Low-T Transport Induced by Coulomb Interaction in an Anderson Insulator

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The Problem

$D \leq 2:$ ***ANDERSON LOCALIZATION***

$+$ ***ELECTRON-ELECTRON INTERACTION***

at vanishing coupling to the external world (phonons, etc.)

Temperature $T \neq 0:$ **Conductivity** $\sigma(T) = ?$

NO INTERACTION $\Longrightarrow \sigma(T) \equiv 0$ FOR $\forall\, T$

TEMPERATURE : DISTRIBUTION OVER ***LOCALIZED*** STATES

$\sigma(T)$ **is (possibly) nonzero due to e-e interaction only !**

Electron-Electron Interactions in 2D and 1D

- **High T + interaction: strong dephasing $L_\phi \ll \xi$**

▷ **singular weak localization corrections $\Delta\sigma_{\rm WL} \propto L_\phi^{2-d}$**
cut off by e-e scattering, L_ϕ

- **Low T + interaction: strong localization**

Key issue: $\sigma(T)$, or the fate of dephasing in the strongly localized regime

▷ ***Variable-Range Hopping ?***

History of the Problem: First There Was ...

- "Phononless Hopping" $\equiv$ VRH due to e-e scattering

  Fleishman, Licciardello & Anderson '78

- No VRH with e-e scattering for short-range interaction

  Fleishman & Anderson '80

Interaction between electrons $V(r) \propto r^{-\gamma}$

$$\gamma + 2 > d: \quad \sigma = 0$$

Coulomb: $\gamma = 1 \rightarrow d = 3$ – critical dimensionality

Short-range:	$\sigma = 0$
Long-range:	standard VRH
Critical:	?

Fleishman & Anderson '80

Outline:

- ▷ ***Localization vs interaction-induced dephasing in 2D and quasi-1D: $\sigma(T)$ in the regime of strong localization***
- ▷ ***Localization & dephasing in 1D: disordered Luttinger liquid***
- ▷ ***Role of the "long-rangeness" of Coulomb interaction***

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Model and Parameters

- **Weak** ***short-range e-e interaction,***

$$\alpha \equiv \nu V_{q=0} \ll 1 \qquad \longleftarrow \text{ our main parameter}$$

- **Weak** ***white-noise disorder*** $\langle UU \rangle_{q=0} = \frac{1}{2\pi\nu\tau}$, $\quad \epsilon_F \tau \gg 1$

- $\Delta_\xi \ll \tau^{-1} \ll \epsilon_F$, $\quad \Delta_\xi = 1/\nu\xi^d$ – ***level spacing in the localization volume***

For definiteness $\rightarrow$ Thick (many-channel) quantum wire

On the metallic side: $|\Delta\sigma_{\mathrm{WL}}|/\sigma_{\mathrm{D}} \sim L_\phi/\xi \sim (T/T_1)^{-1/3}$

$$\tau_\phi^{-1} \sim \Delta_\xi \, (T/T_1)^{2/3}$$

$T < T_1 = \Delta_\xi/\alpha^2 \longrightarrow \tau_\phi^{-1} < \Delta_\xi$: localization strong

Intermediate T: Power-Law Hopping

cf. Gogolin, Mel'nikov & Rashba '75 (phonons)
Basko '03 (dynamical localization)

$T_3 < T < T_1$: Strong localization, but $\sigma(T)$ – *power-law* (not exponential) function of T

$T_1 = \Delta_\xi/\alpha^2$: $\tau_\phi^{-1} \sim \Delta_\xi$ – *single-particle level spacing*

$T_3 = \Delta_\xi/\alpha$: $\tau_\phi^{-1} \sim \Delta_\xi^{(3)} = \Delta_\xi^2/T$ – *three-particle level spacing in the localization volume*

Diffusion over strongly overlapped localized states:

$$\sigma(T) \sim \sigma_{\rm ac}(\Omega = i/\tau_\phi) \sim e^2\nu\,\xi^2/\tau_\phi$$

$$\tau_\phi^{-1} \sim |V|^2/\Delta_\xi^{(3)} \sim \alpha^2 T$$ $\leftarrow$ *still from the Golden Rule*

$T < T_3$: Breakdown of the Perturbative Scheme

(Lifetime of localized states)$^{-1}$: $\qquad \tau_\phi^{-1} \sim V^2/\Delta_\xi^{(3)} < \Delta_\xi^{(3)}$

Matrix element of interaction
with energy transfer $\sim \Delta_\xi$: $\qquad V \sim \alpha\Delta_\xi < \Delta_\xi^{(3)}$

No decay at the Golden Rule level !

