2D electronic phases intermediate between the Fermi liquid and the Wigner crystal (electronic micro-emulsions)

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Electron interaction can be characterized by a parameter $r_s = E_{pot} / E_{kin}$

$$E_{kin} \propto n \qquad E_{pot} \propto n^{g/2}$$

$$r_s \propto n^{\frac{g-2}{2}}$$

(e-e interaction energy is $V(r) \sim 1/r^g$)

Electrons ($g=1$) form Wigner crystals at T=0 and small n

when $r_s >> 1$ and $E_{pot} >> E_{kin}$

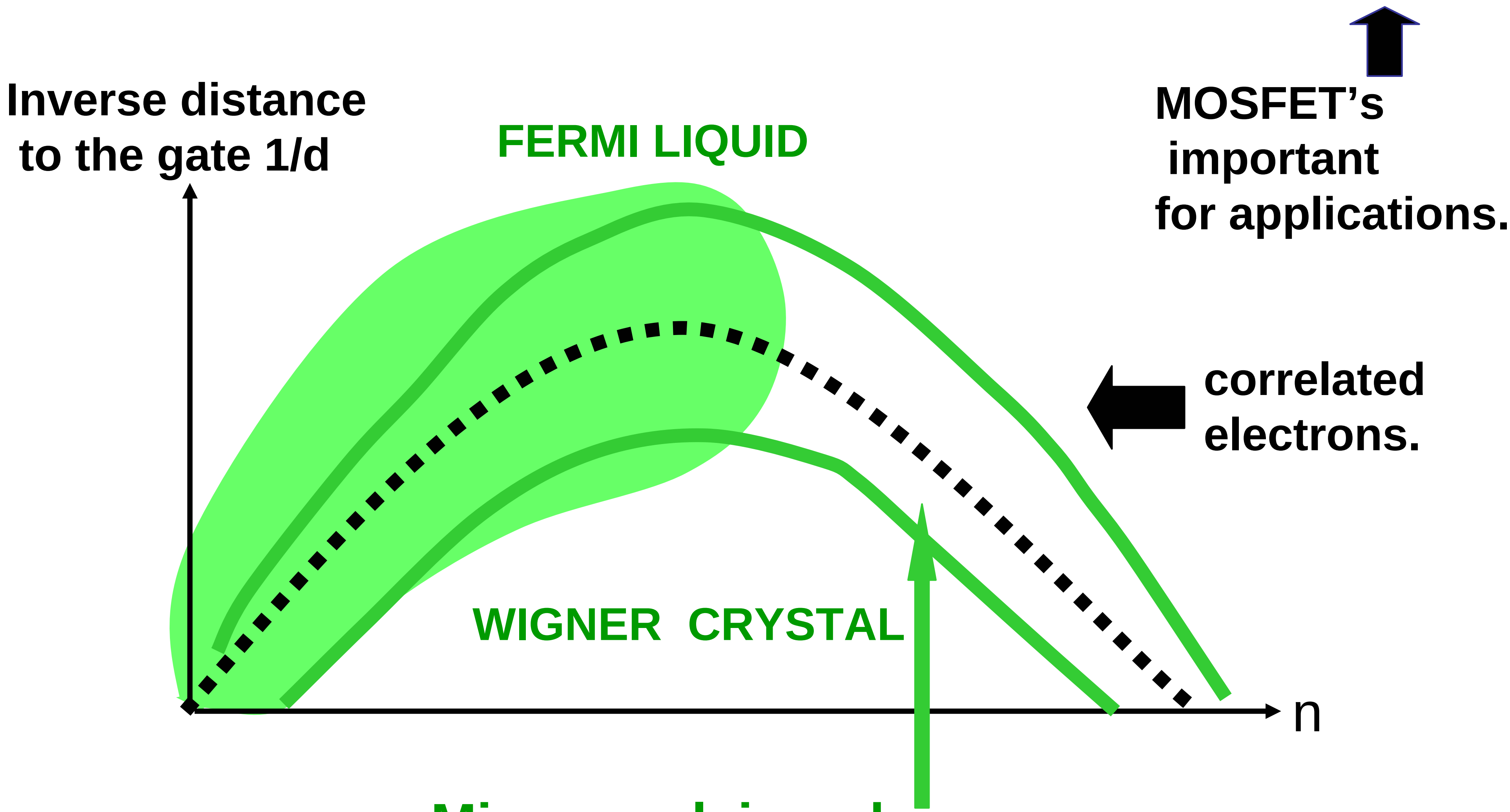
^3He and ^4He ($g>2$) are crystals at large n

a. Transitions between the liquid and the crystal should be of first order.

b. As a function of density 2D first order phase transitions in systems with dipolar or Coulomb interaction are forbidden.

There are 2D electron phases intermediate between the Fermi liquid and the Wigner crystal (micro-emulsion phases)

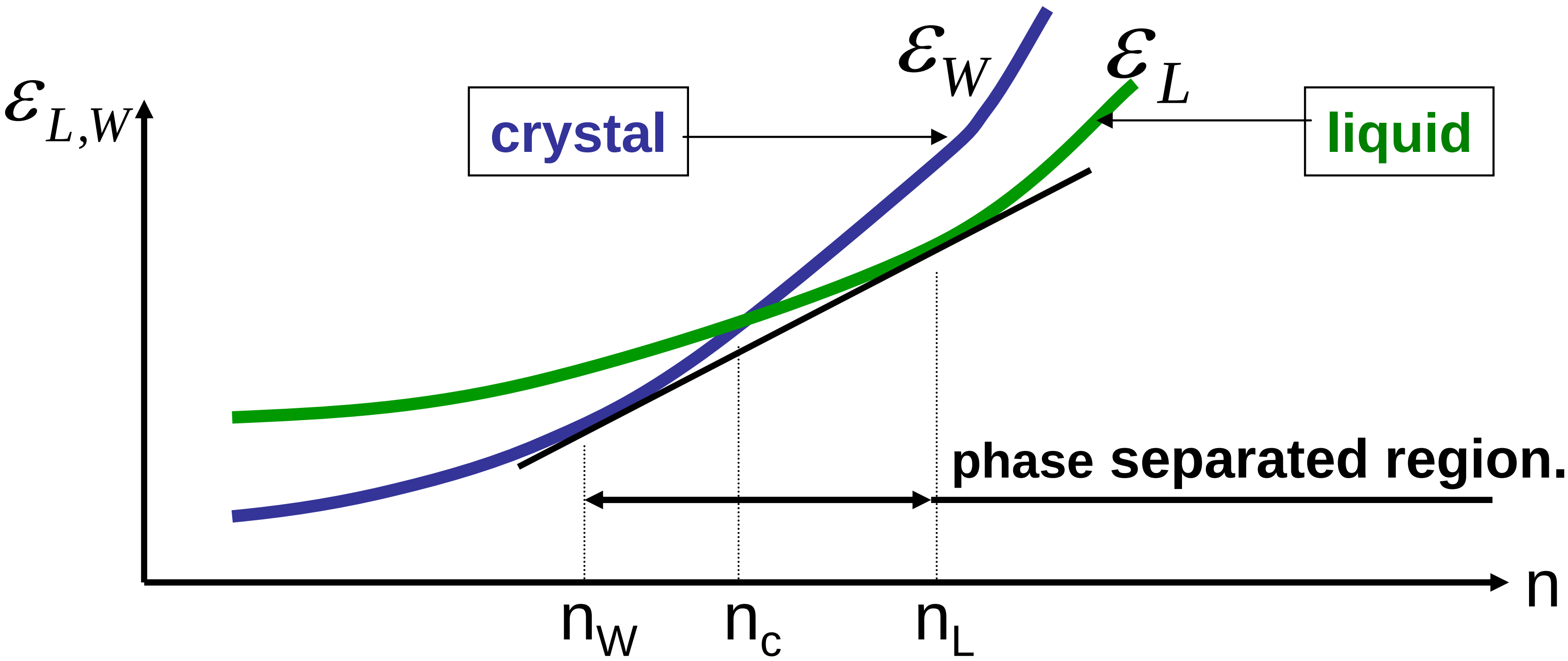
Phase diagram of 2D electrons in MOSFET's . *(T=0)*



Microemulsion phases.

In green areas where quantum effects are important.

Phase separation in the electron liquid.



There is an interval of electron densities $n_W<n<n_L$ near the critical n_c where phase separation must occur

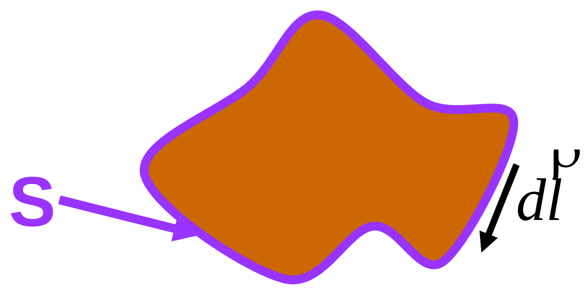
To find the shape of the minority phase one must minimize the surface energy at a given area of the minority phase

the case of dipolar interaction

$$E_{surf} = \int_S \gamma(\theta)\, dl - a \int_S \frac{d\vec{l}\, d\vec{l}\,'}{|\vec{l} - \vec{l}\,'|} \approx$$

$$\gamma L - a\, L \ln \frac{L}{d}$$

$\gamma > 0$ is the microscopic surface energy



At large *L* the surface energy is negative!

Elementary explanation: Finite size corrections to the capacitance

$$C = \frac{R^2}{d} + R \ln \frac{16\pi R}{d}$$

R is the droplet radius

$$E_C = \frac{Q^2}{2C} = \frac{(enR^2)^2}{2C} \propto (en)^2 R^2 d - (en)^2 R \ln \frac{R}{d}$$

This contribution to the surface energy is due to a finite size correction to the capacitance of the capacitor. It is negative and is proportional to –R ln (R/d)

Coloumb case

$$E = \mu[n_0 - n_c][S_+ - S_-] + \int d\vec{l}\;\gamma(\theta) - \frac{\mu^2}{e}\int \frac{d\vec{l}\,d\vec{l}}{|\vec{l} - \vec{l}\,'|},$$

S_+ and S_- are area of the minority and the majority phases,

n_0 and n_c are average and critical densities,

$$\mu = \frac{\mathrm{d}(\varepsilon_{\mathrm{L}} - \varepsilon_W)}{d\,n}$$

At large area of a minority phase the surface energy is negative.

