Charge transfer statistics beyond second moment

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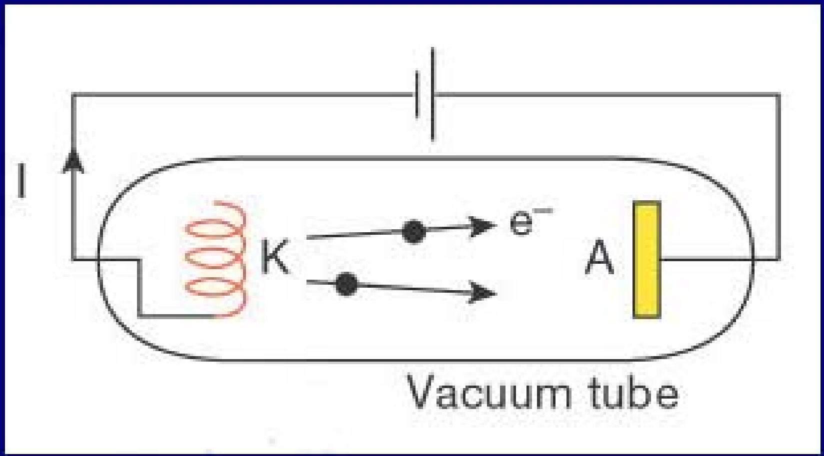
Outline

- History of the Shot noise
- Motivation for studying counting statistics
- Previous work, importance of the environment
- Our measurements in tunneling junctions

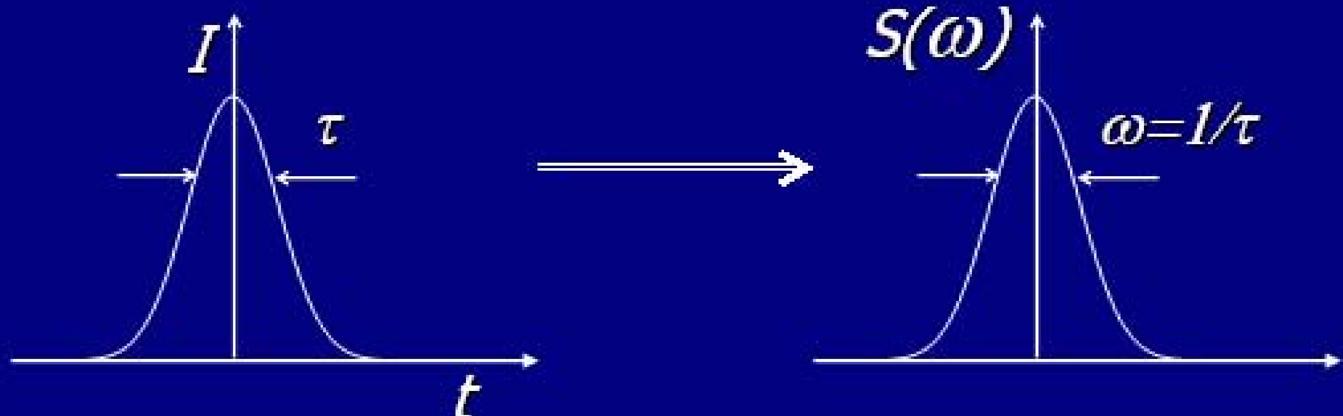
Classical Shot Noise Schottky, 1918

$$P(n) = \frac{(n)^{n} e^{-\bar{n}}}{n!}$$ probability

for n electrons to be transmitted



$$S(\omega) = e\langle I \rangle$$

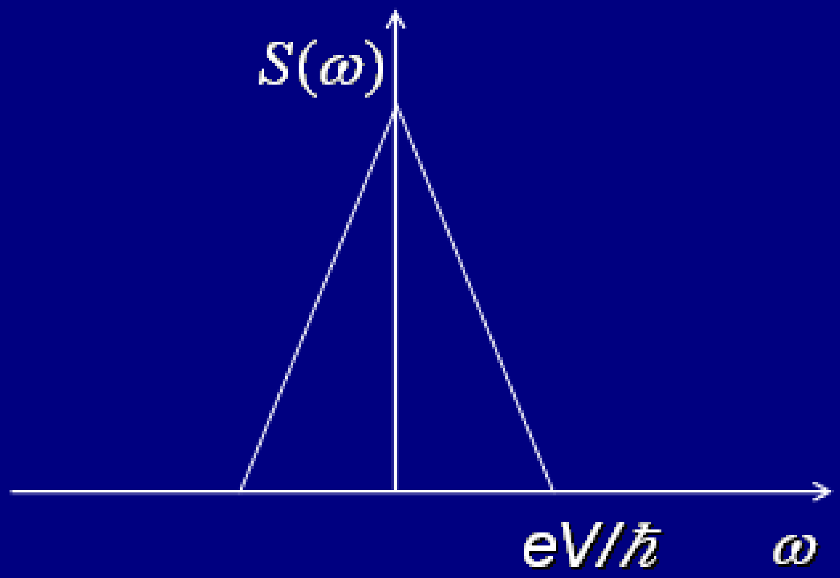


Excess noise at finite transmission

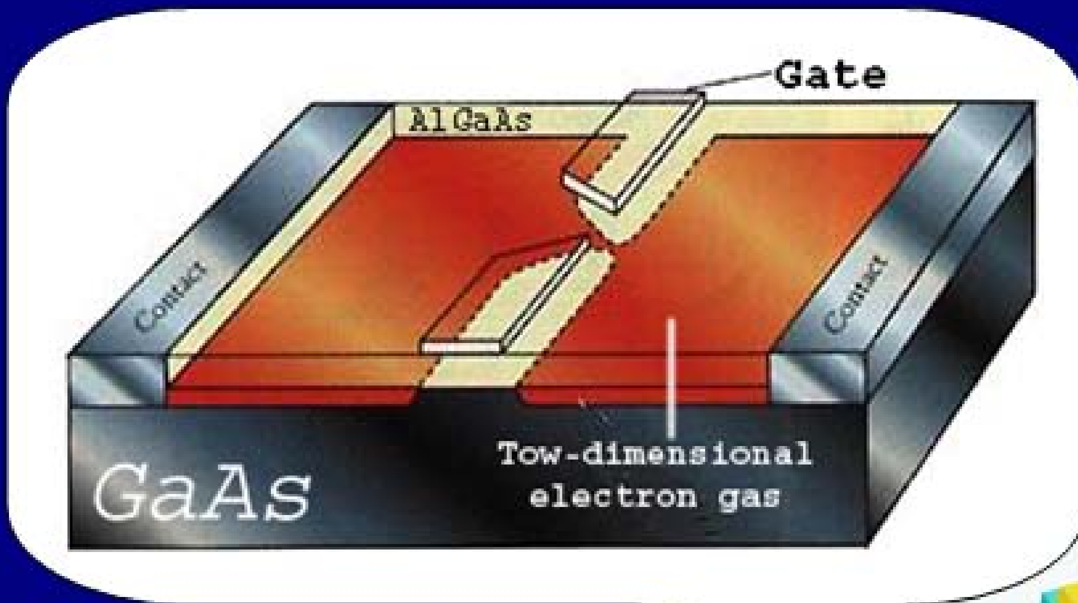
Khlus (1987), Lesovik (1989), Yurke and Kochansky (1989)

$$G = \frac{e^2}{2\pi\hbar} \cdot \sum_{n\alpha} \Gamma_{n\alpha} = g_0 \sum_n \Gamma_{n\alpha} \qquad \text{Landauer formula}$$

