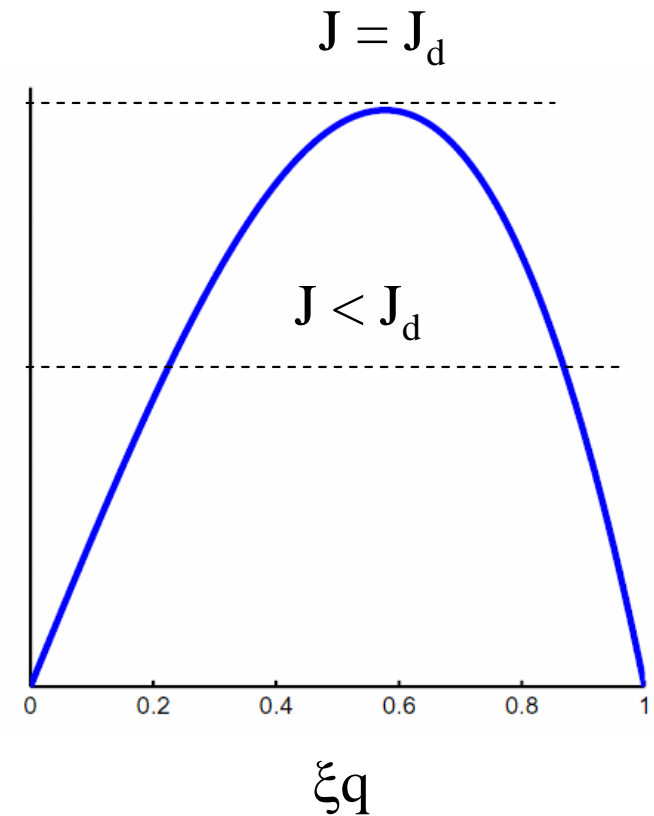
GL depairing current density

- GL current-carrying state with $\psi = \psi_0 \exp(-iqx)$, in a thin filament.

$$\psi_0^2 = 1 - \xi^2 q^2, \quad J = \frac{c \psi_0^2 \phi_0 q}{8 \pi^2 \Lambda^2}$$

- Current density as a function of q :

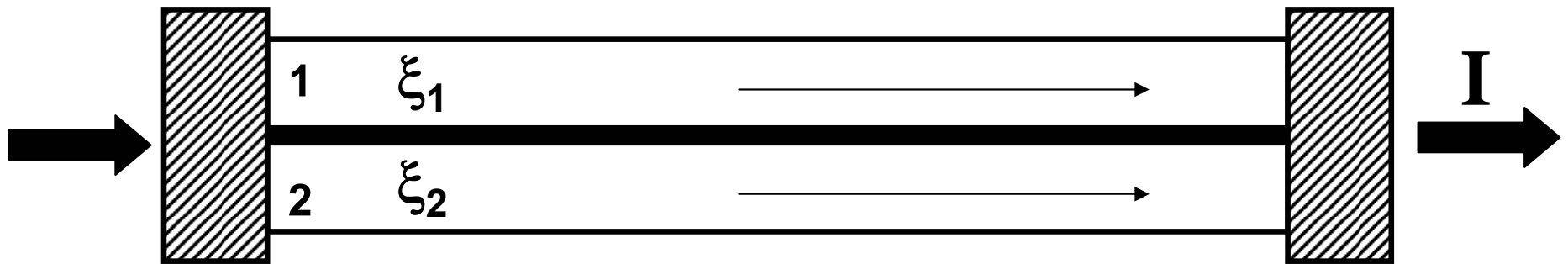
$$J = \frac{c \phi_0 q}{8 \pi^2 \Lambda^2} (1 - \xi^2 q^2) \leftarrow \text{Suppression of } \psi \text{ by current}$$