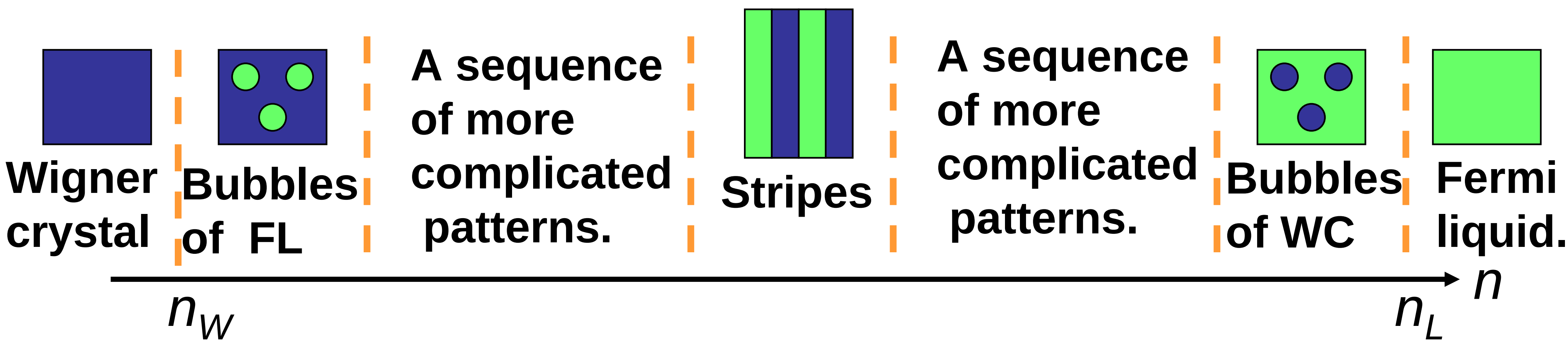
Single connected shapes of the minority phase are unstable. Instead there are new electron micro-emulsion phases.

The characteristic size of the droplets is

$$R \propto d e^{\alpha},$$

$$\alpha = \frac{4\pi\gamma}{e^2 (n_W - n_L)^2} \geq 1, \quad \gamma = \frac{d}{dn}\left[\varepsilon_L - \varepsilon_W\right]$$

Mean field phase diagram of microemulsions



Transitions are continuous.

They are similar to **Lifshitz points.**

T and $H_{||}$ dependences of the crystal's area. (Pomeranchuk effect).

The entropy of the crystal is of spin origin and much larger than the entropy of the Fermi liquid.

$$S_W >> S_L; \qquad M_W >> M_L$$

a. **As T and $H_{||}$ increase, the crystal fraction grows.**

b. **At large $H_{||}$ the spin entropy is frozen and the crystal fraction is T- independent.**

Several experimental facts suggesting

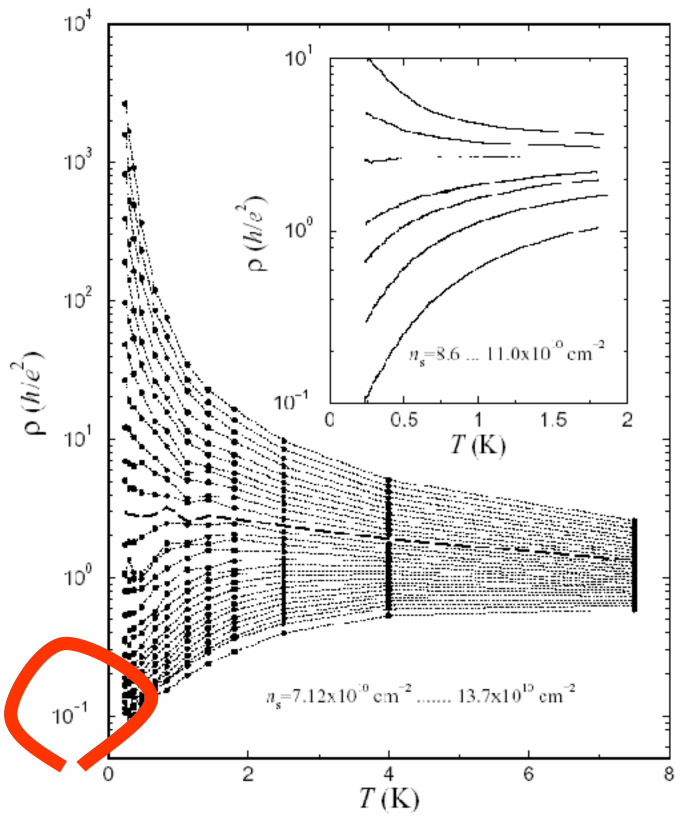
non-Fermi liquid nature 2D electron liquid

at small densities :

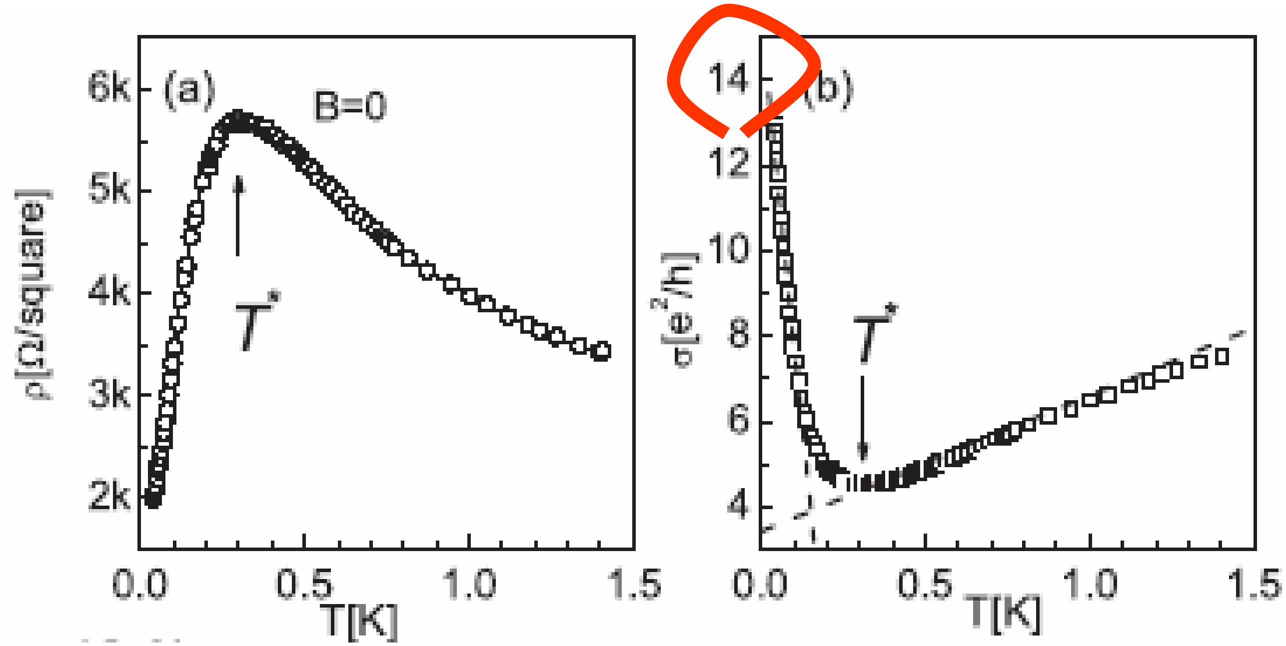
T-dependence of the resistance of 2D electrons at large r_s in the "metallic" regime ($G >> e^2/h$)

Kravchenko et al

Gao at al, Cond.mat 0308003



Si MOSFET



p-GaAs, p=1.3 10 cm^{-2} ; r_s=30

T-dependence of the resistance of 2D p-GaAs layers at large r_s in the "metallic" regime .

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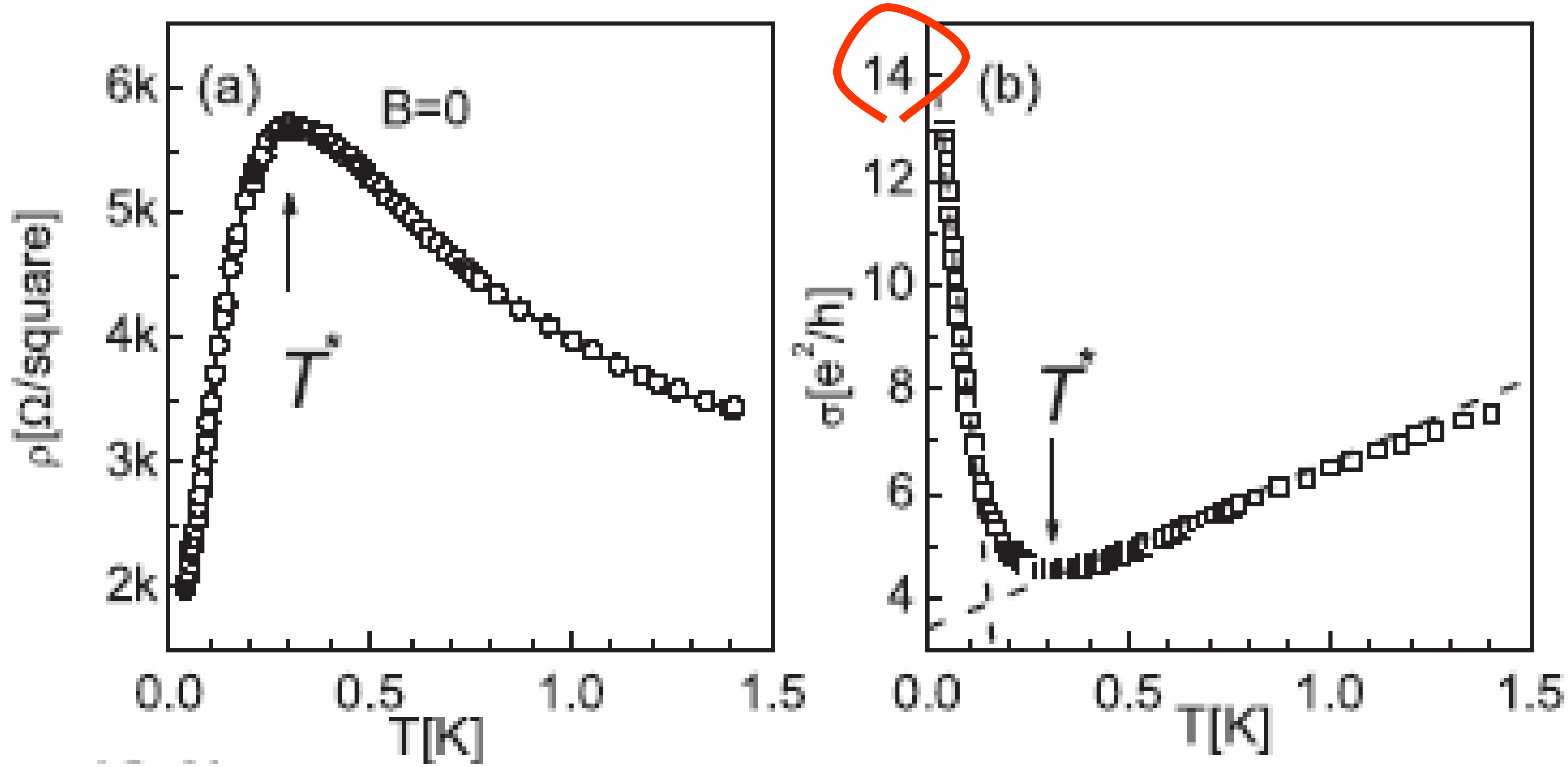

