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# Weak Localization Effects in Granular Metals

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# Granular Metals

- A collection of metallic islands (grains) buried in an amorphous insulating matrix is called GRANULAR METAL
- We suppose that each grain has the same diameter d and discrete energy spectrum.
- Dimensional quantization scale is determined by the Thouless energy :

$$E_T = \frac{4\pi^2 \mathcal{D}}{d^2}$$

- Diffusion coefficient:

$$\mathcal{D} = \lim_{t\to\infty} \frac{\overline{\mathbf{r}^2(t)}}{t}$$

$$E_T^{(d)} \approx \frac{v_F^2 \tau_{imp}}{d^2}$$

$$E_T^{(bl)} \approx v_F / d$$

- Mean level spacing :

$$\delta = \frac{1}{\nu d^D}$$

# Dimensionless Conductance

The fundamental parameter in the description of transport properties is the “dimensionless conductance”. It can be determined from the Einstein’s relation:

$$\sigma = e^2 \nu D = g L^{2-D}$$

The dimensionless conductance in metal corresponds to the number of single-particle energy levels in a “stripe” of width of the order of the Thouless energy centered around the Fermi level:

$$g = \left(\frac{e^2}{\hbar}\right)\frac{E_T}{\delta} \gg 1$$

The tunneling dimensionless conductance $g_T$, corresponds to the number of electron states in the stripe of the width of tunneling amplitude $t \ll E_T$ centered at the Fermi level:

$$g_T = 2\pi\left(t/\delta\right)^2$$

# Electron motion in good granular metal

• We suppose that inside a single grain electrons can move freely in a quasi-ballistic fashion, i.e the intra-grain dimensionless conductance

$$g = \frac{E_T}{\delta} \approx p_F d \gg 1$$

• In a good granular metal electrons can tunnel from one grain to the nearest neighbor with high probability, This means that the tunneling dimensionless conductance $g_T$, is large:

$$g_T = 2\pi\,(t/\delta)^2 \approx (\frac{t}{E_F})^2 g^2 \gg 1$$

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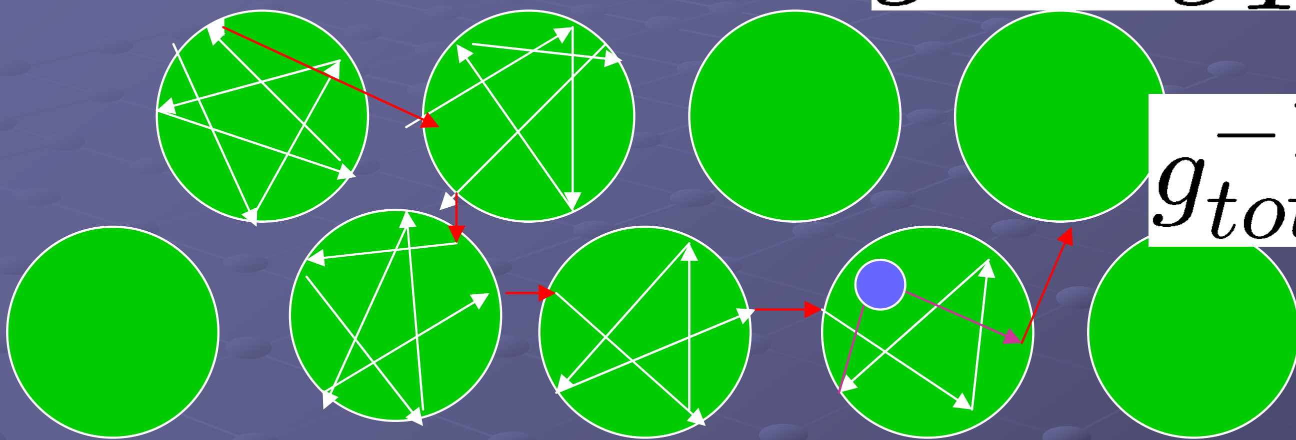
• The macroscopic transport is determined by tunneling processes:

