Two-level fluctuators in Josephson Junctions - mechanisms and diagnostics

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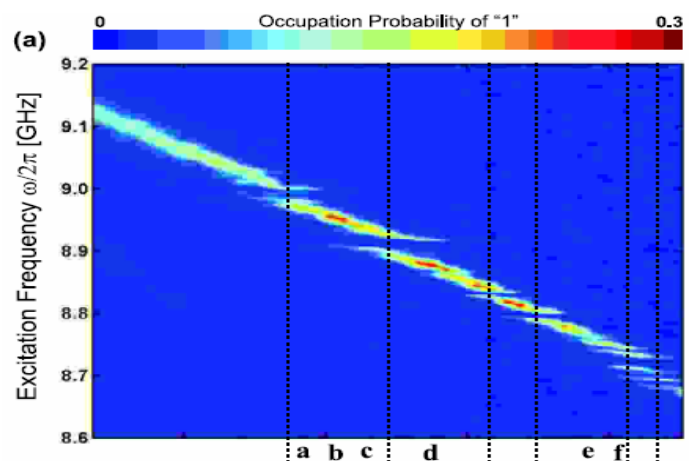
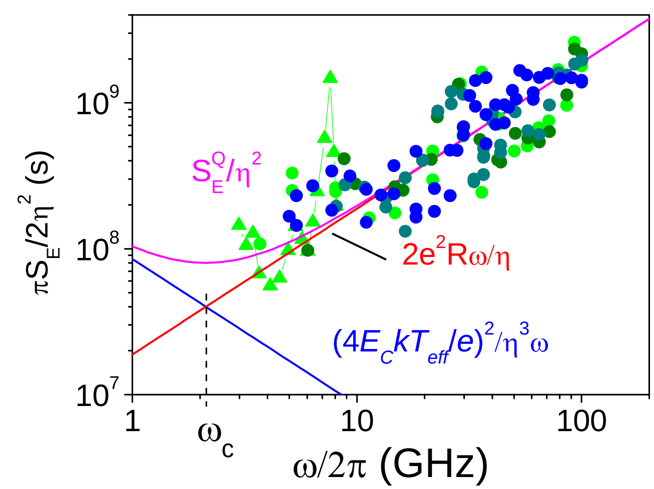


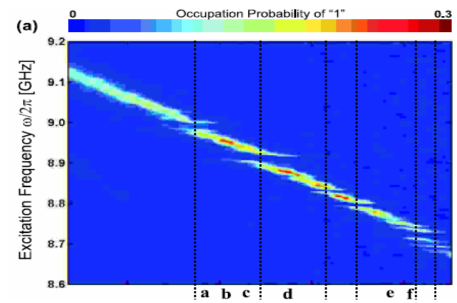


Argonne, Nov. 17, 2005

Outline

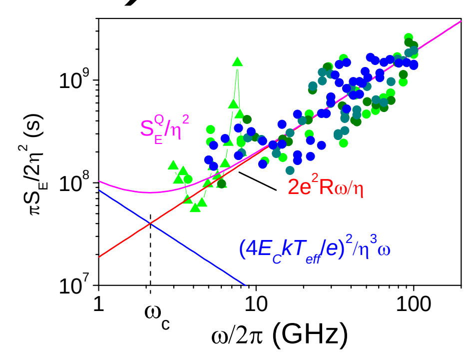
- *Part I:*

  Microscopic models for individual 2-level fluctuators ***inside*** *Josephson junctions (flux qubits ala Martinis):*
  - *Josephson-type (couples to* $\cos\phi$*)*
  - *Dipolar (couples to electric field,* $d\phi/dt$*)*

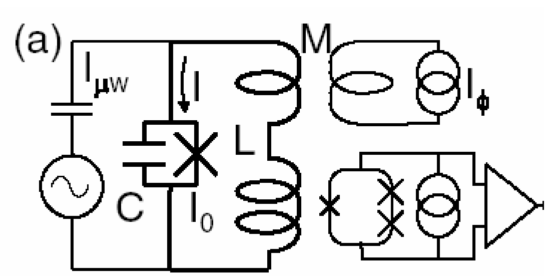


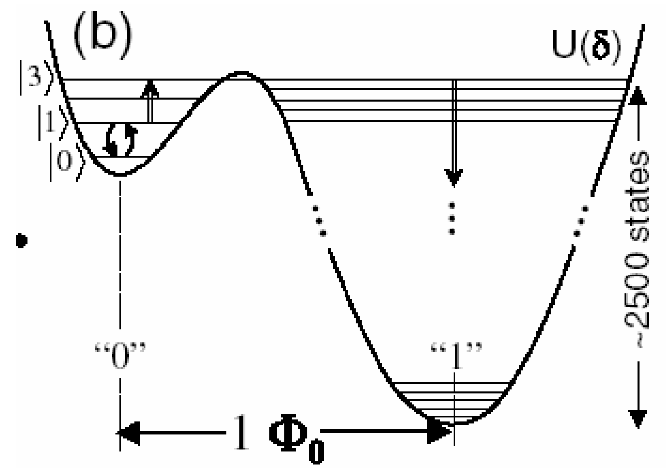
- *Part II:*

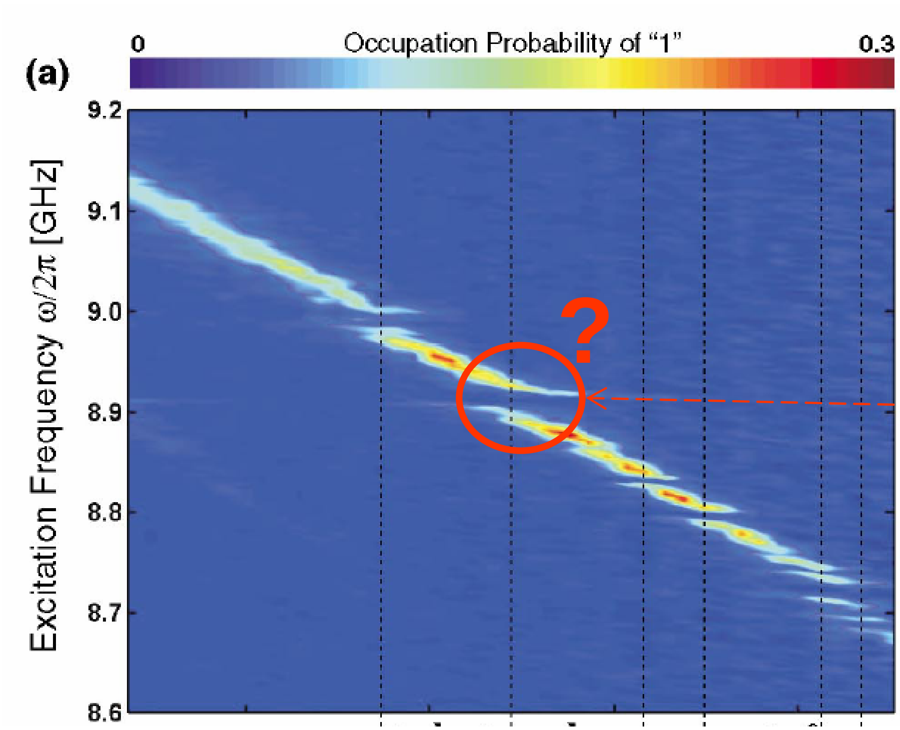
  Statistical influence of many ***weak*** *charge fluctuators on Cooper pair boxes (charge qubits ala Nakamura):*
  - *Nearly coherent 2-level fluctuators, examples*
  - *Connection between low and high frequency noises*