$P=1.3\ 10^{10}\ cm^{-2}$; $r_s=30$

X. P. A. Gao,[1,*] G. S. Boebinger,[2] A. P. Mills Jr.,[3] and A. P. Ramirez, L. N. Pfeiffer, K. W. West[4]

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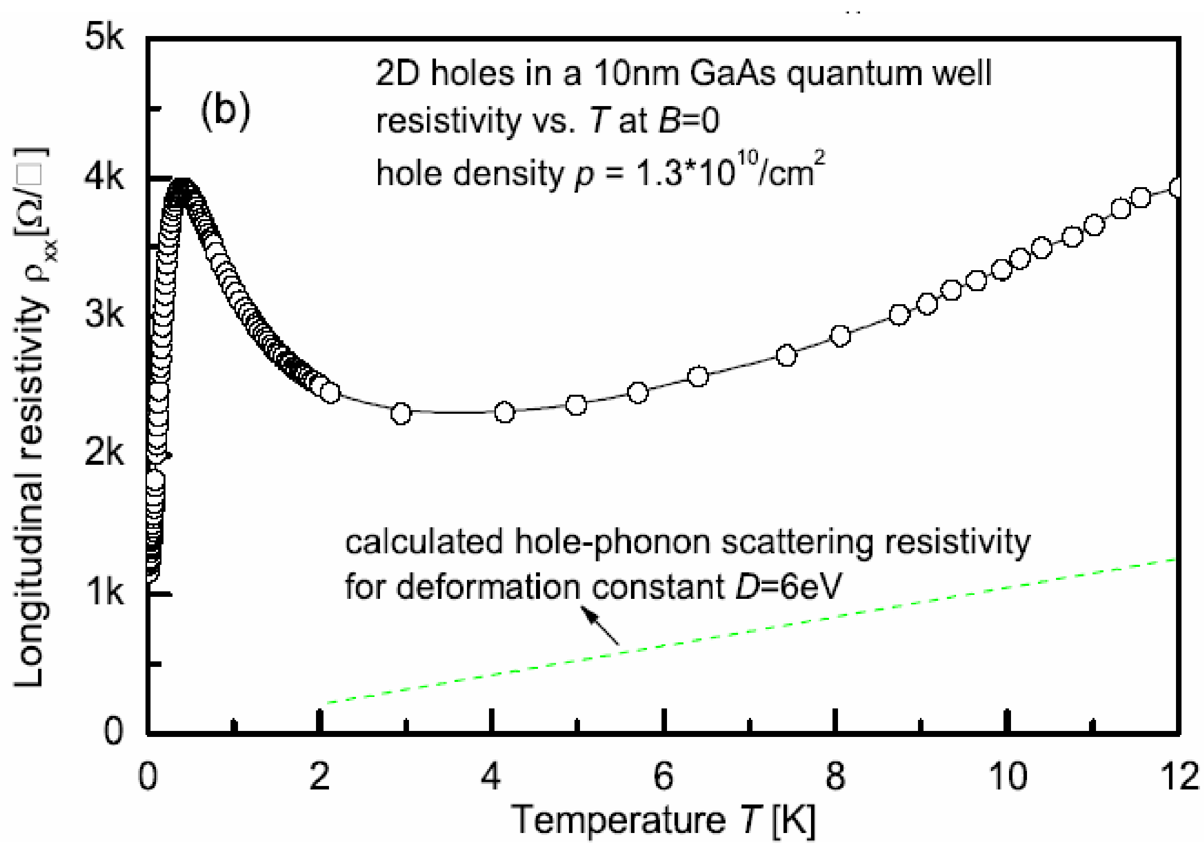
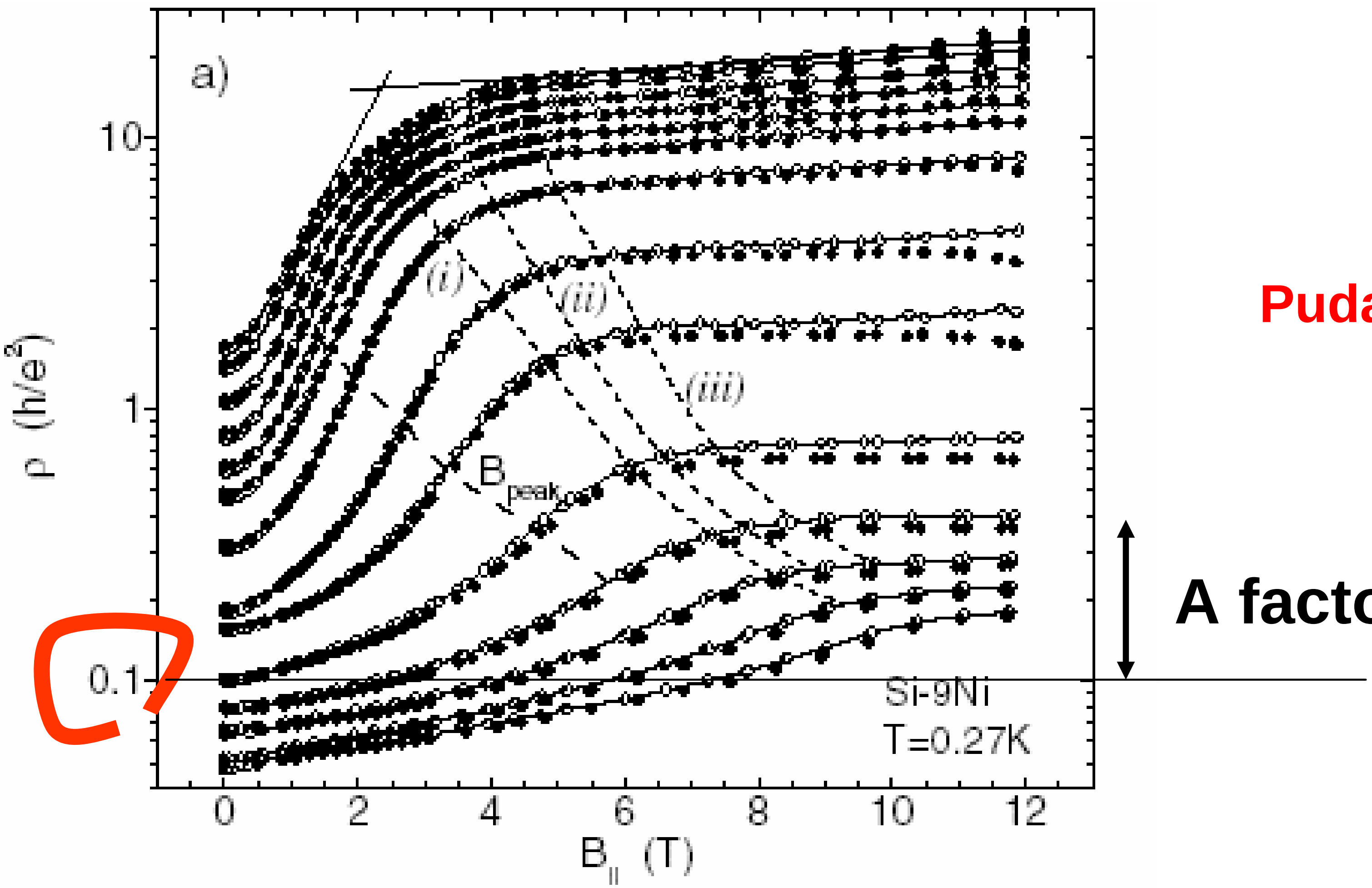


$B_{||}$ dependences of the resistance of Si MOSFET's at different electron concentrations.