$\sigma_{VRH} = 0$ *Fleishman & Anderson '80*

Metal-Insulator Transition ?

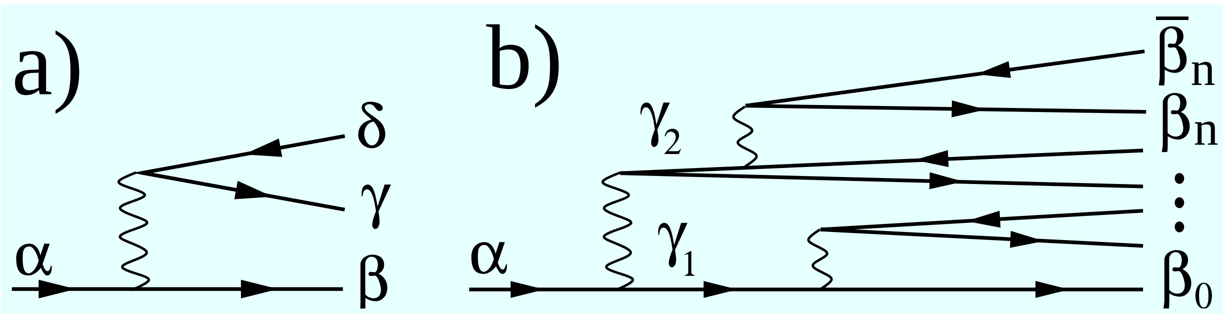
Many-Particle Hopping ?

cf. ***Altshuler, Gefen, Kamenev & Levitov '97 (quantum dots)***

$$T < T_3 \longrightarrow \text{higher-order } V^{(n)}/\Delta^{(2n+1)}$$

$|V^{(n)}|/\Delta^{(2n+1)} \ll 1$: shifts energy levels (ReΣ) , ***but no real transitions***

$|V^{(n)}|/\Delta^{(2n+1)} > c \sim 1$, all $n > n_*$: ImΣ appears $\Leftrightarrow$ ***real transitions***



$$V^{(n)} = \sum_{\text{diagrams}} \sum_{\gamma_1,\dots\gamma_{n-1}} V_1 \prod_{i=1}^{n-1} \frac{V_{i+1}}{E_i - \epsilon_{\gamma_i}}$$

Ballistic Electron-Hole Strings

Parametrize the n-*th* order process by (n, m):

$m\xi$ – **size of the region occupied by n e-h pairs** $(1 \lesssim m \lesssim n)$

$$\Delta^{(2n+1)} \sim \Delta_\xi \left(\frac{n}{m}\frac{\Delta_\xi}{T}\right)^n$$

$$V^{(n)} \sim \alpha^n \Delta_\xi \left[M_m^{(n)}\right]^{1/2}$$

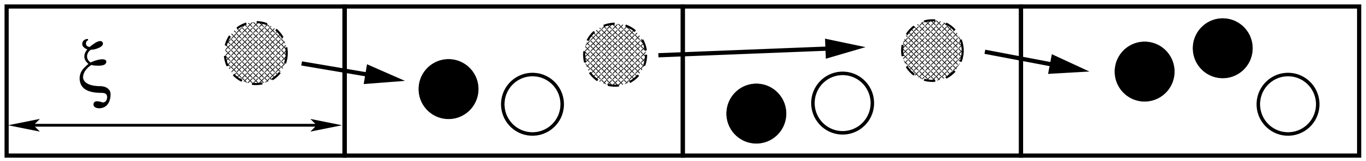
Multiplicativity $M_m^{(n)} \sim (n/m)^n$

$$\frac{V^{(n)}}{\Delta^{(2n+1)}} \sim \left[\alpha \left(\frac{m}{n}\right)^{1/2} \frac{T}{\Delta_\xi}\right]^n$$