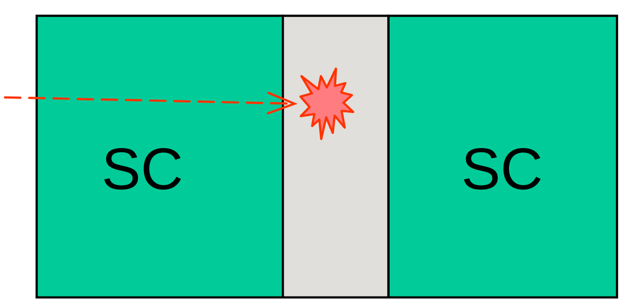
TLS Spectroscopy
Simmonds et al, PRL 2004





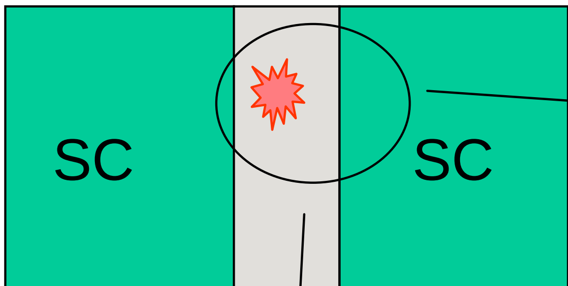


JJ

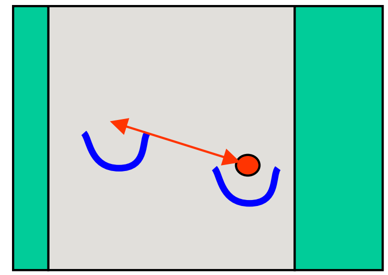


Models- coupling to pseudospin S

JJ

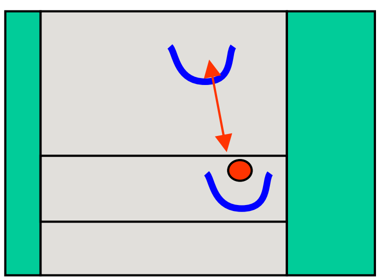


Electric Dipolar



$$\delta H = -\frac{Q_{\mathrm{TL}}\, q}{C} S_x$$

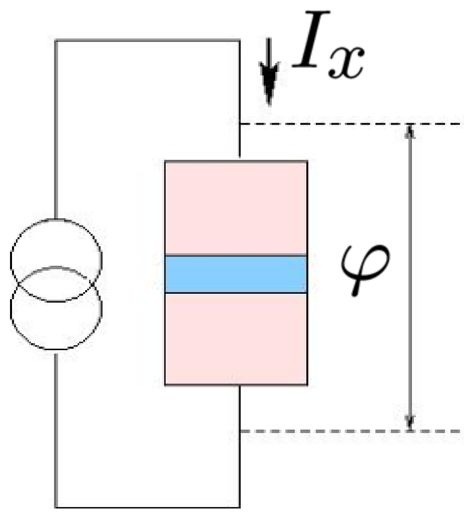
Channel blocking

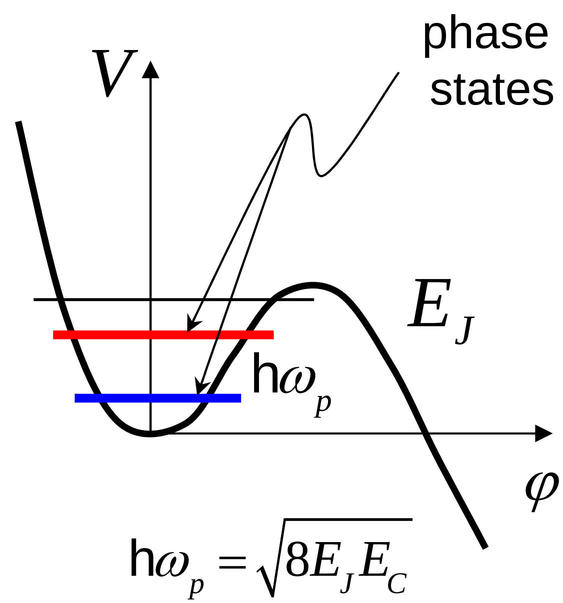


$$H_J = -E_J(1 + \mathbf{j}\cdot\mathbf{S})\cos 2\pi\phi/\Phi_0$$

Both could explain expt.
How to distinguish?

Why hard to distinguish?



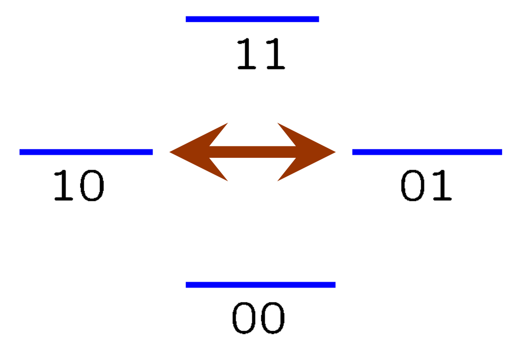


$$H = \hbar\omega_p a^\dagger a$$

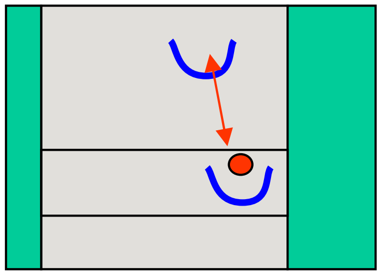
Few quantum levels: $I \approx I_c \rightarrow 2\pi\phi/\Phi_0 \approx \pi/2$

$$S_x \cos 2\pi\phi/\Phi_0 \sim S_x\delta\phi \sim (S_+ + S_-)(a + a^\dagger)$$

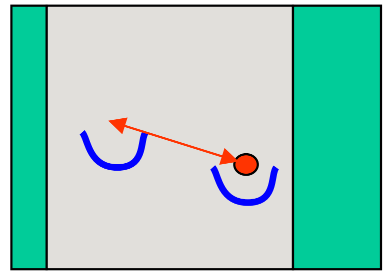
$$S_x\dot{\phi} \sim S_x q \sim (S_+ + S_-)(a - a^\dagger)$$