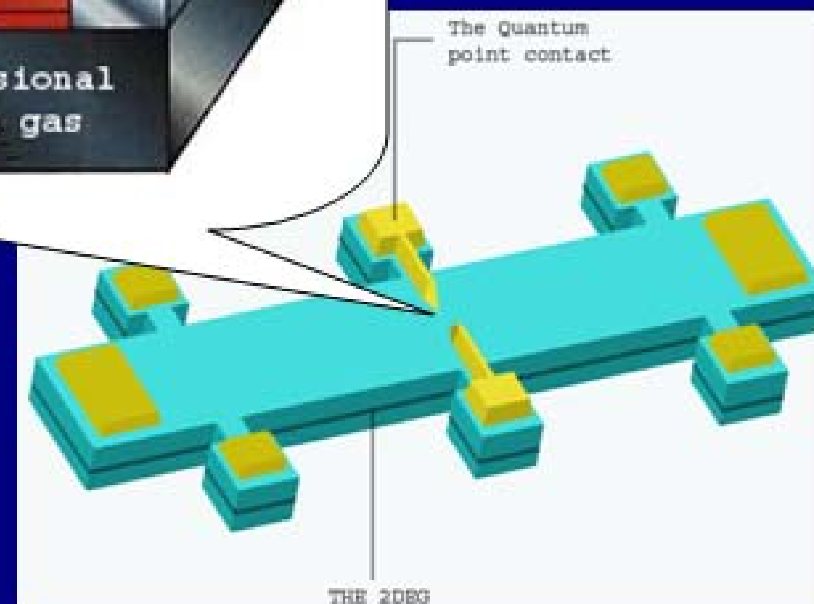
$$S(0) = e g_0 V \sum_{n\alpha} \Gamma_{n\alpha}(1 - \Gamma_{n\alpha}), \qquad T=0$$



Future implementation: QPC



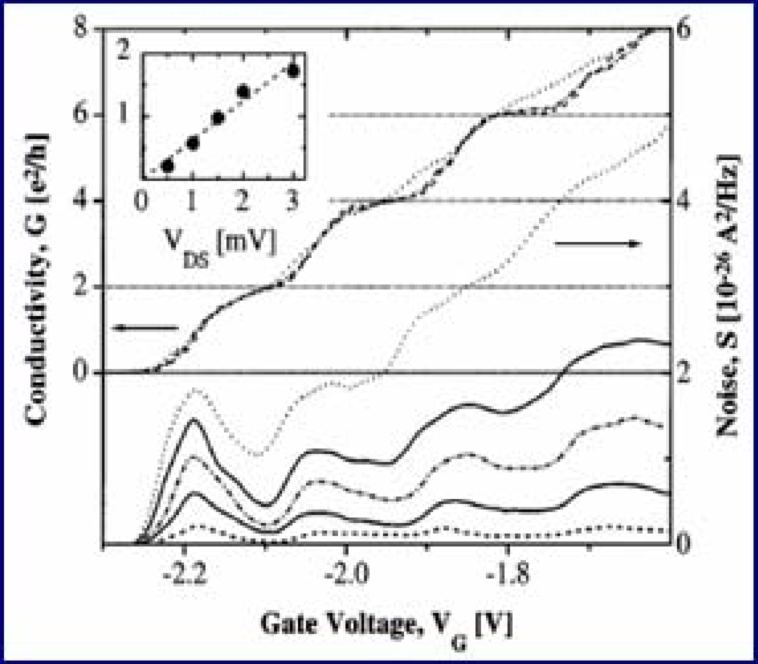
$$L < l_{elastic}$$

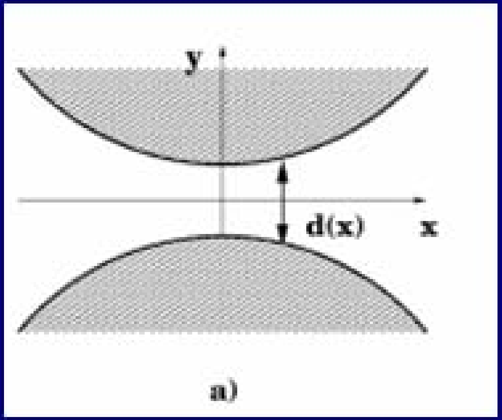


$$S_I^{(3)} = \frac{e^2}{h} e^2 V \sum_n \Gamma_n (1-\Gamma_n)(1-2\Gamma_n)$$

Experimental verification: variable transmission

$$S_I(\omega = 0, T = 0) = \frac{e^2}{2\pi\hbar} eV \sum_n \Gamma_n (1 - \Gamma_n)$$





M. Reznikov, M. Heiblum,
H. Shtrikman and D. Mahalu, 1995

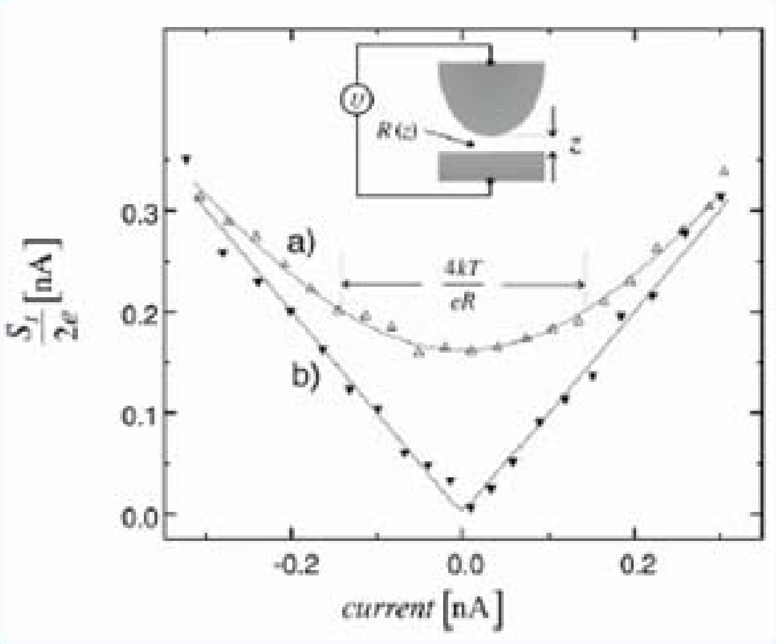
A. Kumar, L. Saminadayar,
D. C. Glattli, Y. Jin and B. Etienne, 1996

Measurements of the second comulant in a Tunneling barrier by Birk et al. (1995)

$$S_I^{(2)}\Big|_{\Gamma\to 0} = \frac{e^2}{2\pi\hbar}\, eVcth\left(\frac{eV}{2K_BT}\right)\sum_n \Gamma_n$$

