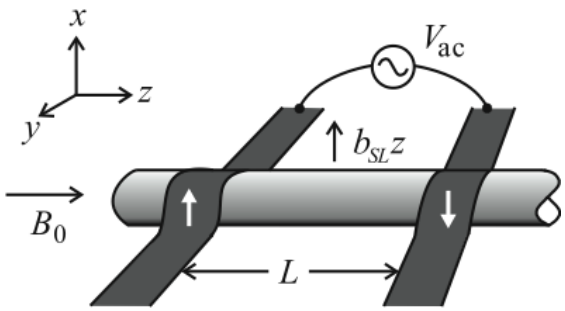
Electron spin resonance and two qubit operation in a slanting Zeeman field

Wilfred van der Wiel (Twente U.)
Toshiaki Obata (ICORP JST)
Michel Pioro-Ladrière (ICORP JST)
Seigo Tarucha (ICORP JST, Tokyo U.)
Yasuhiro Tokura (NTT BRL)

"Almost" spin qubit



cond-mat/0510411

Plan of the talk

- Motivation of this work
  - Difficulty of single electron spin resonance
- Proposed scheme of orbital-spin hybridization
  - Single pseudo-spin qubit manipulation by rf-field
  - Read-out scheme
- Two qubit operation
  - Anisotropic coupling J : C-NOT operation
- Dephasing mechanism
  - Electron-phonon interaction – spin-boson model
  - The quality factor Q of π operations
- Summary and Outlook

Need for one spin-qubit operation

Qubit: coherent two-level system with controllable parameters $\varepsilon_x(t)$, ε_z:

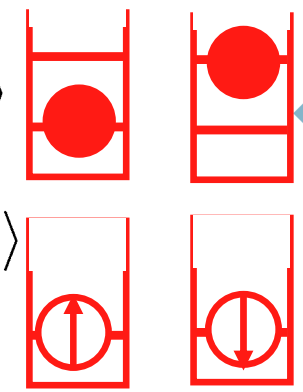
$$H_{2level} = \frac{\varepsilon_z}{2}\sigma_z + \frac{\varepsilon_x(t)}{2}\sigma_x$$

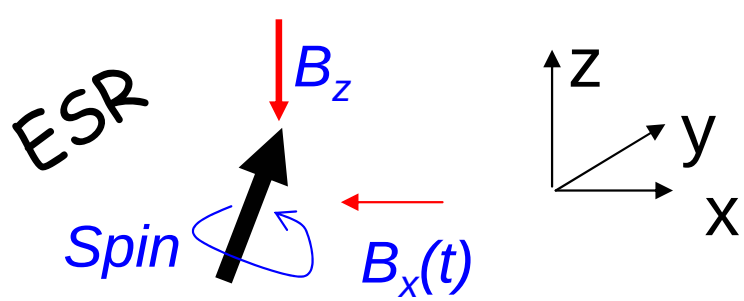
- Charge qubit $\alpha|1\rangle + \beta|2\rangle$

(Hayashi et al., 2003)

- Spin qubit $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$

(Petta et al.,2005)





- Localized electron-spin as qubit
  - Long coherence time (~*1.2μs Petta 2005*) and scalability
  - Two-qubit operation is feasible by controlling exchange *J*
- One-qubit operation is
  - difficult because…

we need to control magnetic field $B_x(t)$

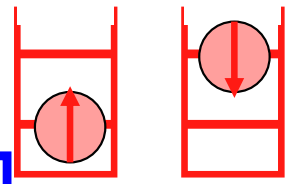
- individually (sub-μm)
- high-frequency (~GH_z)

- Generate $B_x(t)$ by on chip microscopic coil
  - Effect of oscillating electric field, problem of heating

Hybridized system

- Pseudo-spin qubit:

$$\alpha|a\rangle + \beta|b\rangle$$



Orbital-spin hybrid state

$$|p\rangle = \sum_{n=1,2,\sigma=\uparrow/\downarrow} c^{p}_{n\sigma}|n\sigma\rangle$$

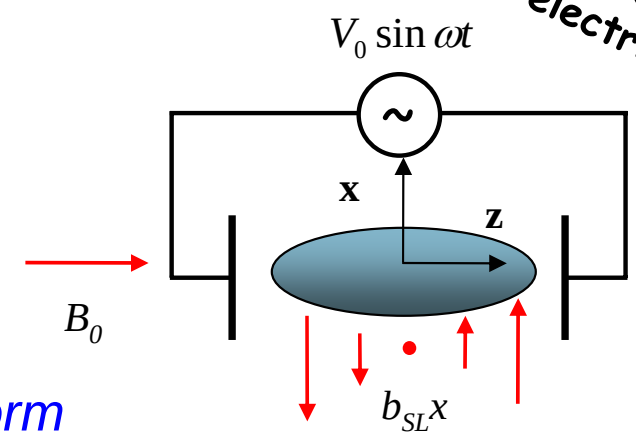
- Engineer *g*-factor tensor to induce *internal field $B_x(t)$* with gate voltage