Testing the mechanism (running phase regime)



or



$$H_S = -\Omega_0 S_z - (\alpha_x S_x + \alpha_z S_z) \times \begin{cases} \cos\phi & \text{Josephson} \\ q & \text{dipolar (same as } \phi\text{)} \end{cases}$$

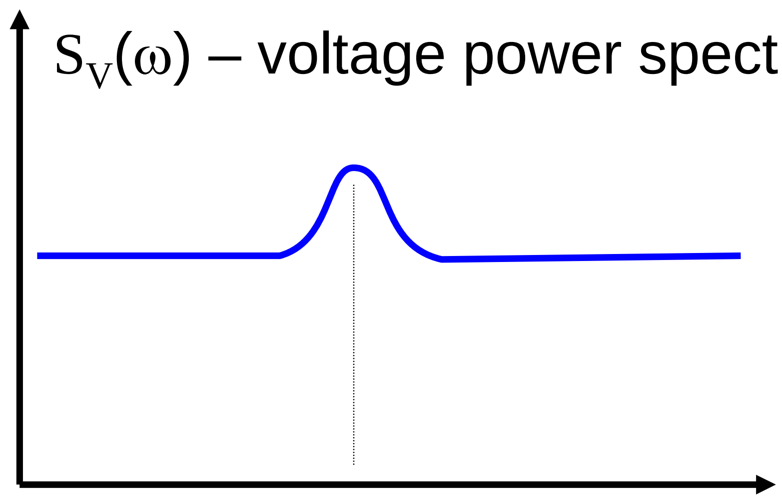
If $V/R \gg I_c$ and $\omega_J \equiv 2eV/\hbar = \Omega_0$

$\cos\phi \to \cos\omega_J t$

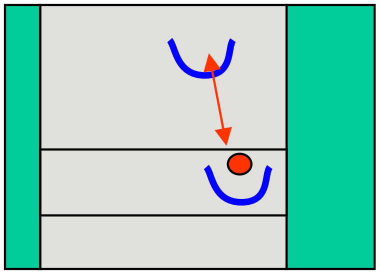
$q \to \cos\omega_J t$

$\Rightarrow$Rabi oscillations

(see also V. Kozub JETP 84)



First mechanism: the Hamiltonian



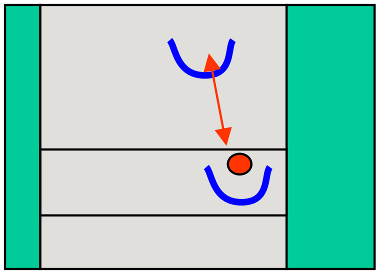
$$H = \frac{q^2}{2C} + H_{\mathrm{R}}(-\phi) + H_J(\phi + Vt) - \Omega_0 S_z$$

„Hamiltonian“ of resistor $H_{\mathrm{R}}(\phi) = \sum_j \left(\frac{q_j^2}{2C_j} + \frac{(\phi_j - \phi)^2}{2L_j} \right)$

$$H_J(\phi) = -E_J(1 + \mathbf{j} \cdot \mathbf{S}) \cos\left(\frac{2\pi\phi}{\Phi_0} \right)$$ Modified Josephson term

For simplicity $\mathbf{j} = (j_x, j_y, j_z) = (j, 0, 0)$

Rotating frame



$$U = \exp\left[2\pi i\,(\phi + Vt)\,S_z\,/\Phi_0\right]$$

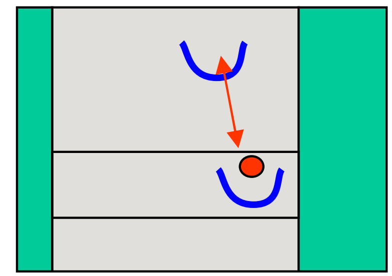
$$H' = UHU^{-1} + i\dot{U}U^{-1}$$

$$H' = \frac{q^2}{2C} + H_R(-\phi) - E_J \cos\left(\omega_J t + 2\pi\phi/\Phi_0\right)$$

$$+(\Omega_0 - \omega_J)S_z - \Omega_R S_x - \frac{2eqS_z}{C}$$

$$\Omega_R \equiv j_x I_c/(4e)$$

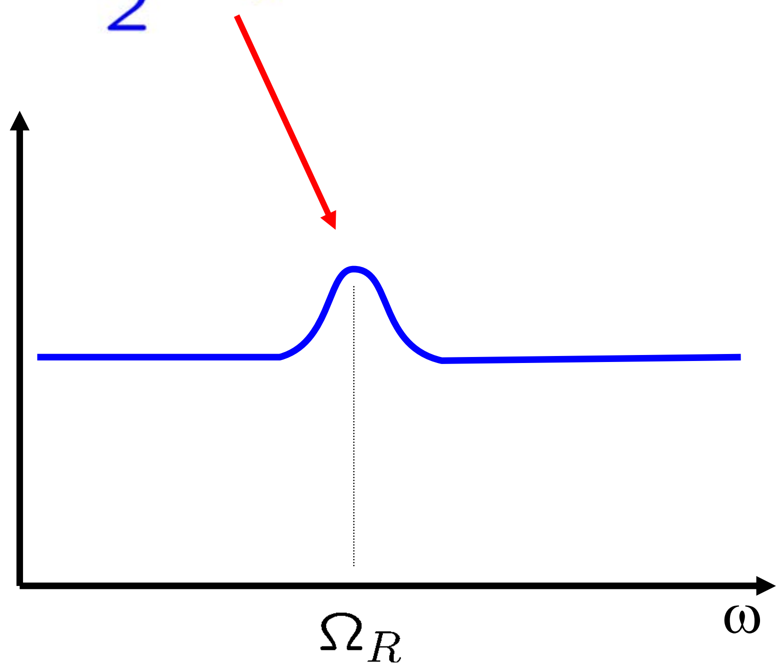
Rotating frame



$$q' = UqU^{-1} = q - 2eS_z$$

$$I'_{\mathsf{J}} = I_c \sin\left(\omega_{\mathsf{J}} t + 2\pi\phi/\Phi_0\right) - \frac{jI_c}{2} S_y$$

$S_V(\omega)$:
voltage power spectr.

