Domain wall superconductivity

in superconductor/ferromagnet bilayers

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Nanoscale superconductivity and magnetism

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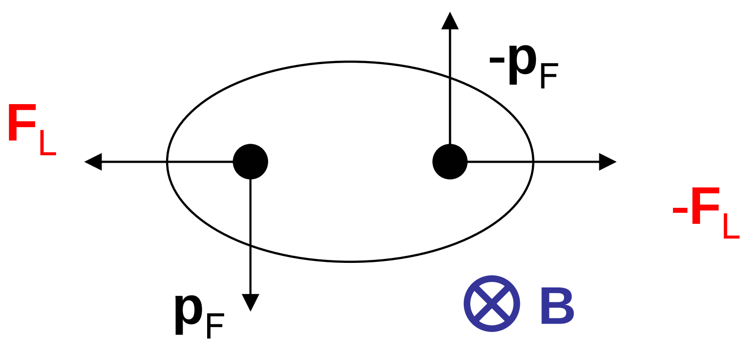
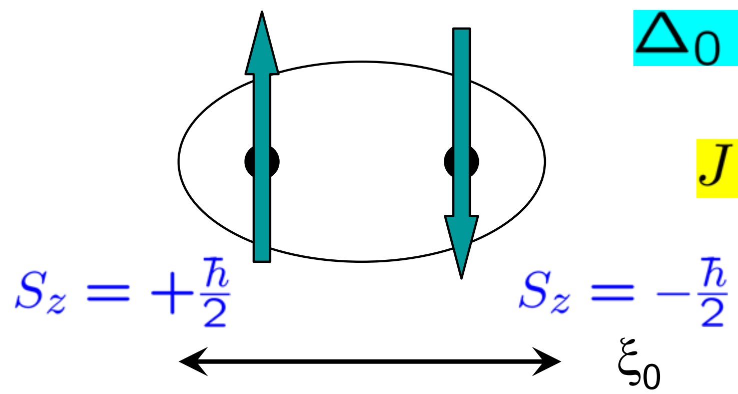
Coexistence of superconductivity and magnetism:

► **Ferromagnetism and superconductivity are antagonisitic orders**

orbital effect
(Lorentz force)



paramagnetic effect
(singlet pairs)



$\Delta_0 \sim T_{c0} = \mu_B H$

$J(\mathbf{s}.\mathbf{S}) \sim T_{c0}$

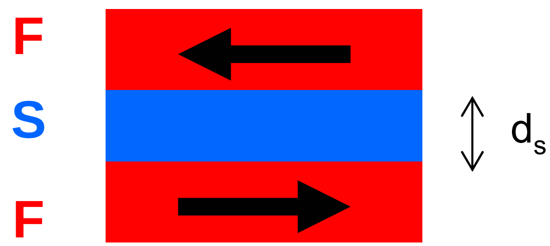
► **Antiferromagnetism and superconductivity coexist easily** $1/Q_{AF} \ll \xi_0$

These effects can be explored in nanoscale hybrid structures.

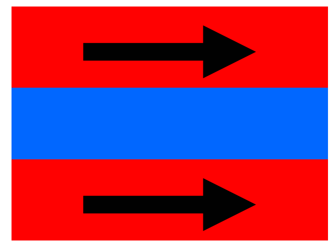
Proximity effect in F/S/F trilayers

If $d_s \sim \xi_s$, the critical temperature is controlled by proximity effect

Antiparallel (AP) configuration



Parallel (P) configuration



Theory in metallic systems

$$\ln \frac{T_{AP}}{T_{c0}} = Re\left[\psi(\frac{1}{2}) - \psi(\frac{1}{2} + \frac{1}{2\pi T_{AP}\tau_s})\right]$$

spin-flip pair breaking

$$\ln \frac{T_P}{T_{c0}} = Re\left[\psi(\frac{1}{2}) - \psi(\frac{1}{2} + \frac{1}{2\pi T_P}(ih + \frac{1}{\tau_s})\right]$$

Spin-flip + exchange pair breaking

experiments with insulating ferromagnets
(FeNi/In/Ni sandwich) after prediction by
de Gennes in 1966