Optimal paths $\longrightarrow$ ballistic $(m \sim n)$ strings of e-h pairs

Gornyi, Mirlin & Polyakov: cond-mat/0407305; PRL '05

Error in $(n!)$ in the earlier version: ***Acknowledgment to D.Basko, I.Aleiner & B.Altshuler***



Metal-Insulator Transition

Main contribution to the higher-order coupling constant:

Ballistic strings of e-h pairs $\longrightarrow$ $|V^{(n)}|/\Delta^{(2n+1)} \sim (T/T_3)^n$

Critical temperature $T_c \sim T_3$: $\sigma(T < T_c) = 0$

Gornyi, Mirlin & Polyakov: PRL* 95, *206603 (2005)

Basko, Aleiner & Altshuler: cond-mat/0506617
for weakly coupled granules

Critical behavior of $\sigma(T)$: **Ballistic strings** $\equiv$ **Bethe lattice**

$$\ln \sigma(T) \propto 1/(T - T_c)^{1/2}$$

Gornyi, Mirlin & Polyakov: PRL* 95, *206603 (2005)

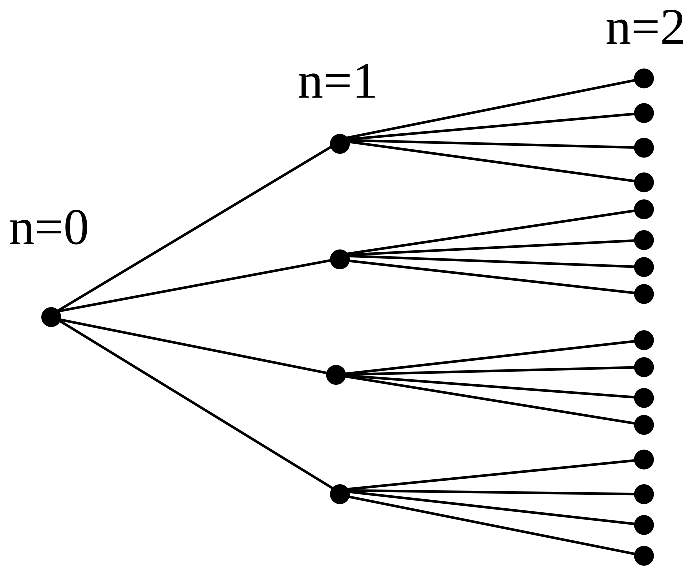
Anderson-Fock glass: Mapping onto the Bethe lattice

Interacting problem in Fock space

$\longrightarrow$ ***Ballistic e-h strings***

$\longrightarrow$ ***nonint. Anderson problem on Bethe lattice*** *Abou-Chacra et al. '73*

$\longrightarrow$ ***Metal-Insulator Transition*** ***at*** $\Delta/V = 4\ln K$



Branching # : $K \sim \frac{\Delta_\xi}{\Delta_\xi^{(3)}} \sim \frac{T}{\Delta_\xi} \gg 1$