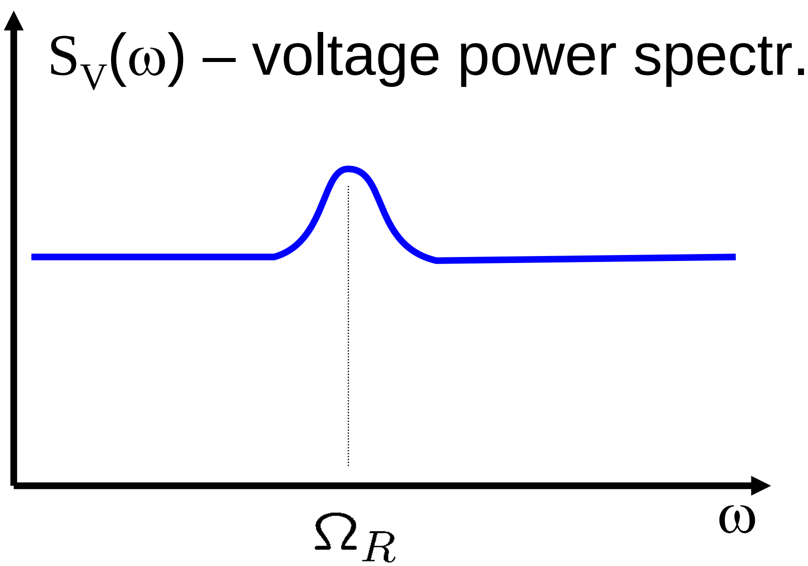


Results

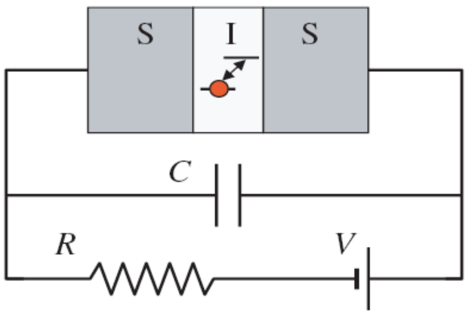
$$\Omega_{\mathrm{R}} \equiv j_x I_c/(4e)$$

$$S/N = \left[\coth \frac{\Omega_{\mathrm{R}}}{2k_{\mathrm{B}}T} + \frac{eR^2 I_c^2}{\Omega_{\mathrm{R}} V}\right]^{-2}$$

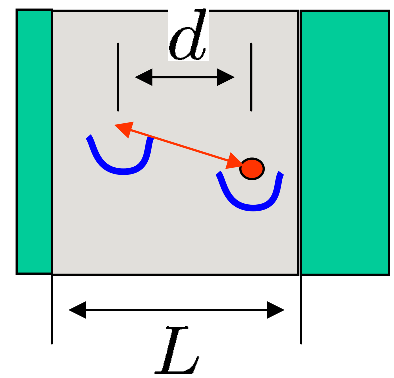
$$\left[\int_{\Omega_{\mathrm{R}}} \frac{d\omega}{2\pi} S_V^{\mathrm{peak}}(\omega)\right]^{1/2} \approx \frac{jRI_c}{4\sqrt{2}}$$



Second (dipolar) mechanism



$$Q_{\mathsf{TL}} = ed/L$$



$$H = \frac{q^2}{2C} + H_{\mathsf{R}}(-\phi) - E_{\mathsf{J}} \cos \frac{2\pi(\phi + Vt)}{\Phi_0} - \Omega_0 S_z - \frac{Q_{\mathsf{TL}}\, q}{C} S_x$$

$$\Omega_{\mathsf{R}} = R I_c Q_{\mathsf{TL}}/(2\hbar)$$

$$\left[\int_{\Omega_{\mathsf{R}}} \frac{d\omega}{2\pi} S_V^{\mathsf{peak}}(\omega) \right]^{1/2} \sim \frac{Q_{\mathsf{TL}} R^2 I_c}{e R_Q}$$

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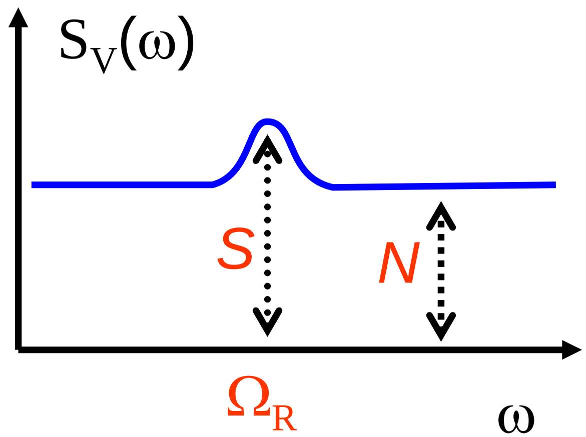
Predictions

	Josephson $E_J(\mathbf{j}\cdot\mathbf{S})\cos 2\pi\phi$	*Dipolar* $\frac{Q_{\rm TL}\, q}{C} S_x$
Rabi Frequency Ω_R	$j_\perp I_c/(4e)$	$R I_c Q_{\rm TL}/\ 2\hbar$
Signal/Noise *S/N*	$\left(\frac{\eta\Omega_R}{2k_B T}\right)^2$	$\left(\frac{\eta\Omega_R}{2k_B T}\right)^2$



$T \approx 10$ mK, or $k_B T/\hbar = 2\pi \times 200$ MHz

$I_c \approx 10\ \mu$A

$C \sim 1$ pF.

$R \sim 0.1\Omega \ll R_N$, $R_N \sim 30\ \Omega$.

$I_c R_N(0.5\text{GHz}) < \omega_J < I_c R_N(150\text{ GHz})$

$j_\perp \approx 6.5 \cdot 10^{-5}$, $j_\parallel < 10^{-3}$

$\Omega_R \approx 2\pi \times 200$ MHz

Peak width 5 kHz (intrinsic)

$\left[\int_{\Omega_R} \frac{d\omega}{2\pi} S_V^{\rm peak}\right]^{1/2} \approx \frac{j_{\rm eff} R I_c}{4\sqrt{2Y(\Omega_R)}} \approx 10^{-2}$ nV.