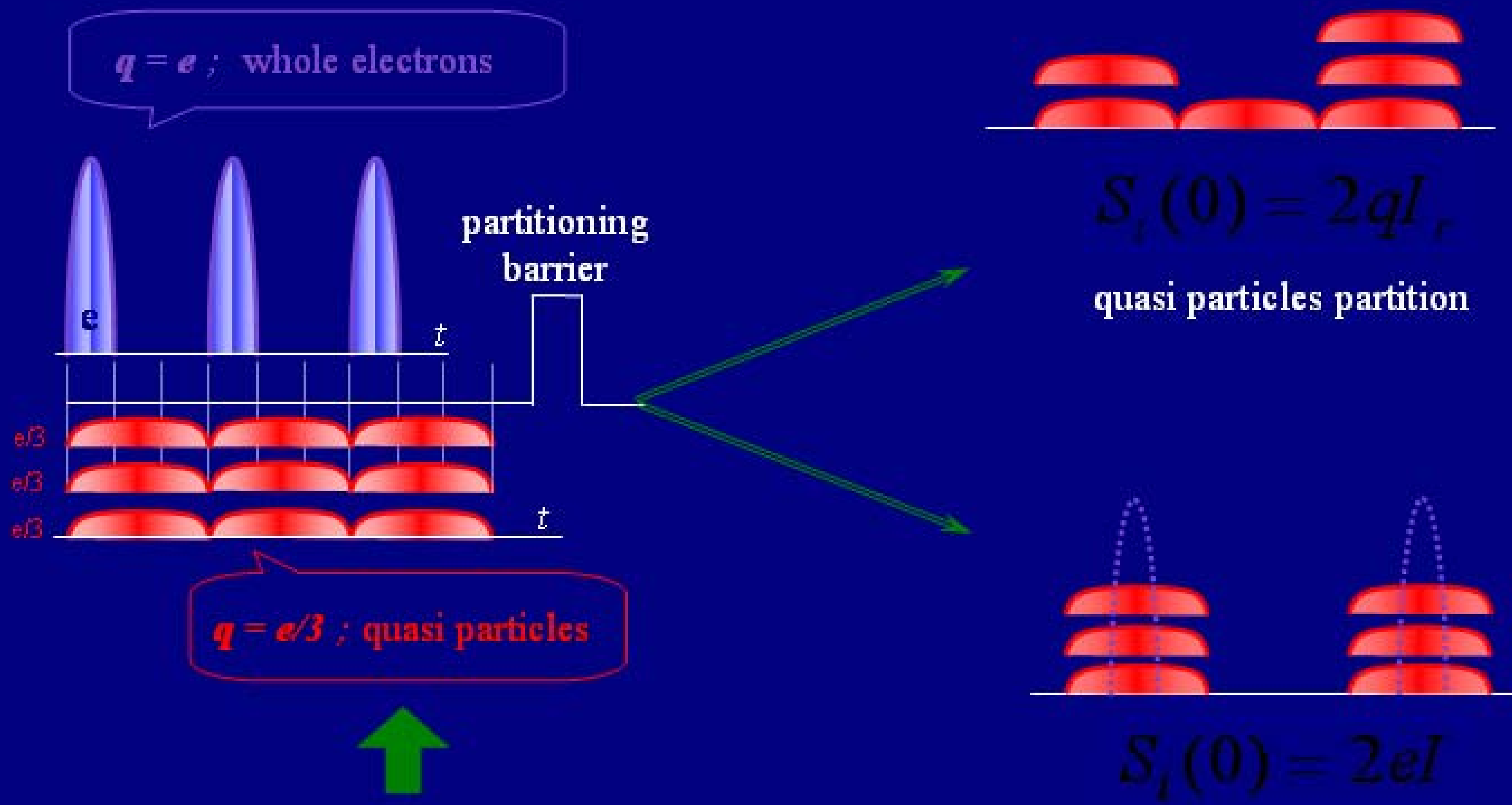
$$= eVGcth\left(\frac{eV}{2K_BT}\right)$$

$$\underset{eV>>k_BT}{=} \begin{cases} e\langle I\rangle & k_BT << eV \\ 2K_BTG & eV << k_BT \end{cases}$$



Expected Noise…..(intuitively)

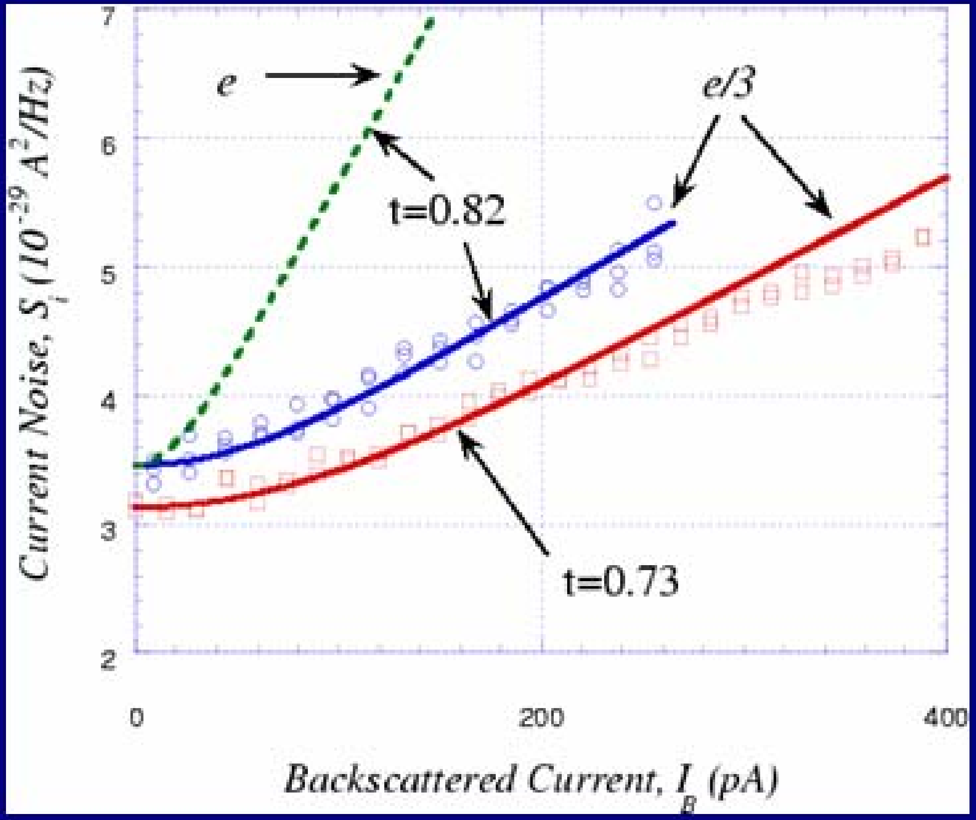
$\nu = 1/3$



Both, e or $e/3$ lead to the same conductance !

Qausiparticle charge measurements in the Fractional QHE regime

$$S(\omega) = qI(1-\Gamma)\coth\left(\frac{qV}{2K_BT}\right) + 2k_BT\cdot G\cdot\Gamma, \qquad \omega << eV,\ T$$



R. de-Picciotto, et. al. 1997

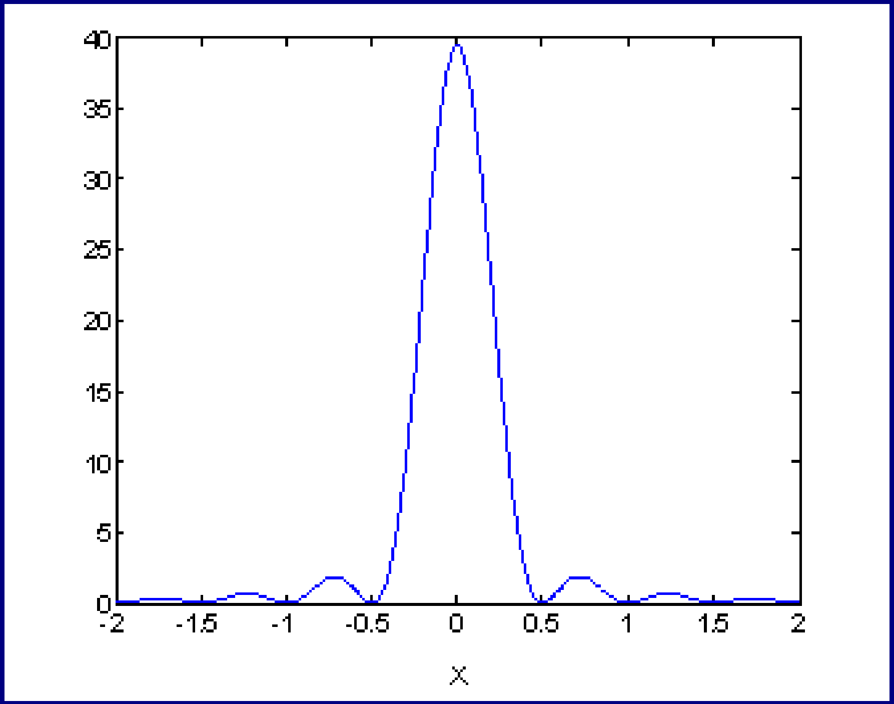
L. Saminadayar et. al. 1997

Justification for week backscattering:

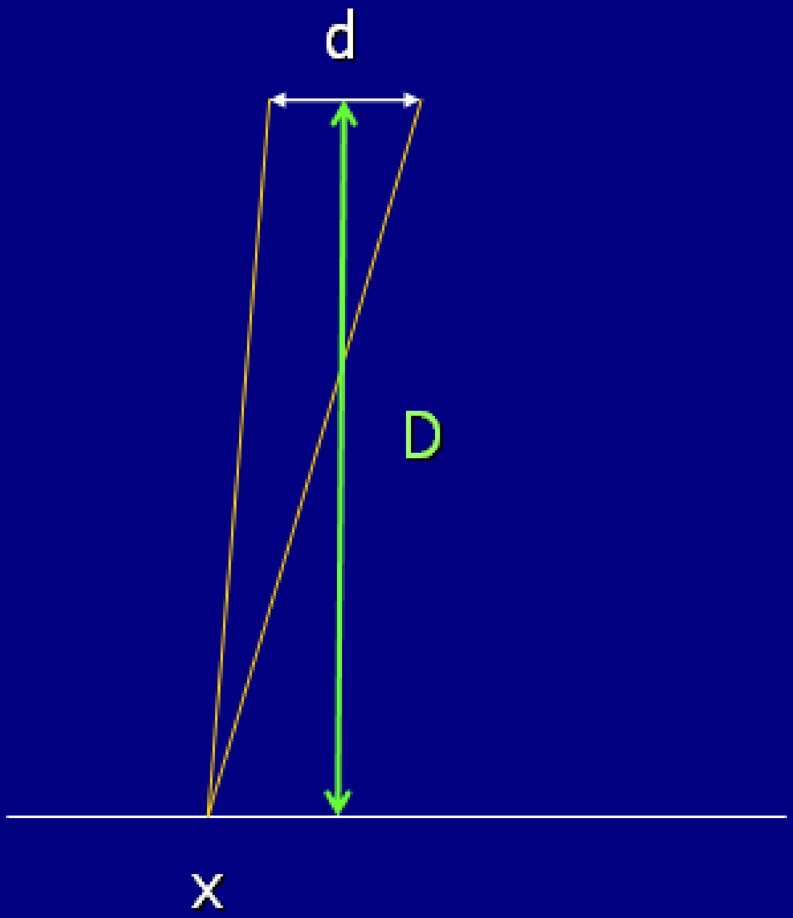
Sukhorukov, Loss, 2001

L. Levitov, M. Reznikov, 2001

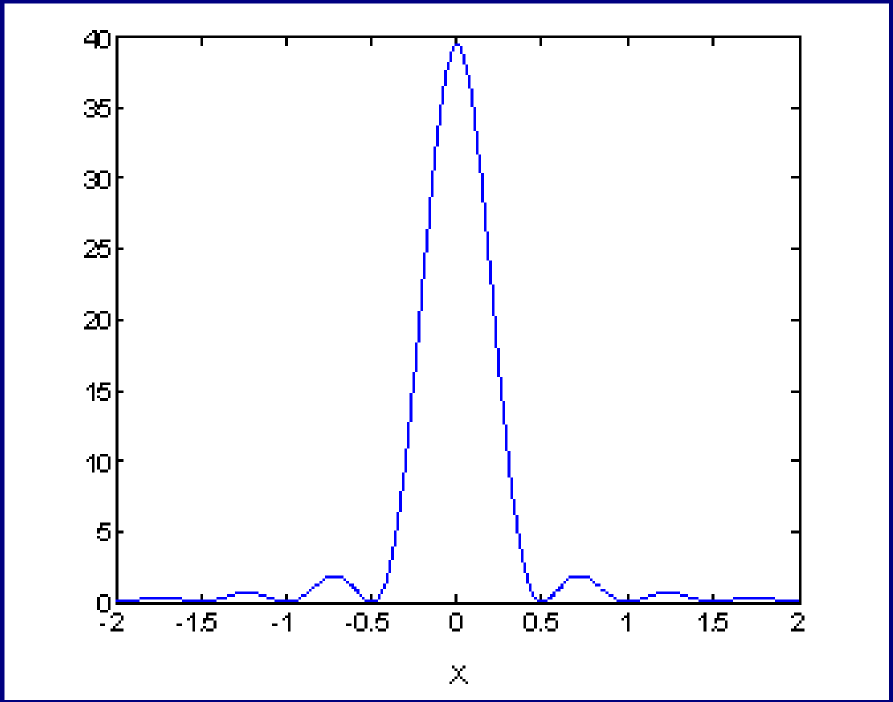
Diffraction on a slit



$$I \propto \frac{\sin^2\left(k\frac{d}{D}x\right)}{\left(k\frac{d}{D}\right)^2}$$



Hanbury Brown and Twiss, 1954



$$\left\langle P(x)P(0)\right\rangle \propto$$

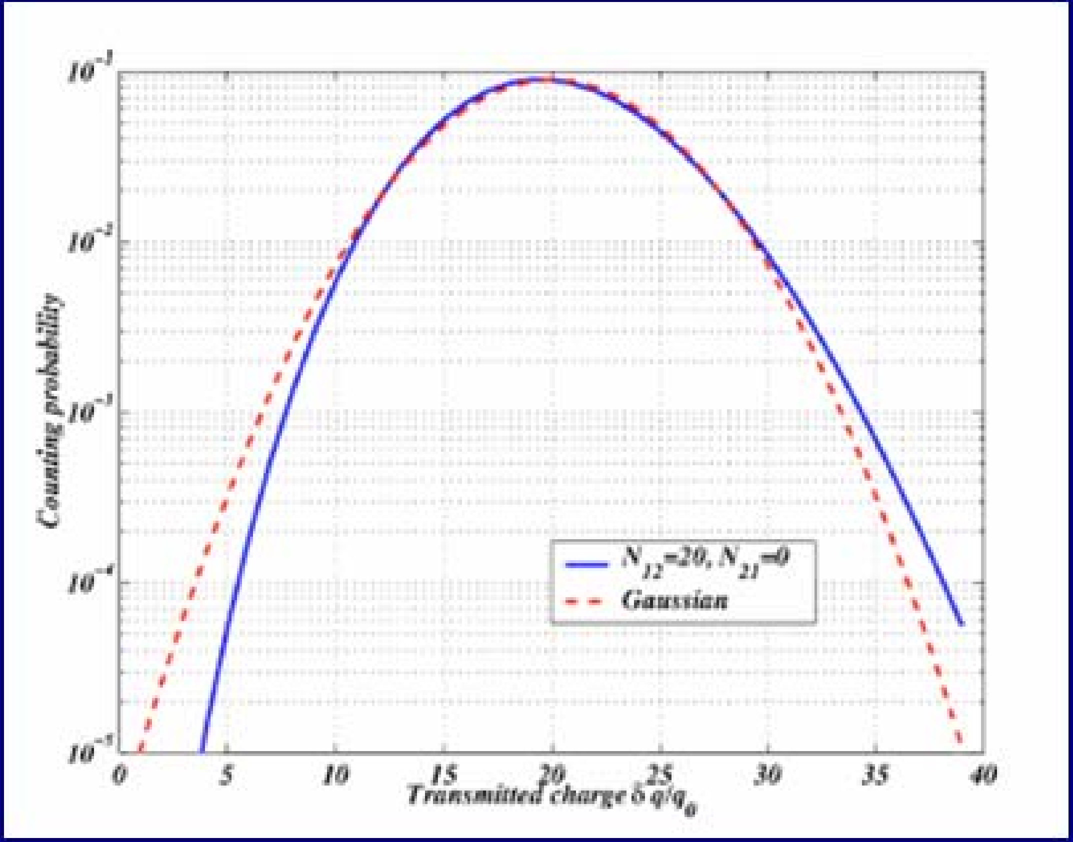
$$\propto \left\langle E(x)E^{*}(x)E(0)E^{*}(0)\right\rangle \propto$$

$$\propto \frac{\sin^{2}\left(k\dfrac{d}{D}x\right)}{\left(k\dfrac{d}{D}\right)^{2}}$$

Important: $\delta n^{2} = \bar{n} + \bar{n}^{2}$ for thermal radiation

Counting statistics

Statistics of the charge transferred during some long time interval τ



$$P_m(\tau) - ?$$

The full counting statistics

moment generation function: $\chi(\lambda) = \sum_m P_m e^{i\lambda m}$

Taylor expansion gives moments

Taylor expansion of $\ln \chi(\lambda)$ gives cumulants

$$\ln \chi(\lambda) = \sum_{k=1}^{\infty} \frac{(i\lambda)^k}{k!} \left\langle \left\langle m^k \right\rangle \right\rangle$$