  (Rashba and Efros 2003)

  need controllable inhomogeneus or anisotropic *g*-factor tensor

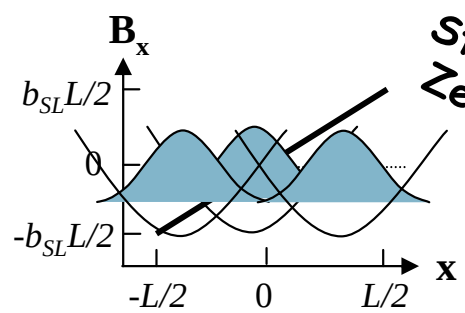
- This proposal: *Stern-Gerlach*

Parabolic 1D dot in electric field



Non-uniform
•magnetic field
•nucl spin pol
•g-factor

Static slanting Zeeman field



Effective spin dynamics

$$H = \frac{1}{2}\varepsilon_z\sigma_z + \frac{1}{2}\varepsilon_x(t)\sigma_x$$

$$\varepsilon_z = -g\mu_B B_0$$

$$\varepsilon_x(t) = -g\mu_B b_{SL}\langle X(t)\rangle$$

$\langle X(t)\rangle$ *Average position*

Hybridization by a slanting field

The Hamiltonian

- 1D quantum dot

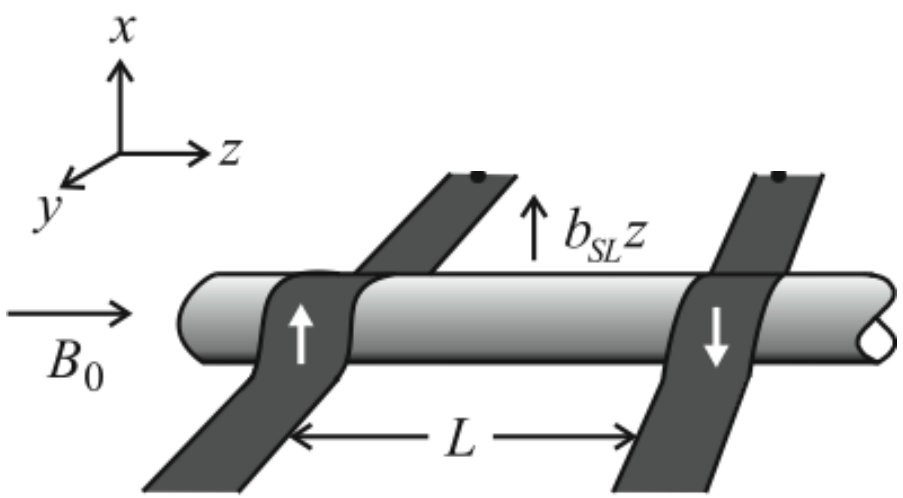
$$H_{00} = \frac{p^2}{2m} + V(z) - g\mu_B B_0 S_z$$

$$\epsilon_{n\sigma} = \epsilon_n - \frac{1}{2} g\mu_B B_0 \sigma$$

where *n=1,2,..,* σ*=+/- 1.*

- Slanting Zeeman field

$$H_{0s} = -g\mu_B b_{SL} z S_x$$



One possible implementation using ferromagnetic (FM) material

$$\boldsymbol{B} = b_{SL} z \boldsymbol{i}_x + (B_0 + b_{SL} x)\boldsymbol{i}_z$$