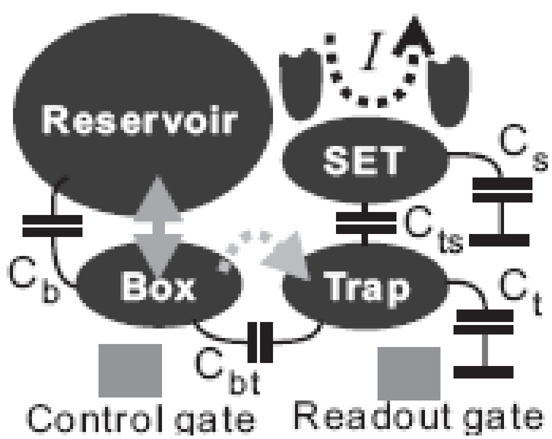
Proposal for TLS identification

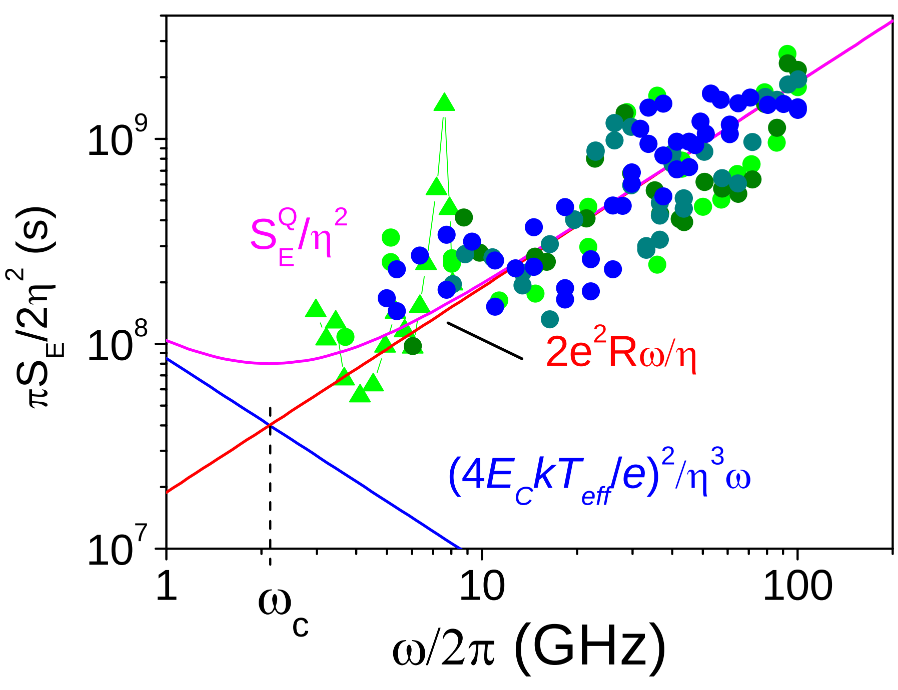
- *Two possible mechanisms: Josephson and dipolar ($\dot{\phi}$)*
- *Measure JJ in the running phase regime at voltage corresponding to TLS splitting:*
  - *Find peak in voltage power spectrum at the Rabi frequency (corresponding to the qubit-TLS coupling)*
  - *Measure the signal to noise at Rabi frequency as a function of parameters (e.g. R, C).*
  - *Two mechanisms, while indistinguishable in the quantum (qubit) regime, in the running phase have different SNR dependence on parameters (see Martin, Bulaevskii, Shnirman, Phys. Rev. Lett. **95**, 127002 (2005))*

Part II

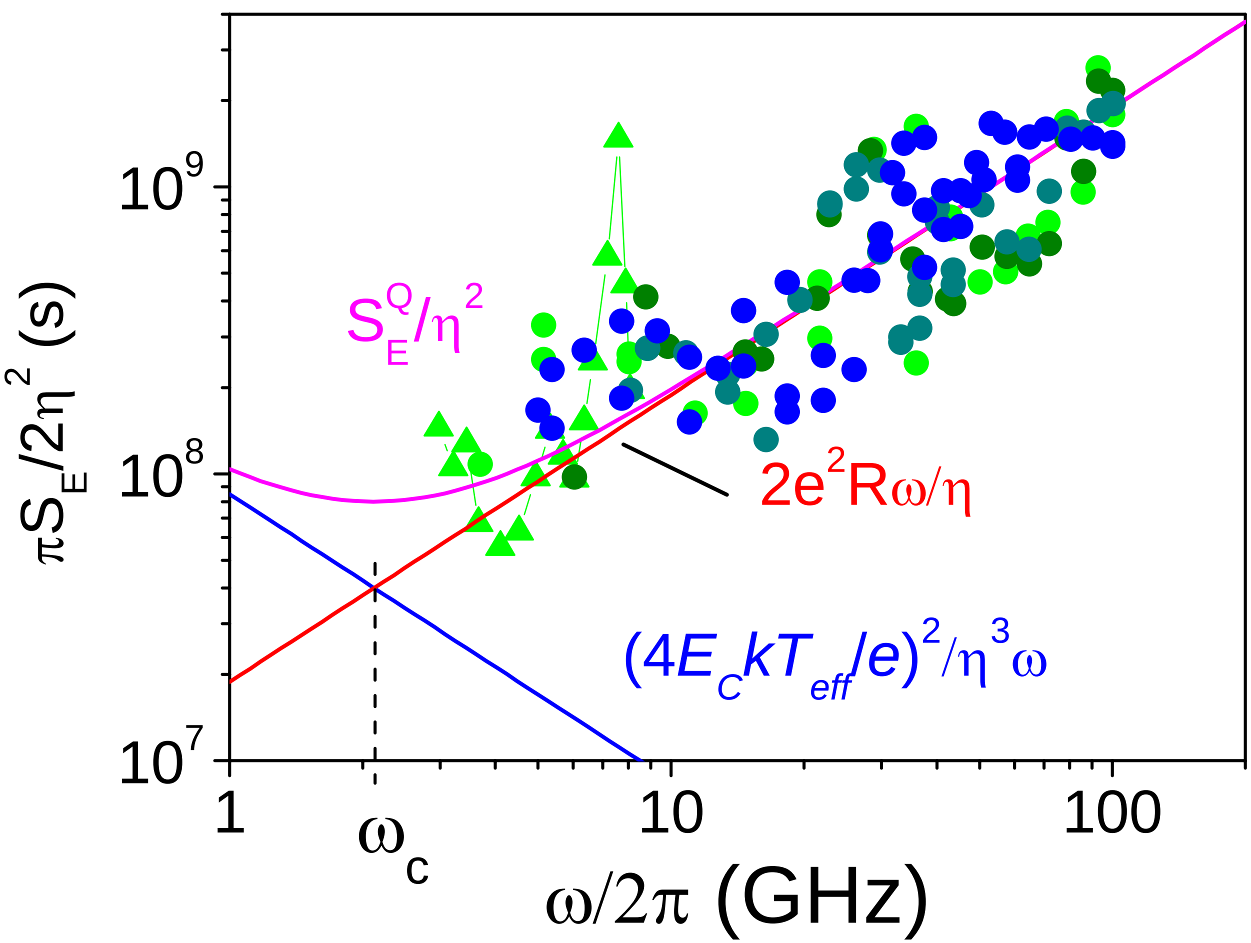
*Statistical influence of many **weak** charge fluctuators on Cooper pair boxes (charge qubits ala Nakamura)*

- Nearly coherent 2-level fluctuators, examples
- *Connection between low and high frequency charge noises*





Experimental data of Astafiev et al. (NEC)



Source of noise: Charge fluctuations

Astafiev et al. (PRL 04)