$$g \gg g_T$$



$$g_{tot}^{-1} = g^{-1} + g_T^{-1} \approx g_T^{-1}$$

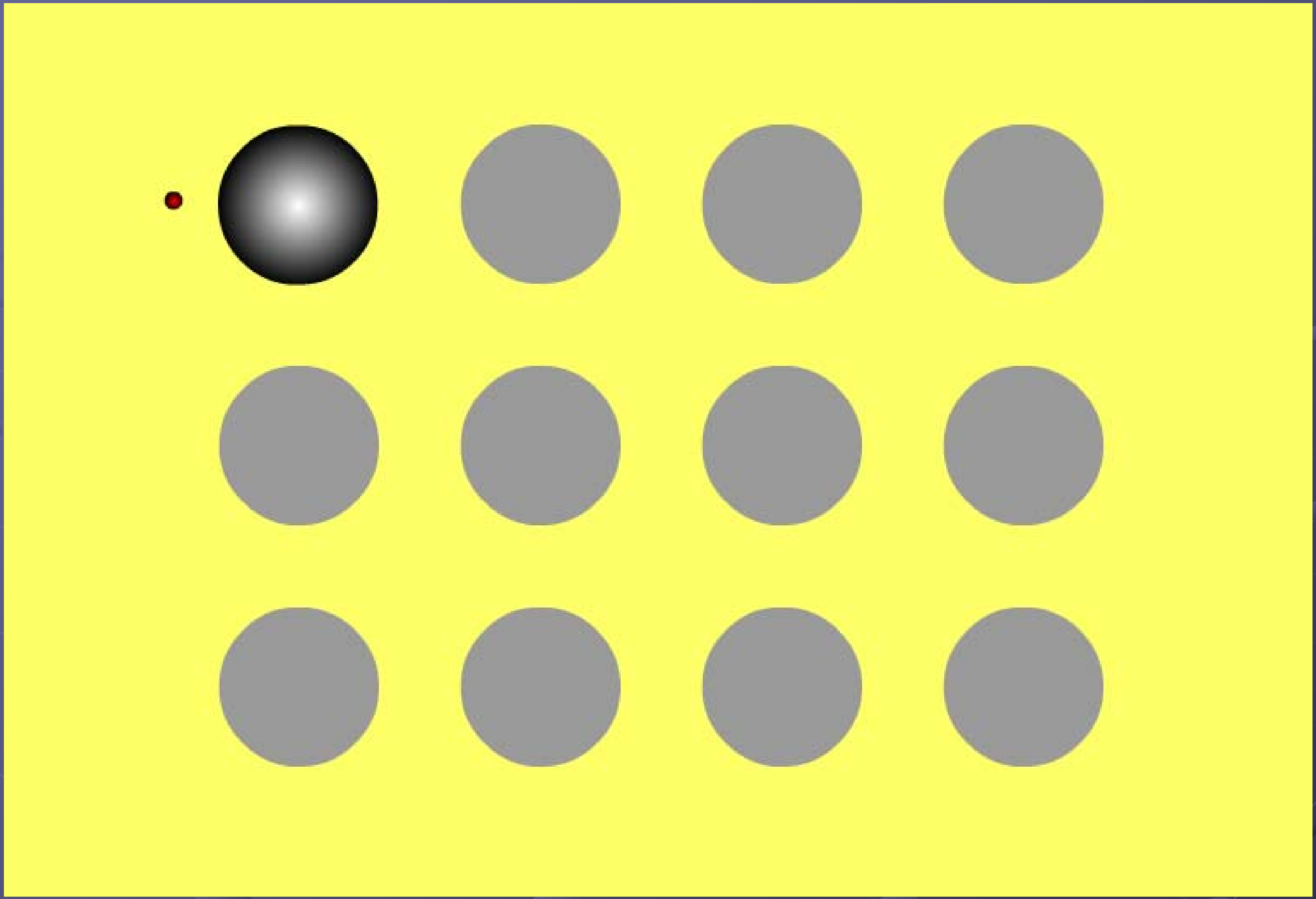
• The elastic scattering time is given by

$$\frac{1}{\tau_{el}} = \frac{1}{\tau_{imp}} + E_T + \Gamma \approx \max\{\tau_{imp}^{-1}, E_T\} = E_T$$