Cumulants do not always exist,
but when they do, they determine the PDF

Moments vs. cumulants

$$X=\sum_{1}^{n}x_i - \text{ the sum of independent stochastic variables } x_i$$

$$\chi_X(\lambda)=\int e^{i\lambda\sum_{i=1}^{n}x_i}\prod_i P(x_i)d^n x=\left(\chi_x(\lambda)\right)^n$$

$$\ln\chi_X(\lambda)=n\ln\chi_x(\lambda)\sim n$$

Central moments and cumulants

$$q = em$$

$$\langle q \rangle_\tau = \langle\langle q \rangle\rangle_\tau = I\tau \sim \tau \qquad \mathrm{I} = \frac{\langle\langle q \rangle\rangle_\tau}{\tau}$$

$$\langle \delta q^2 \rangle_\tau = \langle\langle q^2 \rangle\rangle_\tau \sim \tau \qquad \mathrm{S}^{(2)} = \frac{\langle\langle q^2 \rangle\rangle_\tau}{\tau}, \quad \omega << \frac{1}{\tau}$$

$$\langle \delta q^3 \rangle_\tau = \langle\langle q^3 \rangle\rangle_\tau \sim \tau \qquad \mathrm{S}^{(3)} = \frac{\langle\langle q^3 \rangle\rangle_\tau}{\tau}, \quad \omega << \frac{1}{\tau}$$

$$\langle \delta q^4 \rangle = \langle\langle q^4 \rangle\rangle + 3\langle\langle q^2 \rangle\rangle\langle\langle q^2 \rangle\rangle$$

Photon counting statistics

Glauber, 1963

$$P_m = \frac{(\eta\tau)^m}{m!}\left\langle :\left(a^+a\right)^m e^{-\eta\tau a^+a} : \right\rangle, \quad P_m \text{ is the probability}$$

to measure m photons by a counter with efficiency η during the time τ in a given single mode state

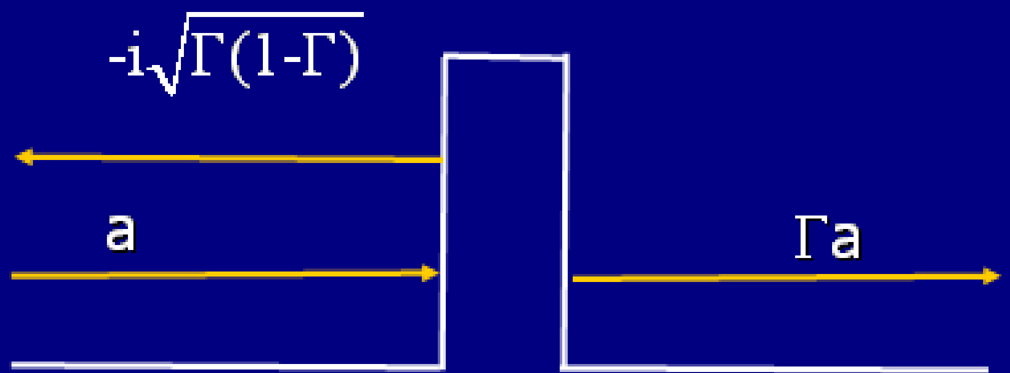
Normal ordering is important and results from the fact that the electron absorbed (counted) by a detector cannot be count once again!

Photon statistics leads to nontrivial effects

e.g. Hanberry Brown and Twiss (1956)

Naïve calculations

$$q(\tau) = \left\langle \int_0^\tau \hat{I}(t)dt \right\rangle \qquad \left\langle \delta q^k \right\rangle_\tau = \left\langle \left(\int_0^\tau \delta\hat{I}(t)dt \right)^k \right\rangle$$



$$I = (a^+ \ \mathrm{b}^+) \begin{pmatrix} \Gamma & -\mathrm{i}\sqrt{\Gamma(1-\Gamma)} \\ \mathrm{i}\sqrt{\Gamma(1-\Gamma)} & \Gamma \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

Naïve calculations

$$q(\tau) = \left\langle \int_0^\tau \hat{I}(t)dt \right\rangle \qquad \left\langle \delta q^k \right\rangle_\tau = \left\langle \left(\int_0^\tau \delta\hat{I}(t)dt \right)^k \right\rangle$$

Consider $\langle\langle I^3 \rangle\rangle$. We show that it does not contain Γ^1 :

The lowest power in Γ occurs from matrix elements which mix a and b :

$$i\sqrt{\Gamma(1-\Gamma)}a^+b \text{ or } -i\sqrt{\Gamma(1-\Gamma)}b^+a$$

The lowest order in Γ terms contain:

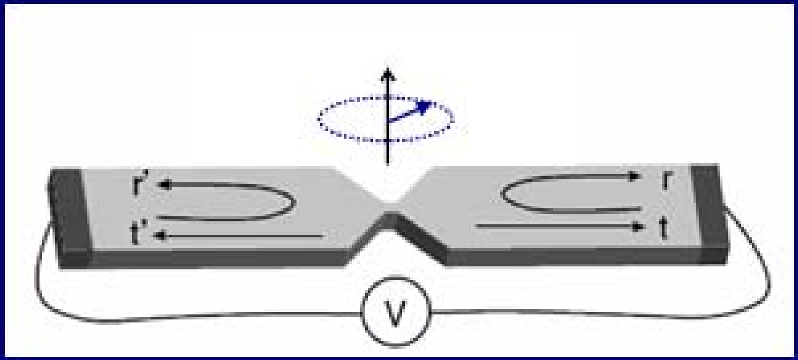
$$\langle a^+bb^+aa^+a \rangle \quad \text{or} \quad \langle a^+bb^+ab^+b \rangle$$

both proportional to $\Gamma^2(1-\Gamma)$

Spin 1/2 as a galvanometer

L.S. Levitov and G.B. Lesovik (1993)

L.S. Levitov and H. Lee (1996)



$$\chi(\lambda) = \left\langle e^{i\hat{H}_{-\lambda}\tau} e^{i\hat{H}_{\lambda}\tau} \right\rangle$$

$$\hat{H}_{\mathrm{int}} = -\frac{\lambda\hbar}{2e}\sigma_z \hat{I}$$

One can still discuss the statistics of the transferred charge without resorting to the current operators (Klich, 2002)