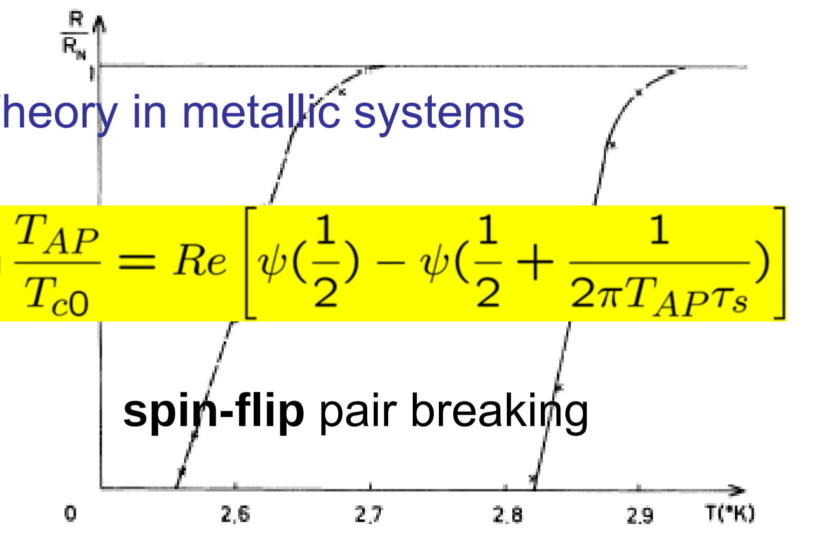


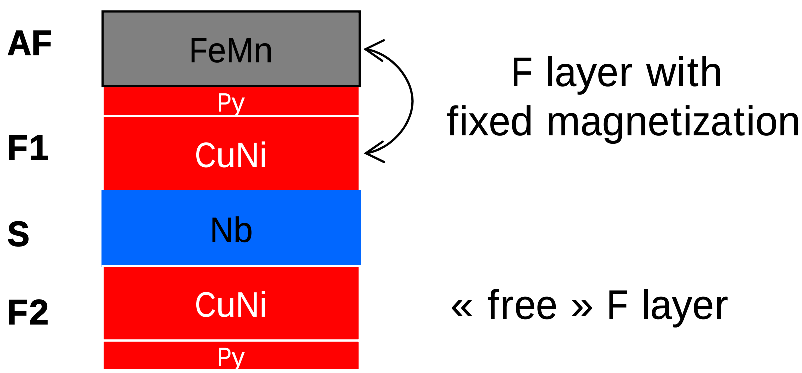
FIG. 1. Resistive measurements of the critical temperatures (R_N = resistance in the normal state) in zero field after the following: dashed line, application of 10 000 G ($T_{c\uparrow\uparrow}$) (all fields are applied parallel to the plane of the films); solid line, application of −10 000 G and subsequently +300 G to return the magnetization of the FeNi layer (H_1 < 300 G < H_2) ($T_{c\uparrow\downarrow}$).

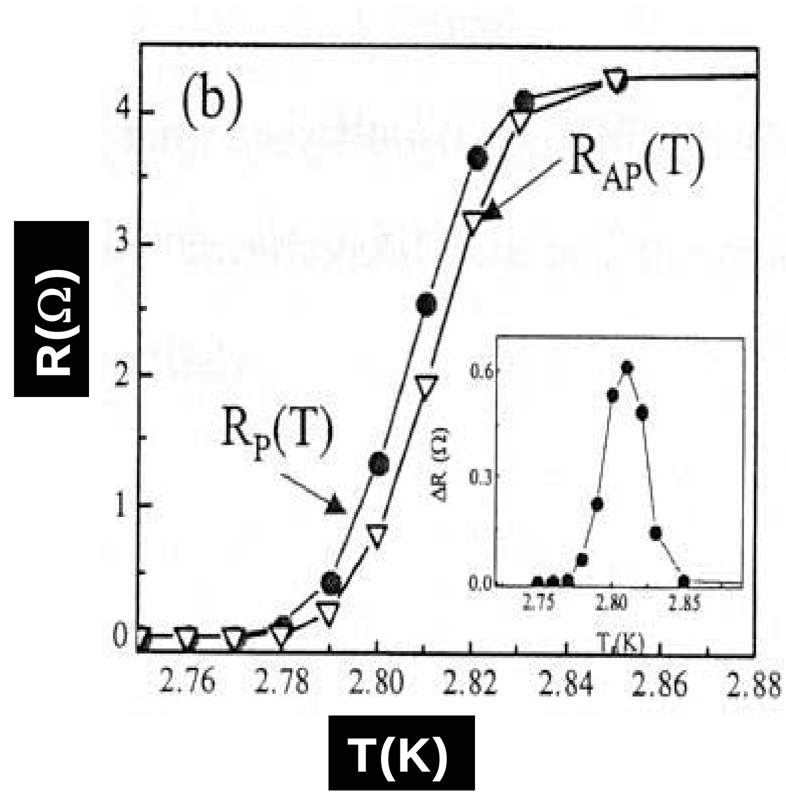
Deutscher, Meunier, 1969
Buzdin, Vedyayev, Ryazhanova, 1999

Tagirov, 1999

Recent experimental verification

Py(4nm) / $Cu_{0.47}Ni_{0.53}$(5) / Nb(18) / CuNi(5) / FeMn(6)



