Geometrical and Andreev Quantization in Mesoscopic Superconductors

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Outline

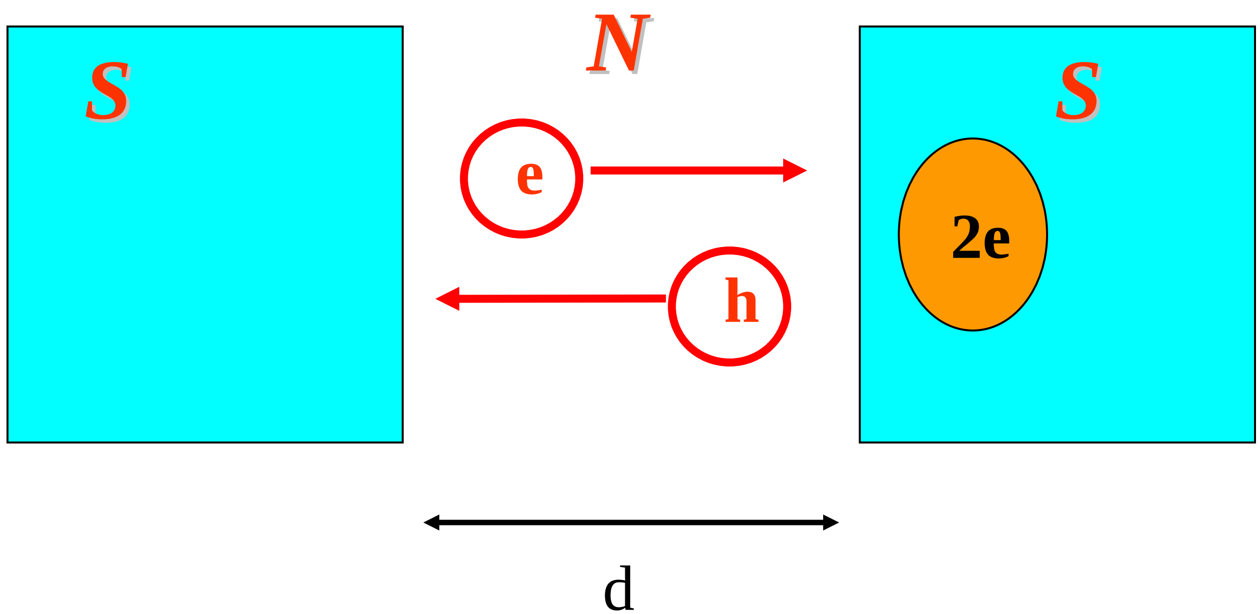
- **Introduction.**
  Andreev states. Examples.
  Geometrical and Andreev quantization.
  Influence of boundary conditions.

- **Quantization rules for Andreev states on closed trajectories.**

- **1D Andreev states in a quantum box. Mesoscopic oscillations of spectrum and Josephson current.**

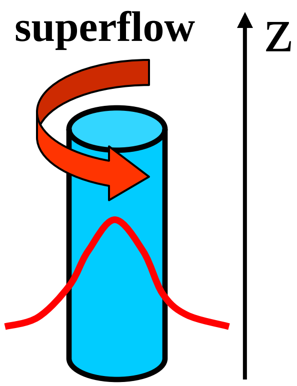
- **Vortex in a mesoscopic cylinder. Spectrum oscillations. Density of states.**

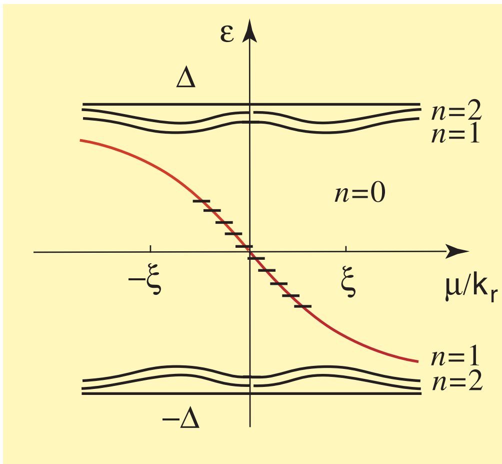
Andreev bound states.



Quasiparticle bound states: examples.

Vortex line



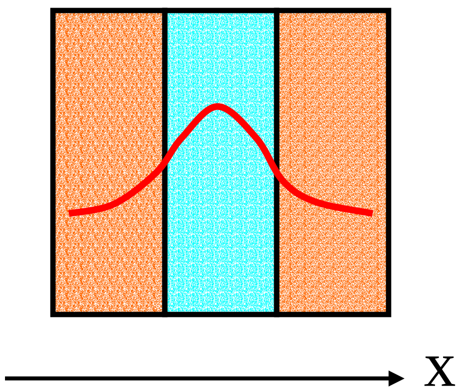


C.Caroli, P.G.de Gennes, J.Matricon (1964)

$$\varepsilon_\mu(k_r) \approx \frac{\mu\Delta}{k_r\xi}$$

$$k_r = \sqrt{k_F^2 - k_z^2}$$

S N S

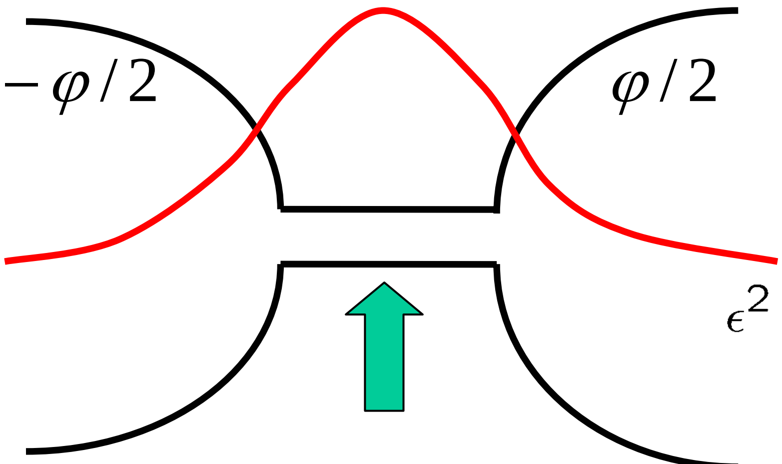


A.F.Andreev (1965)