$$\Delta \to \Delta_\xi^{(3)} \qquad V \to \alpha\Delta_\xi$$

Transition at $T_c \sim \frac{\Delta_\xi}{\alpha \ln(1/\alpha)}$

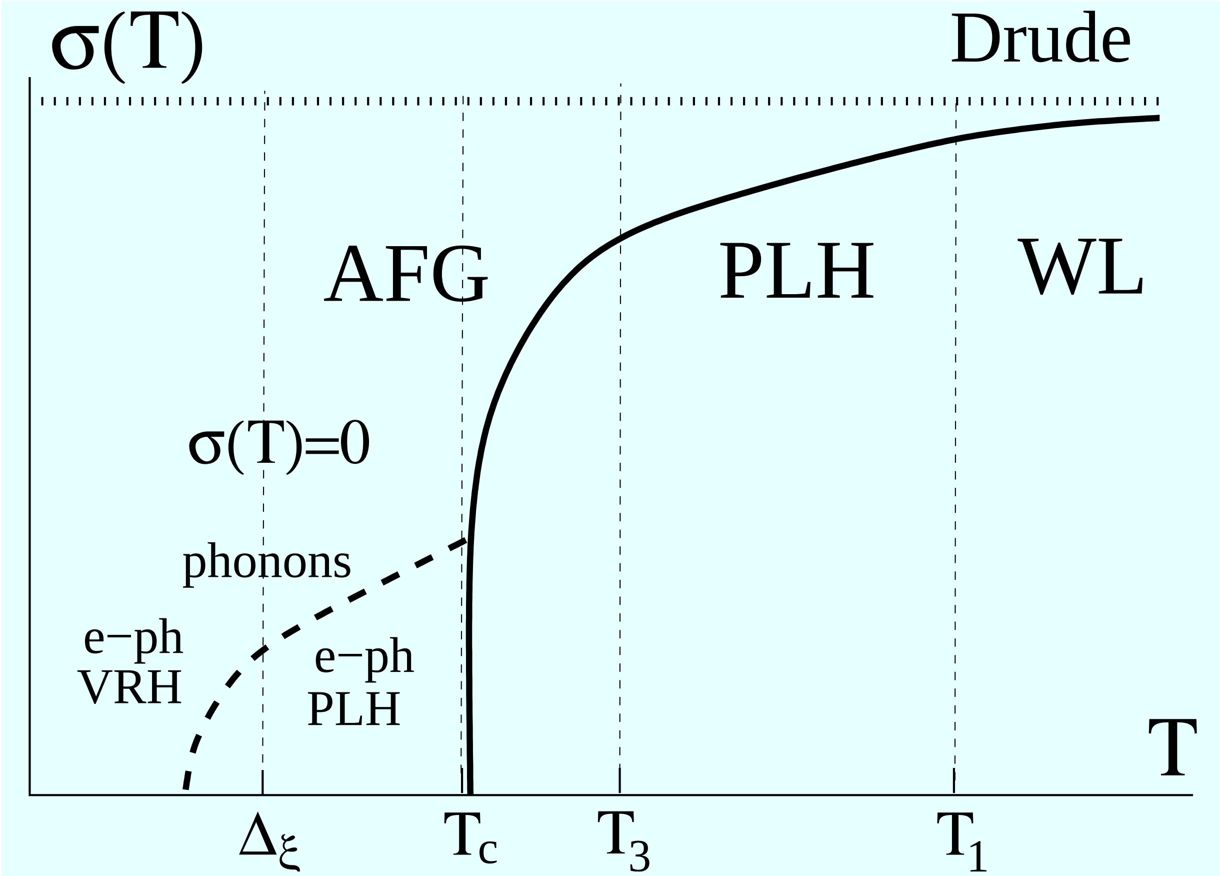
Critical behavior $\sigma(T) \propto \tau_\phi^{-1} \propto \exp\left[-c_\alpha \left(\frac{T_c}{T-T_c}\right)^{1/2}\right]$

Zirnbauer, Efetov, Mirlin/Fyodorov

$c_\alpha \sim \ln \alpha^{-1}$

Absence of Diffusion in Certain Random Lattices,

Even at Finite Temperature



log-log scale

Interaction-induced dephasing rate $\boldsymbol{\tau_\phi^{-1} = 0}$
between $\boldsymbol{T = 0}$ ***and*** $\boldsymbol{T_c}$

Mesoscopics of single-channel quantum wires

- **Interaction, no disorder:** ***Luttinger liquid***
  Conventional wisdom: Non-Fermi liquid
  Proper language = Bosons (density field)

- **Fermi liquid + disorder:** $\boldsymbol{\sigma(T)} \longrightarrow$ ***$T > T_c$: entirely due to dephasing***

- **Luttinger liquid + disorder:** $\boldsymbol{\sigma(T) = ?}$

Fermi-quasiparticle breakdown $\Rightarrow$

Key issue: What is dephasing in Luttinger liquid,
or,
Are the notions of mesoscopics applicable to Luttinger liquid?