Here

$$\Gamma \sim Z g_T \delta$$

is the tunneling scattering rate, Z is the coordination number



(Belongs to  Andreas Glatz)

# Hamiltonian and double momentum representation

- The Hamiltonian in a “mixed” representation (intra-grain momentum space, inter-grain direct space) has the form:

$$\hat{H} = \sum_{i,\mathbf{p}} \varepsilon_{\mathbf{p}} \hat{c}^{\dagger}_{i,\mathbf{p}} \hat{c}_{i,\mathbf{p}} + \frac{1}{2} \sum_{\langle i,j \rangle} \sum_{\mathbf{p},\mathbf{p}'} [t^{\mathbf{p},\mathbf{p}'}_{ij} \hat{c}^{\dagger}_{i,\mathbf{p}} \hat{c}_{j,\mathbf{p}'} + \mathrm{H.c.}],$$

Intra-grain dynamics

Inter-grain dynamics

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# Double momentum representation:

$$\hat{H} = \sum_{\mathbf{K},\mathbf{p}} [\varepsilon_{\mathbf{p}} + tZ\gamma_{\mathbf{K}}]\hat{c}^{\dagger}_{\mathbf{K},\mathbf{p}}\hat{c}_{\mathbf{K},\mathbf{p}} + \frac{tZ}{2}\sum_{\mathbf{K}}\sum_{\mathbf{p}\neq\mathbf{p}'}\gamma_{\mathbf{K}}[\hat{c}^{\dagger}_{\mathbf{K},\mathbf{p}}\hat{c}_{\mathbf{K},\mathbf{p}'} + \mathrm{H.c.}].$$

with

$$\gamma_{\mathbf{K}} = Z^{-1}\sum_{\mu=1}^{Z} e^{i\mathbf{K}\cdot\mathbf{d}_{\mu}};$$

# Conductivity in linear response theory

$$\sigma_{\alpha\beta}(0) = \lim_{\omega\to 0^+} \frac{\mathcal{Q}_{\alpha\beta(\omega)}}{-i\omega}$$

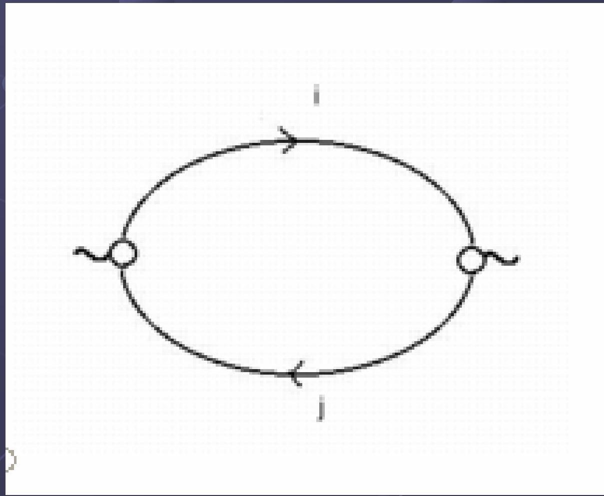
$$\mathcal{Q}_{\alpha\beta(\tau)} = -\frac{1}{\mathcal{Z}[0]}\frac{\delta \mathcal{Z}[\mathbf{A}]}{\delta A_\alpha(\tau)\delta A_\beta(0)}\Bigg|_{\mathbf{A}\to 0}$$

$$\mathcal{Z}[\mathbf{A}] = \int \mathcal{D}\hat{c}_{\mathbf{K},\mathbf{p}}(\tau)\,\mathcal{D}\hat{c}^\dagger_{\mathbf{K},\mathbf{p}}(\tau)\, e^{-\int_0^\beta d\tau \hat{H}\left[c_{\mathbf{K},\mathbf{p}}(\tau),c^\dagger_{\mathbf{K},\mathbf{p}}(\tau)\right]}$$

$$\boxed{\mathcal{Q}_{\alpha\beta}(\tau) = \Pi_{\alpha\beta}(\tau) + \frac{Ne^2}{m}\delta_{\alpha\beta}}$$

with

$$\boxed{\Pi_{\alpha\beta}(\tau) = \left\langle \hat{j}_\alpha(\tau)\,\hat{j}_\beta(0)\right\rangle_0}$$



