Surface Criticality and Multifractality in the Spin Quantum Hall System

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Abstract

We examine the surface of a two-dimensional system exhibiting the spin quantum Hall transition. A supersymmetric (SUSY) formalism enables us to map our surface problem to a classical percolation model, similar to the scheme of Gruzberg, Ludwig and Read [Phys. Rev. Lett. **82**, 4524 (1999)], with reflecting boundary conditions but with the full intact supersymmetry at the surface. Through this mapping, we are able to calculate the surface multifractal exponents, Δ_2^s and Δ_3^s, of the wavefunctions. In addition, we also extract the surface scaling exponents of thermodynamic and transport quantities.

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Outline

- L-D transitions and Disordered Electronic Systems.
- Surface Criticality and Multifractality.
- Analyzing The Spin Quantum Hall Transition using SUSY.
- Calculation of Surface Exponents.
- Summary.

Localization-Delocalization (LD)Transitions

Metal $\leftrightarrow$ Insulator - induced by disorder.

No obvious broken symmetry.

No formal order parameter.

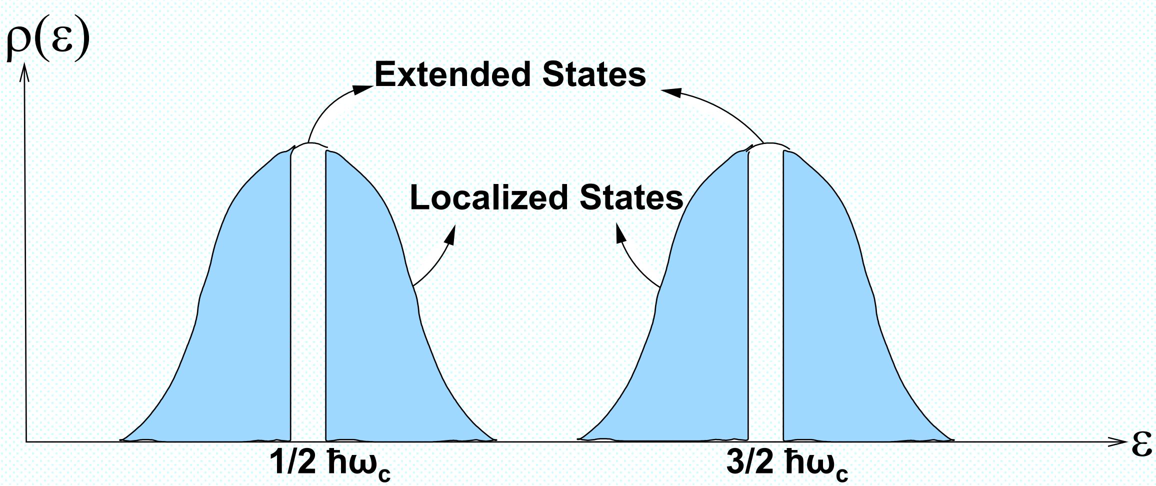
Examples:

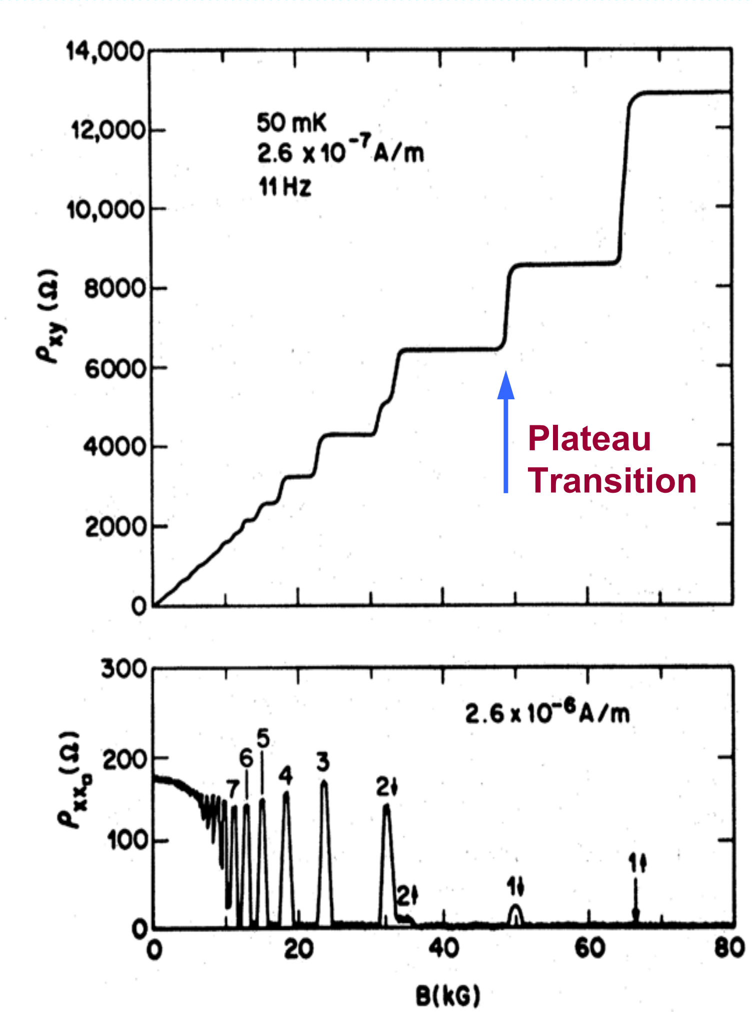
Anderson Transition

Integer Quantum Hall Effect.

Integer Quantum Hall Effect

Electrons in 2D – strong ⊥ $\vec{B}$ field.





(Paalanen, Tsui, Gossard 1982)

Plateau Transition Theory ?

i. Pruisken's Non-linear Sigma Model

ii. Chalker-Coddington Network Model

iii. Antiferromagnetic Superspin Chain (N. Read)

Disordered Electronic Systems

Symmetry Class	Time Reversal	Spin Rotation	Additional Symmetry
A (GUE)	×	×	
AI (GSE)	✓	×	
AII (GOE)	✓	✓	
AIII	×	×	Particle-hole
BDI	✓	✓	Particle-hole
CII	✓	×	Particle-hole
C	×	✓	Particle-hole
CI	✓	✓	Particle-hole
D	×	×	Particle-hole
DIII	✓	×	Particle-hole

IQHE

Anderson Localization

Spin Quantum Hall

(Senthil et al 1998)

(Altland and Zirnbauer 1997)

Multifractality

$$\boxed{\langle|\psi(\mathbf{r})|^{2\mathbf{q}}\rangle \sim \mathbf{L}^{-\tau_\mathbf{q}-\mathbf{d}}}$$

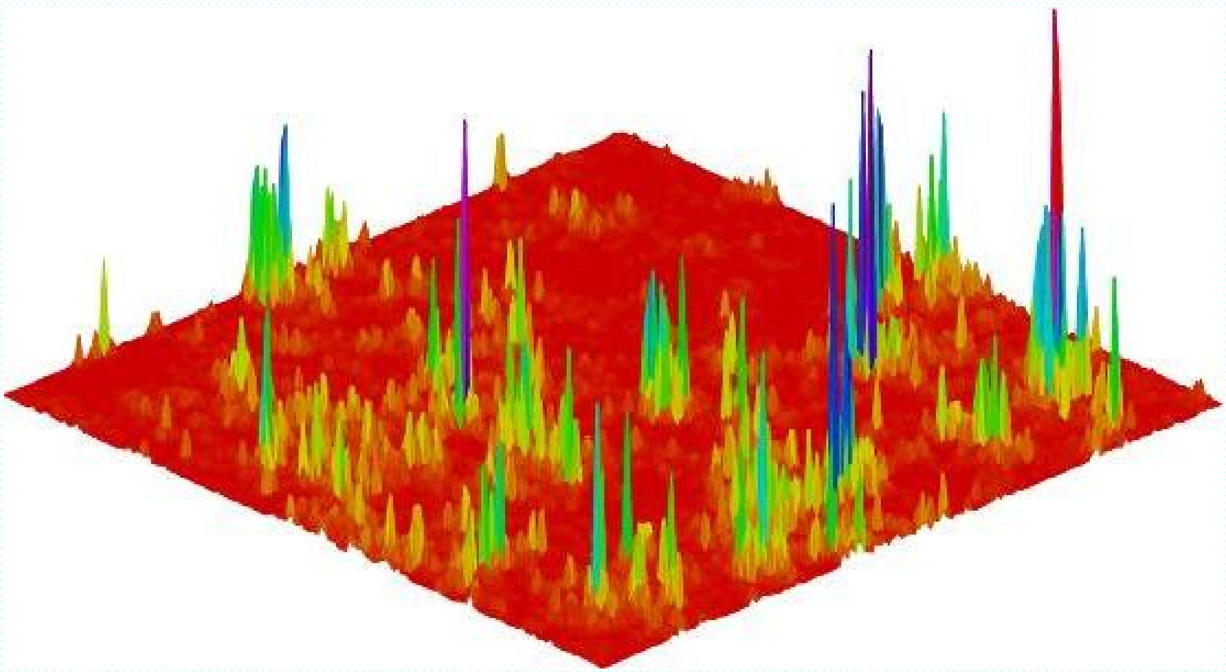
$$\boxed{\tau_\mathbf{q} = \mathbf{d}(\mathbf{q}-\mathbf{1}) + \boldsymbol{\Delta}_\mathbf{q}}$$