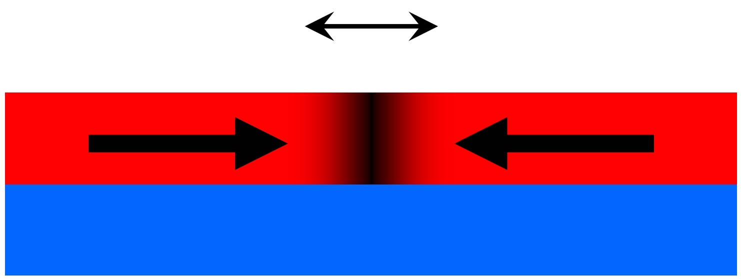


Gu, You, Jiang, Pearson, Bazaliy, Bader, 2002

Similar physics in F/S bilayers

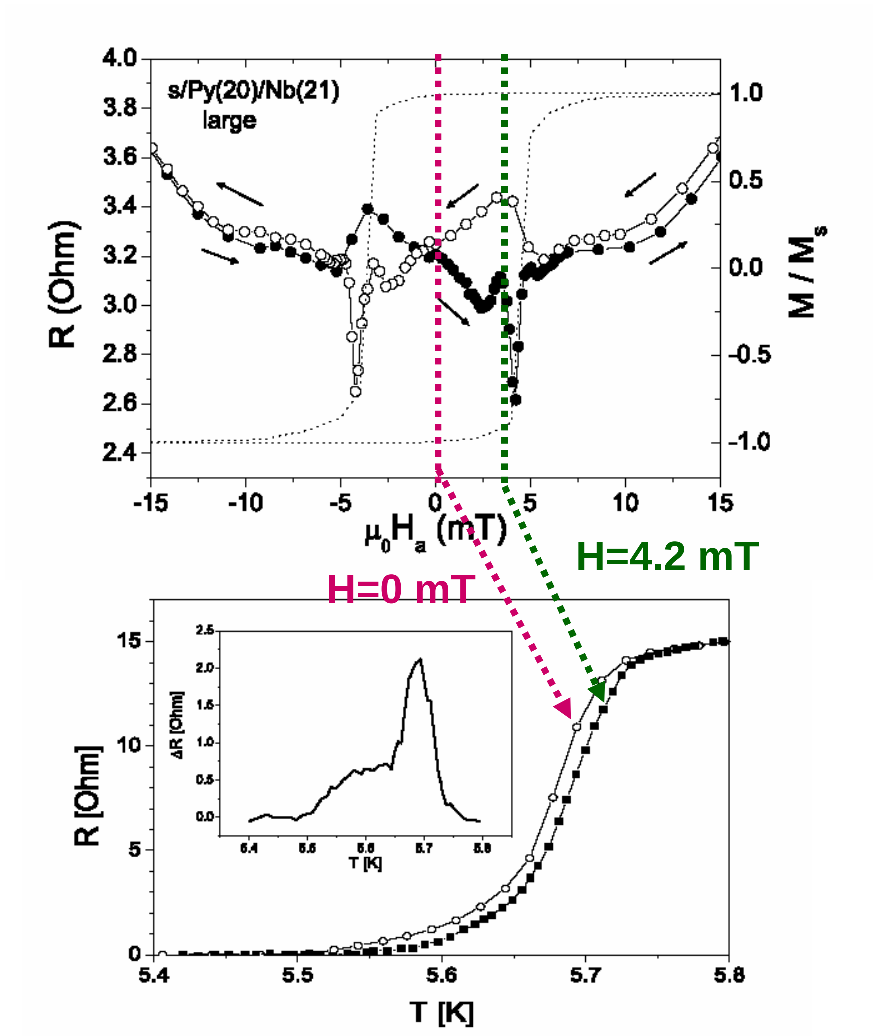
In practice, magnetic domains appear quite easily in ferromagnets

w: width of the domain wall



$Ni_{0.80}Fe_{0.20}$/Nb (20nm)

Thin films : Néel domains



Localized (domain wall) superconducting phase

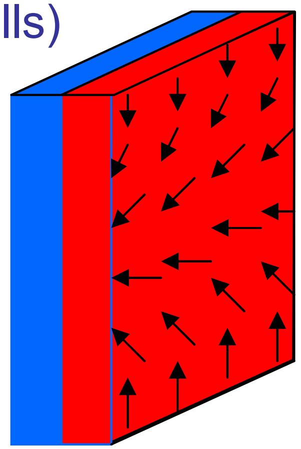
Rusanov, Hesselberth, Aarts, Buzdin, 2004

Description of the domain wall superconductivity

We consider :

• thin films with in-plane easy axis and Néel walls (rather than Bloch walls)

orbital effect is neglected



• The magnetic film is metallic, the contact is good

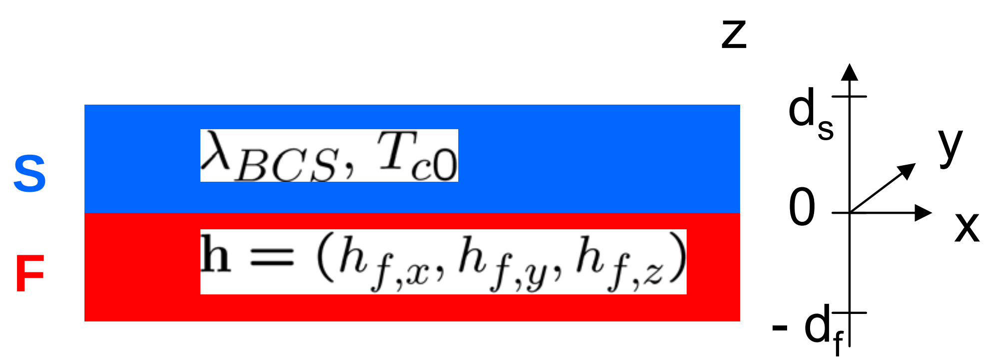
Physics is controlled by proximity effect.