$$\mathrm{div}\boldsymbol{B} = 0, \mathrm{rot}\boldsymbol{B} = 0$$

$$\text{Define: } E_{SL} \equiv -g\mu_B b_{SL} L$$

negligible

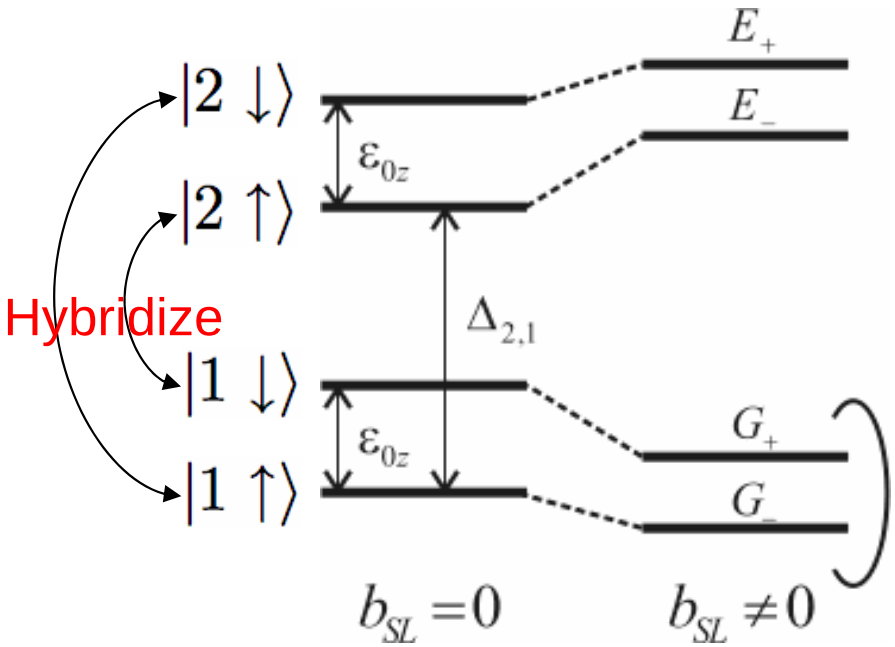
Energy spectrum

Non-zero matrix elements of H_{0s}

Form factor

$$\langle m, -\sigma | H_{0s} | n, \sigma \rangle = \frac{1}{2} E_{SL} \langle m | \frac{z}{L} | n \rangle \equiv \frac{1}{2} E_{SL} \Upsilon_{m,n}$$

For *symmetric potential V(z)=V(-z)*, form factors with different parity remain.



ε_{0z} Bare Zeeman energy

$\Delta_{2,1}$ Confinement energy

Wave function

$$|G_-\rangle = |1\uparrow\rangle + C_{1\uparrow}|2\downarrow\rangle$$

qubit

ε_z

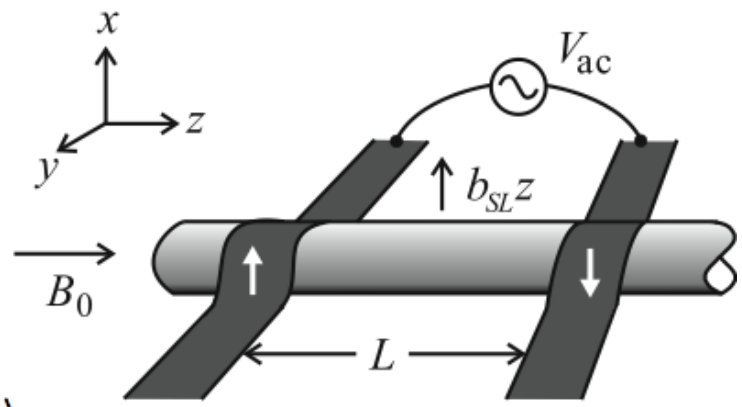
The lowest two states constitute a *qubit* with an effective Zeeman energy

ε_z

Time dependent potential

Time-dependent voltage V_{ac}

$$H_1 = eV_0 \sin \omega t \cdot \frac{z}{L}$$



Being odd function, H_1 couples the qubit states:

$$\langle G_+|H_1|G_-\rangle \equiv \frac{1}{2}\varepsilon_x(t)$$

$$\varepsilon_x(t) = -g\mu_B b_{SL}\langle X(t)\rangle$$

$$\text{Parabolic conf.} \quad \langle X(t)\rangle = \frac{eV_0}{\sqrt{\hbar m \omega_0^3}} \sin \omega t$$

Generic ESR Hamiltonian:

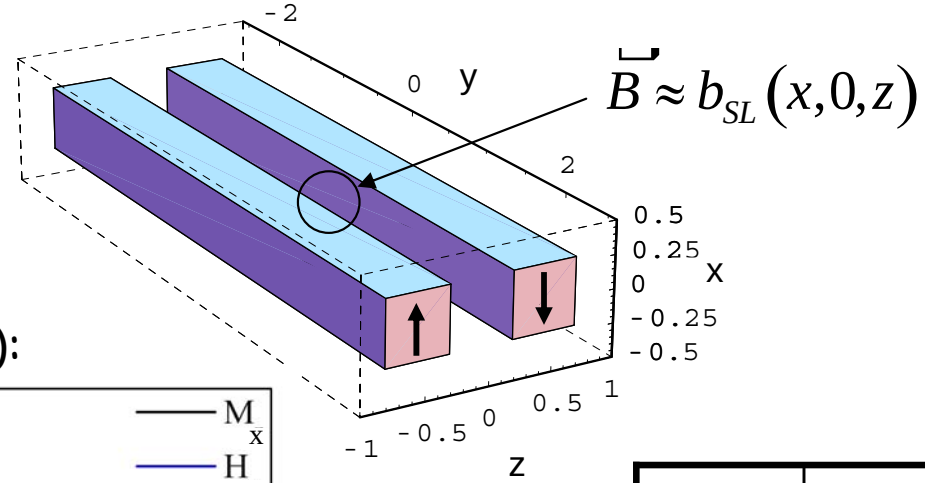
$$H_{2level} = \frac{\varepsilon_z}{2}\sigma_z + \frac{\varepsilon_x(t)}{2}\sigma_x$$

