Interaction effects on magnetooscillations in a 2D electron gas

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Quantum magnetooscillations

- Magnetization, magnetic susceptibility, thermodynamic density of states:
  de Haas – van Alphen oscillations

- Conductivity:
  Shubnikov – de Haas oscillations

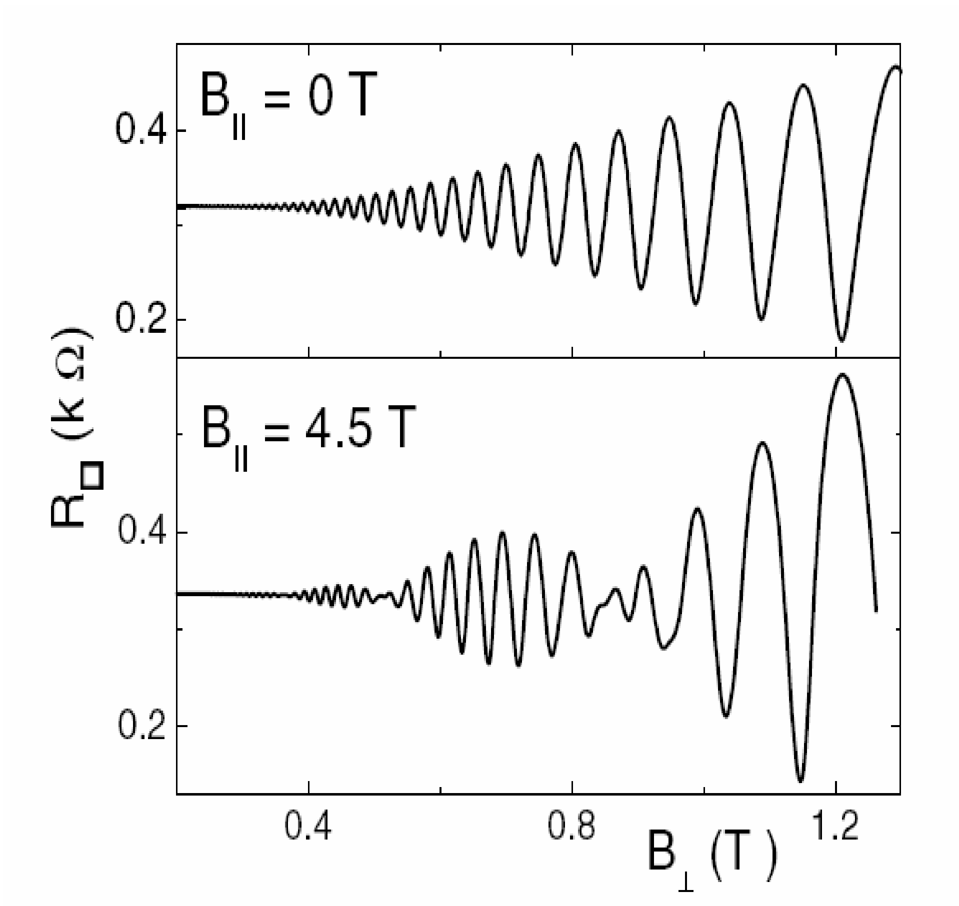
Motivation I. Experiment

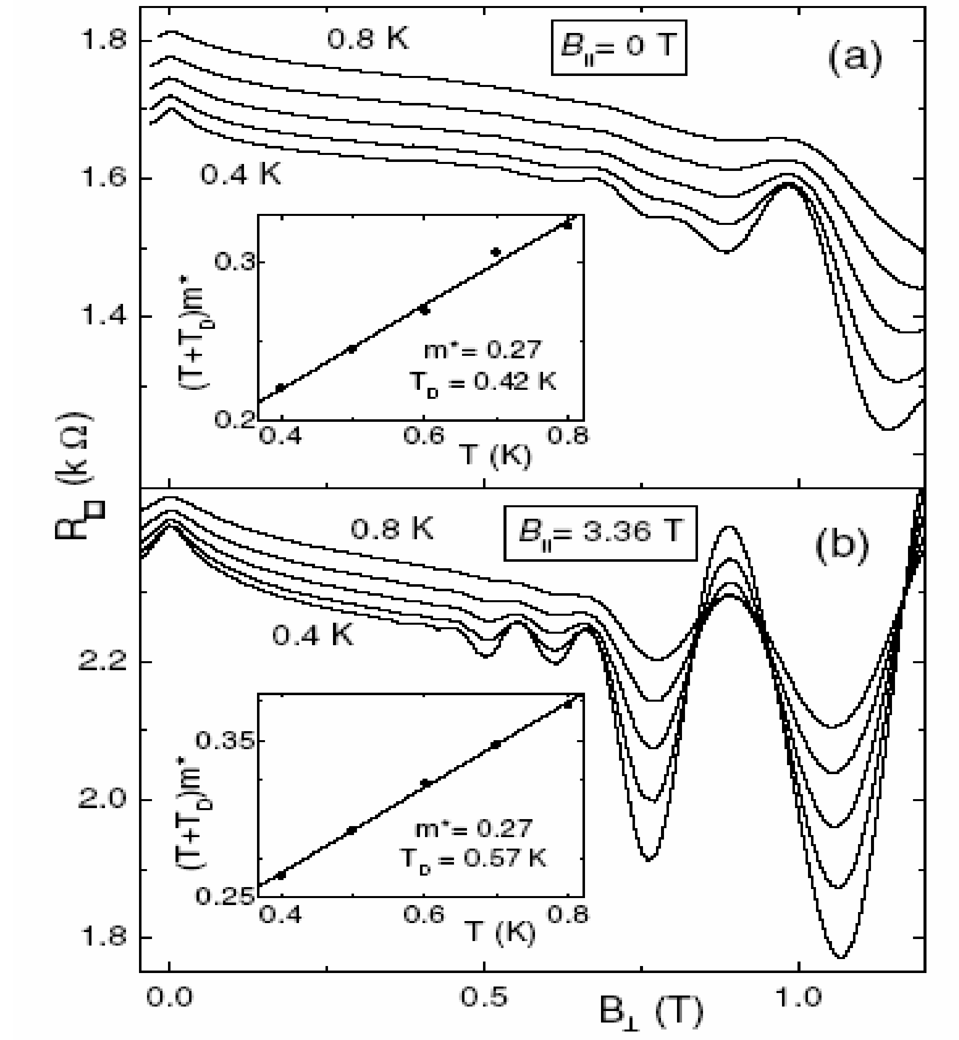
Experiments on low-density high-mobility 2D electron systems ⇒ apparent 2D MIT

- physics on the metallic side?
- divergent susceptibility?
- effective mass or g-factor?