We will describe it within quasiclassical theory

•. The magnetic structure is given

We ignore the influence of superconductivity on it

Linearized Usadel equations



dirty limit

$$\ell_{imp} \ll \xi_s = \sqrt{D_s/\Delta_0}, \quad \xi_f = \sqrt{D_f/h}$$

Weak exchange fields

$$h \ll \epsilon_F$$

The orientation of exchange field varies in space.
Therefore, the spin structure of Green's functions must be kept:

$$\hat{f} = \begin{pmatrix} f_{\uparrow\uparrow} & f_{\uparrow\downarrow} \\ f_{\downarrow\uparrow} & f_{\downarrow\downarrow} \end{pmatrix}$$

In the superconducting layer

$$-D_s \nabla^2 \hat{f} + 2\omega_n \hat{f} = 2\Delta\sigma_z$$

where $\Delta = i\pi T \lambda_{BCS} \sum_{\omega_n} f_{\uparrow\uparrow}$

In the ferromagnetic layer

$$-D_f \nabla^2 \hat{f} + 2\omega_n \hat{f} + i\left(h_{f,x}[\sigma_x, \hat{f}] + h_{f,y}[\sigma_y, \hat{f}] + h_{f,z}\{\sigma_z, \hat{f}\}\right) = 0$$

Boundary conditions

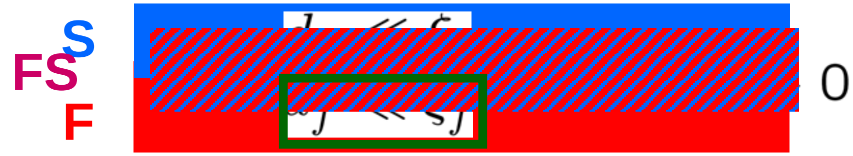
$$\partial_z \hat{f}(z = d_s) = \partial_z \hat{f}(z = -d_f) = 0$$

$$\sigma_f \partial_z \hat{f}(z = 0^-) = \sigma_s \partial_z \hat{f}(z = 0^+) \qquad \sigma_s \partial_z \hat{f}(z = 0^+) = \gamma[\hat{f}(z = 0^-) - \hat{f}(z = 0^+)]$$

Current conservation

γ: interface conductance per unit area

Thin bilayers: effective description



Usadel equations can be averaged over the thickness of the layers

Questionable for real samples

$$\hat{f}_s(x,y) = \frac{1}{d_s}\int_0^{d_s} dz\,\hat{f}(x,y,z) \qquad \text{and} \qquad \hat{f}_f(x,y) = \frac{1}{d_f}\int_{-d_f}^{0} dz\,\hat{f}(x,y,z)$$

$$-D\nabla^2\hat{f} + 2\omega_n\hat{f} + i\left(h_x[\sigma_x,\hat{f}] + h_x[\sigma_y,\hat{f}] + h_z\{\sigma_z,\hat{f}\}\right) = 2\Delta\sigma_z$$

is the Usadel equation for a magnetic superconductor with effective:

diffusion constant — exchange field — attractive pairing

$$D = \frac{D_s\eta_s + D_f\eta_f}{\eta_s+\eta_f}, \qquad \mathbf{h} = \frac{\eta_f}{\eta_s+\eta_f}\mathbf{h}_f, \qquad \lambda = \frac{\eta_s}{\eta_s+\eta_f}\lambda_{BCS}$$

(Fominov and Feigelman, 2001; Fominov, Chtchelkatchev Golubov, 2002)

We assume that $\eta_f = \sigma_f d_f/D_f$ is much smaller than $\eta_s = \sigma_s d_s/D_s$

(roughly $d_f \ll d_s$)

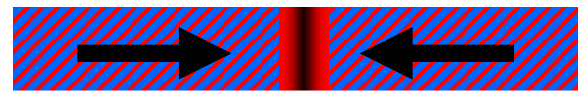
Then $D=D_s$ and the critical temperature (T_{c0}) is hardly affected by proximity effect at h=0.

On the other hand, $h \ll h_f$ and may compete with Δ_0

Second order transition line at uniform magnetization is given by

$$\ln\frac{T}{T_{c0}} + 2\pi T \,\mathrm{Re}\sum_{\omega_n>0}\frac{1}{\omega_n} - \frac{1}{\omega_n + ih} = 0$$

Narrow domain wall $h_z(x) = h\mathrm{sgn}(x)$



$w \ll \xi$

$$-\frac{D}{2}\partial_x^2 f_{\uparrow\uparrow} + (\omega_n + ih_z(x))f_{\uparrow\uparrow} = \Delta$$

The selfconsistency equation transforms into linear integral equation in Fourier space

$$\left\{ \ln\frac{T}{T_{c0}} + 2\pi T \mathrm{Re}\sum_{\omega_n>0}\frac{1}{\omega_n} - \frac{1}{\omega_n + ih + \frac{Dp^2}{2}} \right\}\Delta_p = \sum_{\omega_n}\alpha_p(\omega_n)\int\frac{dk}{2\pi}\alpha_k(\omega_n)\Delta_k$$