Weak-localization correction to the conductivity of Luttinger liquid :

$$\sigma_{\mathrm{WL}} = -2\sigma_{\mathrm{D}} \int_0^\infty dt \int_0^\infty dt_a \, P(t, t_a) \; \exp[-S(t, t_a)]$$

$P \sim 1/\tau^2$ – *probability density of return*

$S \sim (t/\tau_\phi)^2$ – *dephasing action*

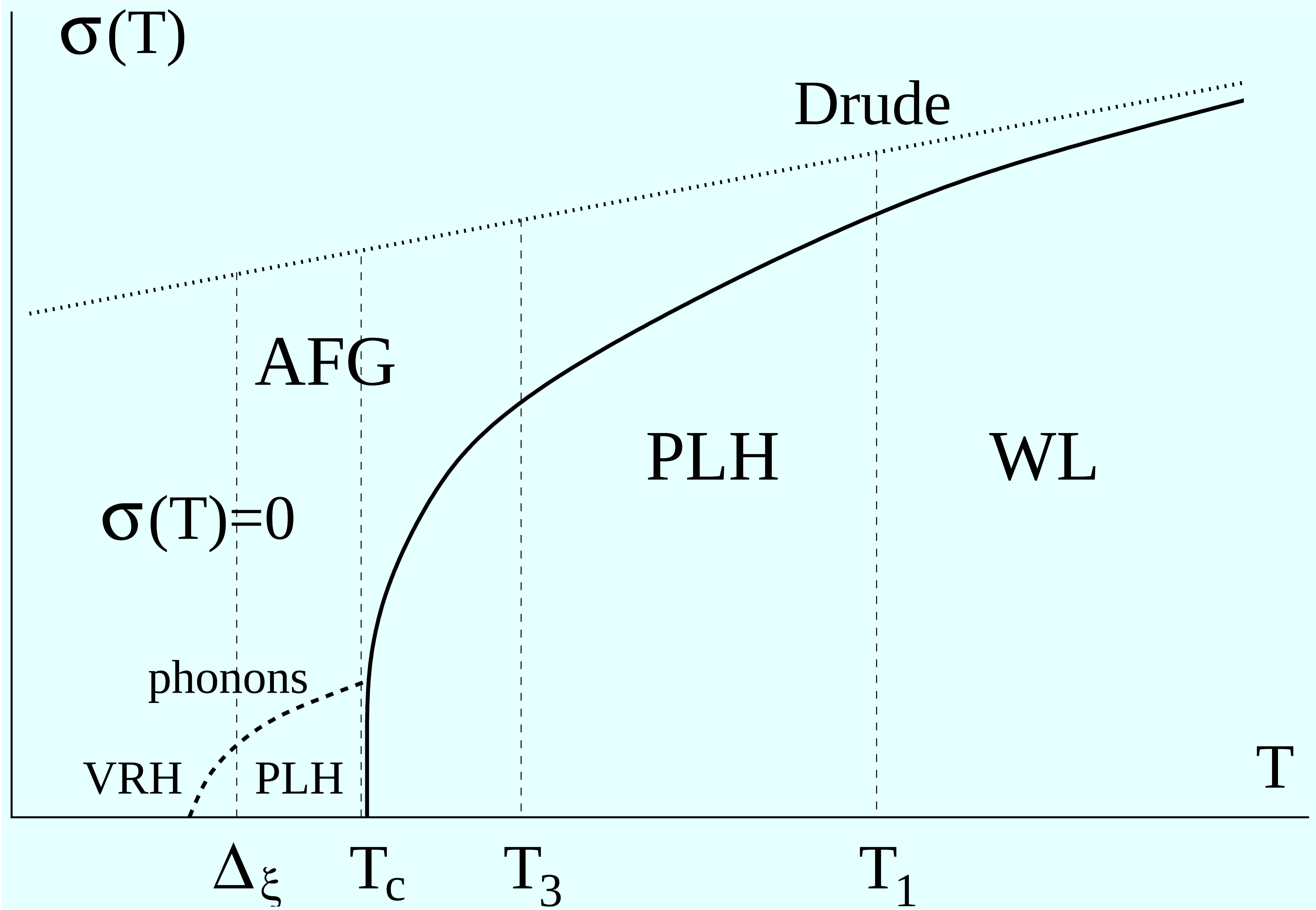
Dephasing of weak localization in Luttinger liquid: Results

Gornyi, Mirlin & Polyakov, PRL 95, 046404 (2005)

Localization correction : $$\sigma = -\frac{1}{4}\sigma_{\mathrm{D}}\left(\frac{\tau_\phi}{\tau}\right)^2 \ln\frac{\tau}{\tau_\phi} \propto \frac{1}{\alpha^2 T}\ln(\alpha^2 T)$$