Wegner (1980)

$\mathbf{Metal:} \quad \boldsymbol{\Delta}_\mathbf{q} = \mathbf{0}$

$\mathbf{Insulator:} \quad \tau_\mathbf{q} + \mathbf{d} = \mathbf{0}$

Multifractal Wavefunction Intensity



from Huckestein

Surface Criticality

• Surface breaks translation symmetry.

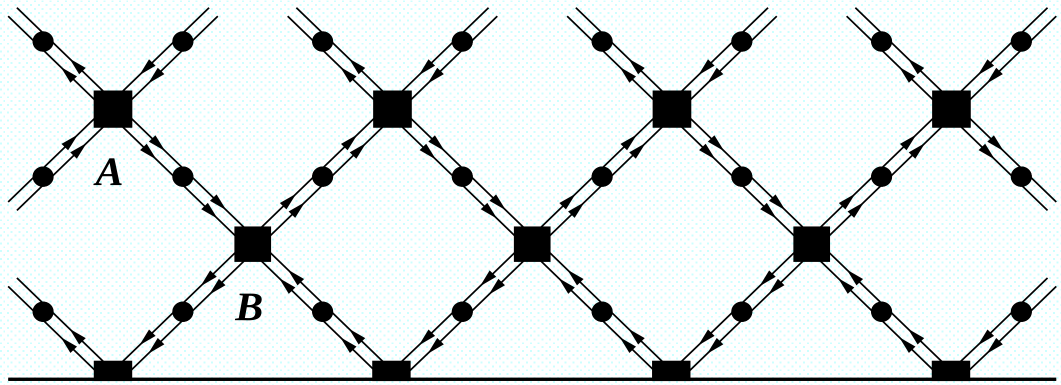
• When correlation length diverges, large scale geometry of the system is important.

• Even mean field theory gives different exponents in the presence of a boundary.

Eg. Bulk : $\mathbf{G}(r_1, r_2) \sim r_{12}^{-d+2}$

Surface : $\mathbf{G}(r_1, r_2) \sim r_{12}^{-d}$

SQH Network Model



REFLECTING BOUNDARY

Class C Network models:

Hamiltonian SU(2) invariant.

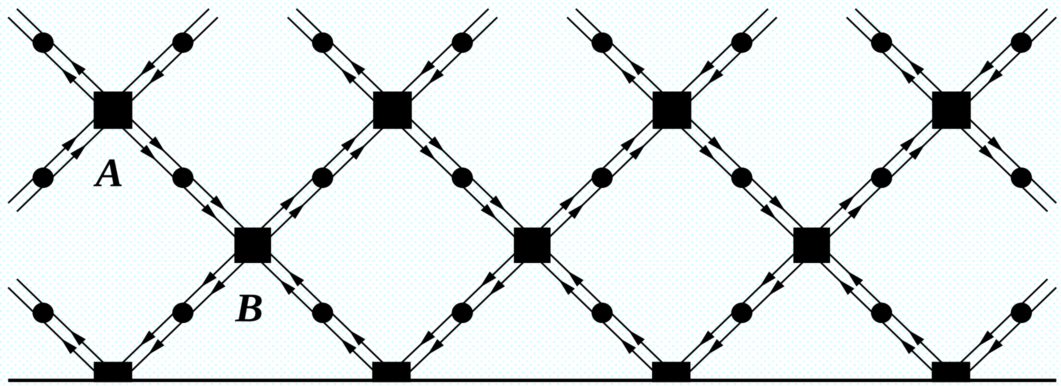
$U^T \sigma_y U = \sigma_y \rightarrow$ Sp(2n) scattering matrices. Our case n = 2.