Cumulants at T=0

$$\langle\langle q \rangle\rangle_\tau = \Gamma \frac{e^2}{2\pi\hbar} V\tau$$

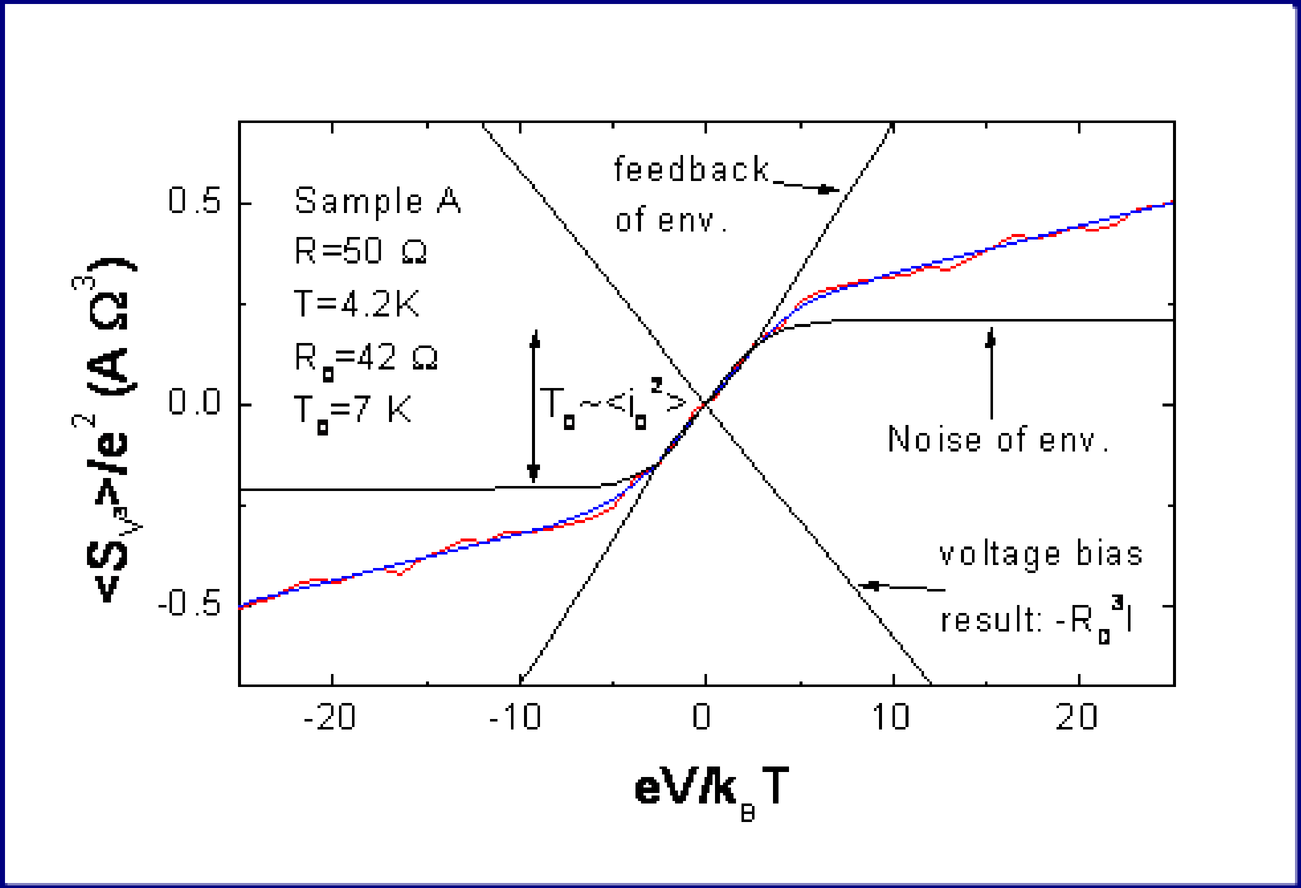
$$\langle\langle q^2 \rangle\rangle_\tau = e\Gamma(1-\Gamma) \frac{e^2}{2\pi\hbar} V\tau$$

$$\langle\langle q^3 \rangle\rangle_\tau = e\Gamma(1-\Gamma)(1-2\Gamma) \frac{e^2}{2\pi\hbar} V\tau$$

T>>eV

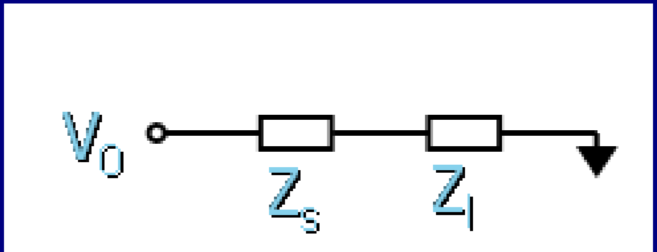
$$\langle\langle q^3 \rangle\rangle_\tau = e^2\Gamma(1-\Gamma) \frac{e^2}{2\pi\hbar} V\tau$$

Experimental results from Yale



B. Reulet, J. Senzier and D. E. Prober, 2002

Voltage bias



$Z_s >> Z_l$ – voltage bias

number of attempts fixed: $\phi = V_0 \tau e / h$

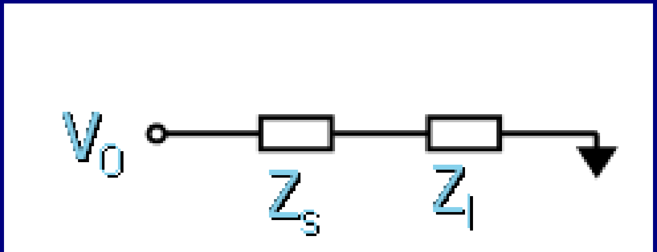
measured: fluctuations of $q = en = \int_0^\tau dt I(t)$

Binomial statistics of charge:

$$P_{\phi_0}(n) = \binom{\phi_0}{n} \Gamma^n (1-\Gamma)^{\phi_0 - n}$$

Kindermann, Nazarov, Beenakker (2002)

Current bias



$Z_s << Z_l$ – current bias

Current and therefore transmitted charge q is fixed

Measured: fluctuations of $\phi = \frac{e}{h}\int_0^{\tau} dt V(t)$

Flux distribution is Pascal: $P_{q_0}(\phi) = \binom{\phi - 1}{\phi_0 - 1} \Gamma^{\phi_0} (1-\Gamma)^{\phi - \phi_0}$

General case

Voltage bias: $S^{(3)} = \langle\langle \delta q^3 \rangle\rangle / \tau = (e)^2 \langle I \rangle$

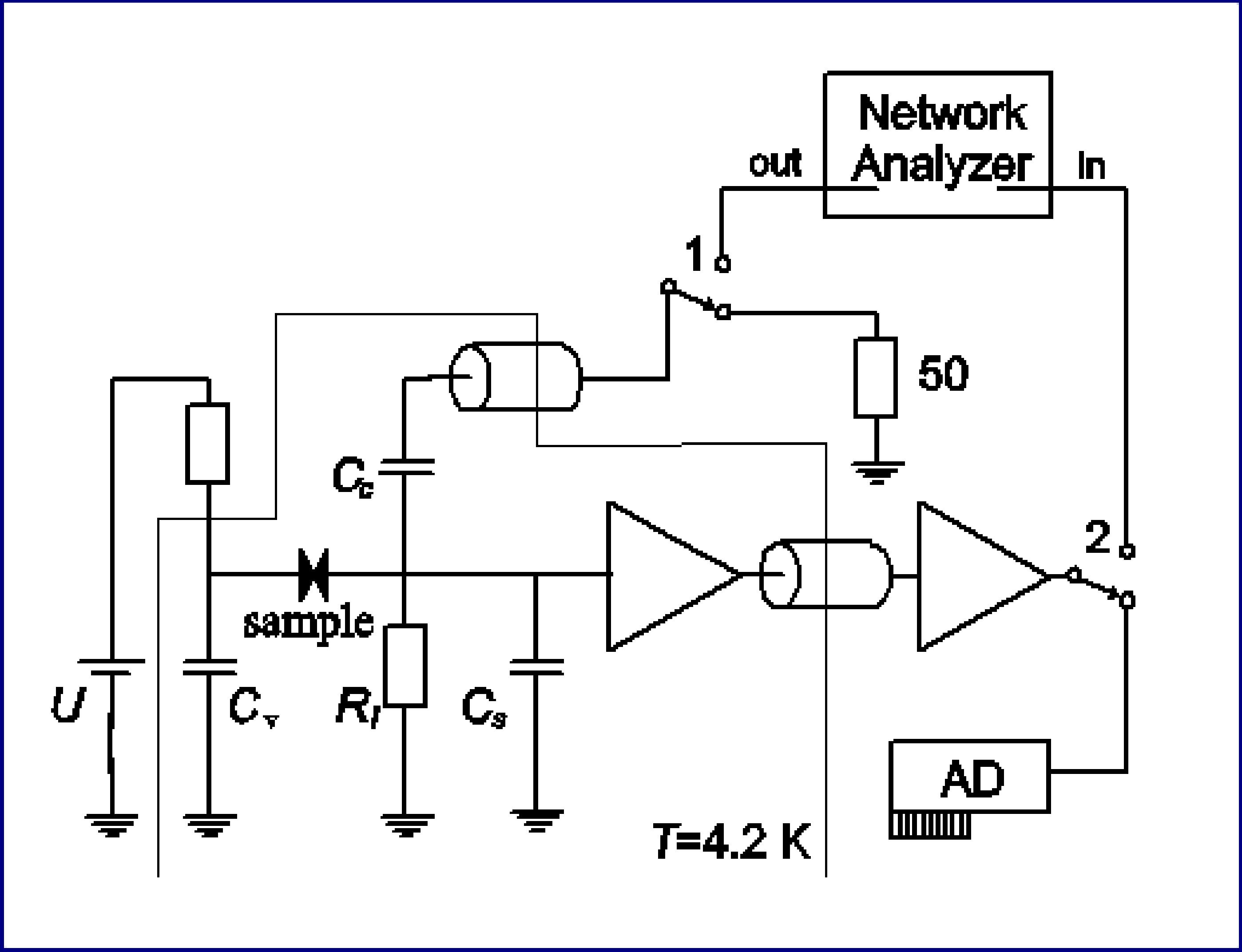
Current bias: $S_V^{(3)} = -R^3 2(e)^2 \langle I \rangle$