Kravchenko, Shashkin, et al.; Pudalov, Gershenson et al.;…

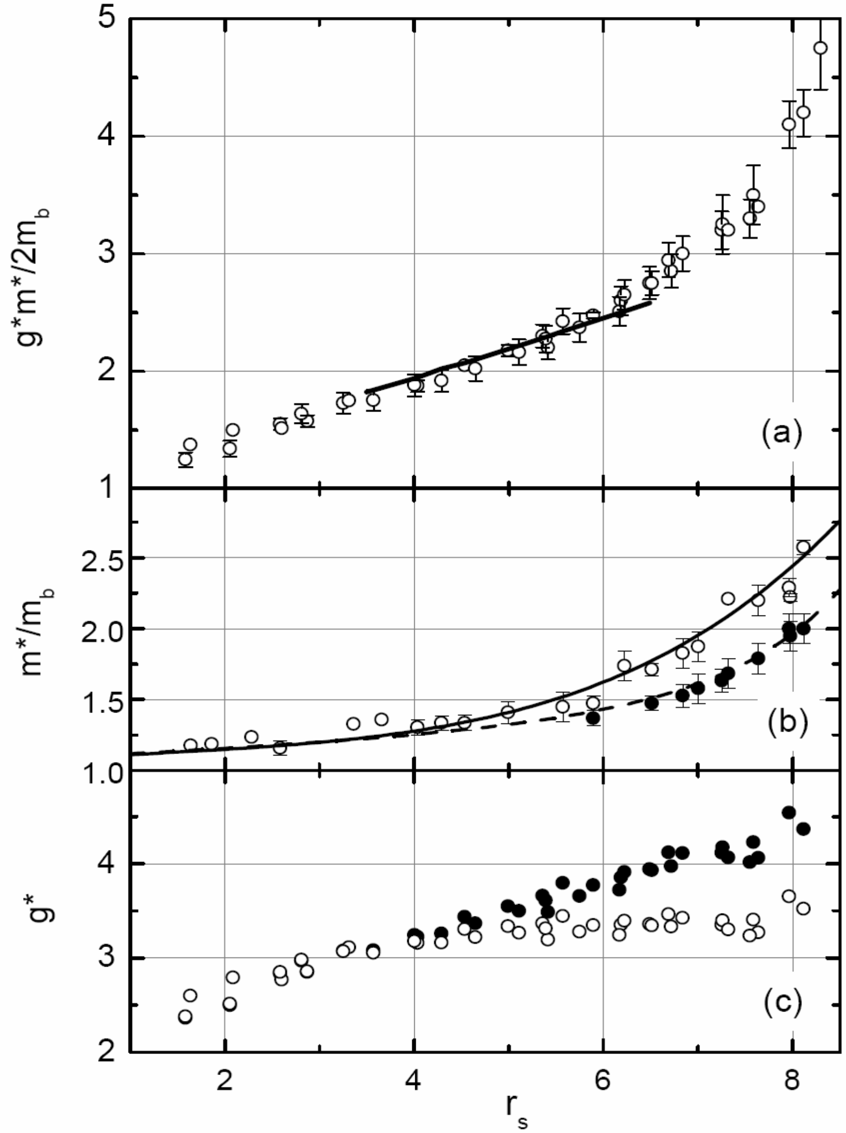
SdH oscillations in Si MOSFETs





Pudalov,Gershenson et al, PRL'02

Mass and g-factor from SdHO



Pudalov et al, PRL'02

- Product gm diverges: how to separate g and m?
- Damping of SdHO

Lifshitz – Kosevich:

$$A_1^{(0)}(T) \equiv \frac{4\pi^2 T}{\omega_c} \exp\left[-\frac{2\pi^2 T}{\omega_c^*} - \frac{\pi}{\omega_c^* \tau^*}\right]$$

Interaction: $\tau(T)$ and $m(T)$

Theory needed !

- strong *T*-dependence of *m* *(V. Pudalov, private commun.)*

Motivation II. Theory

- interaction + disorder corrections

- diffusive: *Altshuler, Aronov*

  conductivity: $\ln T$, tunneling DOS: $\ln^2 T$

- ballistic: conductivity: T

  Gold, Dolgopolov; das Sarma; Zala, Narozhny, Aleiner

  tunneling DOS *Rudin, Aleiner,Glazman*

  magnetotransport *Gornyi, ADM*

Interaction effect on magnetooscillations?

Single-particle property vs. gauge-invariance