Polar Decomposition :
$$\mathbf{S} = \begin{pmatrix} U_1 & 0 \\ 0 & U_2 \end{pmatrix} \begin{pmatrix} \sqrt{1-T^2} & T \\ -T & \sqrt{1-T^2} \end{pmatrix} \begin{pmatrix} U_3 & 0 \\ 0 & U_4 \end{pmatrix}$$

Node

Links

(Kagalovsky 1999)



REFLECTING BOUNDARY

Double links (σ = ↑,↓) connecting nodes of two kinds, A and B.

Nodes diagonal in spin indices. All A (B) nodes have same scattering matrices.

Disorder only on links → random SU(2).

$\mathbf{t}^2_{\mathbf{A}\sigma} + \mathbf{t}^2_{\mathbf{B}\sigma} = 1$ → isotropic network.

$\mathbf{t}_{\mathbf{A}\sigma} = \mathbf{t}_{\mathbf{B}\sigma}$: critical point.

$\mathbf{t}_{\uparrow} = \mathbf{t}_{\downarrow}$ **:** SQH

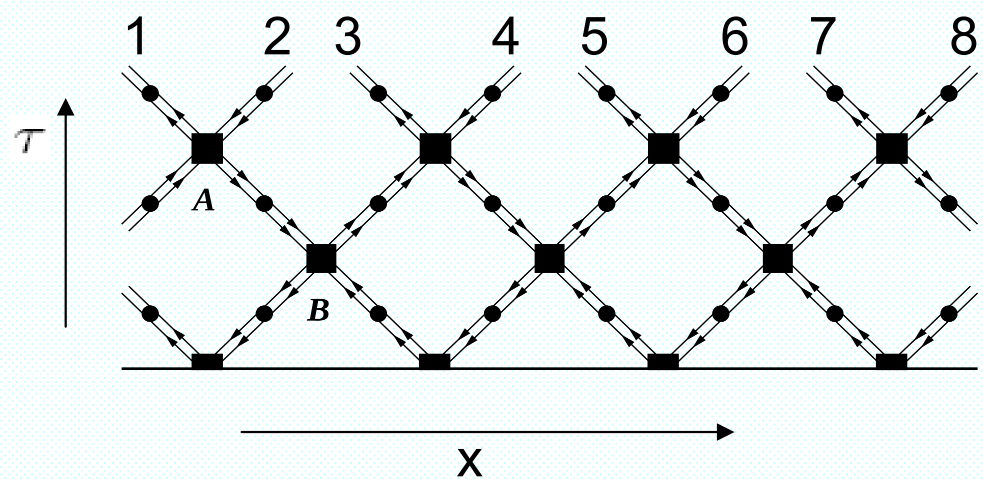
Second Quantized Description

Each uplink (downlink):

2 sets of bosons $\mathrm{b}_\sigma\ \mathrm{b}_\sigma^\dagger\ \ (\tilde{\mathrm{b}}_\sigma\ \tilde{\mathrm{b}}_\sigma^\dagger)$

2 sets of fermions $\mathrm{f}_\sigma\ \mathrm{f}_\sigma^\dagger\ \ (\tilde{\mathrm{f}}_\sigma\ \tilde{\mathrm{f}}_\sigma^\dagger)$

($\sigma = \uparrow$ and $\downarrow$)



Evolution Operator U :

Time ordered ($V_{12}V_{23}V_{34}\ldots$)

Key Point:

No separate operators for advanced Green's functions :$G_A = -\,(\varepsilon G_R\, \varepsilon)^T$