Superconducting kernel for uniform exchange field

due to the presence of the domain wall

The critical temperature $T_c(h)$ corresponds to uniform state (p=0)

Buzdin, Bulaevskii, Panyukov, 1984

Close to T_{c0}

$$\frac{T_{c0} - T_c(h)}{T_{c0}} \approx \frac{7\zeta(3)}{4\pi^2}\frac{h^2}{T_{c0}^2} - \frac{31\zeta(5)}{16\pi^4}\frac{h^4}{T_{c0}^4}$$

$$\frac{T_{cw}(h) - T_c(h)}{T_{c0}} \approx \frac{(8\sqrt{2}-1)^2\zeta(\frac{7}{2})^2}{8\pi^6}\frac{h^4}{T_{c0}^4}$$

Analogy with twinning plane superconductivity

In some samples, pairing may be locally more attractive along twin boundaries :

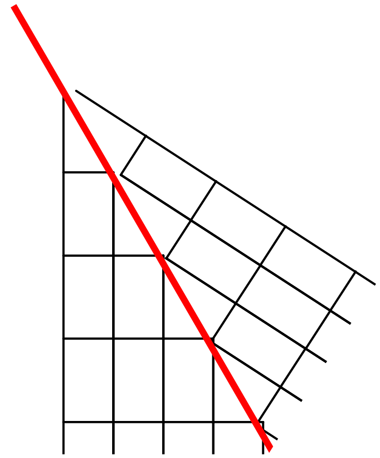
$$\lambda_{BCS}(x) = \lambda_0 + \delta\lambda(x)$$

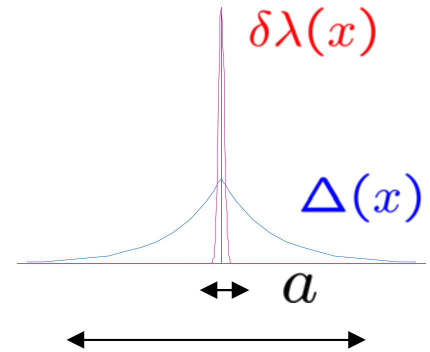
$$\frac{T_c^{TP} - T_c}{T_c} \equiv \tau_0 = \frac{\delta\lambda}{\lambda_0^2} \times \frac{a}{\xi_s(\tau_0)}$$

$$T_c \sim e^{-1/\lambda}$$

Local increase of pairing

Effective volume fraction





$$\xi_s(\tau_0) = \frac{\xi_s}{\sqrt{\tau_0}}$$

$$\frac{T_c^{TP} - T_c}{T_c} = \left(\frac{\delta\lambda}{\lambda_0^2}\frac{a}{\xi_s}\right)^2$$

Khlyustikov and Buzdin, 1987

For domain wall superconductivity

$$\frac{T_{cw} - T_c}{T_c} \equiv \tau_0 = \frac{1}{\tau_s T_{c0}} \times \frac{\xi_s}{\xi_s(\tau_0)}$$

$$\frac{1}{\tau_s} \sim \frac{h^2}{T_{c0}}$$

= Local decrease of pair breaking parameter

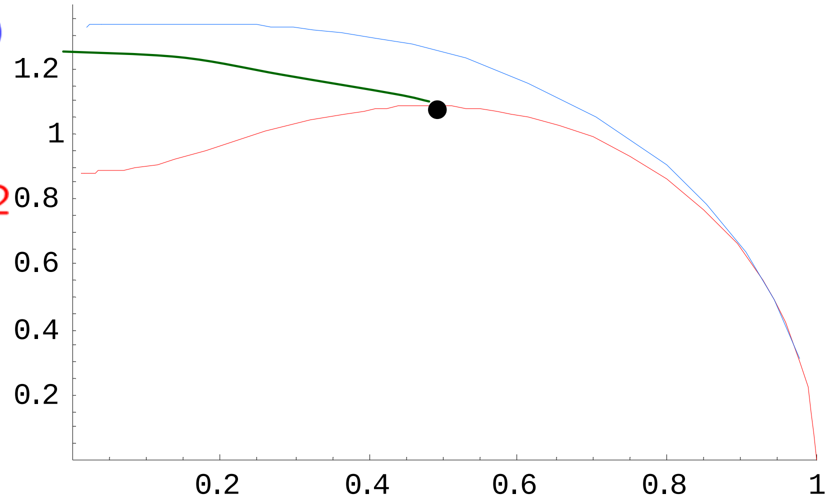
$$\tau_0 \sim \frac{h^4}{T_{c0}^4} \qquad \propto \left(\frac{T_{c0} - T}{T_{c0}}\right)^2$$

Corresponding Guinzburg-Landau equation

$$-\xi_s^2 \Delta''(x) + \tau_0 \Delta(x) = \frac{1}{\tau_s T_{c0}} \xi_s \Delta(x=0)\, \delta(x)$$