# Electromagnetic response operator

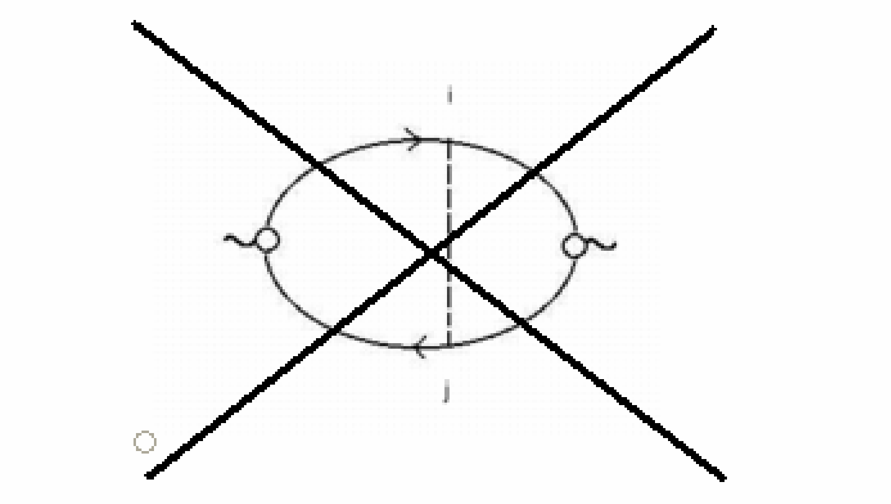
$$\Pi_{\alpha,\alpha}(\omega_\nu) = 2e^2 d^2 |t|^2 \sum_{\mathbf{K}} \sin^2(K_\alpha d) \times T \sum_{\varepsilon_n} \sum_{\mathbf{p},\mathbf{p}'} G_{\mathbf{K}}(\mathbf{p},\varepsilon_{n+\nu}) G_{\mathbf{K}}(\mathbf{p}',\varepsilon_n),$$

One-electron Green function is:

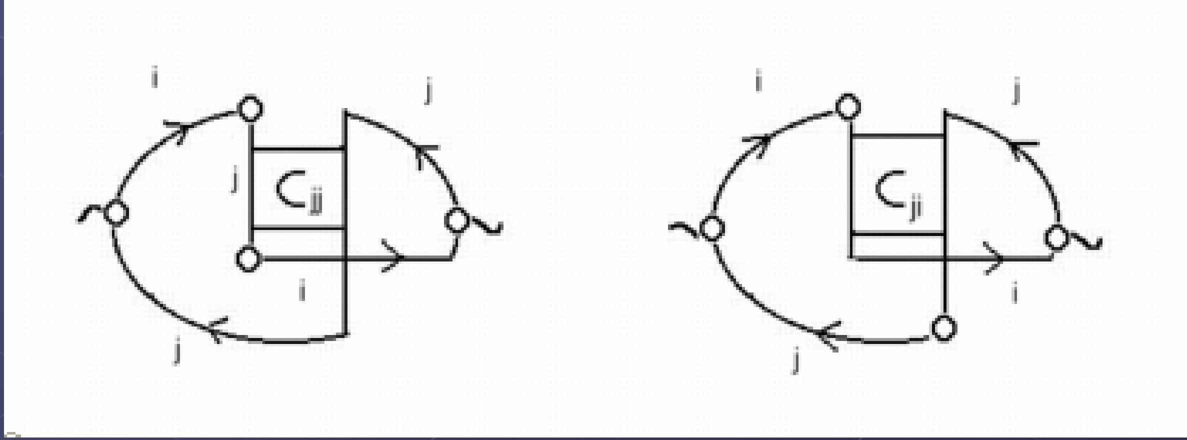
$$G_{\mathbf{K}}(\mathbf{p},\epsilon_n) = \frac{1}{i\tilde{\epsilon}_n - \xi_p - Zt(1-\gamma_{\mathbf{K}})}.$$