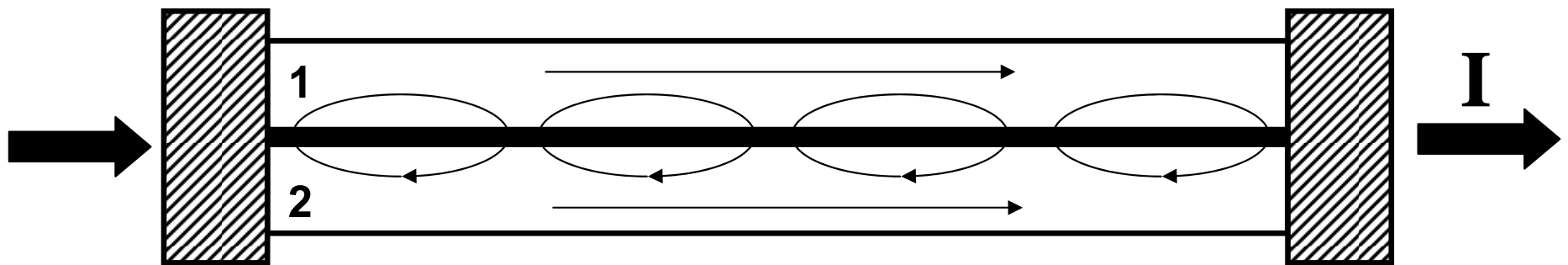
- Maximum J at $\xi q = 1/\sqrt{3}$:

$$J_d = \frac{\phi_0}{12\sqrt{3}\pi^2 \Lambda^2 \xi} \cong 0.54 \frac{H_c}{\Lambda} \propto \left(1 - \frac{T}{T_c}\right)^{3/2}$$

Transition to a phase slip state

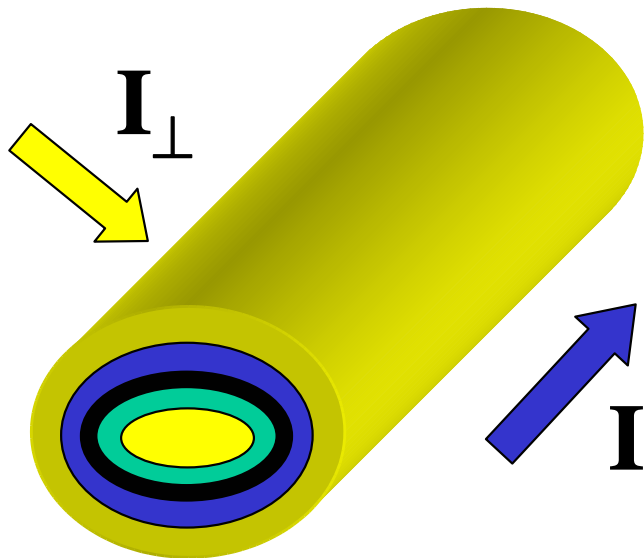
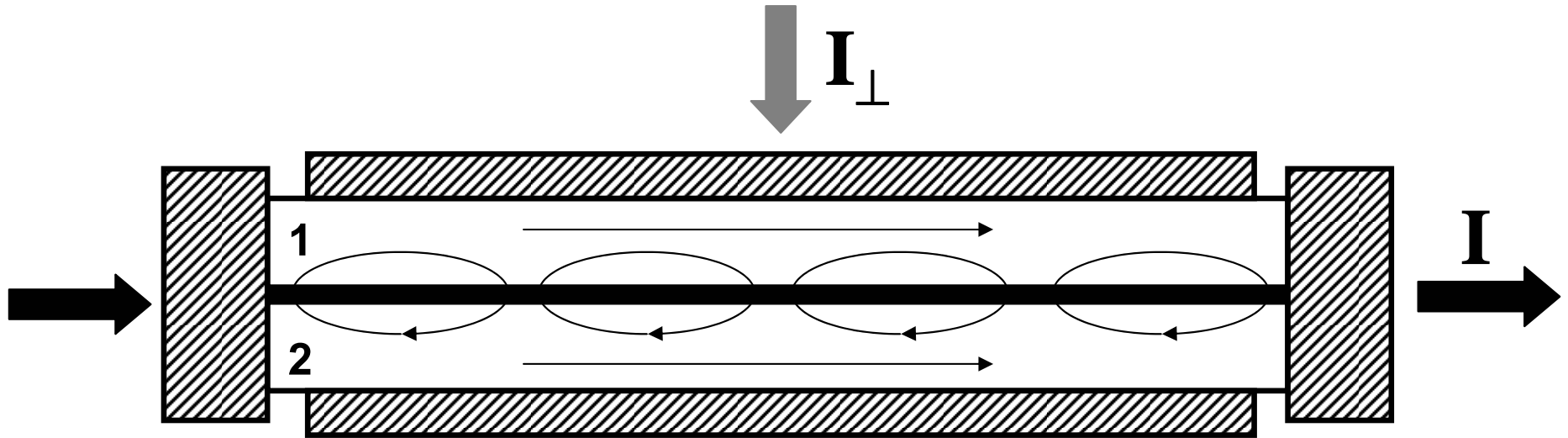