Correction to quantum vs. transport relaxation time

Closely related:

- Non-analyticities in clean Fermi liquids

  Chubukov, Maslov, Glazman, Gangadharajah, Millis;
  Efetov's talk

Martin, Maslov, Reizer PRB'03

Weak magnetooscillations (suppressed by *T*):

- Lifshitz – Kosevich formula works in 2D
- Inelastic *e-e* scattering irrelevant

- interaction + disorder:

treatment incomplete

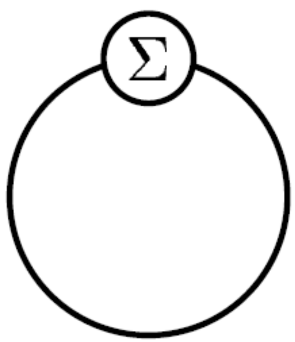
Luttinger – Ward formalism

Thermodynamic potential:

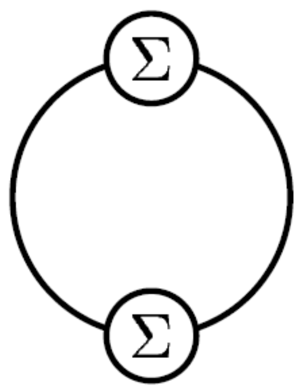
$$\Omega = \boxed{-T\,\mathrm{Tr}\ln(-G^{-1})} - T\,\mathrm{Tr}(G\Sigma) + \Omega'$$

Oscillatory contribution comes from *Tr ln*-term:

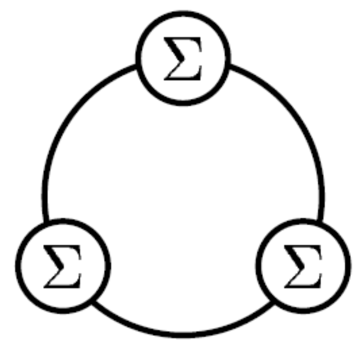
$$-T\,\mathrm{Tr}\ln(-G_0^{-1}) +$$



$$+\frac{1}{2}$$



$$+\frac{1}{3}$$



$$+\cdots$$

Self-energy in FL with disorder

$$\Sigma(i\varepsilon_n, \xi) \simeq \delta\mu + \beta_0(\xi - \tilde{\mu}) - i\alpha_0\varepsilon_n + \delta\Sigma(i\varepsilon_n, \xi)$$