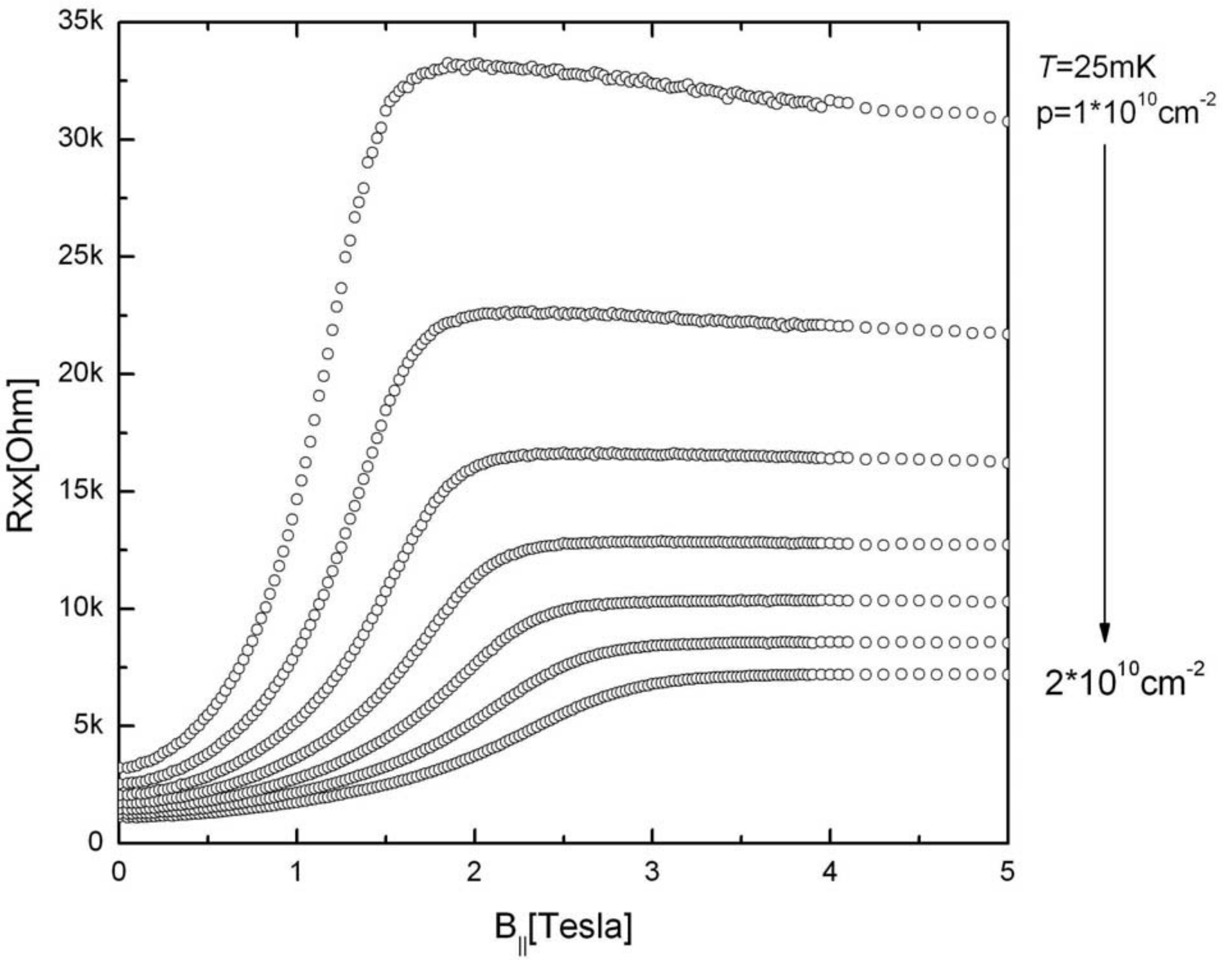
Pudalov et al.

A factor of order 6.

There is a big positive magneto-resistance which saturates at large magnetic fields parallel to the plane.

$B_{||}$ dependence of 2D p-GaAs at large r_s and small wall thickness.

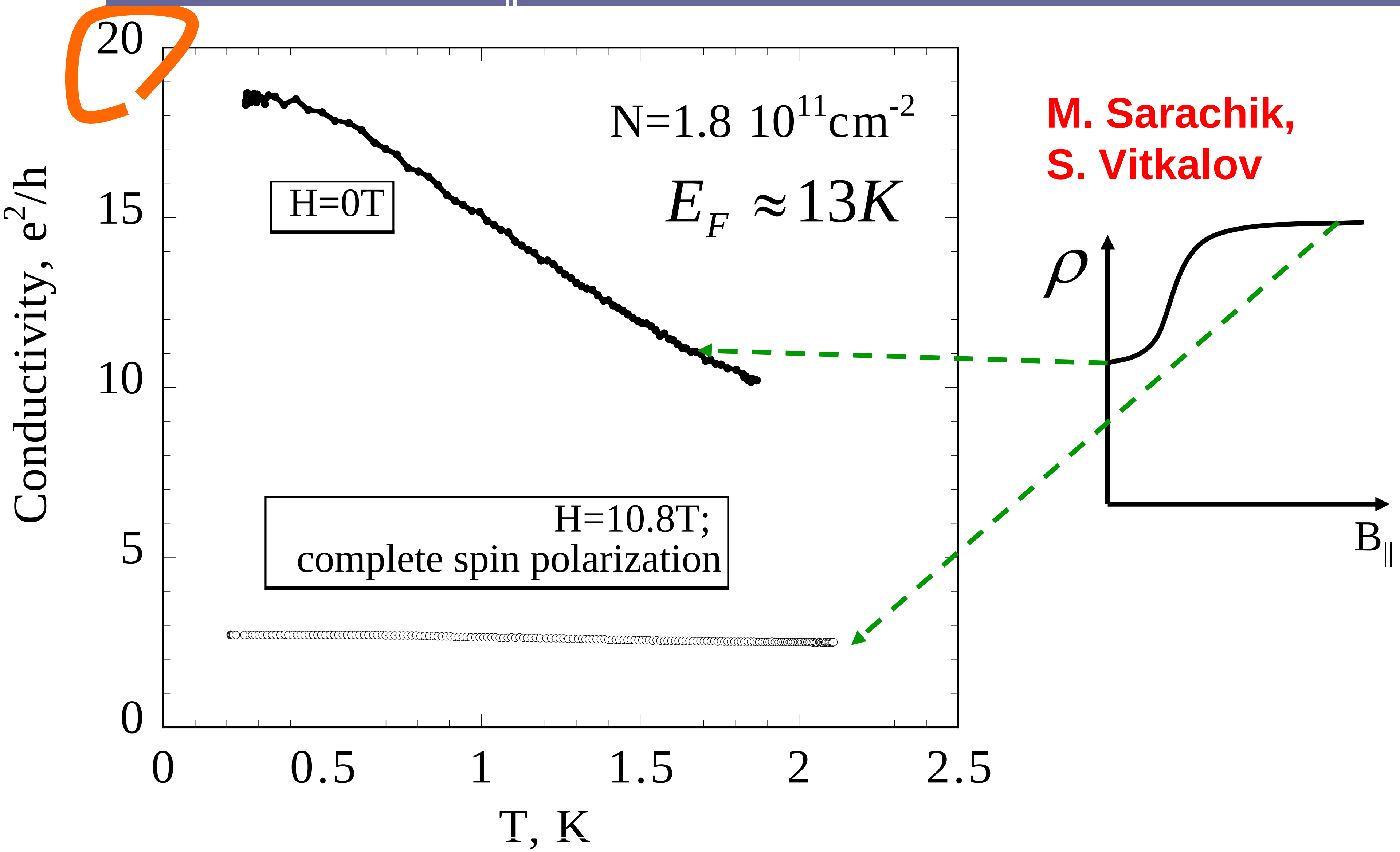
1/3



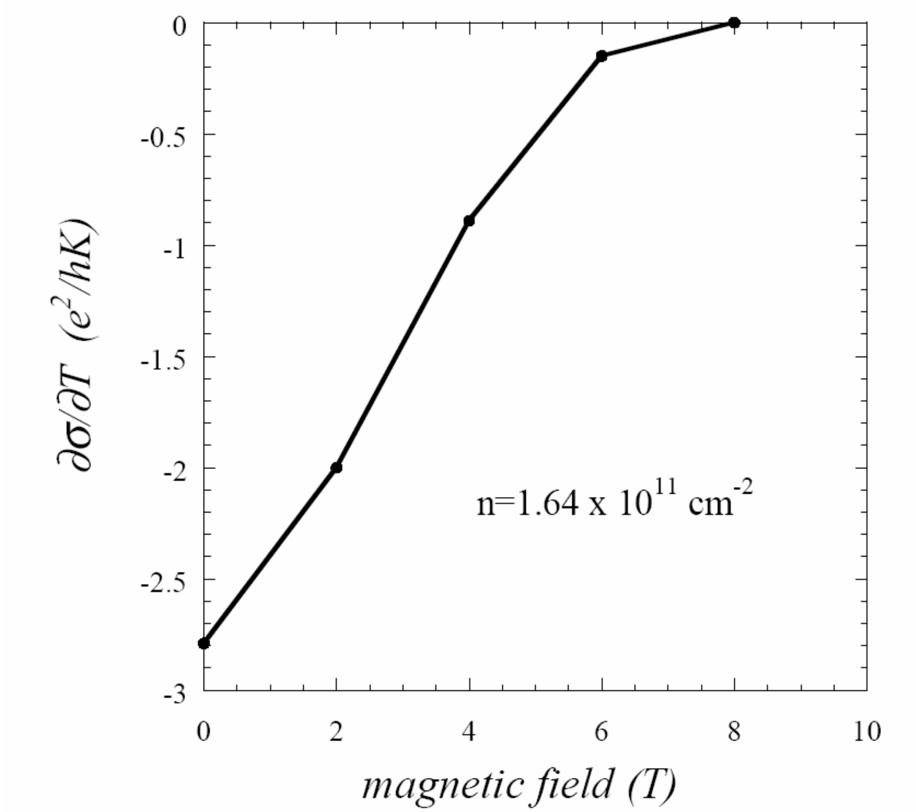
Gao et al

There is a big positive magneto-resistance which saturates at large magnetic fields parallel to the plane.