$|G_+\rangle$ — π — $|G_-\rangle$

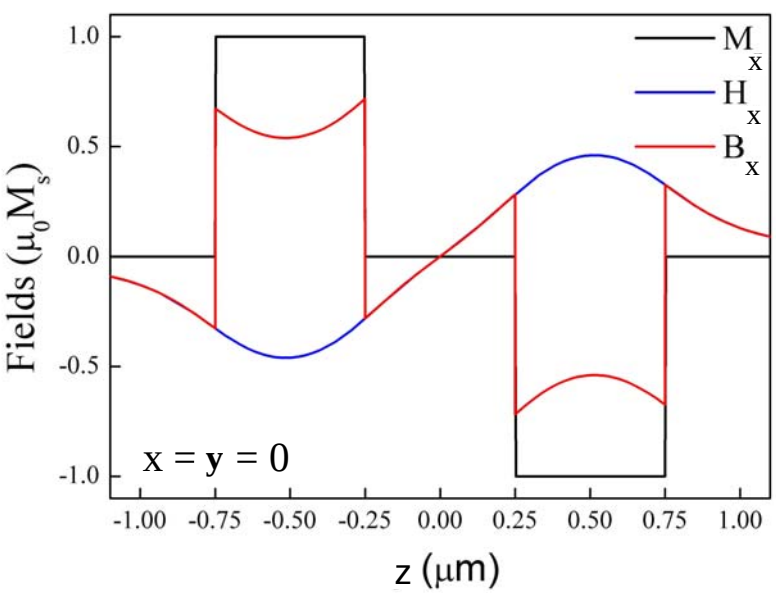
For b_{SL} = *1 T/μm*, GaAs dot *L=35nm*
Driving frequency $\omega = \varepsilon_z$ ~ *6 GHz/T*
Time for π pulse (V_0 = *0.1 mV*) ~ *40 ns*

Slanting field between ferromagnets

2 permanent magnets (5 mm x 0.5 μm x 0.5 μm)
separated by 0.5 μm gap



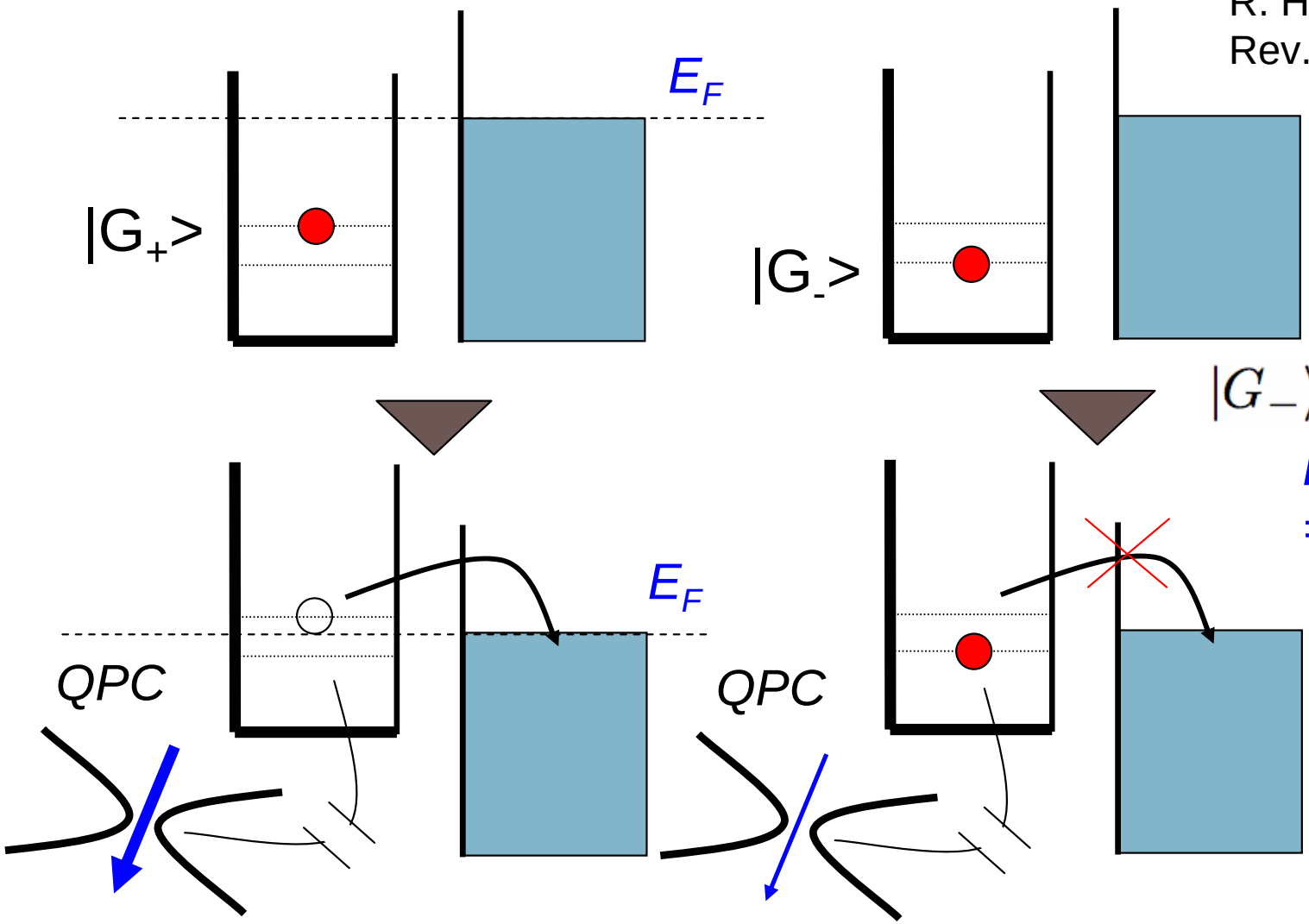
Simulation results
(Radia©+Mathematica©):