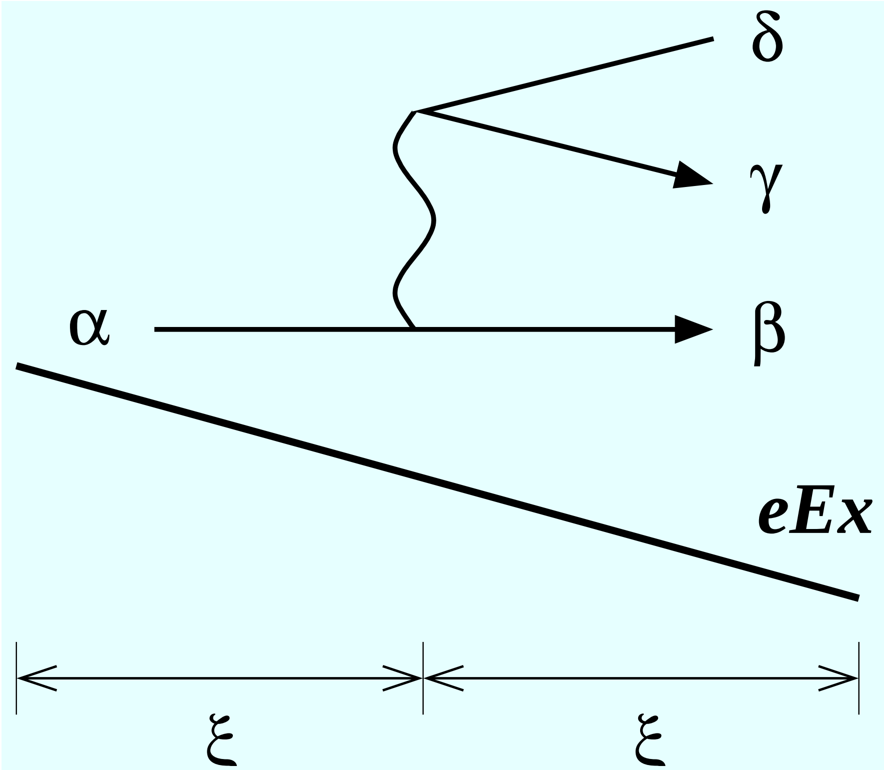
Dephasing rate : $$1/\tau_\phi = \alpha(\pi T/\tau)^{1/2}$$

- *"τ_ϕ^{-1} in 1D: Fermi-liquid approach inapplicable"* $\leftarrow$ *wrong*
- $\tau^{-1}(T) \sim \tau_0^{-1}(\epsilon_F/T)^{2\alpha}$ – *strongly T-dependent τ and ξ*
- *dephasing of spinless electrons:*
  - ▷ $\tau_\phi^{-1} \propto \tau^{-1/2} \ll \tau_{ee}^{-1}$ – *(lifetime)$^{-1}$ of particles in clean Lutt. liquid*
  - ▷ $\delta\tau_\phi^{-1} \sim \alpha^2 T^2/mv_F^2$, $\quad m^{-1} =$ *curvature*
- *dephasing of spinful electrons:*
  - ▷ $\tau_\phi^{-1} \sim \alpha T \propto |u_c - u_s|^{-1}$ – *singular in spin-charge separation*



log-log scale

Creep



"Effective temperature"

$$T \to eE\xi$$

Three-particle DoS
for transitions between
neighboring localization volumes

$$\nu^{(3)} \sim eE/\Delta_\xi^2$$

Threshold in electric field : $$E_c \sim \frac{1}{e\xi}\,\frac{\Delta_\xi}{\alpha \ln(1/\alpha)}$$

Creep for $E > E_c$: $$\ln \sigma(E) \propto 1/(E - E_c)^{1/2}$$

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Transport Mediated by Coulomb Interaction

Screening within the loc. volume: $\kappa(q \sim \xi^{-1}) \sim (\xi/\lambda_s)^{d-1}$

Matrix element between neighboring states :

$$\text{Before:} \quad V \sim \alpha \Delta_\xi$$

$$\text{Now:} \quad V \sim \frac{e^2}{\xi \kappa(q \sim \xi^{-1})} \sim \Delta_\xi \,, \qquad \alpha \sim 1 \ (\textit{caution!})$$