(Gruzberg, Ludwig & Read 1999)

sl (2|1) ≡ osp (2|2) Superalgebra

$$B = \frac{1}{2}(b_\uparrow^\dagger b_\uparrow + b_\downarrow^\dagger b_\downarrow + 1), \qquad Q_3 = \frac{1}{2}(f_\uparrow^\dagger f_\uparrow + f_\downarrow^\dagger f_\downarrow - 1)$$

$$Q_+ = f_\uparrow^\dagger f_\downarrow^\dagger, \qquad Q_- = f_\downarrow f_\uparrow,$$

$$V_+ = \frac{1}{\sqrt{2}}(b_\uparrow^\dagger f_\downarrow^\dagger - b_\downarrow^\dagger f_\uparrow^\dagger), \qquad W_- = (V_+)^\dagger,$$

$$V_- = -\frac{1}{\sqrt{2}}(b_\uparrow^\dagger f_\uparrow + b_\downarrow^\dagger f_\downarrow), \qquad W_+ = -(V_-)^\dagger.$$

Supersymmetry

$$
\begin{aligned}
(J_{2i-1} + \bar{J}_{2i})V_{2i-1,2i} &= V_{2i-1,2i}(J_{2i-1} + \bar{J}_{2i}) \\
(\bar{J}_{2i} + J_{2i+1})V_{2i,2i+1} &= V_{2i,2i+1}(\bar{J}_{2i} + J_{2i+1})
\end{aligned}
$$

SURFACE:

Reflecting boundary conditions: still full supersymmetry

Disorder Averaging

SU(2) averaging projects infinite dimensional Fock space of bosons and fermions onto the fundamental (dual) 3 dimensional representation of sl (2|1) on the uplinks (downlinks).

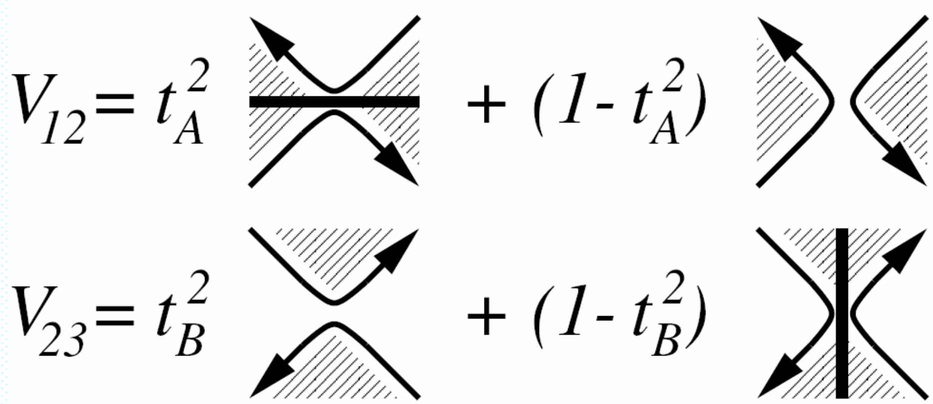
Percolation Mapping

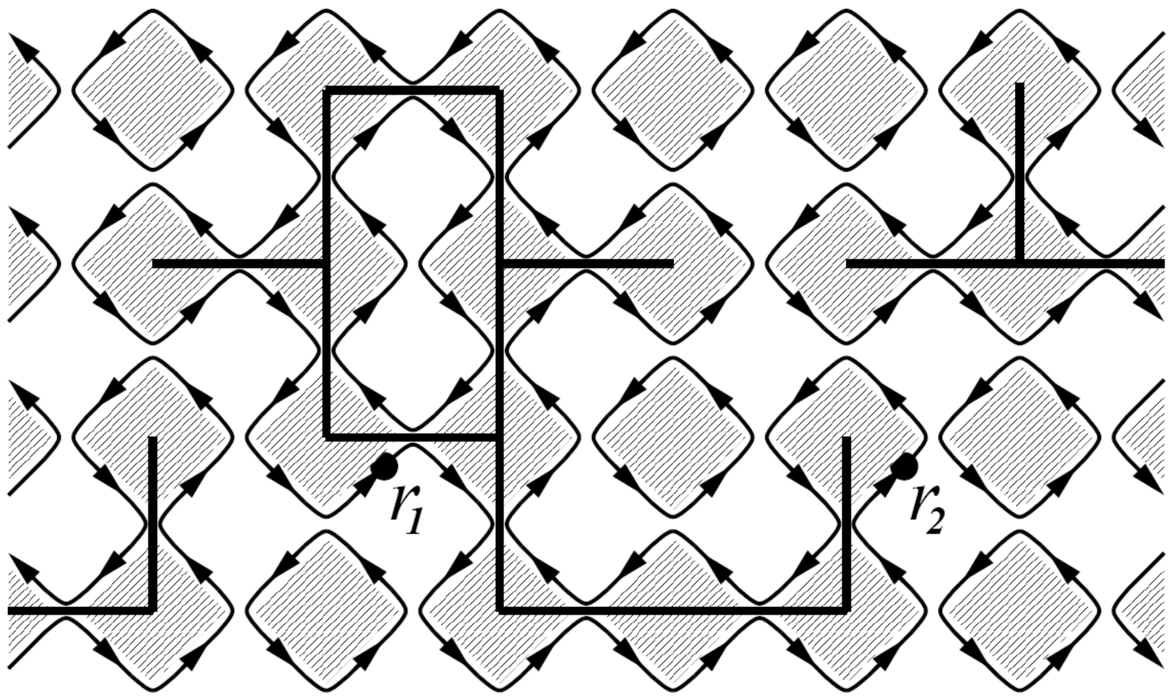
$$\pi(\tfrac{1}{2},\tfrac{1}{2}) \otimes \pi(-\tfrac{1}{2},\tfrac{1}{2}) = \pi(0,0) + \pi(0,1)$$