$$d >> \xi$$

$$\varepsilon_n(k_r) \approx \frac{(n+1/2)\hbar V_F}{k_F d} k_x$$

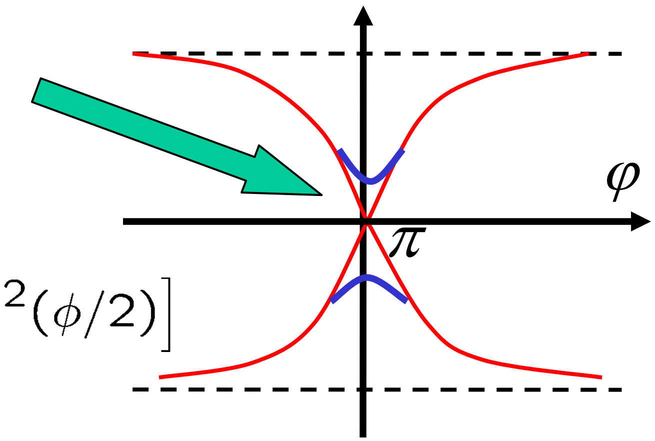
Subgap spectrum in a superconducting constriction



Localized states

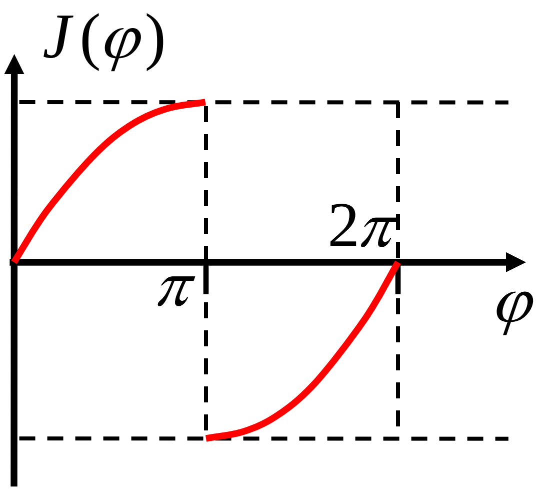
$$E/\Delta = \pm\cos\varphi$$

Minigap induced by normal scattering



$$\epsilon^2 = |\Delta|^2\left[1 - \mathcal{T}\sin^2(\phi/2)\right]$$

Josephson current in ballistic contacts



Quantization of critical current

$$J_c = \frac{e\Delta}{\hbar} N$$

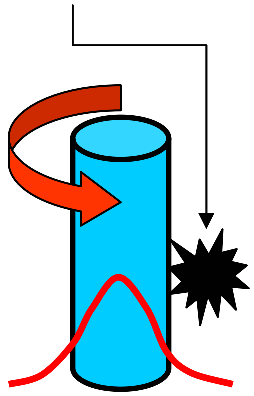
N – number of channels

Contacts with and without barriers:

Kulik-Omel'yanchuk 1977
Habercorn et al 1978
Zaitsev 1984
Beenakker 1991
Bagwell 1992
Beenakker-Houten 1991
Furusaki-Takayanagi-Tsukada 1991

Geometrical and Andreev quantization: examples

Impurity in the vortex core



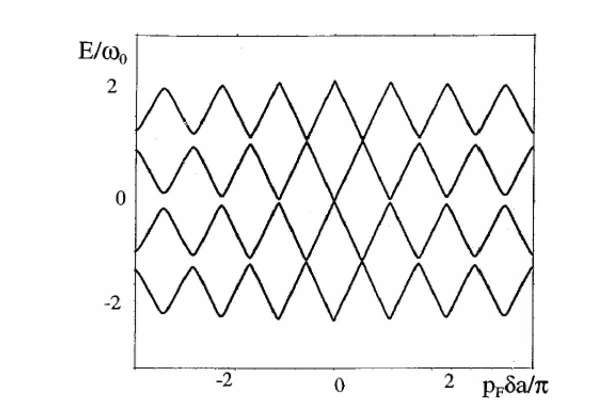


FIG. 1. The excitation spectrum as a function of the impurity distance from the vortex center. The parameter $(\pi^2/$

Larkin, Ovchinnikov,1998

Tomasch oscillations

(i) S slab, DOS oscillations, $\varepsilon > \Delta$

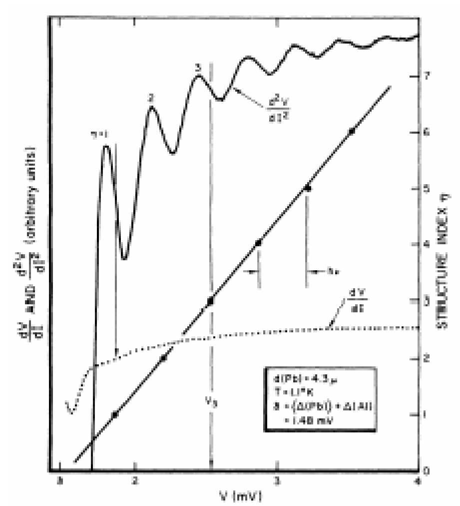


FIG. 1. Voltage dependence of dV/dI and d^2V/dI^2 for an Al-AlO_x-Pb tunnel diode. Modulation levels were 10 μV (rms) and 50 μV (rms), respectively. Integers η index peaks in d^2V/dI^2 with $\eta = 1$ corresponding to the first peak of the series. Voltages V_η denote points of maximum negative slope in d^2V/dI^2 and correspond to local maxima in dI/dV. These are relatively weak, as can be seen from the plot of dV/dI.