Phase diagram

$\Delta_0 = 1.76 T_{c0}$



$$h \approx \frac{d_f}{d_s} h_f$$

The transition into uniform state becomes **1st order** below some critical temperature:

$$2\pi T \Re \sum_{\omega_n} \frac{1}{(\omega_n + ih)^3} > 0 \qquad T^* = 0.56 T_{c0}$$

Instability towards **1st order** transition can be considered for localized state too.
The free energy expands in powers of the gap Δ

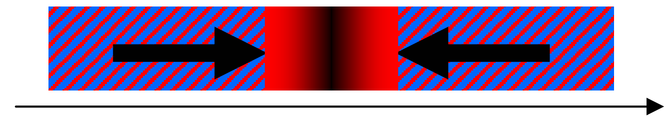
$$\mathcal{F}[\Delta] = \mathcal{F}^{(2)} + \mathcal{F}^{(4)} + \dots$$

Along the second order critical line, $\mathcal{F}^{(2)} = 0$ while $\mathcal{F}^{(4)} = \pi T \nu \Re \sum_{\omega_n} \int dx \left[\frac{D}{2}(\partial_x f)^2 + \Delta f\right] f^2$

$$\mathcal{F}^{(4)}_{dw} > 0$$

Large domain wall

$$\mathbf{h}(x) = h(\cos\phi(x), \sin\phi(x), 0)$$



$$\phi(x \to \pm\infty) = \pm\frac{\pi}{2}$$

$$\begin{cases} -\frac{D}{2}\partial_x^2 f_{\uparrow\uparrow} + \omega_n f_{\uparrow\uparrow} - i\frac{h}{2}\left(e^{-i\phi} f_{\downarrow\uparrow} - e^{i\phi} f_{\downarrow\uparrow}\right) & = & \Delta \\ -\frac{D}{2}\partial_x^2 f_{\uparrow\downarrow} + \omega_n f_{\uparrow\downarrow} + ihe^{-i\phi} f_{\uparrow\uparrow} & = & 0 \\ -\frac{D}{2}\partial_x^2 f_{\downarrow\uparrow} + \omega_n f_{\downarrow\uparrow} - ihe^{i\phi} f_{\uparrow\uparrow} & = & 0 \end{cases}$$

We proceed like in the derivation of Ginzburg-Landau equations in the quasiclassical theory

$$f(x) = f^{(0)}(x) + f^{(1)}(x)$$

Solution which assumes that ϕ and Δ do not vary spatially

Corrections induced by slow variation of ϕ

Inserting the solution f(x) in the selfconsistency equation, we obtain:

$$-\frac{1}{2m}\Delta''(x) + U(x)\Delta(x) = -\ln\frac{T}{T_c}\Delta(x)$$

$$\frac{1}{2m} = 2D\pi T \sum_{\omega_n} \frac{(\omega_n^2 - h^2)}{(\omega_n^2 + h^2)^2}$$

$$U(x) = -2D\pi T(\phi')^2 \sum_{\omega_n} \frac{h^2}{(\omega_n^2 + h^2)^2}$$

Schrodinger equation in 1d with attractive potential $U(x) < 0$.

The corresponding **bond state** negative energy defines $T_{cw} > T_c$

Close to T_{c0}

$$\frac{1}{2m} \sim \xi_s^2 \qquad \xi_s \ll w \ll \xi(T) \qquad U(x) \sim \frac{h^2}{T_{c0}^2}\frac{\xi_s^2}{w}\delta(x)$$

$$\frac{T_{cw}-T_c}{T_c} = \frac{\pi^2}{1152}\frac{h^4}{T_{c0}^4}\frac{\xi_s^2}{w^2}$$

Close to T=0

$$\frac{1}{2m} \to 0 \qquad U_{min} = -\ln\frac{T}{T_c}$$

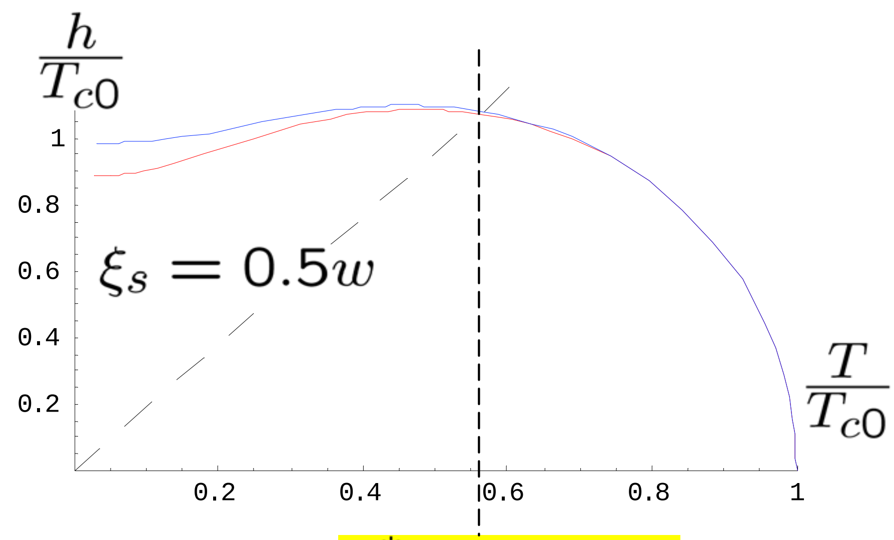
$$h_{cw}-h_c = \frac{\pi D}{16w^2} \sim \frac{\xi_s^2}{w^2}h_c$$