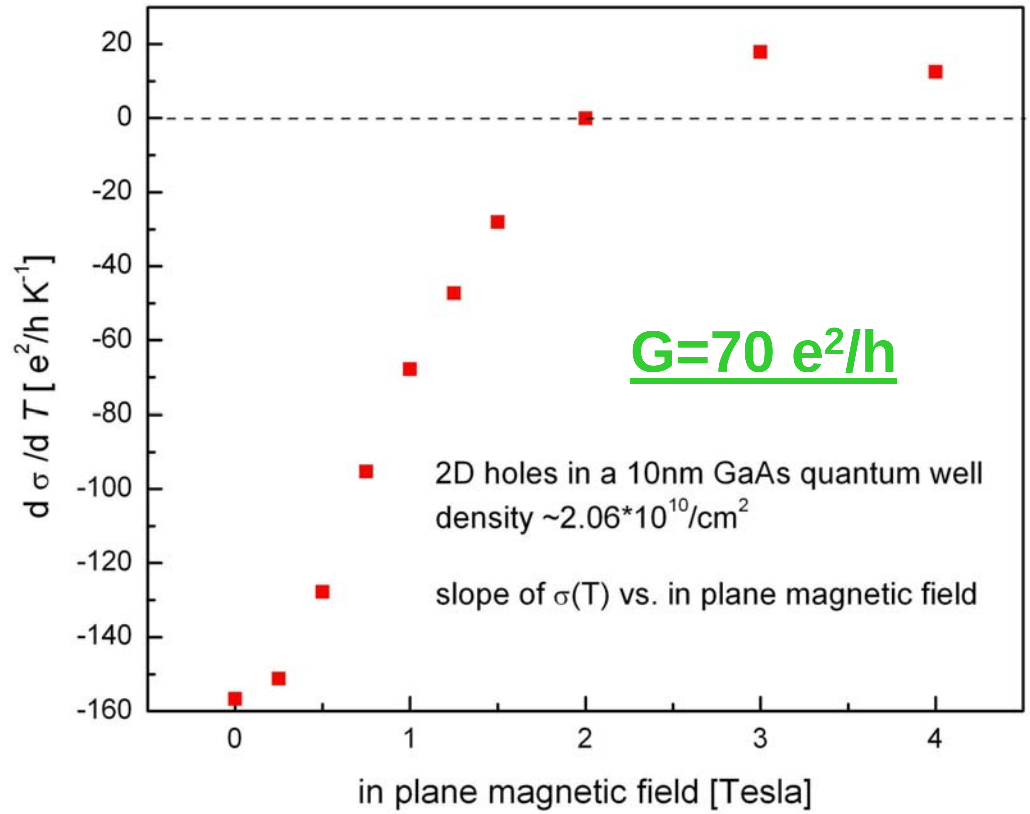
Comparison T-dependences of the resistances of Si MOSFET's at zero and large $B_{||}$



The parallel magnetic field suppresses the temperature dependence of the resistance of the metallic phase. The slopes differ by a factor 100 !!



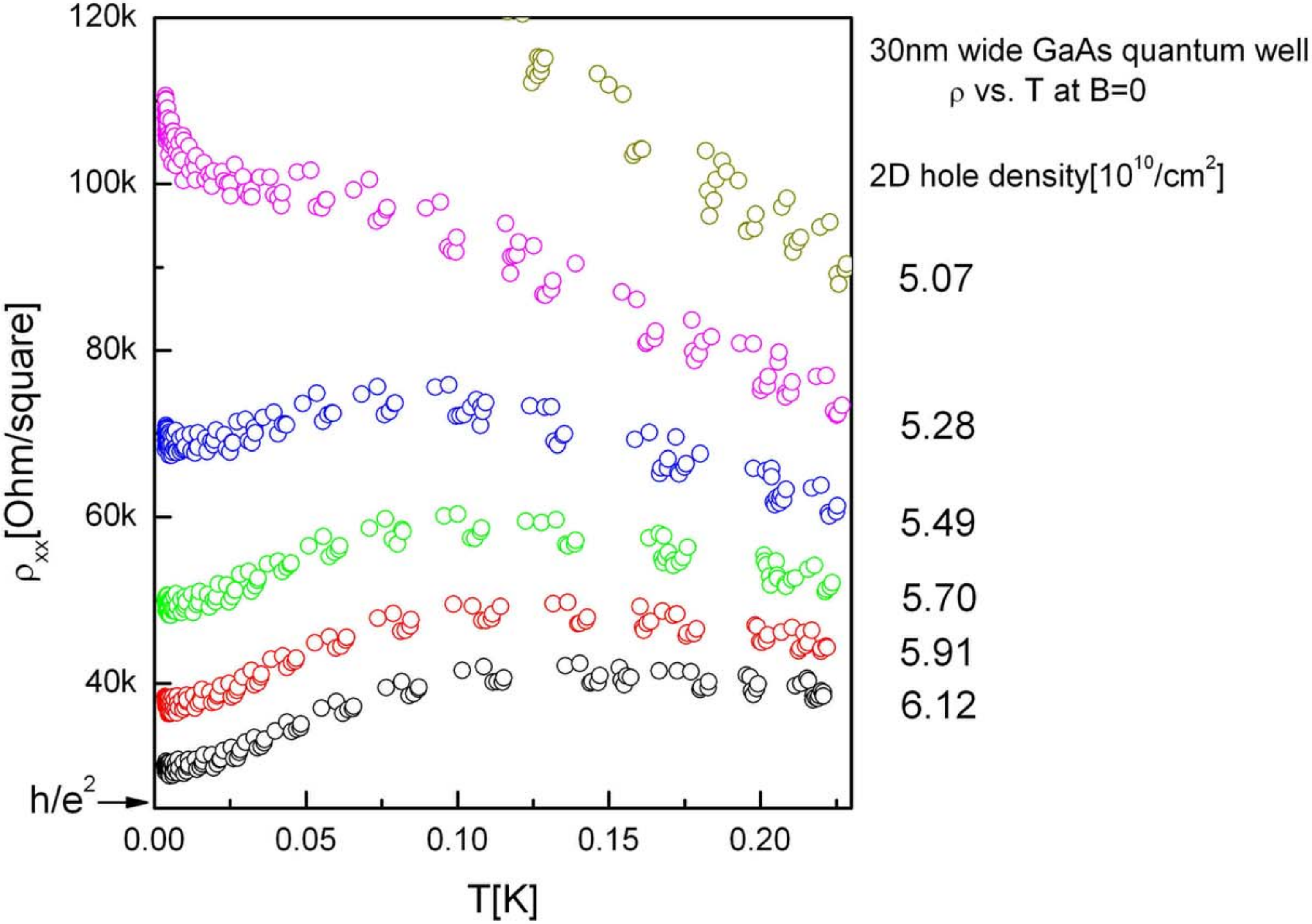
Tsui et al. cond-mat/0406566



Gao et al

The slope of the resistance dR/dT is dramatically suppressed by the parallel magnetic field.
It changes the sign. Overall change can as much as factor 50 in Si MOSFET's and a factor 10-100 in P-GaAs !

Do materials exist where the resistance has dielectric values $R \gg h/e^2$ and yet still increases as the temperature increases ?



<u>If it is all business as usual:</u>

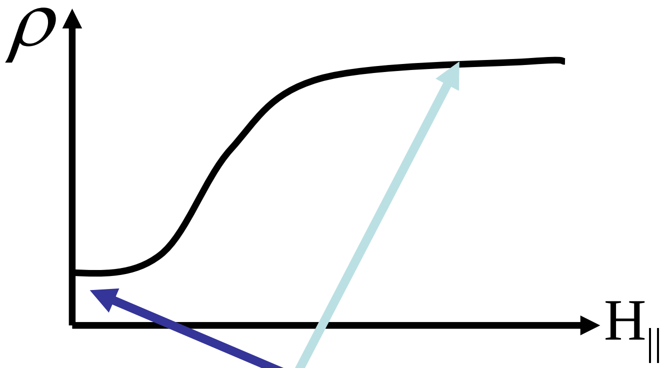
Why is there an apparent metal-insulator transition?

Why is there such strong T and $B_{||}$ dependence at low T, even in "metallic" samples with $G \gg e^2/h$?

Why is the magneto-resistance positive at all?

Why does $B_{||}$ so effectively quench the T dependence of the resistance?

Hopping conductivity regime in MOSFET's
Magneto-resistance in the parallel and the perpendicular tmagnetic field



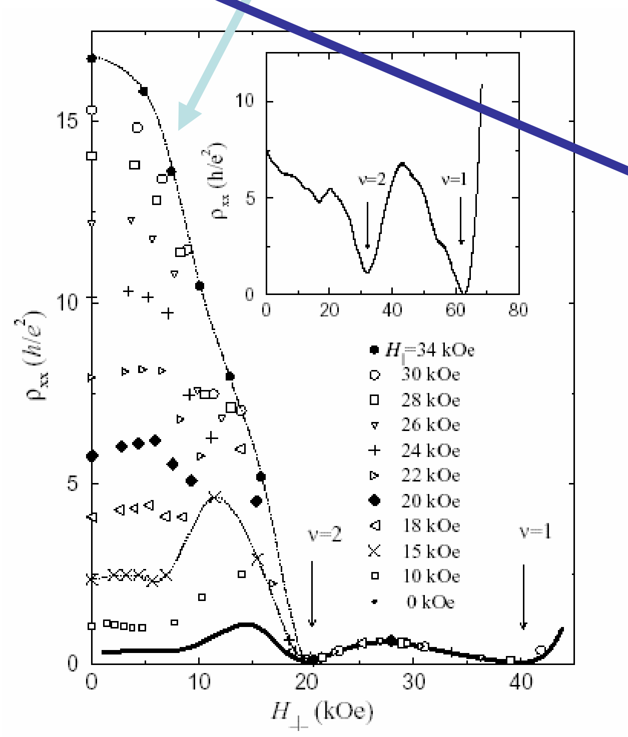
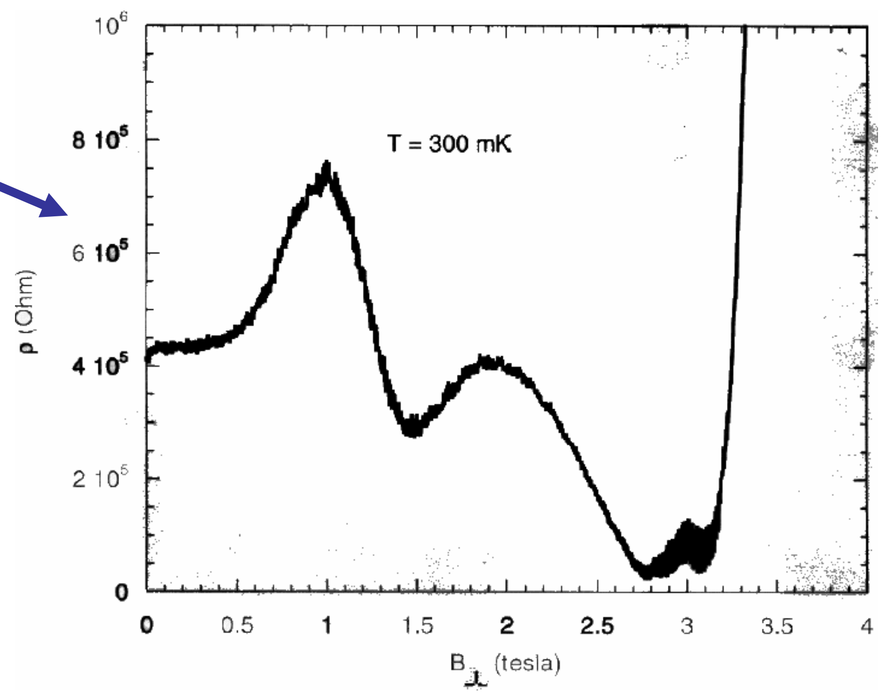