$$[V_{12}] = t_A^2 P_s + (1 - t_A^2) I \otimes \bar{I}$$

$$[V_{12}] = t_A^2 P_s + (1 - t_A^2) I \otimes \bar{I}$$

$$[V_{23}] = t_B^2 P_s + (1 - t_B^2) I \otimes \bar{I}$$





Equivalent Mapping:

- Q=1 Potts Model.

Two relevant scaling exponents:

- $x_{1b} = \frac{1}{4}$
- $x_{1s} = \frac{1}{3}$

Physical Quantities

Green's Function:

$$G_{\alpha\,\beta}(r_1, r_2) = \mathrm{STr}\, b_\alpha(r_1) b^\dagger_\beta(r_2) U$$

Strategy:

Express physical quantities in terms of Green's functions, then write these as disorder averaged sl(2|1) generators.

Evaluate in the fundamental representation to find equivalent percolation probabilities.

Thermodynamic Exponents

Density of States:

$$\langle \rho(r_1,\epsilon)\rangle = \frac{1}{4\pi}\langle \mathrm{Tr}G_R(r_1,r_1,z) - \mathrm{Tr}G_A(r_1,r_1,z)\rangle$$
$$= \frac{1}{2\pi}\langle \mathrm{Tr}G_R(r_1,r_1,z) - 1\rangle$$
$$= \frac{1}{2\pi}\langle 2B(r_1)\rangle$$
$$= \frac{1}{2\pi}\left[1 - \sum_N P\left(r_1;N\right)\cos 2N\epsilon\right]$$

Bulk:

$$\boxed{\rho(\epsilon) \propto \epsilon^{x_1^{\mathrm{b}}/(2-x_1^{\mathrm{b}})} = \epsilon^{1/7}}$$

Surface:

$$\boxed{\rho(r_1,\epsilon) \propto \epsilon^{x_1^{\mathrm{s}}/(2-x_1^{\mathrm{b}})} = \epsilon^{4/21}}$$

Transport Exponents

Diffusion Propagator:

$$\langle \Pi(r_1, r_2) \rangle = \left\langle \mathrm{Tr}\left[G_R(r_1, r_2) G_A(r_2, r_1) \right] \right\rangle = -2 \left\langle V_-(r_1) W_+(r_2) \right\rangle$$

Point-Contact Conductance:

$$\langle g_{point}(r_1, r_2) \rangle = \left\langle f_\uparrow^\dagger(r_1) f_\downarrow^\dagger(r_1) f_\downarrow(r_2) f_\downarrow(r_2) \right\rangle = \left\langle Q_+(r_1) Q_-(r_2) \right\rangle$$

r_1 and r_2 in bulk:	$\sim r^{-2x_{1b}}$	$\sim \mathbf{r}^{-\mathbf{1/2}}$
r_1 and r_2 on surface:	$\sim r^{-2x_{1s}}$	$\sim \mathbf{r}^{-\mathbf{2/3}}$
r_1 in bulk and r_2 on surface:	$\sim r^{-x_{1b}-x_{1s}}$	$\sim \mathbf{r}^{-\mathbf{7/12}}$