- What happens if $Q\xi_2 \sim 1$ in film 2, but $Q\xi_1 \ll 1$ in film 1?
- Film 2 cannot go normal
- Current redistribution enforces different Q_1 and Q_2 competing with the Josephson energy



- **Current-induced** vortex structure (interlayer phase texture) provides current sharing between films 1 and 2
- For weak Josephson coupling, the lock-in transition occurs at $I \ll I_d$

Four terminal devices



- Period of vortex structure is controlled by parallel current I
- Dynamics of vortices is controlled by perpendicular current I_{\perp}
- Parametric amplifier
- **No magnetic field is needed**

GL theory

$$F = -\alpha_1 \Delta_1^2 + \frac{\beta_1}{2} \Delta_1^4 + g_1 \left| \left(\nabla + \frac{2\pi i \mathbf{A}}{\phi_0} \right) \Psi_1 \right|^2 - \alpha_2 \Delta_2^2 + \frac{\beta_2}{2} \Delta_2^4 + g_2 \left| \left(\nabla + \frac{2\pi i \mathbf{A}}{\phi_0} \right) \Psi_2 \right|^2 - \gamma \Delta_1 \Delta_2 \cos \theta$$

- Thermodynamic potential for slow variation of $\theta(\mathbf{r}) = \chi_1 - \chi_2$ and fixed \mathbf{I}

$$G = -\frac{\alpha_1^2 d_1}{2\beta_1} (1 - \xi_1^2 Q_1^2)^2 - \frac{\alpha_2^2 d_2}{2\beta_2} (1 - \xi_2^2 Q_2^2)^2 - \varepsilon_J \cos \theta + cAI$$

$$I = -\frac{\phi_0}{\pi c} \left[\varepsilon_1 Q_1 (1 - \xi_1^2 Q_1^2) + \varepsilon_2 Q_2 (1 - \xi_2^2 Q_2^2) \right]$$

- Energy scales: $\varepsilon_1 = \alpha_1^2 \xi_1^2 d_1 / 2\beta_1$, $\varepsilon_2 = \alpha_2^2 \xi_2^2 d_2 / 2\beta_2$, $\varepsilon_J = \gamma \Delta_1 \Delta_2$

Expand G to quadratic terms in small $\nabla \theta$

Energy of phase textures

$$G_\theta = \varepsilon_J \left[\frac{L_\theta^2}{2} (\nabla \theta)^2 - \cos \theta - (Q \nabla \theta) h(Q) - \beta \theta \right] \quad \beta = I_\perp / I_c$$

Similar to the energy of a long Josephson contact. The effective magnetic field h is due to asymmetric pairbreaking. Both L_θ and h depend on current

$$h = \frac{8\varepsilon_1\varepsilon_2(\xi_2^2 - \xi_1^2)Q^2}{[(1 - 3\xi_1^2Q^2)\varepsilon_1 + (1 - 3\xi_2^2Q^2)\varepsilon_2]\varepsilon_J},$$

$$L_\theta^2 = \frac{4\varepsilon_1\varepsilon_2(1 - 3\xi_1^2Q^2)(1 - 3\xi_2^2Q^2)}{[(1 - 3\xi_1^2Q^2)\varepsilon_1 + (1 - 3\xi_2^2Q^2)\varepsilon_2]\varepsilon_J},$$

$$I = -Q[(1 - \xi_1^2Q^2)\varepsilon_1 + (1 - \xi_2^2Q^2)\varepsilon_2]8\pi c / \phi_0$$

- Nonlinear pairbreaking for $\xi_1 \neq \xi_2$.
- Large h and $L_\theta \gg \xi_2$ due to weak Josephson coupling
- $L_\theta^2(Q)$ changes sign at

$$Q_{c2} = 1 / \sqrt{3}\xi_2$$

Spinodal current instability at Q_{c2}

- Expansion to higher order derivatives:

$$G = \varepsilon_J \left[\frac{\lambda^2 L_0^2}{4} (\nabla^2 \theta)^2 + \frac{L_0^2}{2} (1 - 3Q^2 \xi_2^2) (\nabla \theta)^2 - \cos \theta - (\hat{Q} \nabla \theta) h \right]$$