# Impurities averaging

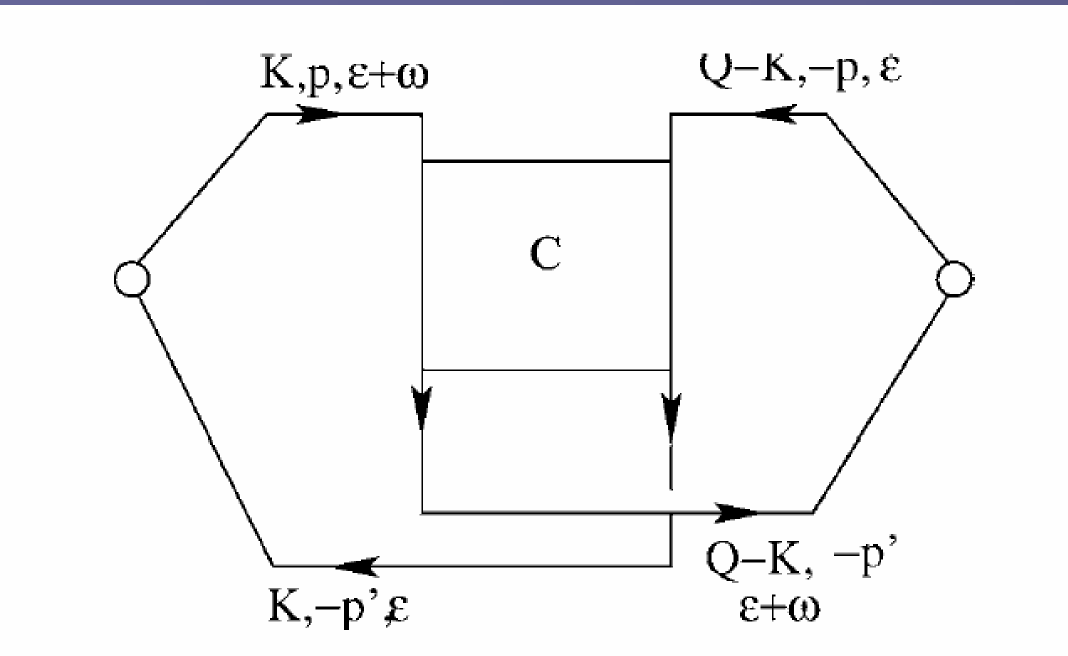
- In the first order in transparency the electrons in different grains are not correlated: they cannot be scattered by the same impurity

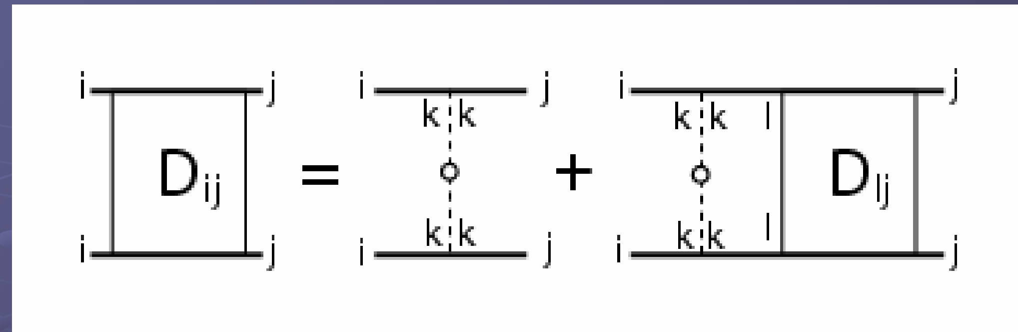



- Assuming higher tunneling corrections, the diagram turns out to be proportional to the tunneling dimensionless conductance