Tomasch, 1965

(ii) Vortex in S disc: DOS oscillations above Δ

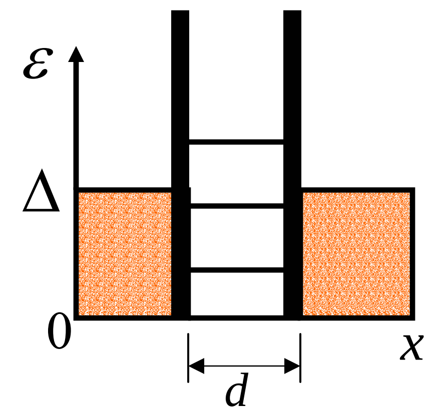
Tanaka, Robel, Janko, 2002

Resonance effects in SINIS junctions

Transmission $T = \dfrac{1}{1+A}$

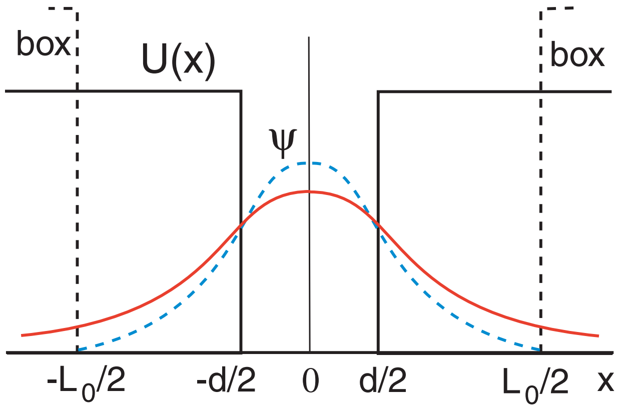
$$A = 4Z^2(1+Z^2)\sin^2(k_F d + \phi)$$

Z is the barrier strength

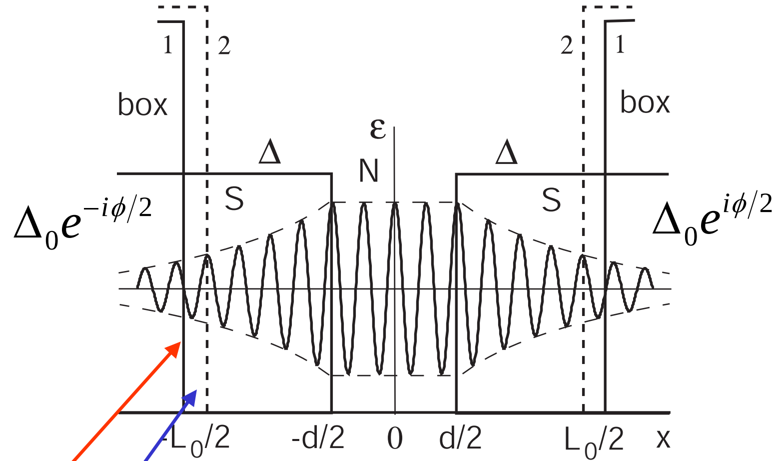


Review: Golubov et al.,RMP 2004

Confinement of Andreev states: role of interference



Usual potential well: Small change in energy



SNS in a box: Huge change in the state

Resonance, $k_F L_0 = \pi n$:
Levels do not feel the box boundaries

Off-resonance, $\sin(k_F L_0) \sim 1$
Levels do feel the box boundaries

Energy change

(a) $d > \xi_0$

$$k = k_F \pm \varepsilon/\hbar v_F \qquad \delta k \propto \varepsilon/\hbar v_F \sim 1/d$$

(b) $d < \xi_0$

$$\delta[\arccos(\varepsilon/\Delta)] \sim 1$$

Amplitude of oscillations

$$\delta\varepsilon \propto \Delta e^{-L_0/\xi}$$

Quantization rules on closed quasiclassical trajectories

Quasiclassical theory of superconductors → Quantum mechanics along beams ∇S ← Normal-state eikonal

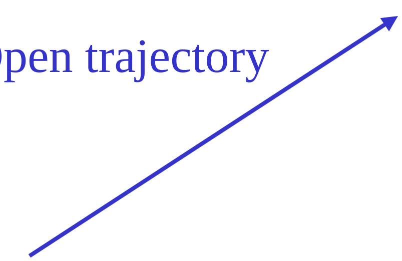
$$|\nabla S| = k_F$$

Quasiclassical wave function:

$$\hat{\Psi} = (u, v) = \hat{\psi} e^{iS}$$

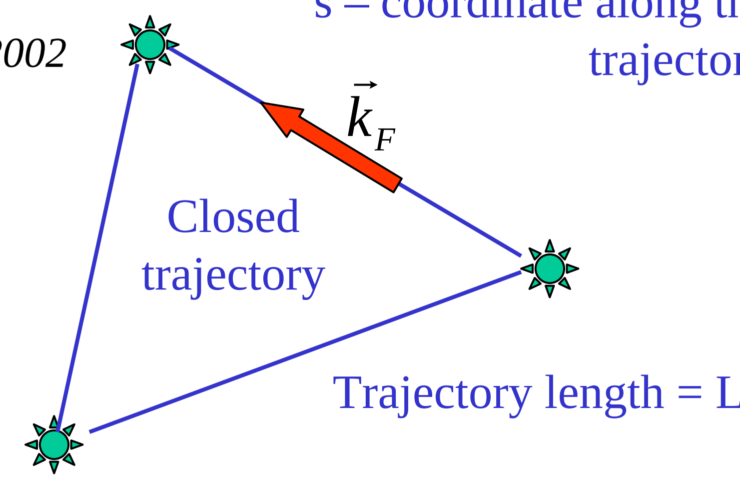
electron (u), hole (v)