Material	T_C (°C)	$\mu_0 M_S$ (T)	b_{SL} (T/μm)
Fe	770	2.19	2.23
Co	1115	1.82	1.86
Py	596	1.70	1.73
Ni	354	0.64	0.65

Read-out scheme

R. Hanson, *et al.*, Phys. Rev.Lett. **94** 196802 (2005).

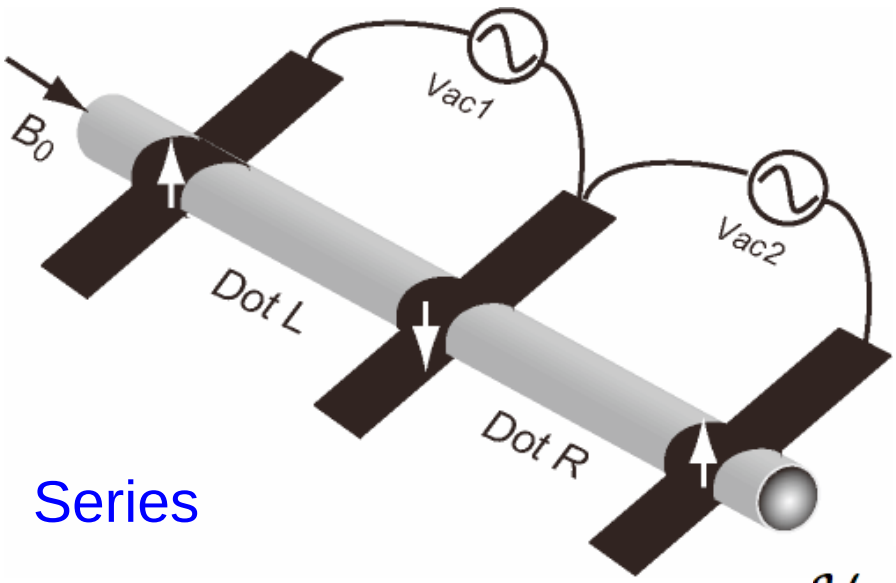


$$|G_-\rangle = |1\uparrow\rangle + C_{1\uparrow}|2\downarrow\rangle$$

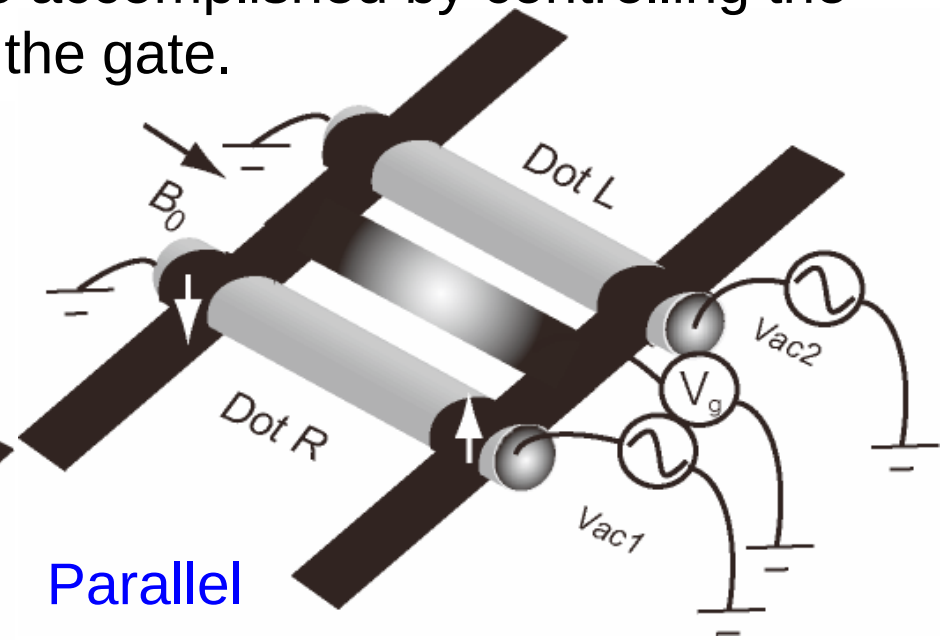
Read-out error
$= |C_{1\uparrow}|^2 \sim 10^{-6}$

Coupled qubits

2 qubit operation, C-NOT, could be accomplished by controlling the tunnel center tunnel barrier by the gate.