FL renormalization: $$Z = \frac{1}{1+\alpha_0}$$

$$m^* = m\frac{1+\alpha_0}{1+\beta_0} \qquad \omega_c^* = \frac{eB}{m^*} = \omega_c\frac{1+\beta_0}{1+\alpha_0}$$

Interplay of interaction and disorder: $\delta\Sigma(i\varepsilon_n, \xi)$

Oscillatory part of Ω vs. Σ

$$\Omega_{\rm osc} \;\simeq\; 2\nu\left(\frac{\omega_c}{2\pi}\right)^2 A_1 \cos\frac{2\pi^2 n_e}{eB}$$

$$A_1 = \frac{4\pi^2 T}{\omega_c}\exp\left[-\frac{2\pi}{\omega_c^*}\;\{\pi T + i\,Z\,\delta\Sigma(i\pi T,\boxed{\xi_0})\}\right]\;\exp\left[-\frac{\pi}{\omega_c^*\tau^*}\right]$$

$$\boxed{\xi_0 = \mu + i\varepsilon_n + \frac{i}{2\tau}\,\mathrm{sgn}\,\varepsilon_n - \Sigma_{\rm ee}(i\varepsilon_n,\xi_0)}$$

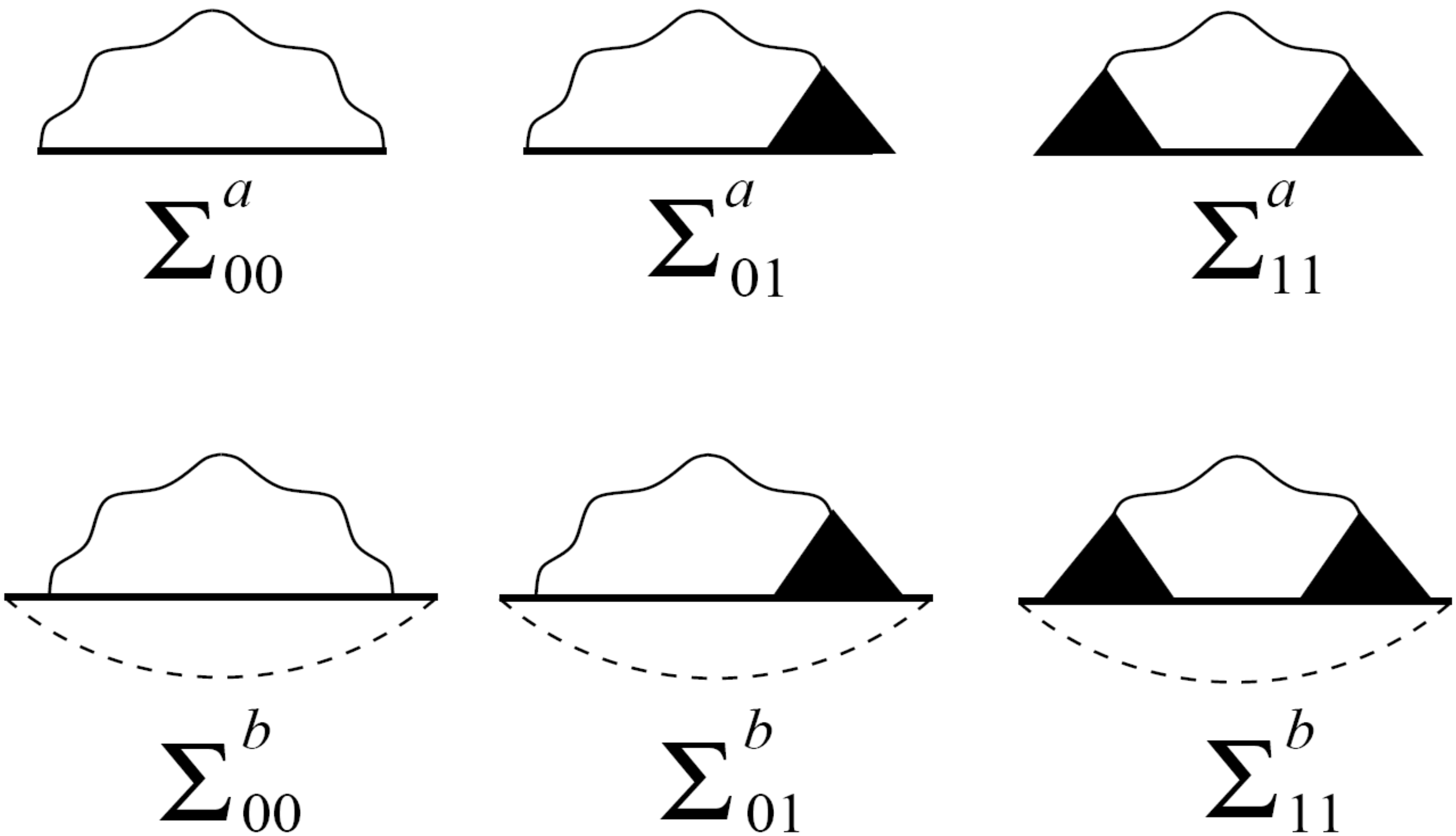
Damping of magnetooscillations

$$A_1 = A_1^{(0)}(T)\,\exp[B(T)]$$

interaction + disorder:

$$B(T) = -\frac{2\pi i\, Z\, \delta\Sigma(i\pi T, \xi_0)}{\omega_c^*}$$

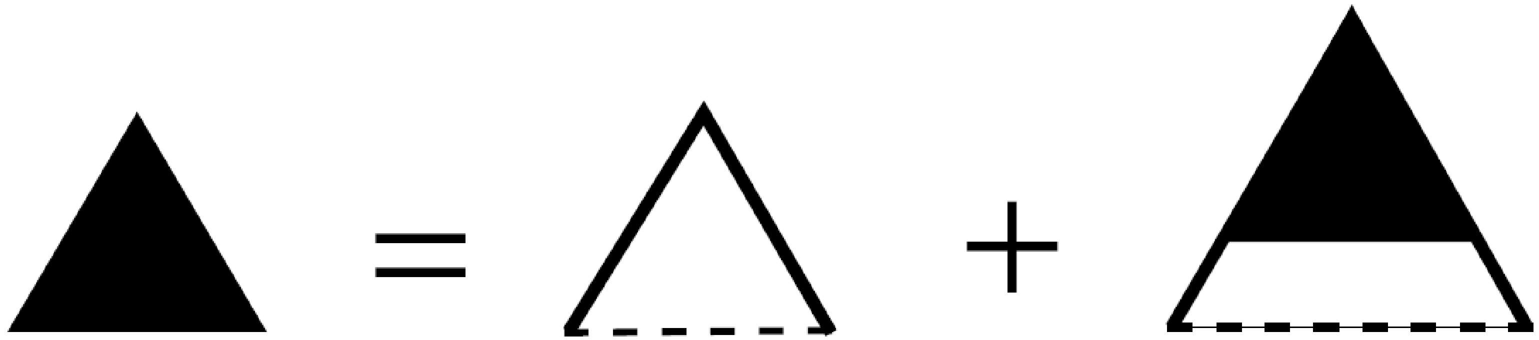
Diagrammatics I.   Self-energy



Martin, Maslov, Reizer

consider only one diagram Σ^a_{01} out of six

Diagrammatics II.
Disorder-dressed interaction vertex



assume white-noise disorder

Impurity ladder: $\Gamma(i\omega_k, \mathbf{q}) = \dfrac{1}{S\tau - 1}$

$$S(i\omega_m, \mathbf{q}) = \sqrt{(|\omega_m| + 1/\tau)^2 + v_F^2 q^2} = \sqrt{W^2 + v_F^2 q^2}, \qquad W = |\omega_m| + 1/\tau$$

General formula for Σ

$$\delta\Sigma(i\varepsilon_n,\xi_0) = -i\,T\sum_{\omega_m>\varepsilon_n}\int\frac{d^2q}{(2\pi)^2}V(i\omega_m,\mathbf{q})K(i\omega_m,\mathbf{q}),$$

$$K(i\omega_m,\mathbf{q}) = \frac{[1+\Gamma(i\omega_m,\mathbf{q})]^2}{S(i\omega_m,\mathbf{q})}\left[1-\frac{W}{\tau S^2(i\omega_m,\mathbf{q})}\right]-\frac{1}{S_0(i\omega_m,\mathbf{q})}$$