FIG. 3. ρ_{xx} of sample B as a function of $H_\perp$ for different values of the parallel magnetic field; $T = 0.36$ K and $n_s = 1.0 \times 10^{11}$ cm^{-2}. The inset shows $\rho_{xx}(H_\perp)$ for a low-mobility sample C; $T = 0.36$ K and $n_s = 2.1 \times 10^{11}$ cm^{-2}.

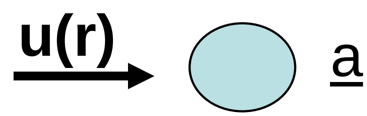
Kravchenko et al (unpublished)



Connection between the resistance and the electron viscosity $\eta(T)$ in the semi-quantum regime.

The electron mean free path $\mathbf{l_{ee} \sim n^{1/2}}$ and hydrodynamics description of the electron system works !

Stokes formula in 2D case: $$F \propto \frac{\eta u}{\ln(\eta / nau)}$$



$$\rho(T) \propto \eta(T) \frac{N_i}{e^2 n^2 \ln(1/N_i a^2)}$$

In classical liquids ***η(T)*** *decreases* exponentially with ***T*****.**

In classical gases η(T) *increases* as a power of **T**.

What about semi-quantum liquids?

If $r_s >> 1$ the liquid is strongly correlated

$$\boxed{E_F \ll \Theta = \frac{E_{pot}}{r_s^{1/2}} \ll E_{pot}}$$

Θ is the plasma frequency

If $E_F << T << h\theta << E_{pot}$ the liquid is not degenerate but it is still not a gas ! It is also not a classical liquid !

Such temperature interval exists both in the case of electrons with r_s >>1 and in liquid He

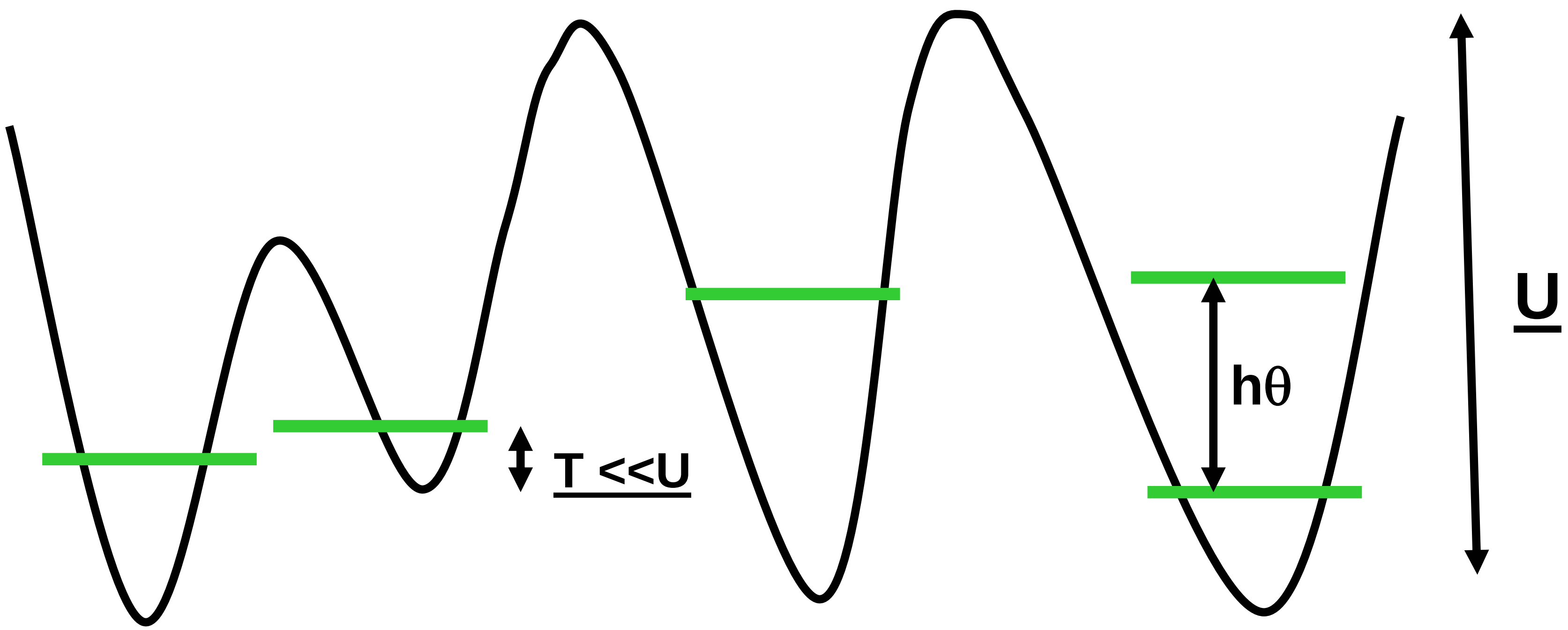
Viscosity of gases (T>>U) increases as T increases

Viscosoty of classical liquids (T_c , $h\Theta_D$ << T<< U) decreases exponentially with T (Ya. Frenkel)

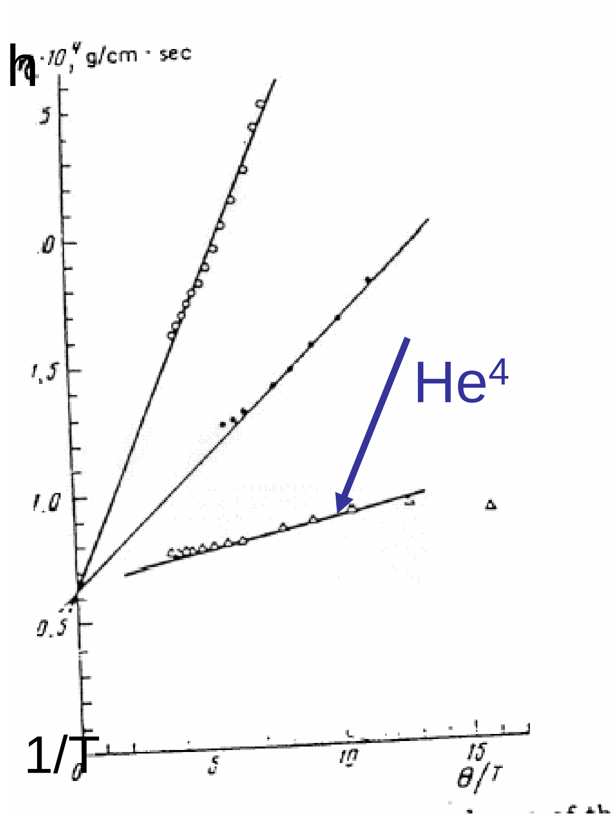
$$\eta \sim \exp(B/T)$$

Semi-quantum liquid: $E_F \ll T \ll h\,\theta \ll U$: (A.F. Andreev)

$$\eta \sim 1/T$$



Comparison of two strongly correlated liquids: He^3 and the electrons at $E_F < T < E_{pot}$



Experimental data on the viscosity of He^3 in the semi-quantum regime (T > 0.3 K) are unavailable!?

A theory (A.F.Andreev): $\eta \propto \frac{1}{T}$

T-dependence of the resistance of 2D p-GaAs layers at large r_s in the “metallic” regime .