Open trajectory



Ozana & Shelankov,2002

s – coordinate along the trajectory



Condition for a general closed trajectory:

Single-valued wave function

$$\hat{\psi}(s) = \hat{\psi}(s + L) e^{iS(L)}$$

Equivalent 1D problem in a periodic gap potential

$$\Delta(s+L) = \Delta(s)$$

Bloch theorem:

$$\hat{\psi}_q(s)e^{iqL} = \hat{\psi}_q(s+L)$$

$$\varepsilon(q) = \varepsilon(q + 2\pi/L)$$

$$q = \frac{2\pi M - S(L)}{L} \in \text{ 1st Brillouin zone}$$

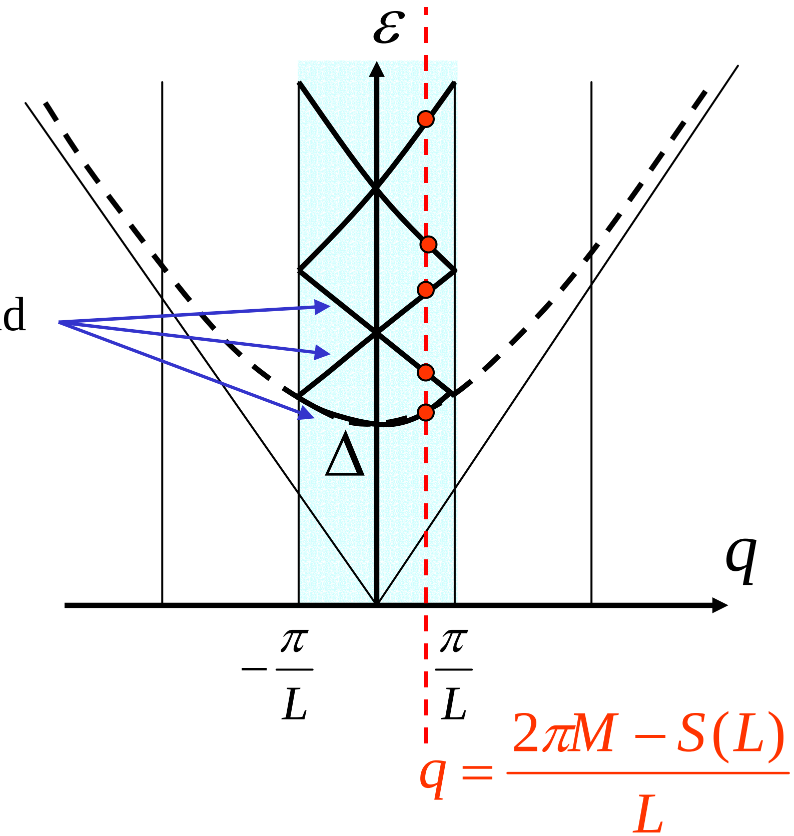
Spectrum for a closed trajectory

States above the gap: Tomash oscillations

$$\varepsilon > \Delta = const$$

$$\varepsilon^2(q) = \Delta^2 + \hbar^2 v_F^2 \left(q - \frac{2\pi N}{L} \right)^2$$

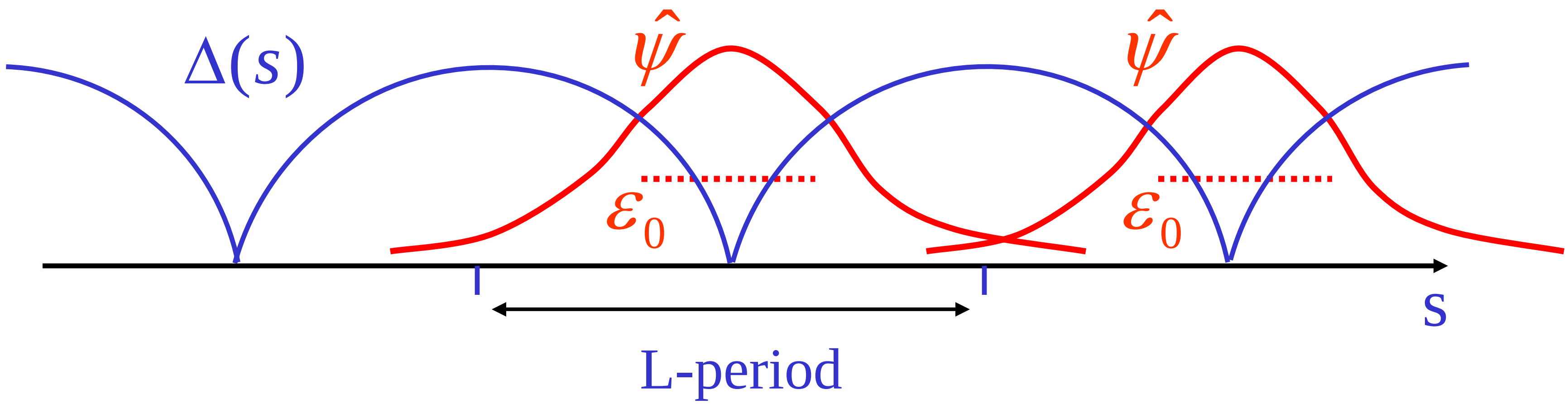
N is the number of the energy band



$$\varepsilon^2(q) = \Delta^2 + \hbar^2 v_F^2 \left(\frac{2\pi M'}{L} - k_F \right)^2$$