Hikami – box contribution

from Σ^a_{00}

$$S_0(i\omega_m,\mathbf{q}) = \sqrt{|\omega_m|^2+v_F^2q^2}$$

$V(i\omega_m,\mathbf{q})$ - e-e interaction

Gauge invariance

$$\boxed{K(i\omega_m, q=0)} = \frac{1}{W}\left[1+\frac{1}{W\tau-1}\right]^2\left[1-\frac{1}{W\tau}\right]-\frac{1}{\omega_m}$$

$$= \frac{1}{W-1/\tau}-\frac{1}{\omega_m}\boxed{=0}$$

Interaction at q=0 can be gauged out

$$\Rightarrow \qquad K(i\omega_m, q=0) = 0$$

(cf. interaction correction to conductivity)

Short-range interaction: Result

$$B(T) = -\mathrm{const}\,\nu U_0 \frac{\pi}{\omega_c \tau} + \frac{\pi T}{\omega_c}\;\frac{\nu U_0}{E_F \tau} \ln \frac{E_F}{T}$$

T logT behavior of damping exponent:

T-dependence of τ *(T)* or *m(T)* ?

➔ analytical continuation to real energies

$$B(T) = -\frac{2\pi^2 T}{\omega_c^*}\frac{\delta m}{m^*} - \frac{\pi}{\omega_c^* \tau^*}\left(\frac{\delta m}{m^*} - \frac{\delta \tau}{\tau^*}\right)$$

Effective mass vs. scattering time

$$A_1(\epsilon, T) \;=\; \exp\left\{\frac{2\pi i}{\omega_c^*}[\varepsilon - \mathrm{Re}\,\delta\Sigma(\varepsilon, \xi_0)]\right\} \exp\left\{-\frac{\pi}{\omega_c^*\tau^*} + \frac{2\pi}{\omega_c^*}\,\mathrm{Im}\,\delta\Sigma(\varepsilon, \xi_0)\right\}$$

$$= \exp\left\{\frac{2\pi i\varepsilon}{\omega_c^*}\left[1 + \frac{\delta m(\varepsilon, T)}{m^*}\right]\right\} \exp\left\{-\frac{\pi}{\omega_c^*\tau^*}\left[1 + \frac{\delta m(\varepsilon, T)}{m^*} - \frac{\delta\tau(\varepsilon, T)}{\tau^*}\right]\right\}$$

$$\boxed{\begin{aligned} \frac{\delta m(\varepsilon, T)}{m^*} &= -\frac{\mathrm{Re}\,\delta\Sigma(\varepsilon, T)}{\varepsilon}, \\ \frac{\delta\tau(\varepsilon, T)}{\tau^*} &= -2\tau^*\,\mathrm{Im}\,\delta\Sigma(\varepsilon, T) + \frac{\delta m(\varepsilon, T)}{m^*} \end{aligned}}$$

T-dependence of *m* and τ

$$\frac{\delta m(T)}{m^*} = -\frac{\nu U_0}{2\pi E_F \tau^*} \ln \frac{E_F}{T},$$

$$\frac{\delta \tau(T)}{\tau^*} = \boxed{-\nu U_0 \frac{T}{E_F}} - \frac{\nu U_0}{2\pi E_F \tau^*} \ln \frac{E_F}{T}$$

agrees with conductivity correction and with Friedel oscillations picture

T logT – dependence of damping due to *m(T)*

Backscattering: $\delta\tau_q$ vs. $\delta\tau_{tr}$

$$\boxed{\delta\left(\frac{1}{\tau_q}\right) = \frac{\nu U_0}{\tau}\left[\text{const} - \frac{T}{E_F}\ln[2\cosh(\varepsilon/2T)]\right]}$$

$$\frac{1}{\tau_q(\varepsilon,T)} = \int \frac{d\phi}{2\pi}S(\phi)$$

$\tau_{\text{tr}} \longrightarrow$ extra factor

$$1-\cos\phi \simeq 2$$

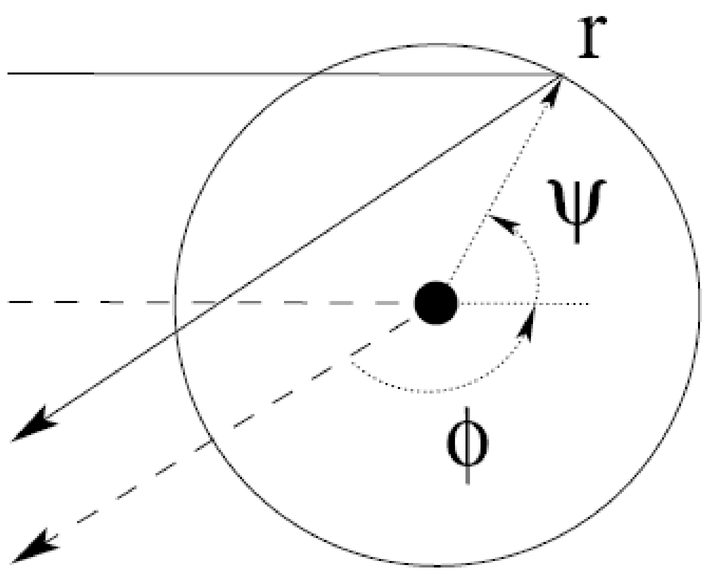
$$\int d\epsilon(-\partial n_F/\partial\epsilon)\ldots$$