Intermediate temperatures

With the specific choice $\phi(x) = 2\,\mathrm{arctan}[\tanh(x/2w)]$,

T_{cw} can be found exactly

(particule in potential well $U(x) \propto 1/\cosh^2(x/w)$)



$T^* = 0.56T_{c0}$

Another analogy with twinning plane superconductivity

Close to T_{c0}

Effective pair-breaking parameter in uniform state $\frac{1}{\tau_s} \sim \frac{h^2}{T_{c0}}$

Rotation angle on scale ξ_s along the domain wall $\alpha = \frac{\xi_s}{w}$

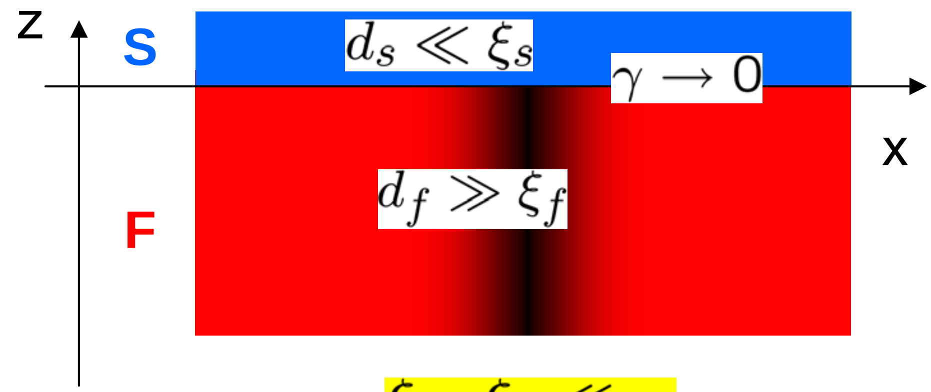
$$\frac{h-h^{av}}{h} \sim \alpha^2$$

Effective decrease of pair breaking parameter along the wall $\delta(\frac{1}{\tau_s}) \sim \frac{\alpha^2}{\tau_s}$

$$\frac{T_{cw}-T_c}{T_c} \equiv \tau_0 = \frac{1}{\tau_s T_{c0}} \frac{\xi_s^2}{w^2} \times \frac{w}{\xi(\tau_0)}$$

$$\frac{T_{cw}-T_c}{T_c} \propto \frac{h^4}{T_{c0}^4} \frac{\xi_s^2}{w^2}$$

Bilayer with thick ferromagnetic layer



$\xi_f,\, \xi_s \ll w$

In F layer

$$\begin{cases} -\frac{D_f}{2}\nabla^2 f_{\uparrow\uparrow} + \omega_n f_{\uparrow\uparrow} - \frac{ih_f}{2}\left[e^{-i\phi} f_{\downarrow\uparrow} - e^{i\phi} f_{\uparrow\downarrow}\right] = 0 \\ -\frac{D_f}{2}\nabla^2 f_{\downarrow\uparrow} + \omega_n f_{\downarrow\uparrow} + ih_f e^{-i\phi} f_{\uparrow\uparrow} = 0 \\ -\frac{D_f}{2}\nabla^2 f_{\uparrow\downarrow} + \omega_n f_{\uparrow\downarrow} - ih_f e^{i\phi} f_{\uparrow\uparrow} = 0 \end{cases}$$

In S layer $\qquad \hat{f}_s(x) = \frac{1}{d_s}\int_0^{d_s} dz\, \hat{f}(x,z)$

$$-\frac{D_s}{2}\partial_x^2 \begin{pmatrix} f_{\uparrow\uparrow}(x,0) \\ f_{\uparrow\downarrow}(x,0) \\ f_{\downarrow\uparrow}(x,0) \end{pmatrix} + \omega_n \begin{pmatrix} f_{\uparrow\uparrow}(x,0) \\ f_{\uparrow\downarrow}(x,0) \\ f_{\downarrow\uparrow}(x,0) \end{pmatrix} + \alpha\partial_z \begin{pmatrix} f_{\uparrow\uparrow}(x,0) \\ f_{\uparrow\downarrow}(x,0) \\ f_{\downarrow\uparrow}(x,0) \end{pmatrix} = \begin{pmatrix} \Delta(x) \\ 0 \\ 0 \end{pmatrix} \qquad \alpha = \frac{\sigma_f D_s}{2 d_s \sigma_s}$$

We proceed similarly. We get similar formulas with

effective **exchange field** $h = \alpha\sqrt{h_f/D_f}$ + **pair breaking** $\tau_s^{-1} = \alpha\sqrt{h_f/D_f}$