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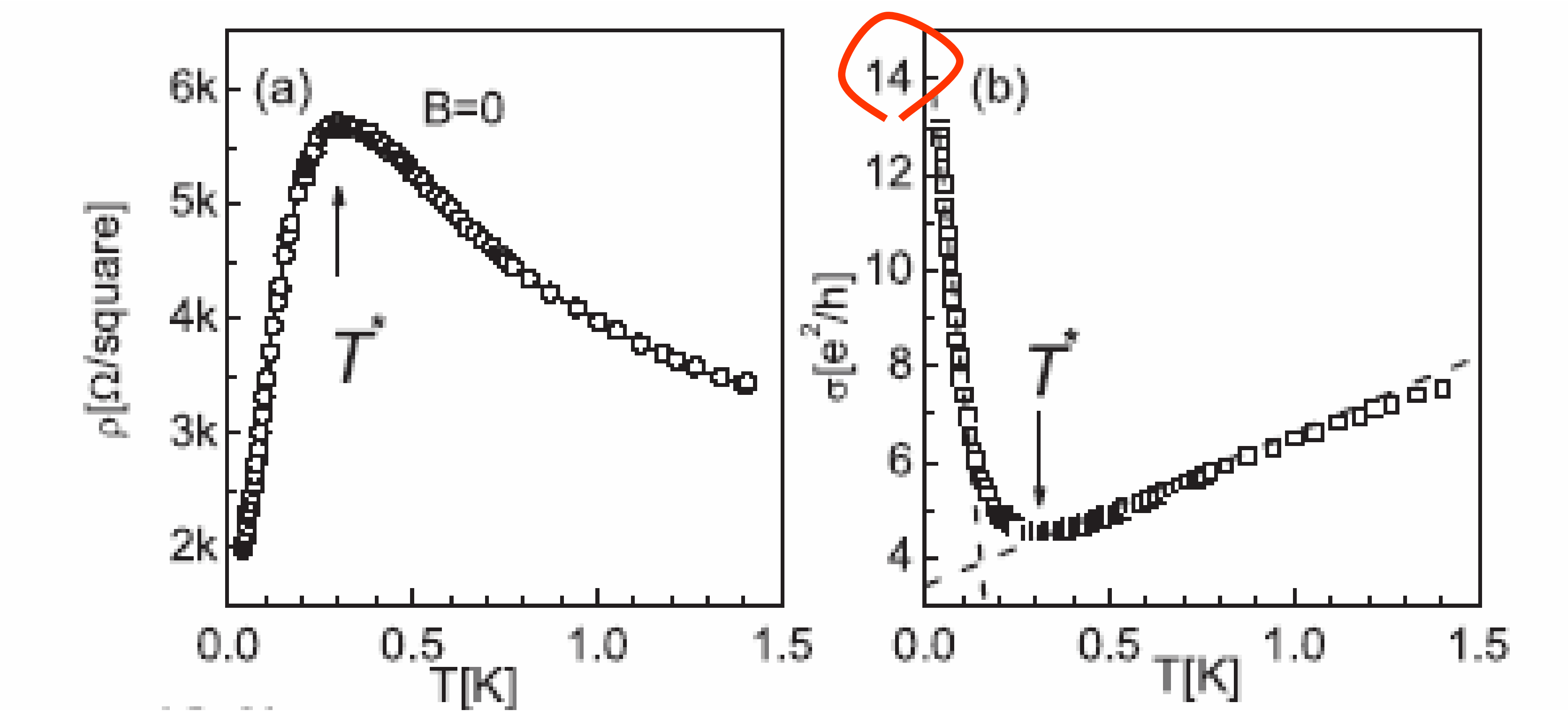


$P=1.3\ 10^{10}\ cm^{-2}$; $r_s=30$

Experiments on the drag resistance of the double p-GaAs layers.

$B_{\|}$ dependence of the resistance and drag resistance of 2D p-GaAs at different temperatures

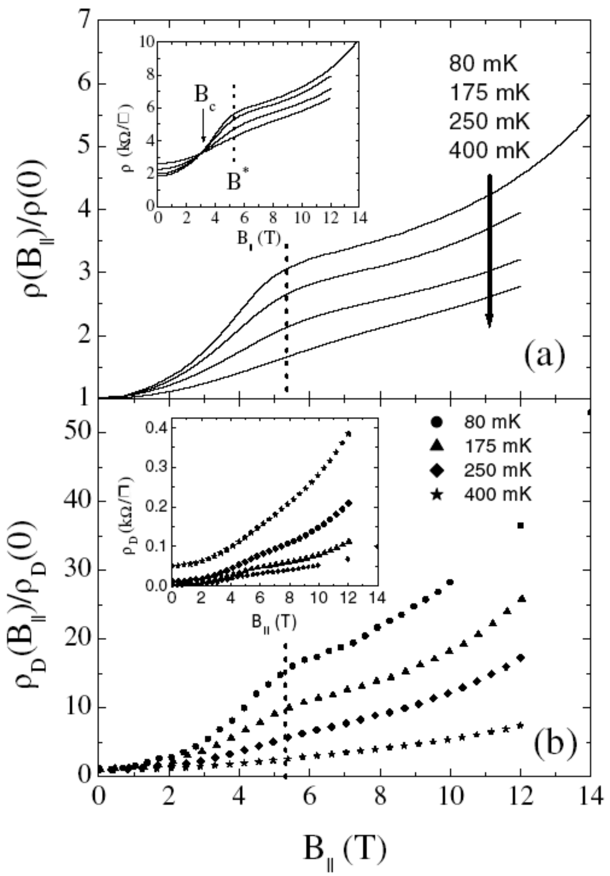


FIG. 1. In-plane magnetotransport data for $p_m = 2.15 \times 10^{10}$ cm^{-2} at $T = 80$, 175, 250, and 400 mK. (a) Inset: ρ vs $B_{\|}$. B_c and B^* are indicated by the arrow and the dashed line, respectively. Main plot: Data from inset normalized by its $B_{\|} = 0$ value. (b) Inset: Corresponding data for ρ_D vs $B_{\|}$. Main plot: Data from inset normalized by its $B_{\|} = 0$ value.

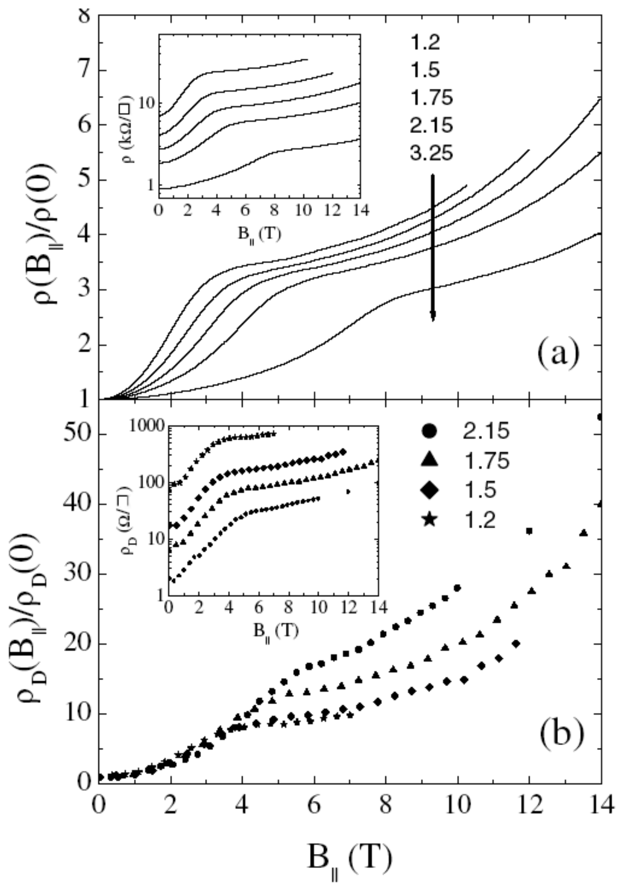
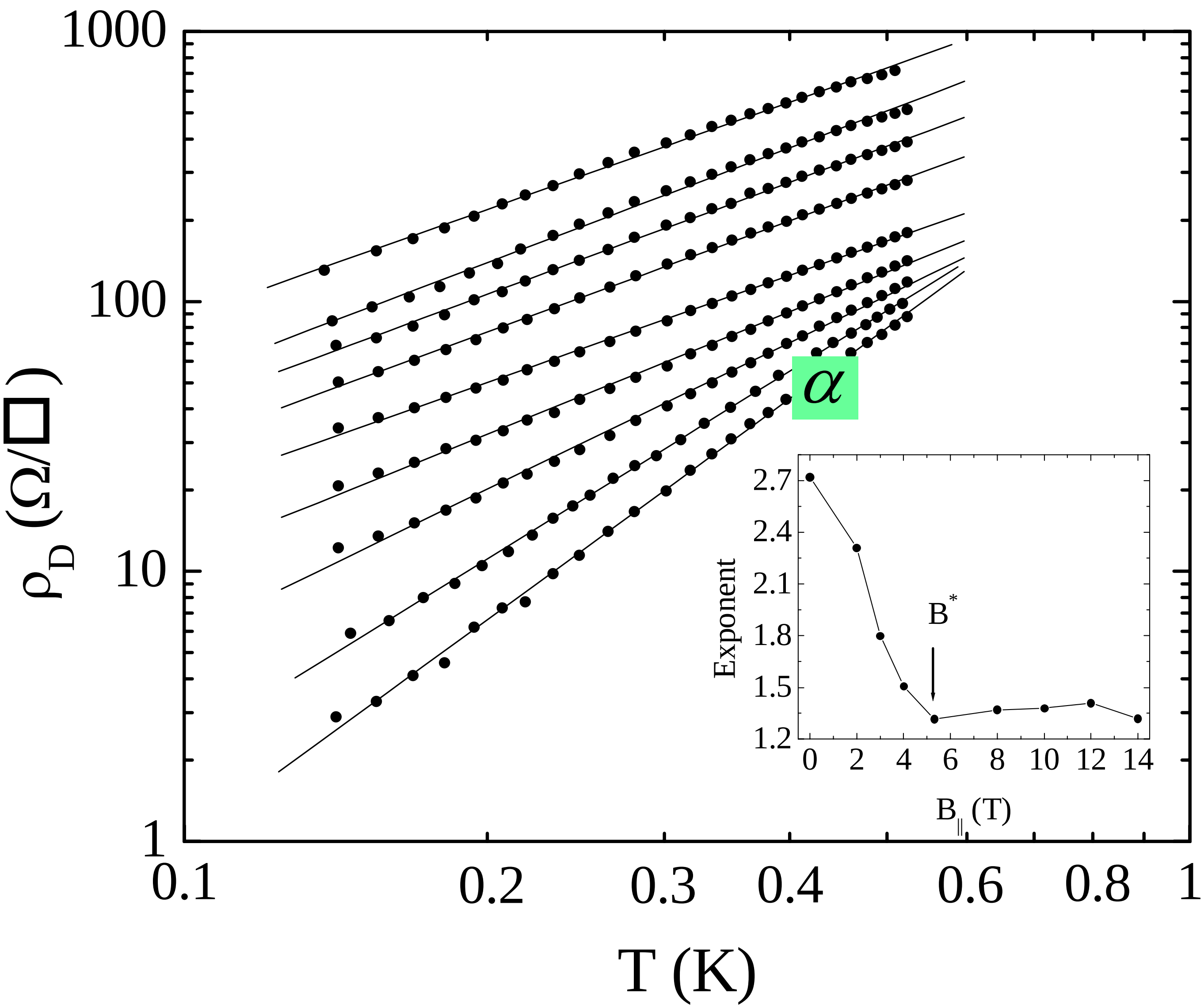


FIG. 2. ρ and ρ_D vs $B_{\|}$ at $T = 80$ mK for different densities. (a) Inset: ρ vs $B_{\|}$ for (from bottom to top) $p = 3.25$, 2.15, 1.75, 1.5, and 1.2×10^{10} cm^{-2}. Main plot: Data from inset normalized by its $B_{\|} = 0$ value. (b) Inset: ρ_D vs $B_{\|}$ for (from bottom to top) $p_m = 2.15$, 1.75, 1.5, and 1.2×10^{10} cm^{-2}. Main plot: Data from inset normalized by its $B_{\|} = 0$ value. Density for each trace is indicated in the legend.

