Spontaneous spin polarization in quantum wires

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A number of recent experiments report spin polarization in quantum wires in the absence of magnetic fields. These observations are in apparent contradiction with the Lieb-Mattis theorem, which forbids spontaneous spin polarization in one dimension. We show that sufficiently strong interactions between electrons induce deviations from the strictly one-dimensional geometry and indeed give rise to a ferromagnetic ground state in a certain range of electron densities. At higher densities, more complicated spin interactions lead to a possibly novel ground state.
Spontaneous Spin Polarization in Quantum Wires

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Why ask this question at all …

**Conductance quantization in Quantum Wires**

- **Theory:** \[ G = k \cdot G_0 \quad (k \text{ integer}) \]
  where \[ G_0 = \frac{2e^2}{h} \]

- **Experiment I:**

  ![Graphs showing conductance vs. gate voltage](image1)

  Berggren & Pepper, Physics World 2002
Motivation

Why ask this question at all …

• **Experiment II:**

  conductance anomalies at low density
  • additional structure
    at $0.7 \, G_0$ (short wires) or $0.5 \, G_0$ (long wires)

• see e.g. Thomas *et al.*, Phys. Rev. B **61**, R13365 (2000)

• spontaneous spin polarization?

BUT …
In 1D, the ground state of an interacting electron system possesses minimal spin.


QUANTUM WIRE:
not a purely one-dimensional system ...

• parabolic confining potential:
  no interactions
  strong interactions?
Quantum wires at low density: Wigner crystal

- at low electron densities $n_e$,
  interaction energy ($\sim n_e$) dominates over kinetic energy ($\sim n_e^2$)
  $\Rightarrow$ formation of (classical) Wigner crystal

- Coulomb interaction:
  $$V_{\text{int}} = \frac{e^2}{\epsilon} \sum_{i<j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}$$

- formation of zig-zag chain favorable when $V_{\text{int}}$ of order $V_{\text{conf}}$

- confining potential:
  $$V_{\text{conf}} = \frac{1}{2} m \Omega^2 \sum_i y_i^2$$

- minimize
  $$E(d) = \frac{e^2}{\epsilon} \sum_{j=1}^{\infty} \frac{1}{\sqrt{\frac{1}{n_e^2} (2j-1)^2 + d^2}} + \frac{1}{2} m \Omega^2 \left( \frac{d}{2} \right)^2$$
  with respect to distance $d$ between rows
• \( V_{\text{int}}(r_0) = V_{\text{conf}}(r_0) \equiv E_0 \Rightarrow \text{characteristic length scale } r_0 \)

\[
r_0 = \left( \frac{2e^2}{\epsilon m \Omega^2} \right)^{1/3}
\]

• dimensionless density \( \nu = n_e r_0 \)

• transition 1D \( \rightarrow \) zig-zag

at \( \nu_c = \left( \frac{4}{7 \zeta(3)} \right)^{1/3} \approx 0.78 \)

• [ crystals with larger number of chains are stable at even higher densities ]

Piacente et al. 04
○ structure ✓ ○ spin properties ?

Spin interactions in a Wigner crystal

- to a first approximation, spins do not interact ...

- BUT:
  weak tunneling through Coulomb barrier

⇒ exponentially small exchange constants $J$
Exchanges in a zig-zag chain

- **1D chain**: (AF) nearest-neighbor exchange
- **zig-zag chain**: in addition, next-nearest neighbor exchange
Frustrated Heisenberg spin chain

\[ H_P = \frac{1}{2} \sum_j [J_1 P_{j,j+1} + J_2 P_{j,j+2}] \]

use \[ P_{ij} = \frac{1}{2} + 2S_i S_j \]

• spin Hamiltonian: \[ H = \sum_j (J_1 S_j S_{j+1} + J_2 S_j S_{j+2}) \]