In general:

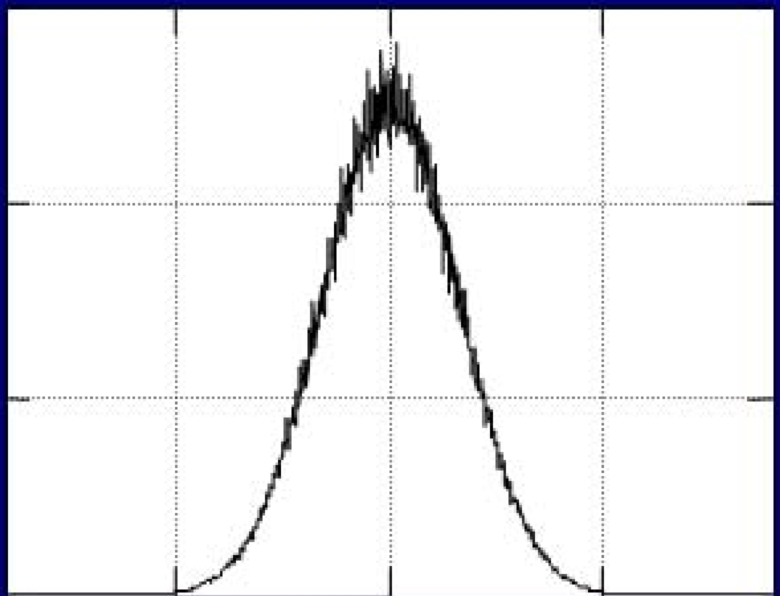
$$S_V^{(3)} = R^3 \left((e)^2 \langle I \rangle - 3RS^{(2)} \frac{\partial S^{(2)}}{\partial V} \right)$$

Kindermann, Nazarov, Beenakker (2002)