Pillarisetty et al. PRL. **90**, 226801 (2003)

$$\rho_D = \frac{V_P}{I_A}$$

T-dependence of the drag resistance in double layers of p-GaAs at different $B_{||}$



Pillarisetty et al.
PRL. **90**, 226801 (2003)

$$\rho_D \propto T^{\alpha}$$

If it is all business as usual:

Why the drag resistance is 2-3 orders of magnitude larger than those expected from the Fermi liquid theory?

Why is there such a strong T and $B_{||}$ dependence of the drag?

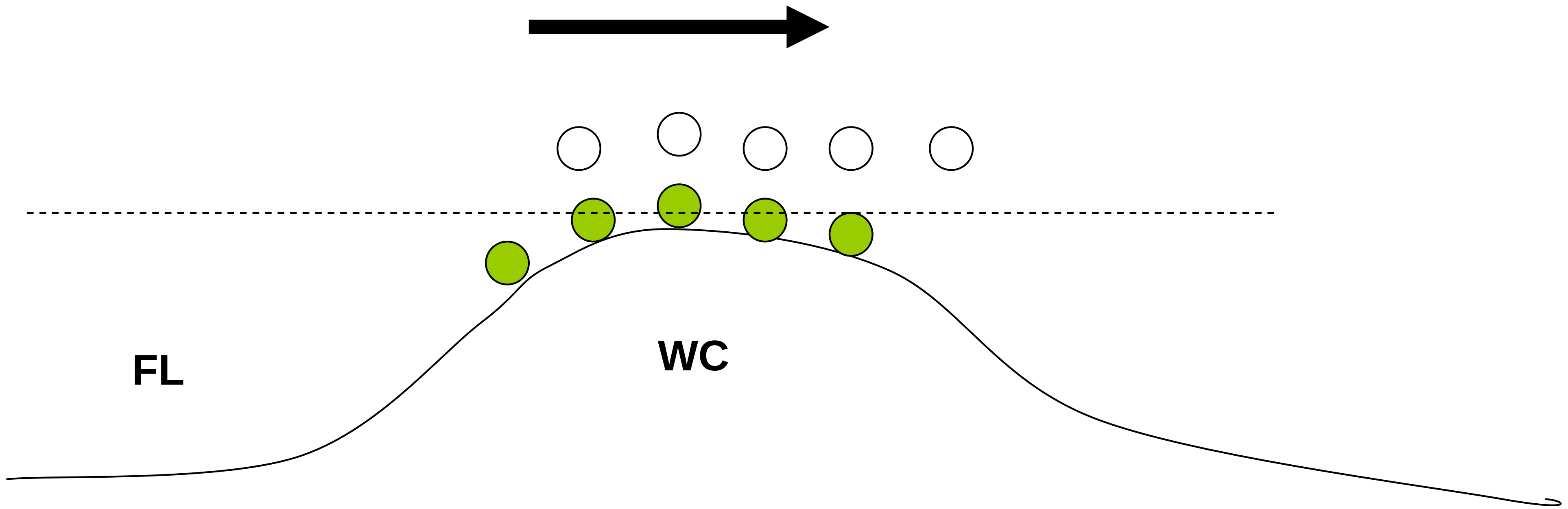
Why is the drag magneto-resistance positive at all?

Why does $B_{||}$ so effectively quench the T dependence of drag resistance?

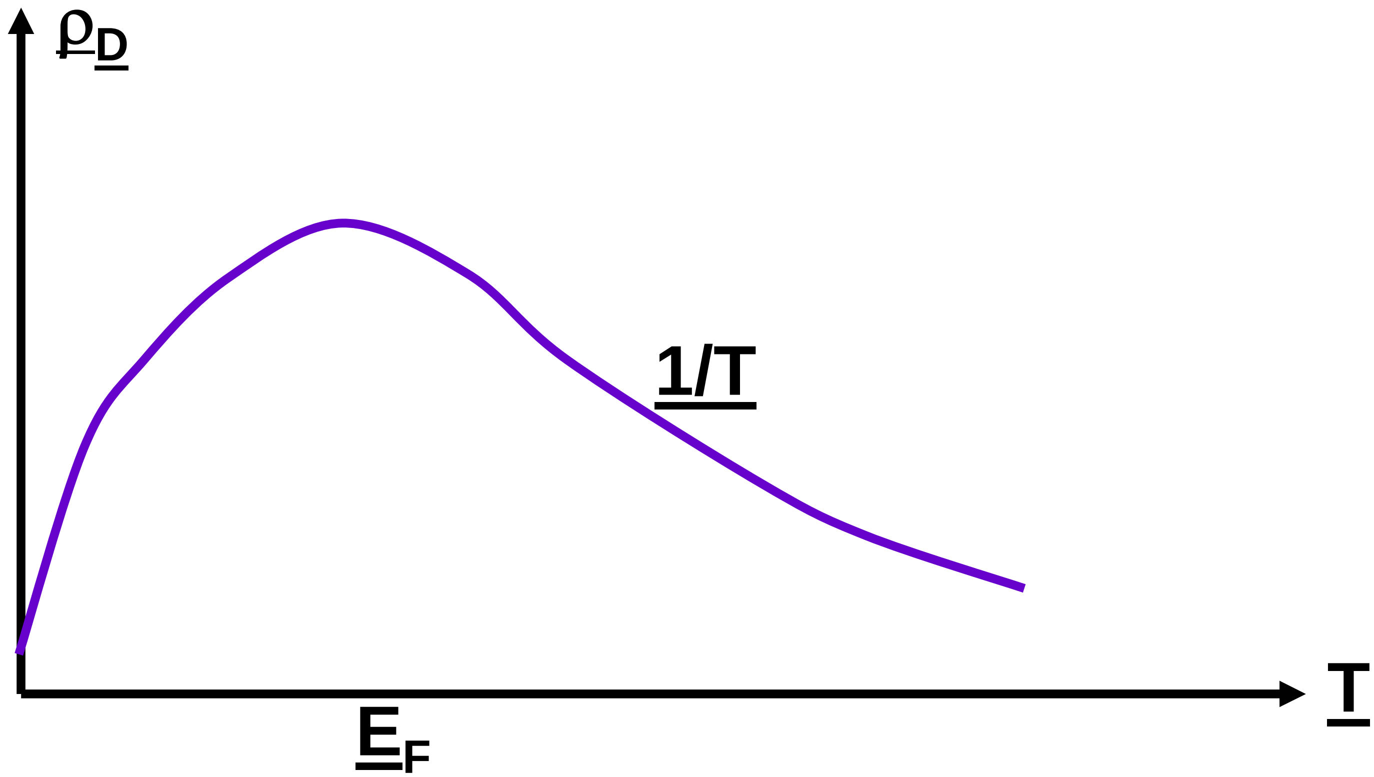
Why $B_{||}$ dependences of the resistances of the individual layers and the drag resistance are very similar

An open question: Does the drag resistance vanish at T=0?

The drag resistance is finite at T=0



A theoretical picture of the T dependence of the drag resistance in pure samples



Questions:

What is the effective mass of the bubbles?

What are their statistics?

Is the surface between the crystal and the liquid a quantum object?

Are bubbles localized by disorder?

effective droplet's mass m*

At T=0 the liquid-solid surface is a quantum object.

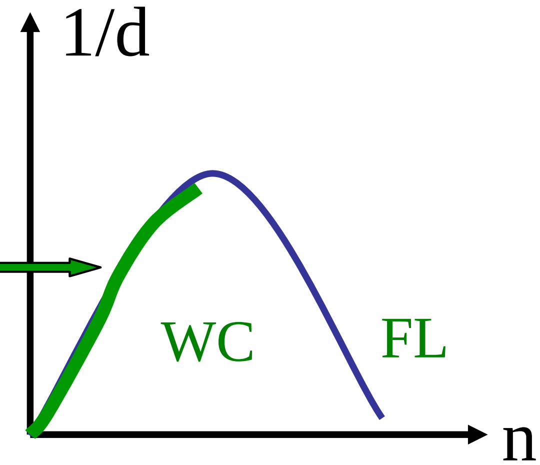
a. If the surface is quantum smooth, a motion WC droplet corresponds to redistribution of mass of order

$$m^* \approx m n_c \pi R^2$$

b. If it is quantum rough, much less mass need to be redistributed.

$$m^* \approx m(n_L - n_W)\pi R^2$$

$$nd^2 \approx 1; \quad m^* \approx m$$



In Coulomb case m ~ m*

Properties of “quantum melted” droplets of Fermi liquid embedded in the Wigner crystal :

Droplets are topological objects with a definite statistics

The number of sites in such a crystal and the number of electrons are different .

Such crystals can bypass obstacles and cannot be pinned

This is similar to the scenario of super-solid *He* (A.F.Andreev and I.M.Lifshitz). The difference is that in that case the zero-point vacancies are of quantum mechanical origin.

Conclusion:

There are pure 2D electron phases which are intermediate between the Fermi liquid and the Wigner crystal .

(Unsolved problems):

1. Quantum hydrodynamics of the micro-emulsion phases.

2. Quantum properties of WC-FL surface. Is it quantum smooth or quantum rough? Can it move at T=0 ?

3. What are properties of the microemulsion phases in the presence of disorder?

4. What is the role of electron interference effects in 2D microemulsions?

5. Is there a metal-insulator transition in this systems? Does the quantum criticality competes with the single particle interference effects ?