Multifractal Exponents

$$\langle |\psi(\mathbf{r})|^{2\mathbf{q}} \rangle \sim \mathbf{L}^{-\tau_{\mathbf{q}}-\mathbf{d}}$$

$$\tau_{\mathbf{q}} = \mathbf{d}(\mathbf{q}-\mathbf{1}) + \boldsymbol{\Delta}_{\mathbf{q}}$$

Away from criticality:

$$L^{2q}\langle |\psi(r_1)|^2 |\psi(r_2)|^2 \cdots |\psi(r_q)|^2 \rangle \sim (r/\xi_\epsilon)^{\Delta_q} \qquad r \lesssim \xi_\epsilon$$

Multifractal Calculation

$$
\begin{aligned}
&(2\pi)^2 \tilde{\mathcal{D}}^2 (r_1, r_2) \\
&\quad = \left\langle \sum_{ij\alpha\beta} |\psi_{i\alpha}(r_1)|^2 |\psi_{j\beta}(r_2)|^2 \, \delta(\epsilon_1 - \epsilon_i)\delta(\epsilon_2 - \epsilon_j) \right\rangle \\
&\quad = 4 \sum_N \left[1 - z^{2N}\right] P\left(r_1, r_2, ; N\right) \\
&\qquad + 4 \sum_{N,N'} \left[1 - z^{2N}\right] \left[1 - z^{2N'}\right] \left[P_-\left(r_1, r_2; N, N'\right)\right]
\end{aligned}
$$

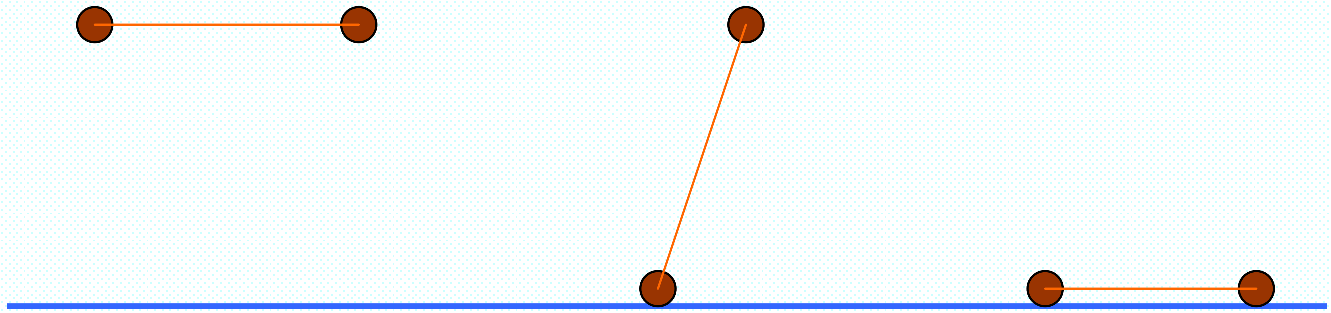
(Mirlin, Evers & Mildenberger 2003)

Δ_2 calculation: $z=e^{-\gamma} \rightarrow 1$: exact cancellation.

$$P(r_1, r_2; N) \sim N^{-25/21} r^{-1/3}, \qquad r \equiv |r_1 - r_2| \lesssim N^{4/7}.$$