Series



Parallel

Hund-Mulliken Hamiltonian

$$\mathcal{H} = \sum_{f=L,R} \mathcal{H}_{0f} + \mathcal{H}_T + \mathcal{H}_V,$$

$$\mathcal{H}_{0f} = \frac{\varepsilon_z}{2} \sum_\sigma c^\dagger_{f\sigma} c_{f\sigma} + U n_{f\uparrow} n_{f\downarrow}$$

$$\mathcal{H}_T = \sum_\sigma [t_\sigma c^\dagger_{L\sigma} c_{R\sigma} + s c^\dagger_{L\sigma} c_{R-\sigma} + \text{H.c.}]$$

$$\mathcal{H}_V = V \sum_{\sigma\sigma'} n_{L\sigma} n_{R\sigma}$$

Parameter *s* depends on the series/parallel configurations.

Projected to the qubit base:

Exchange Hamiltonian

Heisenberg Hamiltonian

$$\mathcal{H} = J\boldsymbol{S}_L \cdot \boldsymbol{S}_R + \varepsilon_{0z}(S_{Lz} + S_{Rz})$$

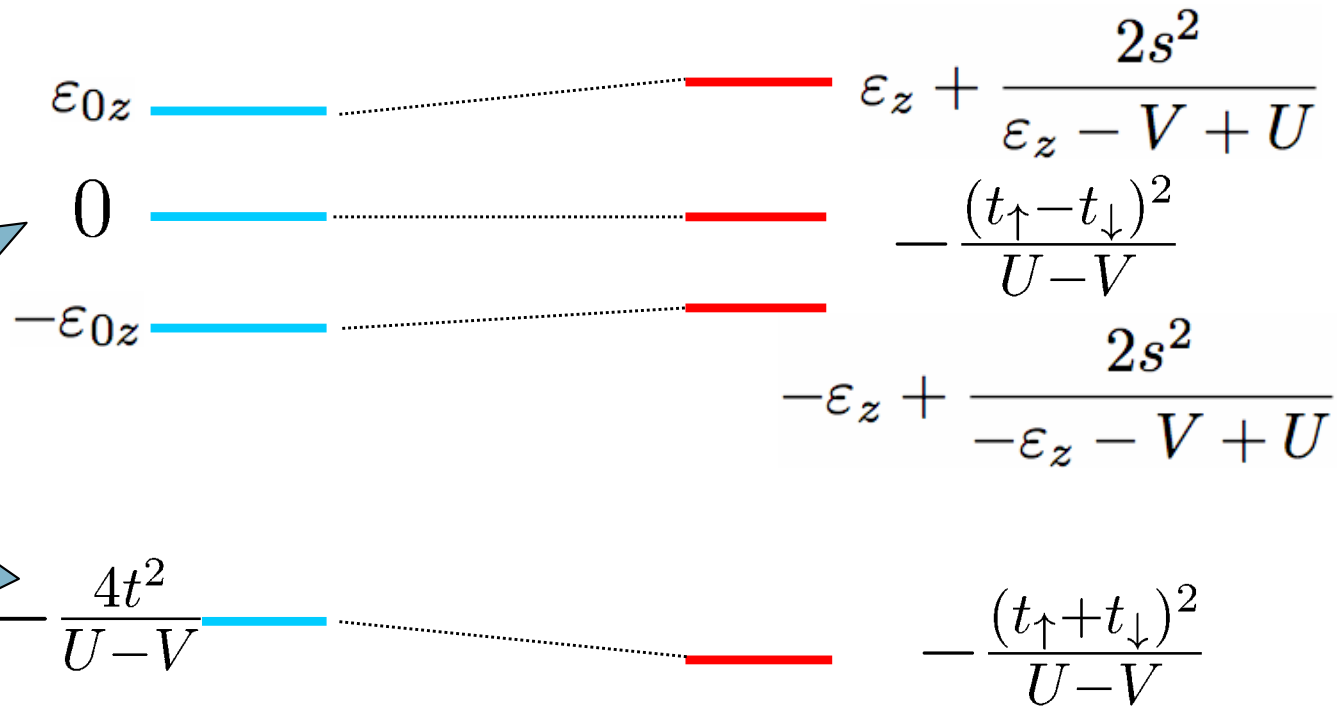
Effective Hamiltonian anisotropic (XXZ)