• next-nearest neighbor exchange \( J_2 \) causes frustration

• phase diagram

  [ Majumdar & Ghosh, Haldane, Eggert, White & Affleck, Hamada et al., Allen et al., Itoi & Qin, … ]

\( J_2 < 0.24... J_1 \): weak frustration
  \[ \rightarrow \text{the groundstate is antiferromagnetic} \]

\( J_2 > 0.24... J_1 \): strong frustration
  \[ \rightarrow \text{the ground state is dimerized} \]

dimerization \[ d = \langle S_j (S_{j-1} - S_{j+1}) \rangle \]
Exchanges in a zig-zag chain

- **1D chain**: (AF) nearest-neighbor exchange
- **zig-zag chain**:
  - in addition, next-nearest neighbor exchange
  - increase distance between rows → equilateral configuration

→ **RING EXCHANGES**

cf. 2D Wigner crystal:

(Roger 84, Bernu, Candido & Ceperley 01, Voelker & Chakravarty 01, ...)

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Ring exchanges

- cyclic exchange of \( l \) particles: 
  \[ P_{j_1 \ldots j_l} = P_{j_1 j_2} P_{j_2 j_3} \ldots P_{j_{l-1} j_l} \]

- ring exchange of even number of particles: antiferromagnetic
- ring exchange of odd number of particles: ferromagnetic
  (Thouless 1965)

- Hamiltonian:
  \[
  H_P = \frac{1}{2} \sum_j \left[ J_1 P_{j \ j+1} + J_2 P_{j \ j+2} - J_3 (P_{j \ j+1 \ j+2} + P_{j+2 \ j+1 \ j}) \\
  + J_4 (P_{j \ j+1 \ j+3 \ j+2} + P_{j+2 \ j+3 \ j+1 \ j}) - \ldots \right]
  \]
Frustrated Heisenberg spin chain + 3-particle ring exchange

\[ H_P = \frac{1}{2} \sum_j [J_1 P_{j,j+1} + J_2 P_{j,j+2} - J_3 (P_{j,j+1} P_{j+1,j+2} + P_{j+2,j+1} P_{j+1,j})] \]

- nearest neighbor exchange: \[ \tilde{J}_1 = J_1 - 2J_3 \]
- next-nearest neighbor exchange: \[ \tilde{J}_2 = J_2 - J_3 \]
- spin Hamiltonian:

\[ H = \sum_j \left( \tilde{J}_1 S_j S_{j+1} + \tilde{J}_2 S_j S_{j+2} \right) \]

- phase diagram

[ Majumdar & Ghosh, Haldane, Eggert, White & Affleck, Hamada et al., Allen et al., Itoi & Qin, … ]
Computation of exchange constants

• strength of interactions is characterized by

\[ r_\Omega = \frac{r_0}{a_B} = 2 \left( \frac{me^4}{2\hbar^2\varepsilon^2} \frac{1}{\hbar\Omega} \right)^{2/3} \]

(where \( a_B \) Bohr’s radius \( \approx 100\text{Å} \) in GaAs)

• use WKB at \( r_\Omega \gg 1 \) [note: \( r_s \sim r_\Omega / \nu \)]

• imaginary-time action \( S = \hbar\eta \sqrt{r_\Omega} \) with

\[
\eta[\{\mathbf{r}_j(\tau)\}] = \int_{-\infty}^{\infty} d\tau \left[ \sum_j \left( \frac{\dot{r}_j^2}{2} + y_j^2 \right) + \sum_{j<i} \frac{1}{|\mathbf{r}_j - \mathbf{r}_i|} \right]
\]
Numerical results I