$$\longrightarrow \boxed{\delta\tau_q(T) = \frac{1}{2}\delta\tau_{\text{tr}}(T) = -\nu U_0 \frac{T}{E_F}\tau}$$

agrees with the conductivity correction

(Zala, Narozhny, Aleiner) !

Friedel oscillations



$\phi=\pi, \;\; \psi=0,\pi$

enhanced backscattering

Rudin, Aleiner, Glazman, 1997

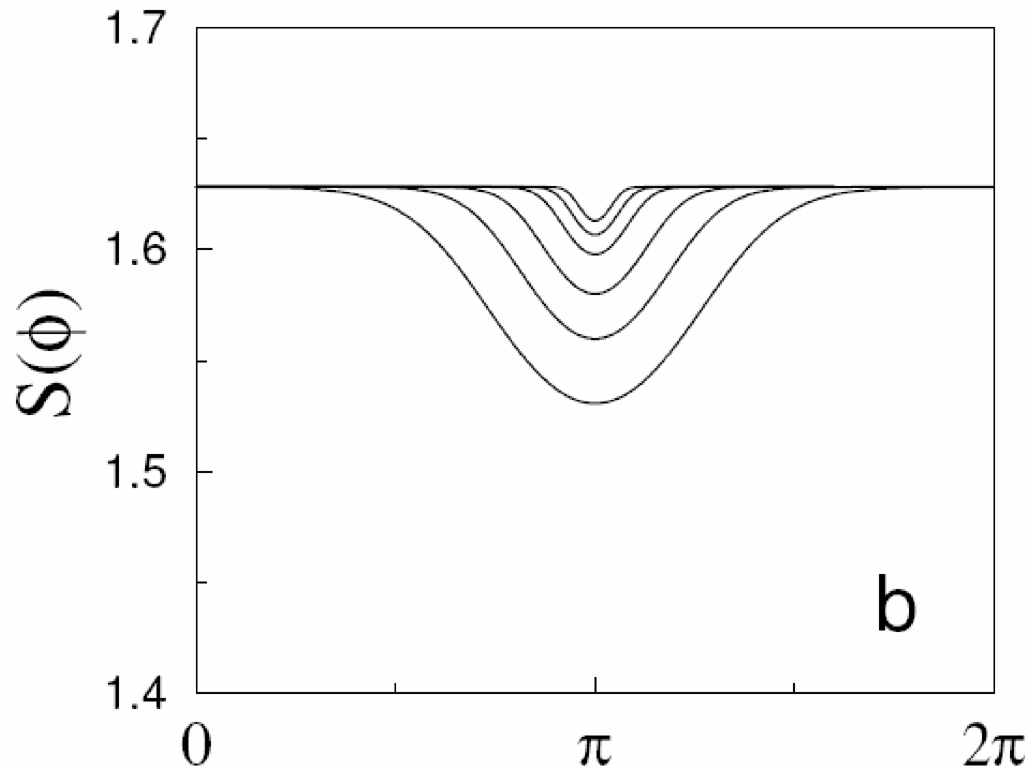
Zala, Narozhny, Aleiner, 2001

Scattering off Friedel oscillations

enhanced back-scattering off an impurity dressed by Friedel oscillations

$\epsilon = 0$

$T/E_F = 0.005, 0.01, 0.02,$
$0.05, 0.1, 0.2$



reproduce $$\delta\left(\frac{1}{\tau_q}\right) = \frac{\nu U_0}{\tau}\left[\mathrm{const} - \frac{T}{E_F}\ln[2\cosh(\varepsilon/2T)]\right]$$