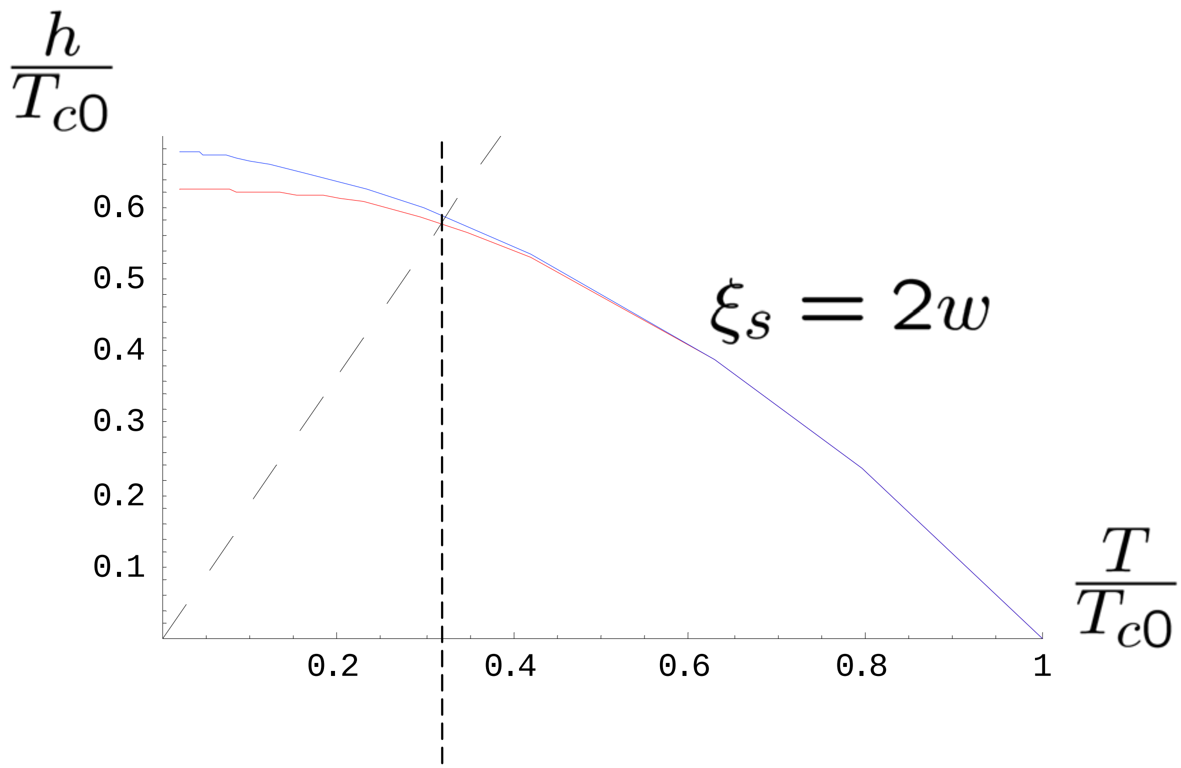
T_c for uniform state $\qquad \ln\frac{T_c}{T_{c0}} + 2\pi T \,\mathrm{Re}\sum_{\omega_n>0}\left\{\frac{1}{\omega_n} - \frac{1}{\omega_n + (1+i)h}\right\} = 0$

T for localized state $\qquad -\frac{1}{2m}\Delta''(x) + U(x)\Delta(x) = -\ln\frac{T}{T_c}\Delta(x)$

$$\frac{1}{2m} = 2D_s\pi T\sum_{\omega_n}\frac{[(\omega_n+h)^2 - h^2]}{[(\omega_n+h)^2+h^2]^2} \qquad U(x) = -2D_s\pi T(\phi')^2\sum_{\omega_n}\frac{h^2}{[(\omega_n+h)^2+h^2]^2}$$

$$\frac{h}{h_f} \propto \frac{\sigma_f}{\sigma_s} \frac{\xi_f}{d_s}$$

$$\frac{h}{T_{c0}} \propto \frac{\sigma_f}{\sigma_s} \frac{\xi_s^2}{d_s \xi_f}$$



$$T^* = 0.31 T_{c0}$$

The instability towards **1st order** transition is given by

$$2\pi T \Re \sum_{\omega_n} \left\{ \frac{1}{(\omega_n + \alpha q)^3} + \frac{1}{4} \frac{\alpha q}{(\omega_n + \alpha q)^4} \right\} > 0, \qquad q = \sqrt{\frac{2ih_f}{D_f}}$$

Related studies

- Nucleation of superconductivity due to orbital effect (in-plane anisotropy)
  (similar mechanism as for surface superconductivity, H_{c3})

Buzdin, Melnikov, 2003

- Helicoidal magnetic order $\phi(x)=Qx$

Here domains and domain walls have the same width

On the other hand, the order parameter does not vary along the domain

Champel and Eschrig, 2005

Perspectives

- Effect of superconductivity on the magnetic structure
- Properties of the domain wall superconducting phase
  transition to the uniform phase
- finite spin polarization

Conclusion

- We determined the conditions for the appeareance of localized superconducting phase in F/S bilayers in the presence of magnetic domains

- Domain wall superconductivity is similar to twinning plane superconductivity

- The experimental control of the domain structure may allow to build superconducting switches