Low frequency 1/*f* noise crosses *f* quantum noise at $\eta\omega_c \approx k_B T$

$$S(\omega) \approx \frac{a\,(k_B T)^2}{\hbar\omega} + a\,\hbar\omega$$

same strength a for low- and high-frequency noise

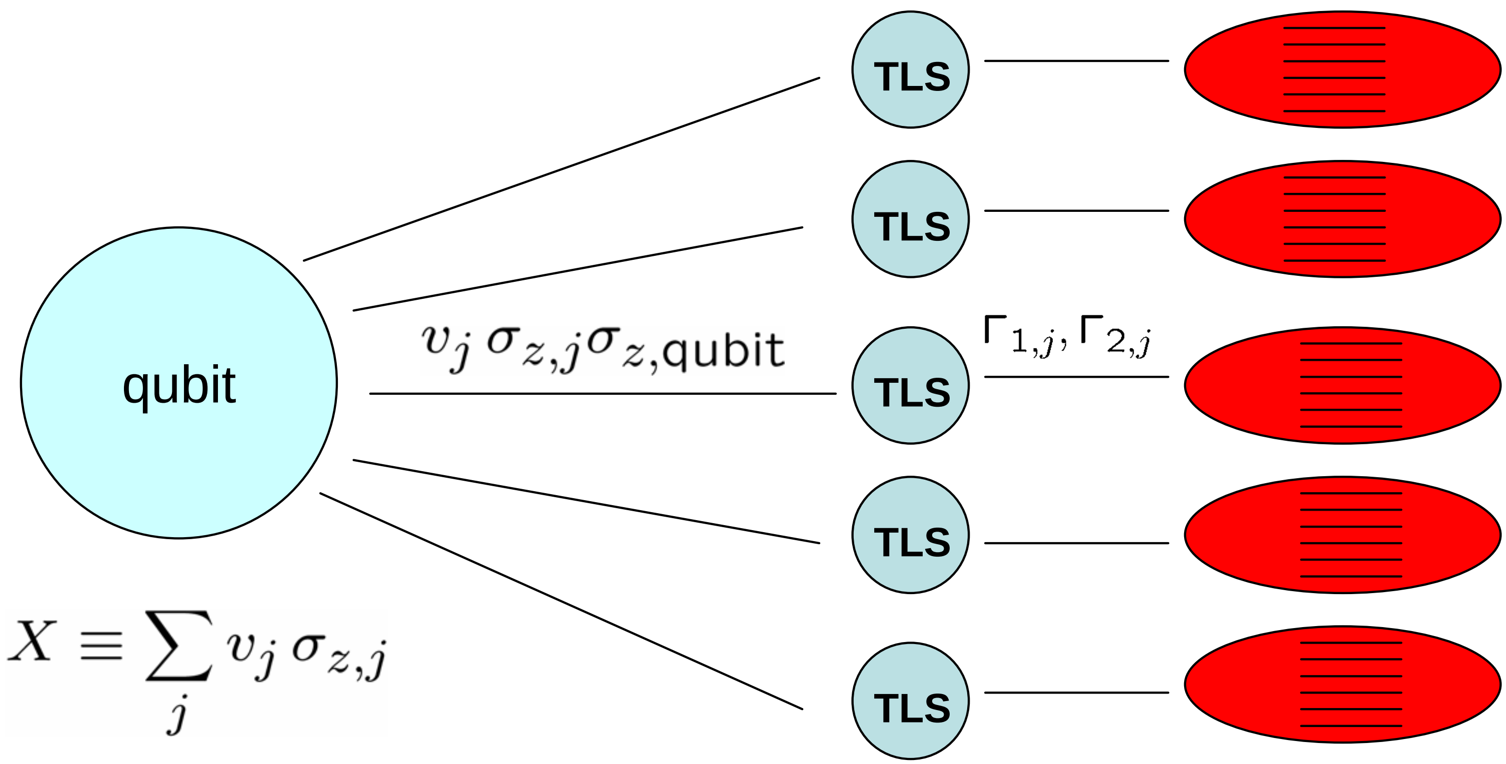
Our Model

- Fluctuations X(t) probed by qubit $H = -\frac{1}{2}\Delta E_{ch}\,\sigma_z \; -\frac{1}{2}\Delta E_J\,\sigma_x - \frac{1}{2} X\,\sigma_z$
- Source of X(t) is an ensemble of two-level systems (TLS)

$$H_{\mathsf{TLS}} = \sum_j \left[-\frac{1}{2}\left(\varepsilon_j \sigma_{z,j} + \Delta_j \sigma_{x,j}\right) + H_{\mathsf{diss},j}\right]$$

- each TLS is coupled (weakly) to dissipative environment $\boldsymbol{H}_{\mathbf{diss.j}}$

$\Rightarrow$ weak relaxation and decoherence $H_{\text{diss},\,j} \rightarrow \Gamma_{1,\,j}\,,\Gamma_{2,\,j} = E_j$ $\quad E_j \equiv [\epsilon_j^2 + \Delta_j^2]^{1/2}$



Noise of a single TLS

In eigenbasis

$$H_{\mathsf{TLS}} = \sum_j \left[-\frac{1}{2} \left(\varepsilon_j \sigma_{z,j} + \Delta_j \sigma_{x,j} \right) + H_{\mathsf{diss},j} \right] = \sum_j \left[-\frac{1}{2} E_j \rho_{z,j} + H_{\mathsf{diss},j} \right]$$

$$\sigma_{z,j} = \cos\theta_j \, \rho_{z,j} - \sin\theta_j \, \rho_{x,j} \qquad E_j = [\epsilon_j^2 + \Delta_j^2]^{1/2} \; , \; \tan\theta_j = \Delta_j / \epsilon_j$$

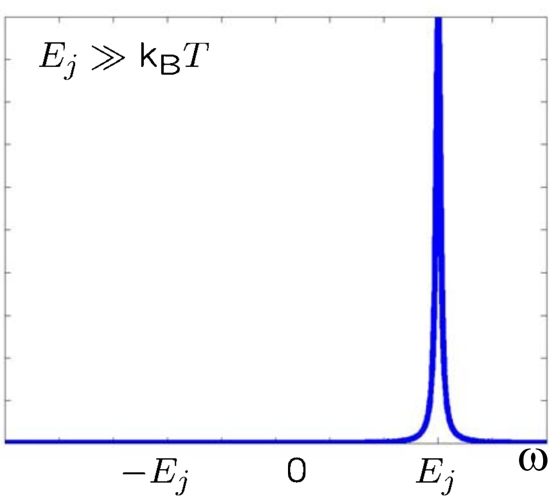
Correlation function $C_j(\omega) \equiv \int dt \left\{ \langle \sigma_{z,j}(t) \sigma_{z,j}(0) \rangle - \langle \sigma_{z,j} \rangle^2 \right\} e^{i\omega t}$