$b_{SL} = 0$

$b_{SL} \neq 0$

Triplet

Singlet



C-NOT operations

$$\begin{aligned}\mathcal{H} &= J_{\|} S_{Lz} S_{Rz} \\ &+ J_{\perp}(S_{Lx}S_{Rx} + S_{Ly}S_{Ry}) \\ &+ \tilde{e}_z(S_{Lz} + S_{Rz})\end{aligned}$$

Coupling constants:

$$\begin{aligned}J_{\|} &= \frac{2(t_{\uparrow}^2 + t_{\downarrow}^2)}{U-V} - \frac{4s^2(U-V)}{(U-V)^2 - \varepsilon_z^2} \\ J_{\perp} &= \frac{4t_{\uparrow}t_{\downarrow}}{U-V} \\ \tilde{e}_z &= \varepsilon_z\left(1 - \frac{2s^2}{(U-V)^2 - \varepsilon_z^2}\right)\end{aligned}$$

cf: spin-orbit interaction

$$\mathcal{H}_{SO} = J\{\boldsymbol{S}_L \cdot \boldsymbol{S}_R + \boldsymbol{\beta} \cdot (\boldsymbol{S}_L \times \boldsymbol{S}_R) + \gamma[\boldsymbol{\beta} \cdot \boldsymbol{S}_L][\boldsymbol{\beta} \cdot \boldsymbol{S}_R]\}$$

β :Dzyaloshinski-Moriya vector

Control-Not operation is possible with $J_{\|} \to J, \quad \tilde{e}_z \to \varepsilon_{0z}$

Burkard & Loss, Phys. Rev. Lett. 2002

Relaxation by phonon

Since we hybridize the spin and orbital degrees of freedom, orbital relaxation processes harm the spin coherence.

Electron-phonon interaction:

Project to pseudo spin qubit state

$$H_{e-ph} = \sum_q \lambda_q (e^{iqz} b_q^\dagger + H.C.)$$

$$= \sum_q (\underline{\Lambda_q^x \sigma_x} + \underline{\Lambda_q^z \sigma_z}) b_q^\dagger + H.C.$$

Spin-boson model

Dissipation *Pure decoherence*

Effective coupling constants:

$$\left|\frac{\Lambda_q^z}{\Lambda_q^x}\right| \sim \frac{\epsilon_{0z} \Upsilon_{2,3} E_{SL}}{\Delta_{3,1} \Delta_{2,1}} \ll 1$$

Pure decoherence process is negligible.

$$\boxed{\frac{1}{T_{2phonon}} \sim \frac{1}{2T_1}}$$

Quality factor

Relaxation rate of the qubit is obtained in a form of Fermi Golden rule

$$\frac{1}{T_1} = \frac{2\pi}{\hbar} \sum_q |\Lambda_q^x|^2 \delta(\varepsilon_z - \hbar\omega_q) \coth \frac{\beta\varepsilon_z}{2}$$

Coherence time by phonons determined

$$\frac{1}{T_{2phonon}} = \frac{1}{2T_1} \propto B_0^3$$

Piezo-electric phonon scattering is dominating

Time needed for π- operation:

$$\tau_\pi = \frac{2\pi\hbar}{\varepsilon_x}$$

Total coherence time:

$$\frac{1}{T_{2total}} = \frac{1}{T_{2phonon}} + \frac{1}{T_{2spin}}$$

Quality factor of π- operation

$$Q \sim \frac{T_{2total}}{\tau_\pi} \sim \frac{2\Delta_{2,1}\tau_p(\varepsilon_z)}{\pi\hbar} \frac{eV_0}{|E_{SL}|}$$

Tunable!

(τ_p is the relaxation time with energy ε_z)

T_1, T_2 estimation of GaAs dot in bulk

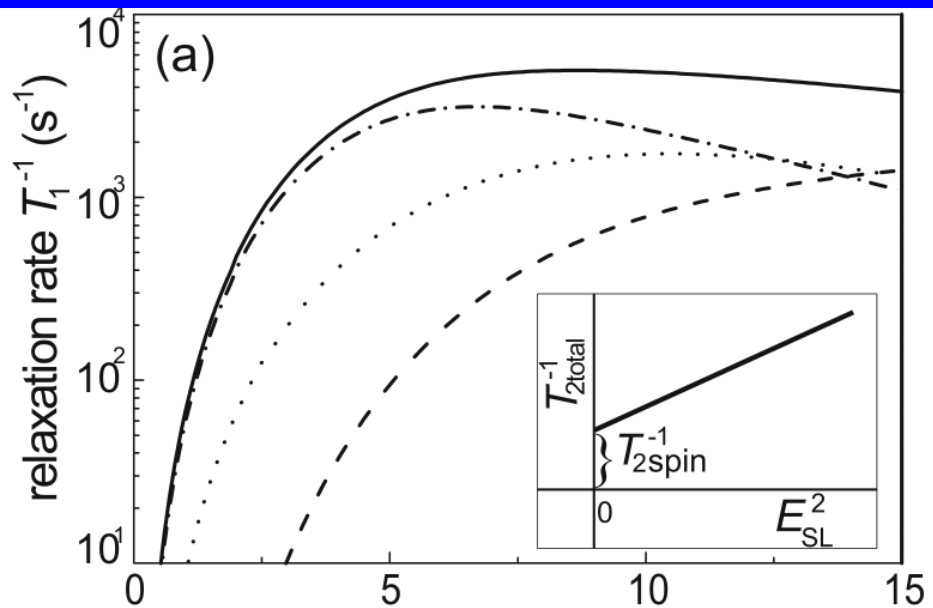
Three 3D acoustic phonon scattering mechanisms:

- Deformation potential (– – –)
- Longitudinal piezoelectric (·····)
- Transversal piezoelectric (– · –)

GaAs system:
Parabolic confinement
Longitudinal *1meV (~35nm)*
Transversal *10meV (~10nm)*

Slanting field:
b_{SL}*~1T/μm*

AC voltage:
10 μV



Decoherence by phonon with spin-orbit interaction (Dresselhaus) is negligible.

Metallic single-wall carbon nanotube

P.Jarillo-Herrero et al., Phys. Rev. Lett. 2005

Strong suppression of electron-phonon coupling for *B=0*.

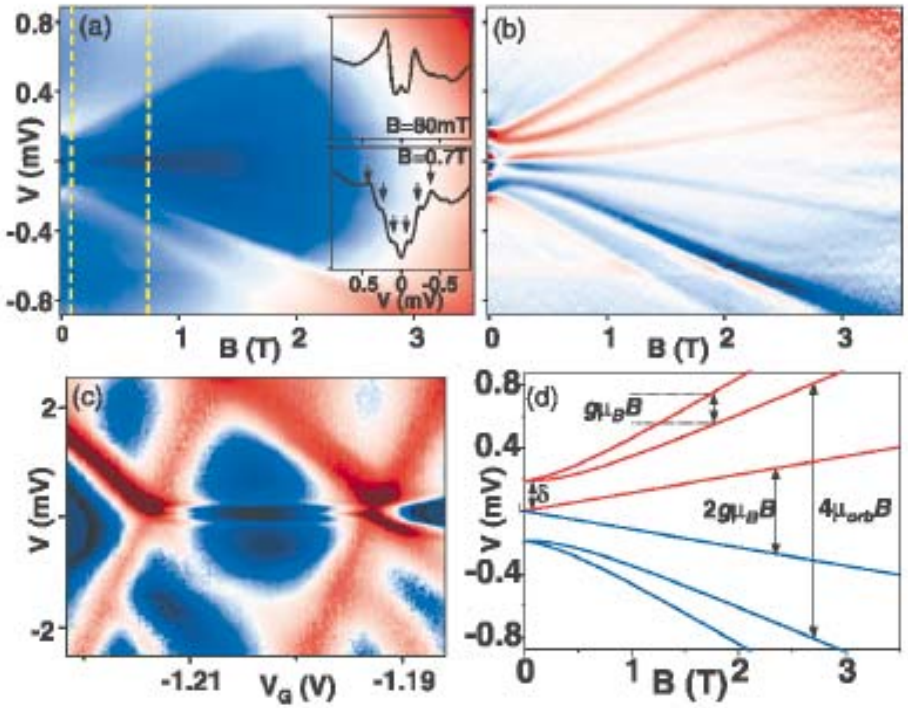
H. Ajiki and T.Ando (1994)

4-fold degeneracy *is flited by* B_0

Lowest four levels with parallel field:

$$\pm\frac{1}{2}g\mu_B B_0, \quad \pm\frac{1}{2}\mu_{orb}B_0$$

$$\frac{g\mu_B}{\mu_{orb}} \sim 0.3$$



What happens for $B_0 <> 0$, $B_{SL} <> 0$?

T. Nakanishi and T. Ando, J. Phys. Soc. Jpn ***74****, 3027 (2005).*

$\sigma(B_0) - \sigma(0) \propto - B_0^2$.

The effect of B_{SL} is negligible

Weak spin-orbit, vanishing nuclear spins:

CNT is an ideal system !

Conclusions and Outlook

- We consider a single electron in 1D quantum dot with a static slanting Zeeman field. This defines a pseudo-spin qubit, which is coherently manipulated by the voltage applied to the gate electrodes.

- Field gradient on the order of 1 T/μm can be achieved with standard ferromagnetic material.

- Adjacent two qubits show anisotropic exchange interaction, and C-NOT operation is feasible.

- Phonon emission process introduces decoherence, but the quality factor of one qubit operation is quite high (~10^4).

  Other sources of decoherence ? Magnon in ferromagnets ?

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