$$M' = M - N$$

Sub-gap states: tight-binding approximation

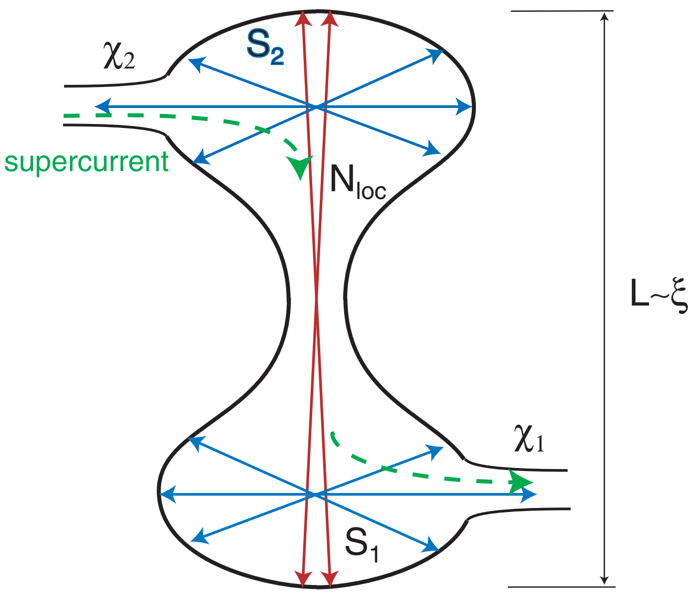


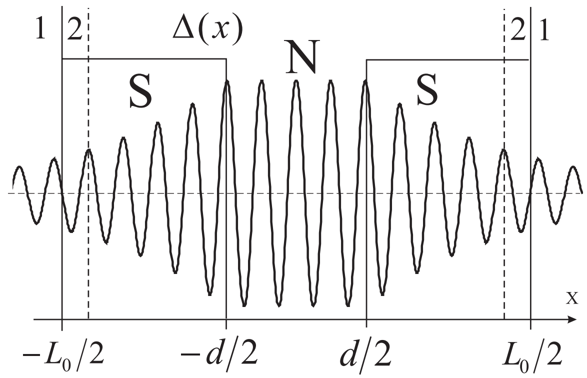
$$\varepsilon = \varepsilon_0 + \delta[\cos(k_F L + \beta) + C]$$

Band width: $\delta \propto \Delta e^{-L/\xi}$

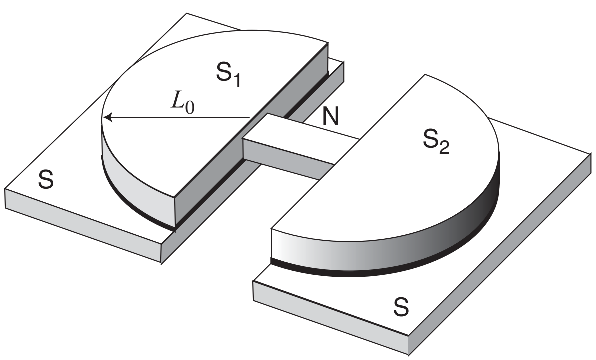
1D Andreev state for SNS in a quantum box

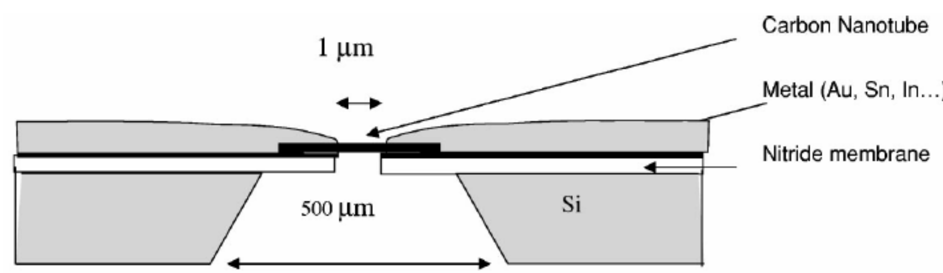
Model device





Possible experimental realizations





A. KASUMOV *et al.* PHYSICAL REVIEW B **68**, 214521 (2003)

Mesoscopic oscillations of spectrum and Josephson current.

Spectrum of bound states for *d*<<ξ: $$\varepsilon^2 = \Delta_0^2\left[1 - T\sin^2(\phi/2)\right]$$

Effective transmission coefficient:

$$T = \frac{1}{1 + \dfrac{\sin^2(k_x L_0)}{\sinh^2\left(\sqrt{\Delta_0^2 - \varepsilon^2}\cdot L_0 / \hbar v_x\right)}}$$

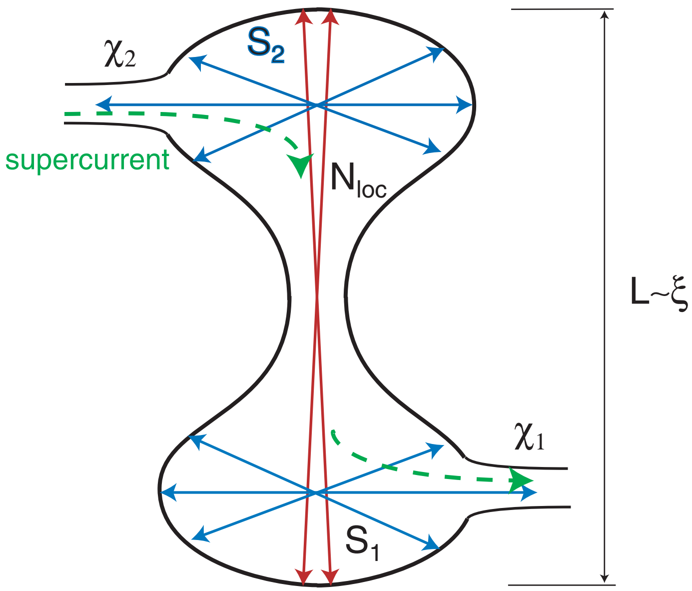
Spectrum for $|\varepsilon| >> |\Delta_0|$ $\Longrightarrow$ $$(\hbar k_x \pm \varepsilon/v_x)L_0 = \pi n\hbar$$