- exchange constants: \[ J_l = J_l^* \exp \left[ -\eta_l \sqrt{r \Omega} \right] \]
- solve equations of motion for various exchange processes numerically
- nearest and next-nearest neighbor as well as 3-, 4-, 5-, 6-, and 7-particle ring exchanges
- dominant exchange: \[ J_1 \rightarrow J_3 \rightarrow J_2 \]
Heisenberg spin chain with nearest and next-nearest neighbor exchange

\[ H = \sum_j (\tilde{J}_1 S_j S_{j+1} + \tilde{J}_2 S_j S_{j+2}) \]

with nearest neighbor exchange: \( \tilde{J}_1 = J_1 - 2J_3 \)

next-nearest neighbor exchange: \( \tilde{J}_2 = J_2 - J_3 \)

\( \tilde{J}_2 > 0 \): frustration

\( \tilde{J}_2 < 0 \): ✓
Numerical results II

• “spectators” participate in exchange process

• 12 spectators included on either side of the exchanging particles

⇒ • smaller values $\eta_l$
Numerical results II

e.g.:

\( v=1.3 \)

![Graph showing the relationship between the number of spectators and \( \eta \) values for different \( v \) values.](image-url)
Numerical results II

e.g.: 

\[ v = 1.6 \]

\[ \eta_1, \eta_2, \eta_3, \eta_4 \]

Number of spectators
• $J_4$ wins over $J_2$ at large densities!

• 4-particle ring exchange generates 4-spin interaction:

$$H_4 \sim (S_j S_{j+1})(S_{j+2} S_{j+3}) + (S_j S_{j+2})(S_{j+1} S_{j+3}) - (S_j S_{j+3})(S_{j+1} S_{j+2})$$
4-particle ring exchange

PRELIMINARY RESULTS:
exact diagonalization
spin
of the ground state
for 12 spins
(periodic boundary conditions)
What about experiment? ...

... Are quantum wires ferromagnetic?

- Are interactions in realistic quantum wires strong enough?
- "strength of interaction" controlled by confining potential:

\[ r_\Omega \propto \Omega^{-2/3} \]

2 types of quantum wires:

- cleaved-edge overgrowth:
  steep confining potential \( r_\Omega < 1 \)

- split gate:
  shallow confining potential \( r_\Omega > 1 \)

( e.g. Thomas et al., Phys. Rev. B 61, R13365 (2000): \( r_\Omega = 3 - 6 \) )
Prefactors (preliminary)

- exchange constants:

\[ J_l = J_l^* \exp \left[ -\eta_l \sqrt{r_\Omega} \right] \text{ where } J_l^* = \frac{e^2}{\epsilon a_B} m_l F_l \left( \frac{\eta_l}{2\pi} \right)^{1/2} r_\Omega^{-5/4} \]

(Gaussian fluctuations around classical exchange path)

- ``phase diagram´´: stability of the ferromagnetic groundstate
A ferromagnetic ground state in quantum wires is possible at strong enough interactions. The interactions induce deviations from one-dimensionality and lead to ferromagnetism in a certain range of electron densities.
Conclusions & Outlook

TO DO ...

EXPERIMENT:
• ideal devices to observe spontaneous spin polarization:
  split-gate wires with widely separated gates
  ⇒ shallow confining potential ⇒ large $r_\Omega$

THEORY:
• compute prefactors (under way)
• explore zig-zag chains with 4-particle ring exchange
• conductance?
Conclusions & Outlook

A ferromagnetic ground state in quantum wires is possible at strong enough interactions. The interactions induce deviations from one-dimensionality and lead to ferromagnetism in a certain range of electron densities.
What about experiment? ...
... Does ferromagnetism lead to $G = 0.5 \ G_0$?

• coupling of spin and charge excitations due to leads

$$\mathcal{H}_s = \sum_{k=1,2} \left[ \sum_j J_k (j + q(t)) S_j S_{j+k} \right]$$

• reflection of spin excitations
Summary

Can the ground state of the electron system in a quantum wire be ferromagnetic?

YES - for sufficiently strong interactions, there is a range of electron densities, where the electrons form a **zig-zag Wigner crystal** and the spin interactions due to **3-particle ring exchange** make the system **ferromagnetic**.