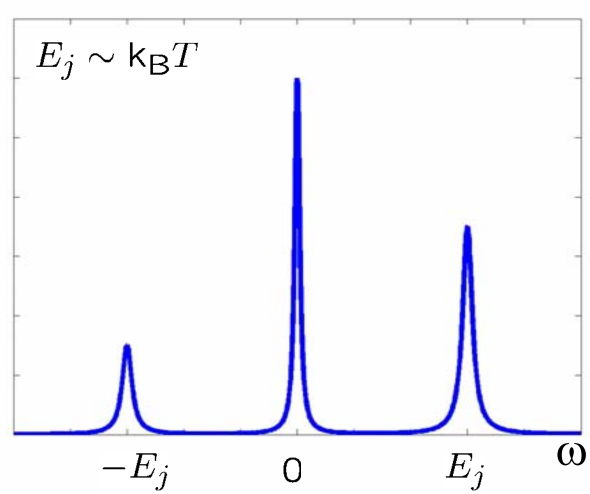
$$C_j(\omega) \approx \cos^2\theta_j \left[1 - \langle \rho_{z,j} \rangle^2 \right] \frac{2\Gamma_{1,j}}{\Gamma_{1,j}^2 + \omega^2}$$ random telegraph noise

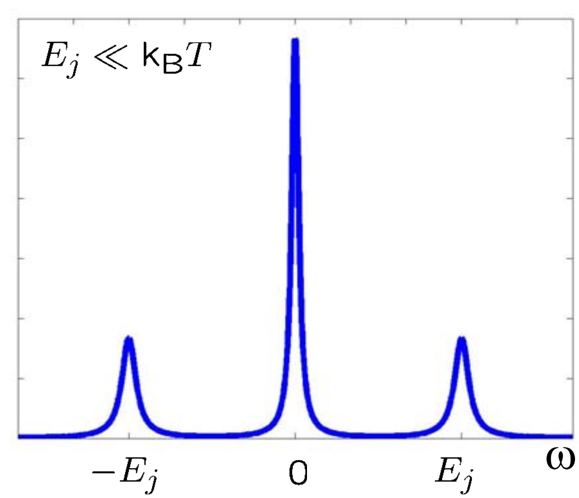
$$+ \sin^2\theta_j \left[\frac{1 + \langle \rho_{z,j} \rangle}{2} \right] \frac{2\Gamma_{2,j}}{\Gamma_{2,j}^2 + (\omega - E_j)^2}$$ absorption

$$\langle \rho_{z,j} \rangle = \tanh(E_j / 2k_{\mathsf{B}}T)$$

$$+ \sin^2\theta_j \left[\frac{1 - \langle \rho_{z,j} \rangle}{2} \right] \frac{2\Gamma_{2,j}}{\Gamma_{2,j}^2 + (\omega + E_j)^2}$$ emission







Spectrum of noise felt by qubit

$$X \equiv \sum_j v_j \, \sigma_{z,j}$$

$$S_X(\omega) = \frac{1}{2}[C_X(\omega) + C_X(-\omega)]$$

$$C_X(\omega) \equiv \int dt \, \left\{ \langle X(t) X(0) \rangle - \langle X \rangle^2 \right\} \, e^{i\omega t} = \sum_j v_j^2 C_j(\omega)$$

depends on distribution $P(\epsilon, \Delta, v)$ of TLS-parameters

High frequencies:

$\hbar\omega \gg k_{\mathrm{B}} T \gg \Gamma_2$

$$S_X(\omega) \approx \sum_j v_j^2 \sin^2\theta_j \, \frac{\Gamma_{2,j}}{\Gamma_{2,j}^2 + (\omega - E_j)^2}$$

$$\approx N \int d\epsilon \, d\Delta \, dv \, P(\epsilon, \Delta, v) \, v^2 \sin^2\theta \cdot \pi \delta(\omega - E) \quad \propto \omega$$

Low frequencies:

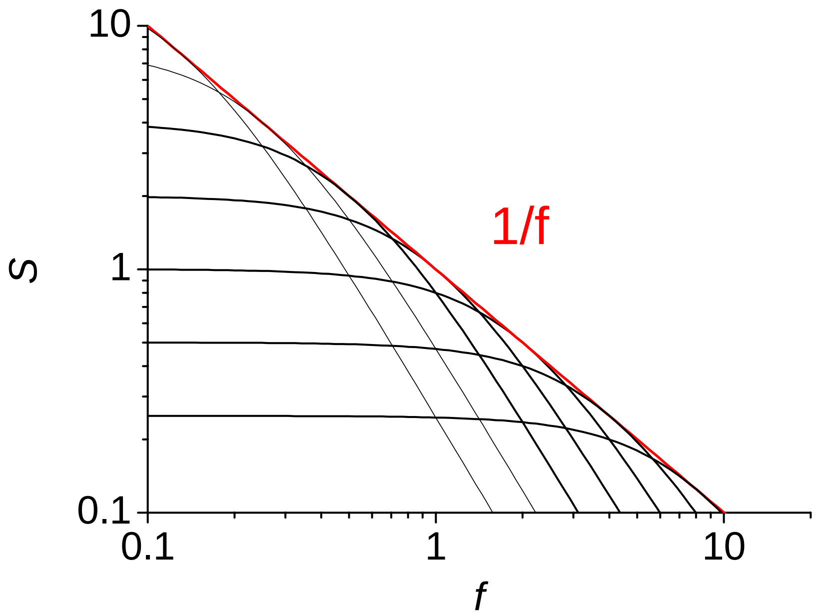
$$\int_{\text{low freq.}} \frac{d\omega}{2\pi} S_X(\omega) \approx \int_{\text{low freq.}} \frac{d\omega}{2\pi} \sum_j v_j^2 \cos^2\theta_j \left[1 - \langle \rho_{z,j} \rangle^2\right] \frac{2\Gamma_{1,j}}{\Gamma_{1,j}^2 + \omega^2}$$

$$\approx N \int d\epsilon \, d\Delta \, dv \, P(\epsilon, \Delta, v) \, v^2 \cos^2\theta \, \frac{1}{\cosh^2 \frac{E}{2T}} \quad \propto T^2$$

1/f noise as sum of many telegraph noises (Dutta-Horn model)

$$S_j \propto \frac{\Gamma_j}{\omega^2 + \Gamma_j^2} \qquad\qquad P(\Gamma) \propto 1/\Gamma$$

$$S = \sum S_j \propto \int d\Gamma \, P(\Gamma) \frac{\Gamma}{\omega^2 + \Gamma^2} \propto \frac{1}{|\omega|}$$