The measurement Setup

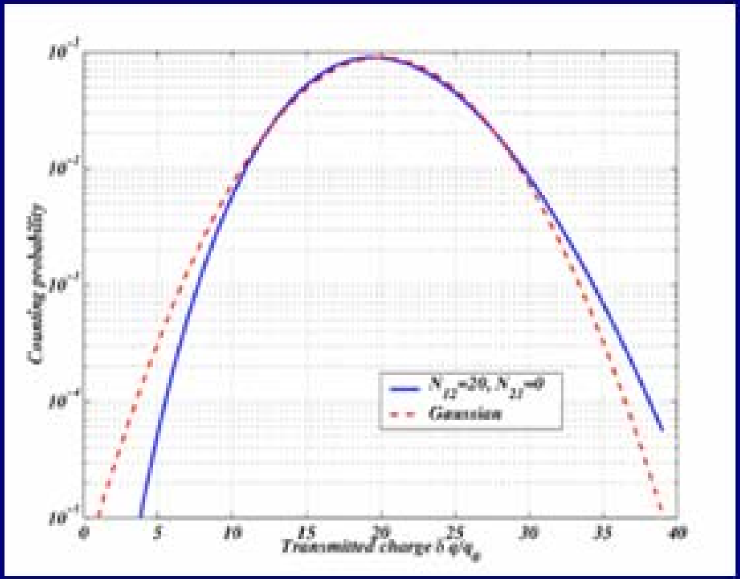


Measured PDF



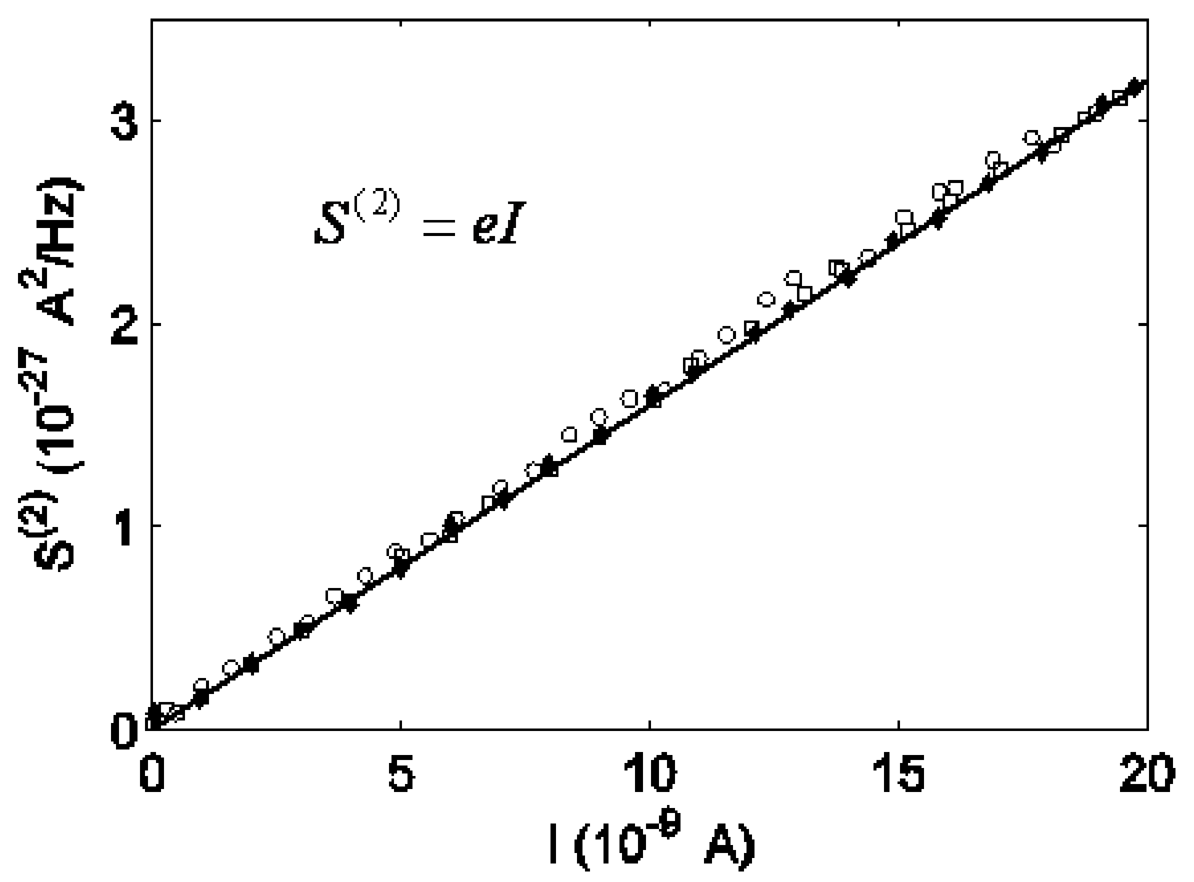
counts

output voltage



Typically 1000 electrons during 20 ns

Shot noise measurements



Origin of the discrepancy

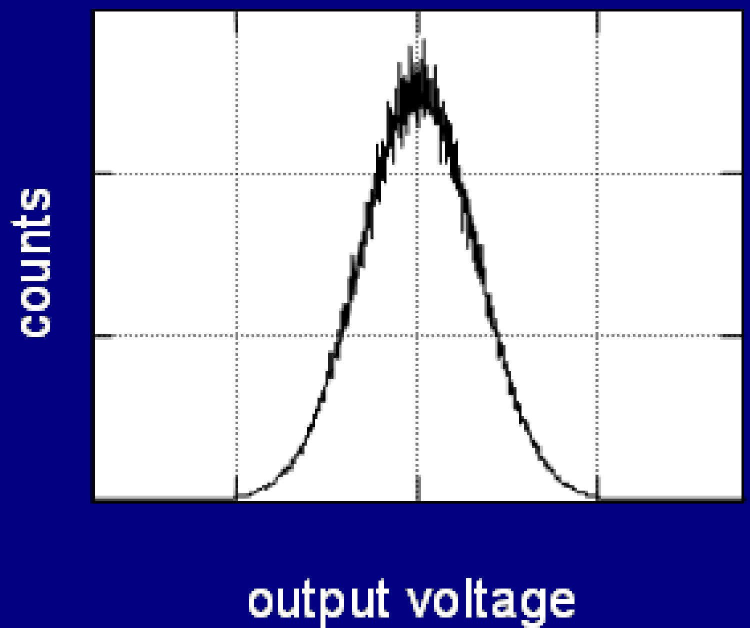
Nonlinearity of the measurement chain

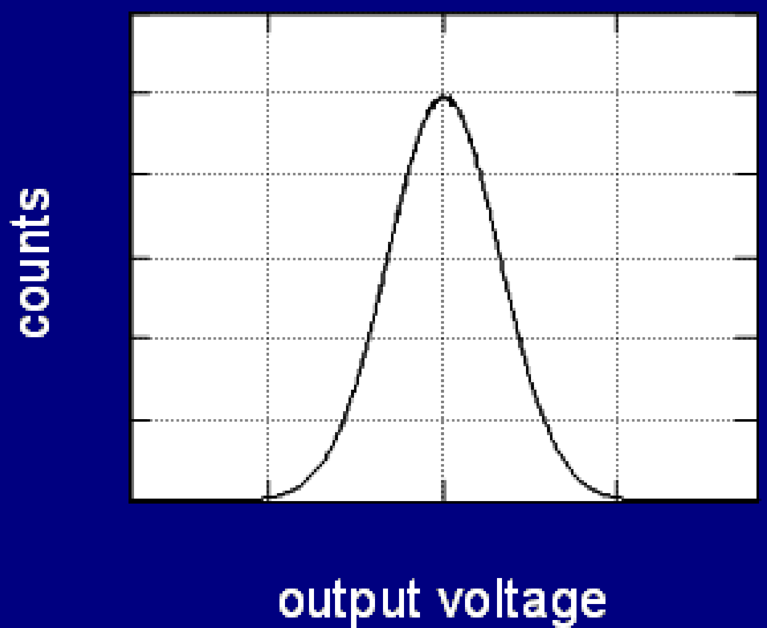
$$V_{out} = k(V_{in} + \frac{1}{u} V_{in}^2)$$

$$\left\langle V_{out}^3 \right\rangle - k^3 \left\langle V_{in}^3 \right\rangle = \frac{9k^3}{u} \left\langle V_{in}^2 \right\rangle^2$$

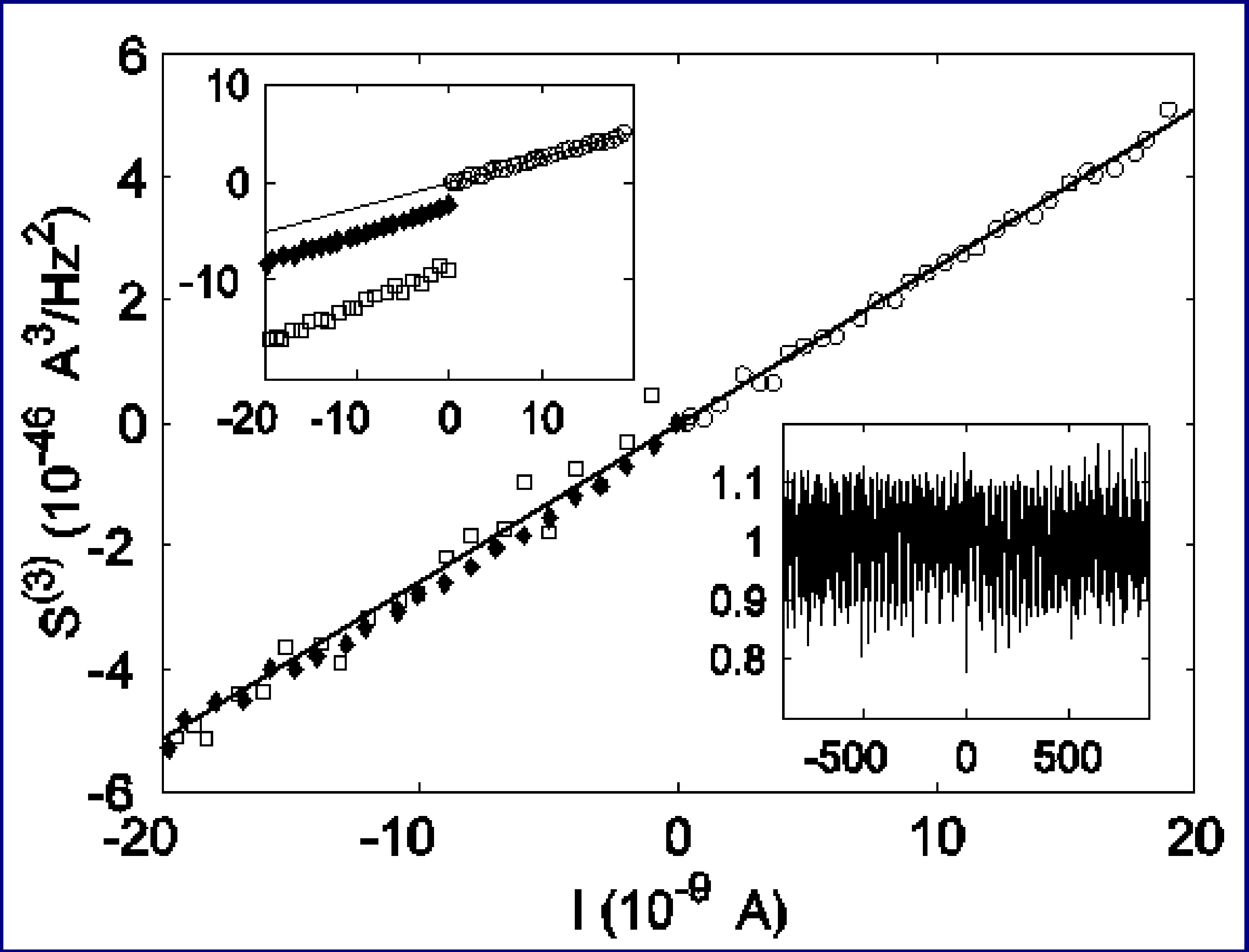
There is no commercial source
that produces a known third correlator

A/D board nonlinearity





$S^{(3)}$ after nonlinearity correction



Third cumulant in QPC

$$U = \frac{eV}{T}$$

$$S^{(3)} = eg_0 T\Gamma\left(1-\Gamma\right)\left(6\Gamma\frac{\sinh U - U}{\cosh U - 1} + \left(1-2\Gamma\right)U\right)$$

$$S^{(3)} = e^2 g_0 V\Gamma\left(1-\Gamma\right)\left(1-2\Gamma\right), \quad \mathrm{eV>>T}$$

$$S^{(3)} = e^2 g_0 V\Gamma\left(1-\Gamma\right), \quad \mathrm{eV<<T}$$

Open question

- $S^{(3)}$ in diffusive conductors $$S^{(3)} \propto \left(\frac{l}{L}\right)^2$$
- Effect of interactions on S(3)
  (D. Gutman, Y. Geffen, A. Mirlin, K. Nagaev)
- Statistics of fractional charge in FQHE beyond $S^{(2)}$.
  Nonabelian statistics?
- Properties of High T_c superconductors above T_c
  Preformed pairs?

Conclusion

WHETHER NOISE IS A NOISENS OR A SIGNAL MAY DEPEND ON WHOM YOU ASK