Contribution to supercurrent:

$$I_n(\phi) = -\frac{2e}{\hbar}\frac{d\varepsilon_n(\phi)}{d\phi}\tanh\frac{\varepsilon_n(\phi)}{2T}$$

Main features

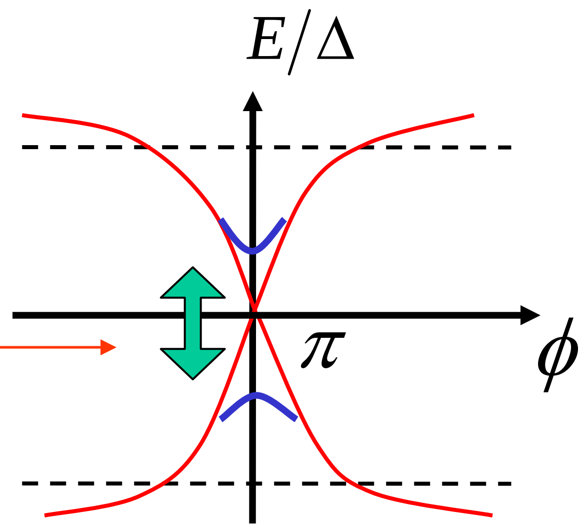
- **Extra conducting modes due to normal-supercurrent conversion**