$$\Gamma \propto \Delta^2 \qquad (\epsilon \gg \Delta)$$

We want

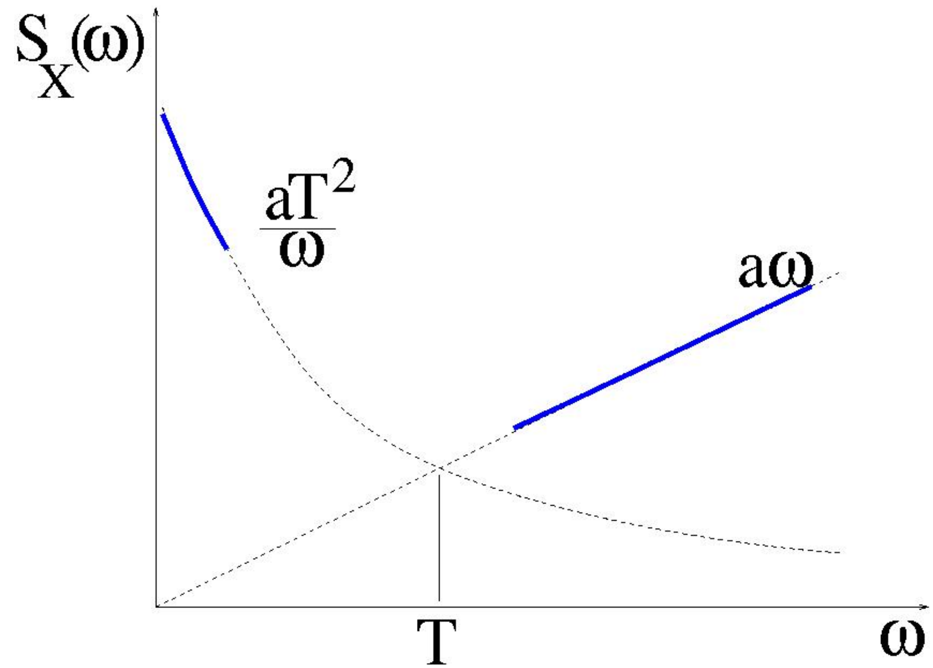
$$P(\Delta) \propto 1/\Delta$$

$$P(\epsilon, \Delta, v) \propto P_v(v) \times \frac{\epsilon}{\Delta}$$

- ϵ: for linear ω-dependence
- Δ: exponential dependence on barrier height
- $P_v(v)$: $< v^2 >$ overall factor

explains observed spectrum $S_X(\omega)$

$$S(\omega) \approx \frac{a\,(k_B T)^2}{\hbar\omega} + a\,\hbar\omega$$



„Andreev fluctuators“ (Faoro et al. cond-mat 2004) might have this distribution of parameters

Microscopic models

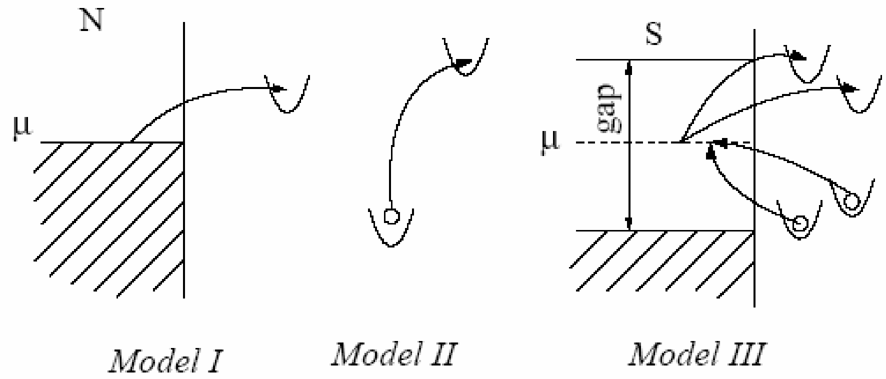


FIG. 1: The three models

Faoro, Bergli, Altshuler, Galperin (2004)

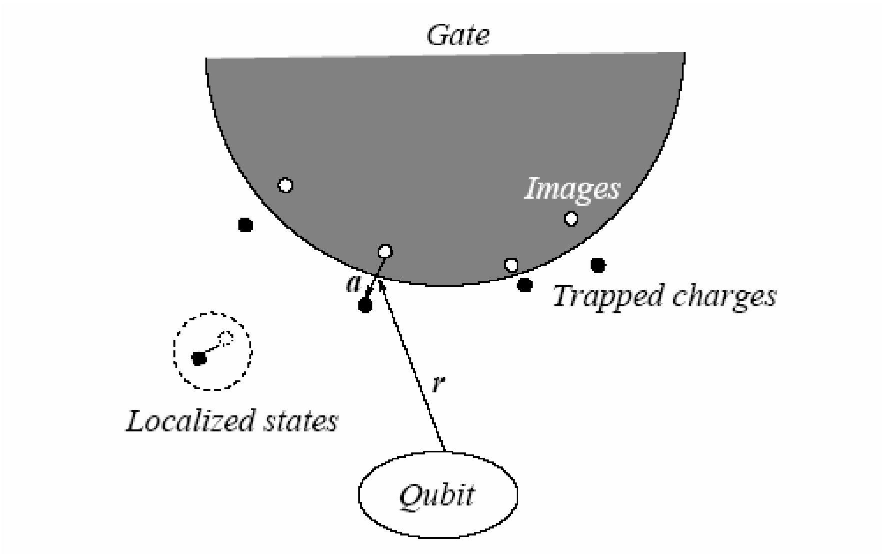
Models 2 and 3

$$\epsilon = \epsilon_1 + \epsilon_2$$

$$\epsilon_1 > 0 \; , \; \epsilon_2 > 0 \quad \longrightarrow \quad P(\epsilon) \propto \epsilon$$

$$P(\epsilon_1) = P(\epsilon_2) = const.$$

Microscopic models



Paladino, Faoro, Falci, Fazio (2002) + ...

Galperin, Altshuler, Shantsev (2003)

Faoro, Bergli, Altshuler, Galperin (2004)

Grishin, Yurkevich, Lerner (2004)

de Sousa, Whaley, Wilhelm, von Delft (2005)

Conclusions, Part II

- Qubit used as spectrum analyzer of noise of environment

  Astafiev et al. (NEC), Martinis et al. (NIST), Vion et al. (Saclay), Schoelkopf et al. (Yale), Kouwenhoven et al. (Delft),….

- High- and low-frequency noise derive from the same ensemble of ‘coherent’ TLS
- Plausible distribution of parameters produces:

  - Ohmic (*f*) high-frequency noise responsible for relaxation
  - Low-frequency (1/*f*) noise scaling as T^2 responsible for decoherence
  - both governed by same parameters

$$S(\omega) \approx \frac{a\left(k_{\mathrm{B}}T\right)^2}{\hbar\omega} + a\,\hbar\omega$$

Conclusions

- *Two possible mechanisms: Josephson and dipolar (ϕ)*

  I. Martin, L. Bulaevskii, and A. Shnirman, "Tunneling Spectroscopy of Two-level Systems Inside Josephson Junction," Phys. Rev. Lett. ***95****, 127002 (2005)*

- *Connection between high- and low-frequency noises from an ensemble of almost coherent 2-level fluctuators*

  Alexander Shnirman, Gerd Schön, Ivar Martin, Yuriy Makhlin "Low- and high-frequency noise from coherent two-level systems," Phys. Rev. Lett. ***94****, 127002 (2005)*

Thanks: DOE, SQUBIT