Coupling between neighboring states only $\rightarrow$

Transition at $T = T_c \sim \Delta_\xi$

Are e_*^2/r **tails of screened Coulomb interaction important ?**

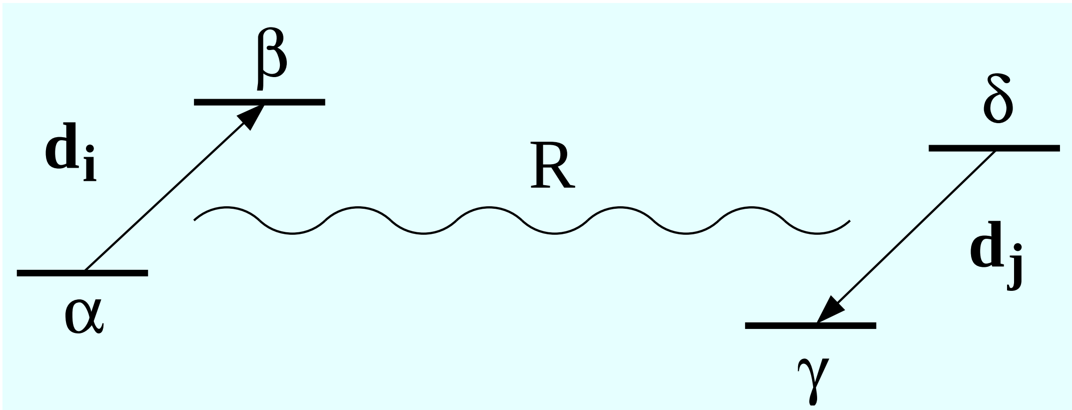
Transport Mediated by Coulomb Interaction (cont'd)

Matrix element between distant states :

particle $\longrightarrow$ 2 (distant) particles + hole: dipole-dipole interaction

$$V_{ij} = \frac{\mathrm{d}_i \mathrm{d}_j}{R^3} - \frac{3(\mathrm{d}_i \mathrm{R})(\mathrm{d}_j \mathrm{R})}{R^5}$$

between two e-h pairs with dipole moments d_i and d_j



$$\mathrm{d}_{\alpha\beta} \to (\mathrm{r}_\alpha - \mathrm{r}_\beta) I_{\alpha\beta}/|\omega_{\alpha\beta}| \propto \exp(-|\mathrm{r}_\alpha - \mathrm{r}_\beta|/\xi)$$

Transport Mediated by Coulomb Interaction (cont'd)

Dipole-dipole interaction $V_{ij} = \frac{\mathrm{d}_i \mathrm{d}_j}{R^3} - \frac{3(\mathrm{d}_i \mathrm{R})(\mathrm{d}_j \mathrm{R})}{R^5}$

Coupling constant for $e \longrightarrow 2e + h$ **decay:** $c(R) = \nu p^2 \int d\mathrm{R}/R^3$

(***probability of a dipole-dipole resonance within radius*** R)

$$p \propto \exp[-(T_0/T)^{1/2}] \ , \qquad T_0 \sim e_*^2/\xi$$

$d = 3$ **– critical dimensionality:** $c(R) = \nu p^2 \ln(R/l_h)$

$l_h \sim \xi (T_0/T)^{1/2}$ – ***hopping length***

Distance R_c **between resonating pairs:** $c(R_c) \sim 1$

$$\tau_\phi^{-1} \sim V \ \propto R_c^{-3} \ \propto \exp(-\#/\nu p^2)$$

$\# \sim 1$ – ***from percolation theory***

$$\ln \ln \sigma \sim -(T_0/T)^{1/2}$$

Summary

TRANSPORT INDUCED BY ELECTRON-ELECTRON INTERACTIONS IN DISORDERED SYSTEMS

- ***Anderson-Fock Glass,*** $\boldsymbol{\sigma(T < T_c) = 0}$
- ***Dephasing in Luttinger liquid:***
  Mesoscopics of strongly correlated systems
- ***Coulomb-interaction mediated transport***
  in 3D Anderson insulators: Double-exp behavior