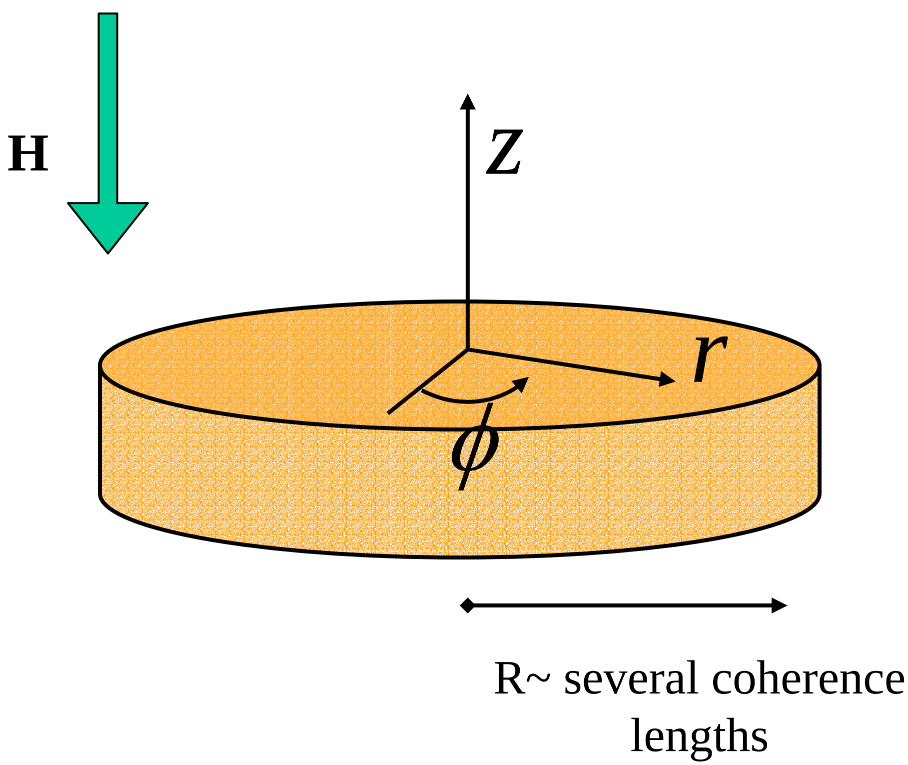
- **Oscillating minigap:**

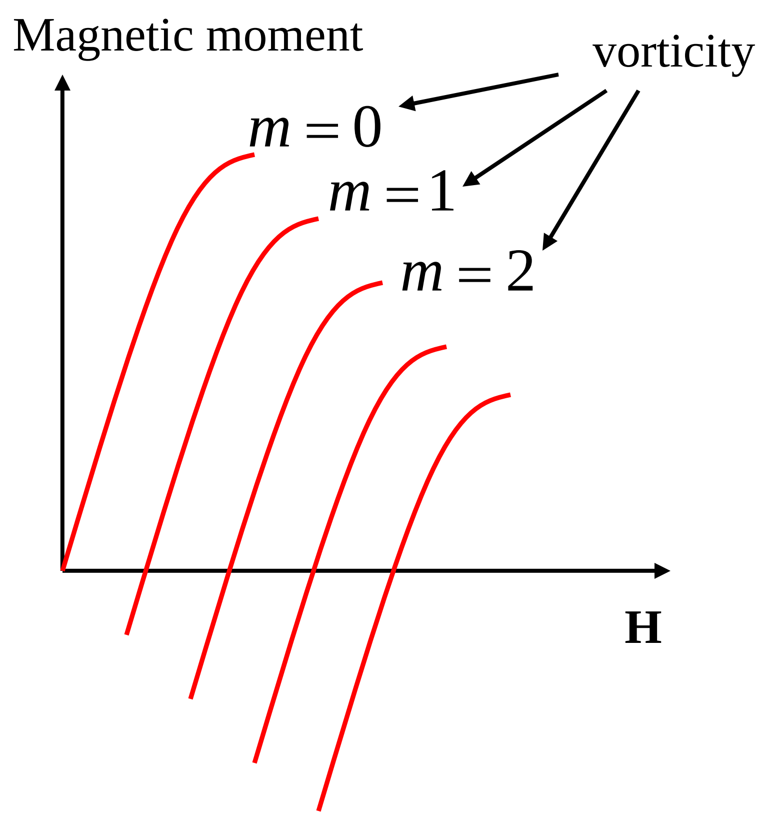
Lowest level for $L_0 \le \xi$

$$\varepsilon_0^2 = \Delta_0^2\left[\cos^2(\phi/2) - \left(\frac{\hbar v_x}{\Delta_0 L_0}\right)^2 \sin^2(k_x L_0)\right]$$



Vortex in a mesoscopic cylinder.





Model: Bogolubov – de Gennes equations

$$-\frac{\hbar^2}{2m}\left(\nabla - \frac{ie}{\hbar c}\mathbf{A}\right)^2 u - \frac{\hbar^2 k_F^2}{2m}u + \Delta v = \epsilon u$$

$$\frac{\hbar^2}{2m}\left(\nabla + \frac{ie}{\hbar c}\mathbf{A}\right)^2 v + \frac{\hbar^2 k_F^2}{2m}v + \Delta^* u = \epsilon v$$

$$\Delta = |\Delta(r)|\, e^{i\phi} \qquad \hat{\Psi} = e^{i(\hat{\sigma}_z/2+\mu)\phi}\hat{U} \qquad \hat{U} = \begin{pmatrix} u \\ v \end{pmatrix}$$

$$\hat{U} = e^{ik_z z} H^{(1)}_{\mu+\hat{\sigma}/2}(k_r r)\hat{w}^{(+)} + e^{ik_z z} H^{(2)}_{\mu+\hat{\sigma}/2}(k_r r)\hat{w}^{(-)}$$

$$k_r^2 + k_z^2 = k_F^2$$

Boundary conditions at the cylinder surface

$$\hat{U}(R,z) = 0$$

Spectrum oscillations

$$\varepsilon = -\omega(k_r)\mu + \frac{\Delta_0 \cos(2k_r R - \pi\mu + \pi/2)}{\Lambda \cosh\left(2K(R)\right)}$$

CdGM spectrum

$$\omega(k_r) = \frac{2m\Delta_0}{\hbar^2 k_r^2 \Lambda} \int_0^\infty \left(|\Delta(r)|/r\right) e^{-2K(r)}\, dr \ ,$$

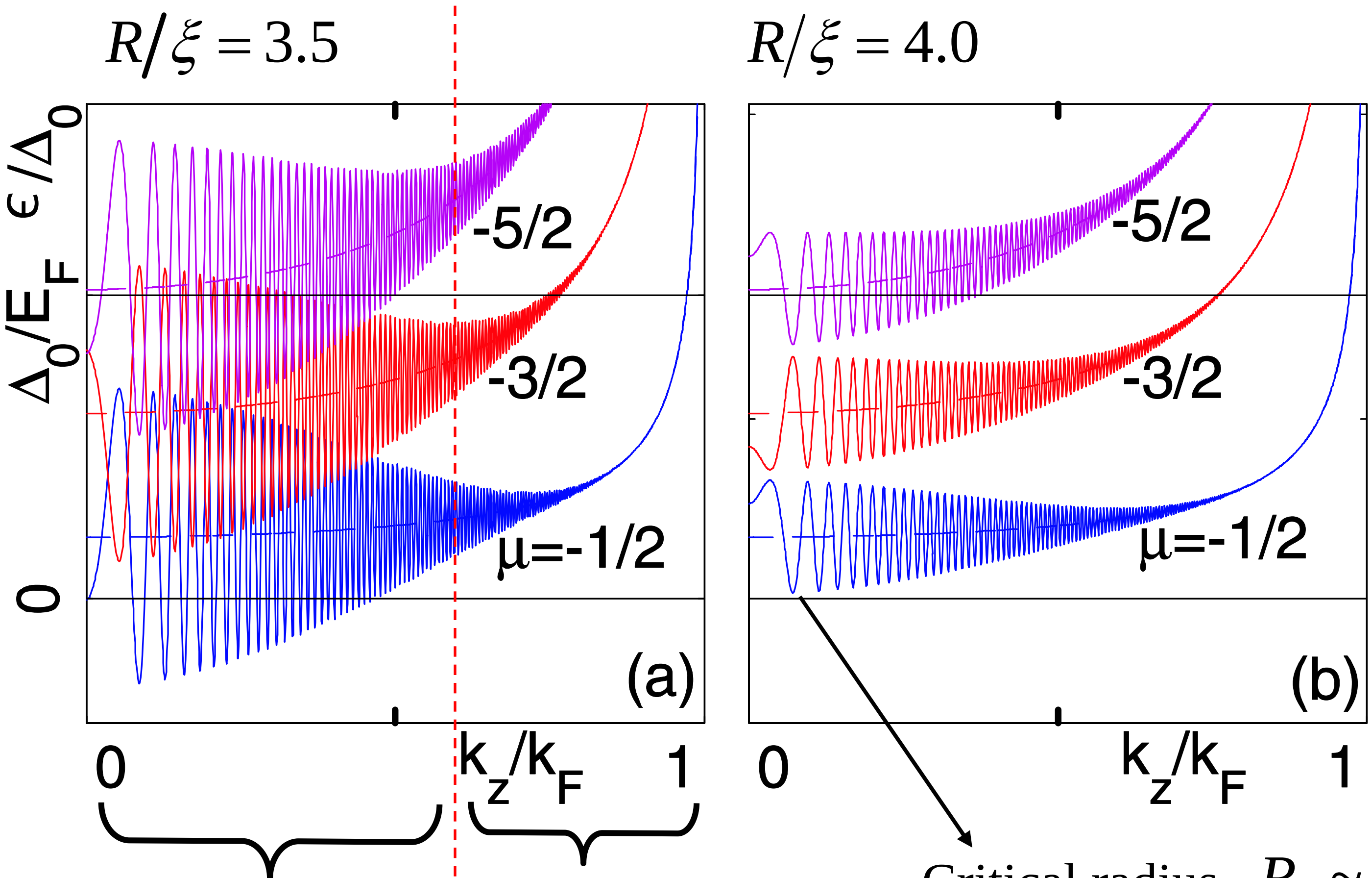
$$K(r) = \frac{m}{\hbar^2 k_r} \int_0^r |\Delta(r')|\, dr', \ \Lambda = \frac{2m\Delta_0}{\hbar^2 k_r} \int_0^\infty e^{-2K(r)}\, dr$$