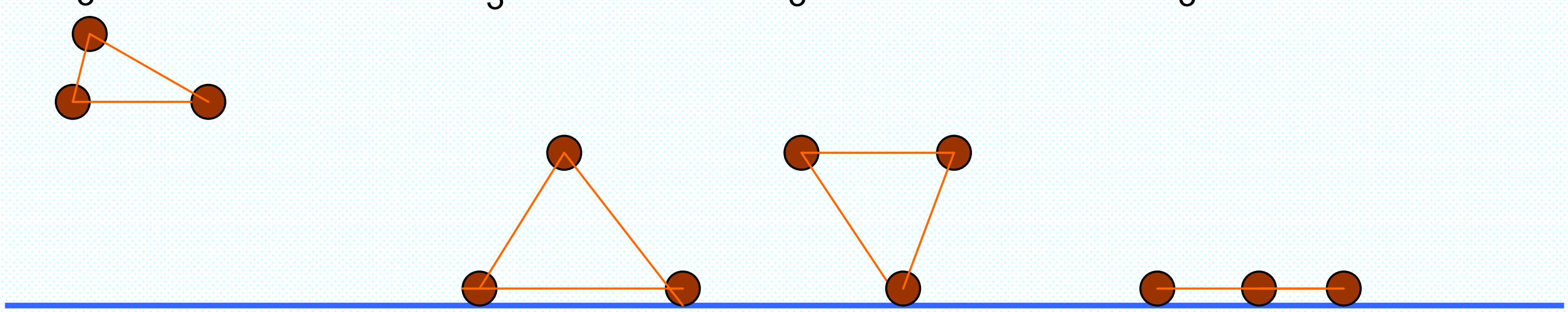
Multifractal Exponents

$\Delta_2 = -1/4$ $\Delta_2 = -1/4$ $\Delta_2 = -1/3$

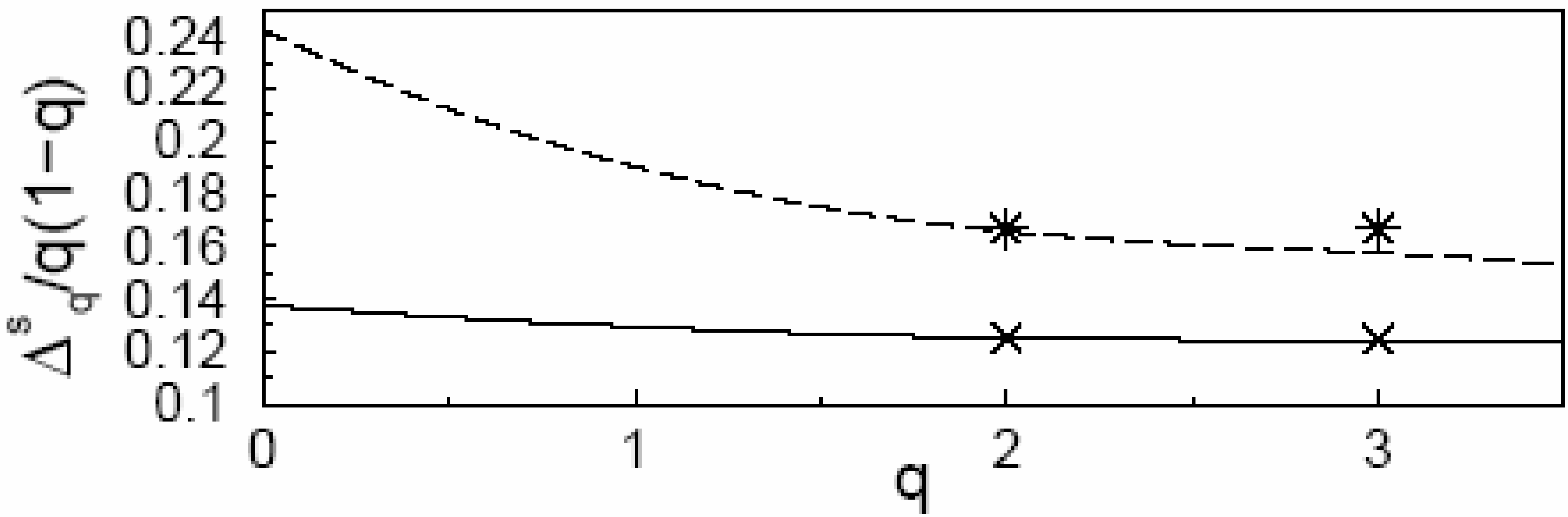


Δ_3 calculation: surprising!

$\Delta_3 = -3/4$ $\Delta_3 = -5/6$ $\Delta_3 = -11/12$ $\Delta_3 = -1$



Network Simulation



Δ_2 – 0.5 % deviation

Δ_3 - 6 % deviation

(*Evers, Mildenberger, Mirlin*)

Summary

- New paradigm – Criticality and Multifractality at the surface in LD transitions.
- Illustration in the spin quantum Hall case.

Current Work

- Surface behavior in other LD transitions, esp. Integer Quantum Hall.
- Understanding surface multifractality.

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Thank You