- where $\ell \sim \xi_2$, and $L_0^2 = 4\varepsilon_2/\varepsilon_J \gg \xi_2$.
- Periodic weak disturbance $\theta_0 \cos(kx)$, with $k^2 = (3Q^2 \xi_2^2 - 1)/\ell^2$
- Linear instability at Q_{c2} at finite wave vector k_m

$$Q_{c2} = \frac{1}{\xi_2 \sqrt{3}} \left(1 + 2\lambda \sqrt{\frac{\varepsilon_J}{\varepsilon_2}} \right), \quad k_m = \frac{1}{2} \left(\frac{\varepsilon_J}{\lambda^2 \varepsilon_2} \right)^{1/4}$$

- No small amplitude solutions above Q_{c2} (analog of H_{c2} for vortices)

Explosive current redistribution at Q_{c2} well below the usual depairing current

Equilibrium phase textures

$$\tau^2 \mathcal{G} + \tau_r \mathcal{G} = L_\theta^2 \nabla^2 \theta - \sin \theta + \beta$$

Textures are energetically favorable for $Q > Q_{c1}$ analogous to H_{c1} for JJ

Different from nonequilibrium textures caused by electric field, [AG & V. Vinokur, PRL 90, 047004 \(2003\)](#)

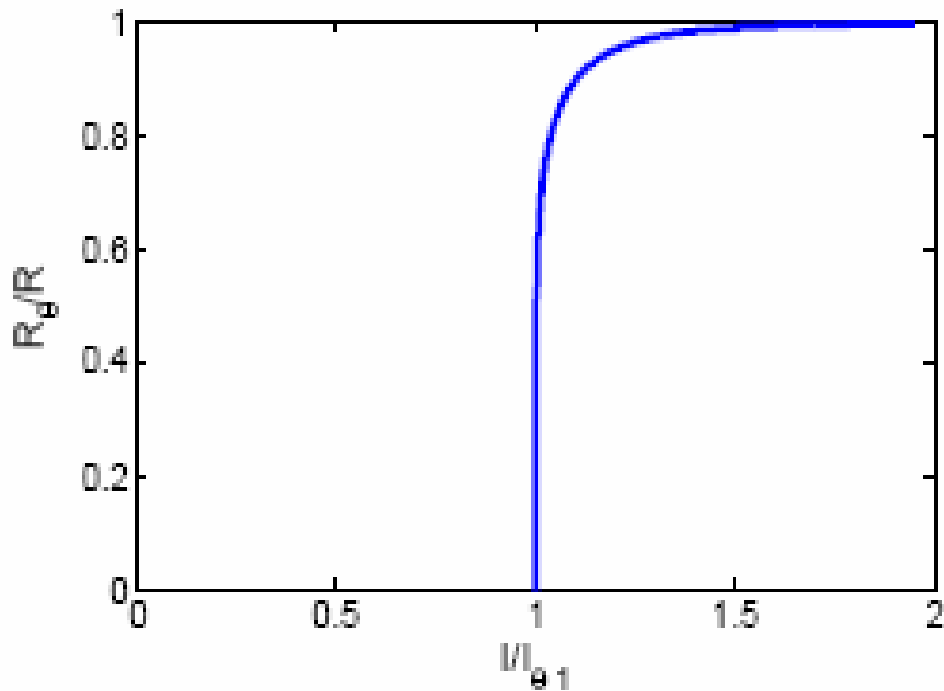
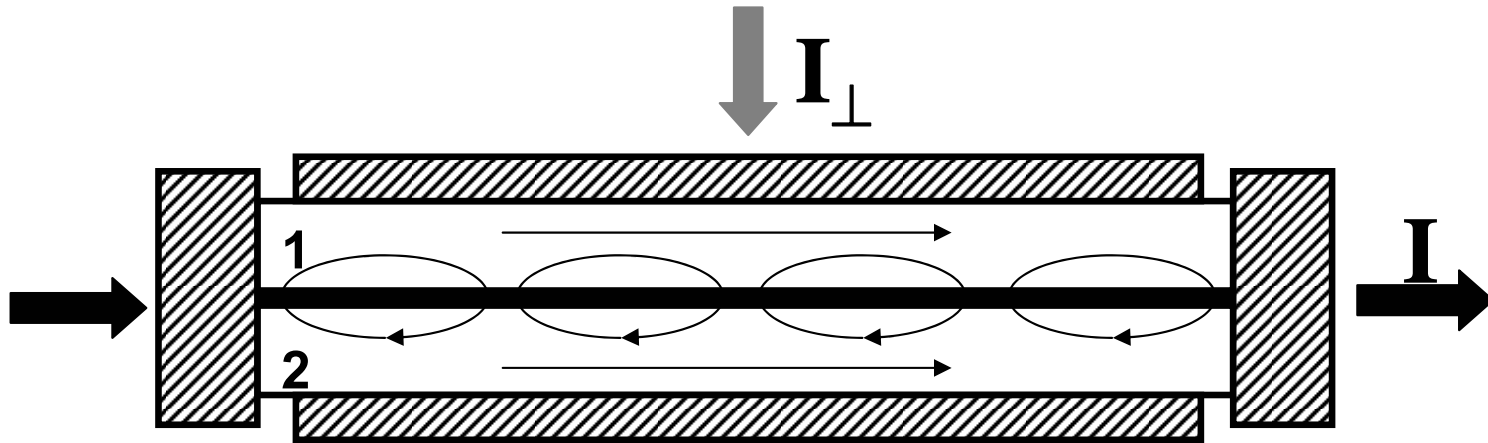
- A single 2π kink, $\theta(x) = 4 \tan^{-1} \exp(x/L_\theta)$ appears if $2\pi Q h(Q) > 8L_\theta$:

$$Q_{c1} = \left[\frac{(\varepsilon_1 + \varepsilon_2) \varepsilon_J}{\pi^2 \varepsilon_1 \varepsilon_2 (\xi_2^2 - \xi_1^2)^2} \right]^{1/6} \ll Q_{c2}, \quad I_{c1} = \frac{8\pi c}{\phi_0} (\varepsilon_1 + \varepsilon_2) Q_{c1}$$

- Periodic phase texture $\theta(x) = am(x/pL_\theta, p)$.
- Parametric relation which defines the period $a(l)$ ($0 < p < 1$):

$$a = 2L_\theta p K(p), \quad I^3 = I_{c1}^3 E(p) / p$$

Phase-unlocked resistive transition

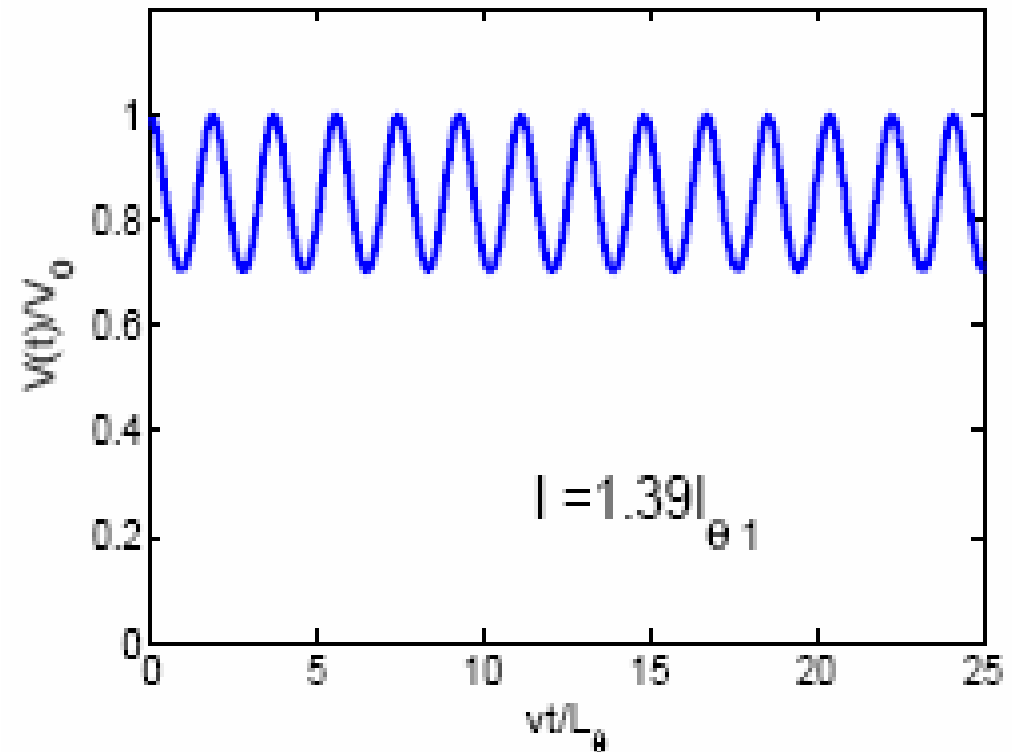
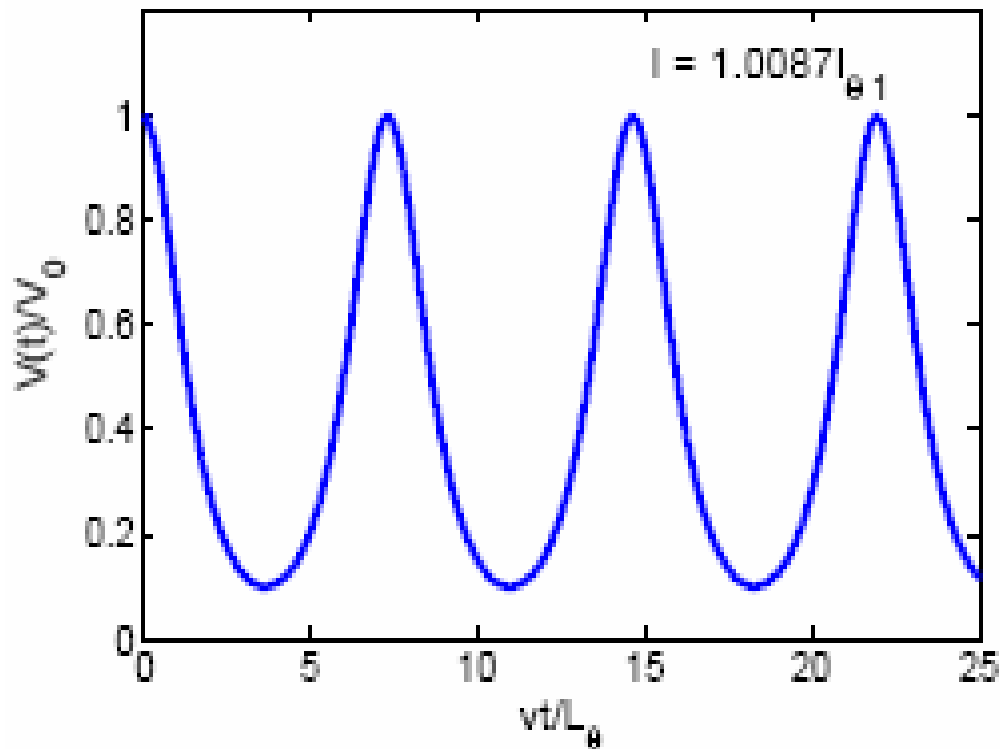


- Switching to ohmic flux flow for $I > I_{c1}$

$$\bar{V} = R_{\theta} I_{\perp}, \quad R_{\theta} = \frac{\pi^2 R_p L_{\theta}}{2E(p)a(p)}$$

- Resistive transition controlled by longitudinal current

Oscillating Josephson voltage



- Josephson ac voltage voltage: $V = (\phi_0 v / 2\pi c) \dot{\theta} / (vt/pL_\theta, p)$
- Washboard frequency proportional to the transverse current I_\perp

Two-gap superconductors

Dirty limit, two-gap Usadel equations:

AG, PRB 67, 184515 (2003)

$$\omega f_1 - \frac{D_{\alpha\beta}^{(1)}}{2} [g_1 \Pi_\alpha \Pi_\beta f_1 - f_1 \nabla_\alpha \nabla_\beta g_1] = \Delta_1 g_1 + (f_2 g_1 - f_1 g_2) \gamma_{12}$$

$$\omega f_2 - \frac{D_{\alpha\beta}^{(2)}}{2} [g_2 \Pi_\alpha \Pi_\beta f_2 - f_2 \nabla_\alpha \nabla_\beta g_2] = \Delta_2 g_1 + (f_1 g_2 - f_2 g_1) \gamma_{21}$$