Simple approximation:

$$\omega(k_r) \approx \frac{k_F}{k_r}\omega_0 \qquad \omega_0 \sim \frac{\Delta_0^2}{E_F}$$

$$K(R) \approx \frac{Rk_F}{\xi_0 k_r} \qquad \Lambda \sim 1$$

Example $|\Delta(r)| = \dfrac{\Delta_0 r}{\sqrt{r^2+\xi_v^2}}$ $\Delta_0/E_F = 0.01 \quad \xi_v = \xi_0$



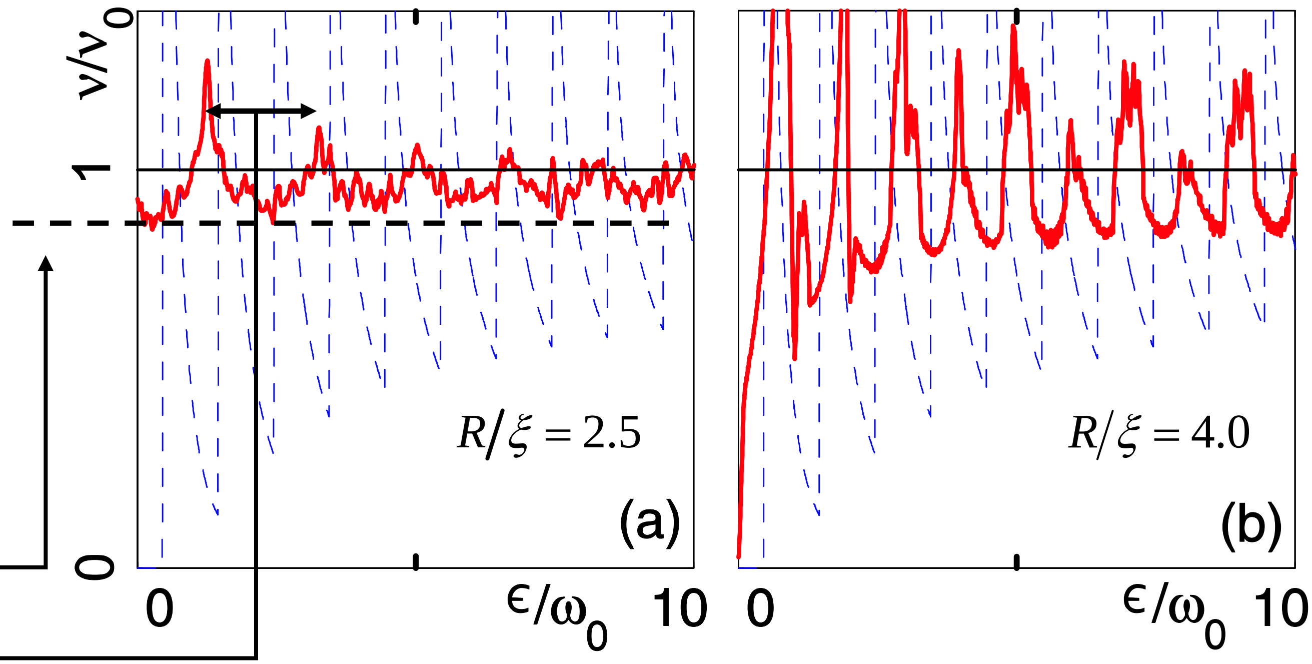
Critical radius $R_c \sim \xi_0 \ln \dfrac{\Delta_0}{\omega_0}$

DOS contribution ν_1

$$k_r^* \approx \frac{Rk_F}{R_c}$$

DOS contribution ν_2

Density of states.



Period of oscillations = $\omega\left(k_r^*\right) \approx \omega_0 / \rho > \omega_0$, where $\rho = R / R_c$

Background (zero energy) DOS

$$\nu_1(k_r^*) = \frac{1}{\pi}\int_{k_r^*}^{k_F} \frac{k_r\, dk_r}{\omega(k_r)\sqrt{k_F^2 - k_r^2}}$$

$$= \nu_0 \left[1 - (2/\pi)\left(\arcsin\rho - \rho\sqrt{1-\rho^2}\right)\right]$$

CdGM DOS $\nu_0 = \nu_1(0)$

CONCLUSIONS

- **Giant mesoscopic oscillations of Andreev levels**
- **Vortices:**
  - **Modified DOS**
  - **Zero modes**
- **Supercurrent:**
  - **Mesoscopic oscillations**
  - **Increased number of conducting modes**