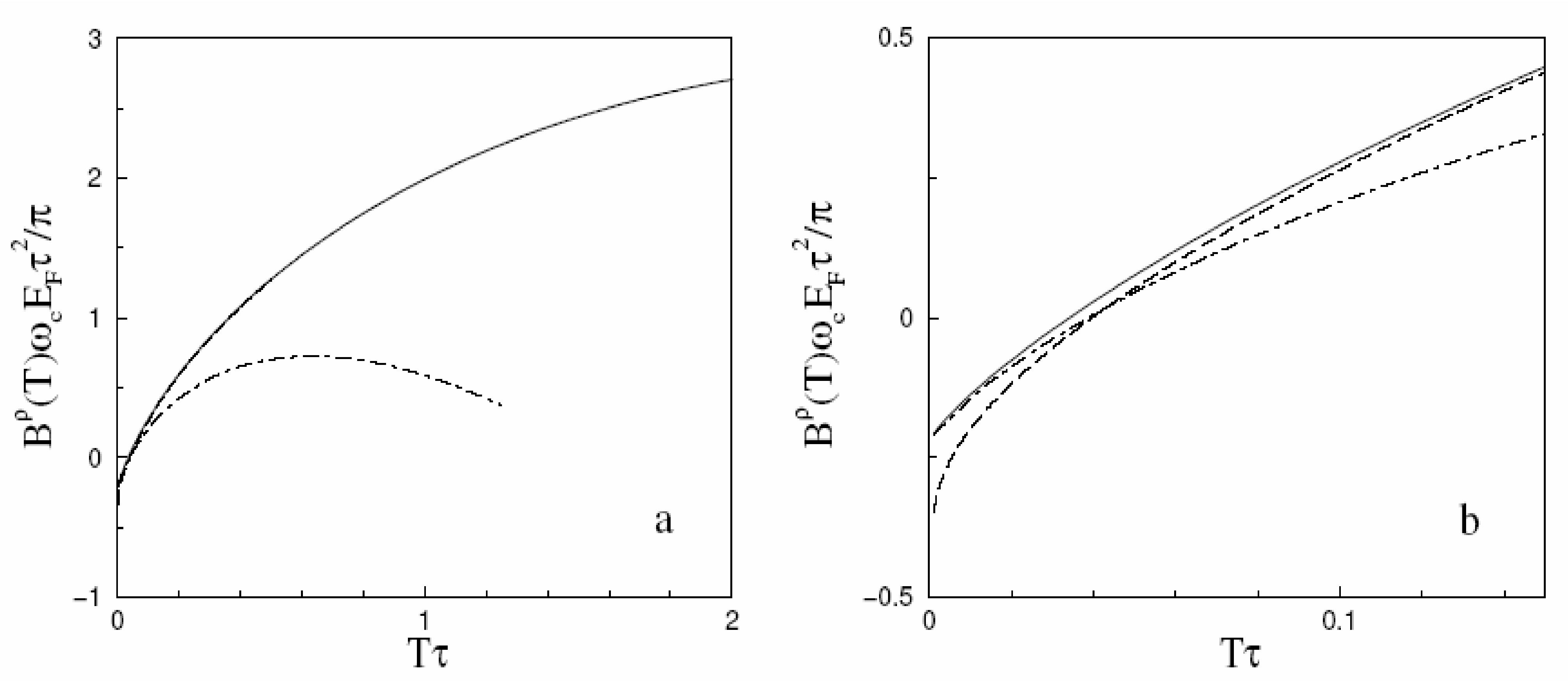
Coulomb interaction

Gauge invariance kills singularity in $V(i\omega_m, \mathbf{q})$

$$B^\rho(T) = \frac{\pi}{\omega_c \tau} \frac{T}{E_F} \left[\left(1 - \frac{1}{8\pi T\tau} \right) \ln \frac{\Delta}{T} + f(4\pi T\tau) \right]$$

$$= \frac{\pi}{\omega_c \tau} \frac{T}{E_F} \times \begin{cases} \frac{3}{2} \ln \frac{\Delta}{T} - \frac{1}{2} \ln \Delta\tau, & 2\pi T\tau \ll 1, \\ \ln \frac{\Delta}{T}, & 2\pi T\tau \gg 1. \end{cases}$$

Ballistics - diffusion crossover



Conclusions

- Interaction makes *m* and τ *T*-dependent

- Dominant effect: *log T* correction to the mass

⟹ *T logT* dependence of damping due to *m(T)*

- Correction to quantum time agrees with that for the conductivity and with enhanced back-scattering off Friedel oscillations

- Gauge invariance kills singularity in $V(\omega,q)$, in contrast to the tunneling density of states