Here $D_{\alpha\beta}$ are intraband electron diffusivities, $\Pi = \nabla + 2\pi i A / \phi_0$, $|f_m|^2 + g_m^2 = 1$, γ are interband scattering rates, λ_{nm} are the BCS coupling constants

• Gap equations:

$$\Delta_m = 2\pi T \sum_{\omega > 0} \sum_{s=1,2}^{\omega_D} \lambda_{ms} f_s(\omega, \Delta_s)$$

• Current density:

$$J_\alpha = -4\pi e T \operatorname{Im} \sum_{\omega} [N_1 D_{\alpha\beta}^{(1)} f_1^+ \Pi_\beta f_1 + N_2 D_{\alpha\beta}^{(2)} f_2^+ \Pi_\beta f_2]$$

Phase textures for all T

$$G_\theta = \varepsilon_J \left[\frac{L_\theta^2}{2} (\nabla \theta)^2 - \cos \theta - (Q \nabla \theta) h(Q) \right]$$

$$\varepsilon_J = \gamma \Delta_1 \Delta_2$$

$$2\gamma = N_1 \lambda_{12}^{-1} + N_2 \lambda_{21}^{-1}$$

Small Q expansion for all T: $h = f(T)Q^3$. Example for T = 0

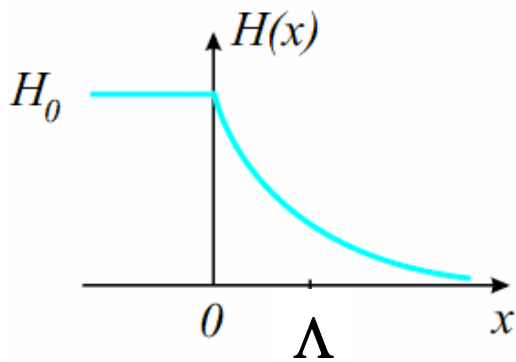
$$L_\theta^2 = \frac{\pi N_1 N_2 \Delta_1 \Delta_2 \xi_1^2 \xi_2^2}{4\gamma (N_1 \Delta_1^2 \xi_1^2 + N_2 \Delta_2^2 \xi_2^2)},$$

$$J_{c1} \cong J_2 \left[\frac{\gamma \xi_2 L_\theta (N_1 \Delta_1^2 \xi_1^2 + N_2 \Delta_2^2 \xi_2^2)}{N_1 N_2 \Delta_1 \Delta_2 (\xi_2^2 - \xi_1^2)} \right]^{1/3}$$

$\xi = (D/\Delta)^{1/2}$ are dirty coherence lengths at T = 0, and $J_2 = c\phi_0/8\pi^2\Lambda^2\xi_2$ is the depairing current density for the weaker band.

Interband phase textures in MgB₂

- For the parameters of MgB₂, J_{c1} is not much smaller than J_{c2} .
- Static interband phase textures $\theta(x)$ along the current direction at $\Omega \approx 1/\xi_\pi \sqrt{3}$



Screening current: $cH/4\pi\Lambda \approx c\phi_0/8\pi^2\Lambda^2 \xi_\pi \sqrt{3}$

Band decoupling magnetic field

$$H_\theta = \frac{\phi_0}{2\sqrt{3}\pi\Lambda\xi_\pi} \cong H_c \frac{\xi_\sigma}{\xi_\pi} \cong 0.3H_c \cong 0.13T$$

- Textures facilitate vortex penetration over the surface barrier
- Breakdown of linear London electrodynamics, mechanism of nonlocality and nonlinearity
- Nonlinearity of the rf surface impedance at $H \approx H_\theta$

Conclusions

- Two - stage superconductivity breakdown by current:
 - $I < I_{c1}$ - phase-locked state
 - $I_{c1} < I < I_d$ - band (layer) decoupling, current redistribution resulting in static phase textures
 - $I > I_d$ - global pairbreaking
- Current - controlled 4-terminal devices
 - Current switching and ac flux flow oscillators
 - Parametric generators and amplifiers
- Interband phase textures in MgB_2
 - Dc current pairbreaking
 - Vortex penetration
 - Effect on vortex lattice and the vortex core structure
 - Pinning and critical currents
 - Nonlinearity of the rf surface impedance