# WL correction; Cooperon




$$\mathcal{C}_{\mathbf{Q}}\left(\omega_\nu\right) = \frac{1}{2\pi\tau_{el}^2\nu_F}\,\frac{1}{|\omega_\nu| + 2\Gamma\left(1-\gamma_{\mathbf{Q}}\right)}.$$

$$\mathcal{D}^{eff} = \left[\frac{2\Gamma(1-\gamma_{\mathbf{K}})}{|\mathbf{K}|^2}\right]_{\mathbf{K}\to 0}$$

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$$\frac{\delta\sigma^{WL}_{(D)}}{\sigma^{n}_{(D)}} = -\frac{1}{\pi Z g_T}\sum_{\mathbf{Q}} \frac{\cos(Q_\alpha d)}{1-\gamma_{\mathbf{Q}}}.$$

$$\frac{\delta\sigma^{\mathrm{WL}}_{(1)}}{\sigma^{n}_{(1)}} \approx -\frac{2}{Z\pi^3 g_T}\frac{1}{\sqrt{\gamma_\varphi}};$$

$$\frac{\delta\sigma^{\mathrm{WL}}_{(2)}}{\sigma^{n}_{(2)}} \approx -\frac{1}{Z\pi^2 g_T}\ln\frac{\pi^2}{\gamma_\varphi}.$$

$$\frac{\xi^{\mathrm{loc}}_{(1)}}{d} \approx \frac{Z\pi^3}{2} g_T;$$

$$\frac{\xi^{\mathrm{loc}}_{(2)}}{d} \approx e^{Z\pi^2/2g_T}/\pi.$$

# Anomalous Magnetoresistance

- No Time - Reversal invariance: the equation for the Cooperon must be solved in the direct space

$$(4\Gamma)\left(1-\gamma_{\mathbf{Q}+2e\mathbf{A}}\right)\psi_i(\mathbf{r}) = E\psi_i(\mathbf{r})$$

- Also the conductivity correction must be written in the direct space

$$\frac{\delta\sigma^{WL}_{(D)}}{\sigma^{n}_{(D)}} = -\frac{2}{Zg_T}\Gamma\frac{\tilde{C}_{i,i+\alpha}+\tilde{C}_{i+\alpha,i}}{2}$$

# Cooperon

$$\widetilde{C}_{ij}(\mathbf{r},\mathbf{r}',\omega)=\sum_{Q_{\parallel},Q_{\perp},n}\frac{\psi_{i,Q_{\parallel}Q_{\perp}n}(\mathbf{r})\,\psi^{*}_{j,Q_{\parallel}Q_{\perp}n}(\mathbf{r}')}{-i\omega+\Omega_c\left(n+\frac{1}{2}\right)+\mathcal{D}Q_{\parallel}^{2}+\frac{1}{\tau_{\varphi}}+\mathcal{E}_0(H)},$$

$$\mathcal{E}_0(H)=\tfrac{2}{5}(\pi\phi/\phi_0)^2E_T$$

$$\Omega_c=4\mathcal{D}/\ell_H^2$$

# WL correction

$$\frac{\delta\sigma^{\mathrm{WL}}_{(2)}}{\sigma^{n}_{(2)}}=-\frac{\cos\left(\frac{\Omega_c}{2\Gamma}\right)}{\pi Z g_T}\sum_{n=0}^{n_{\max}}\frac{1}{n+\frac{1}{2}+\left(\gamma_{\varphi}+\frac{\mathcal{E}_0}{\Gamma}\right)\frac{\Gamma}{\Omega_c}}.$$

# General formula for magnetoconductivity

$$\delta\sigma(H) = -\frac{\sigma_0}{\pi Z g_T}\left(\mathcal{F}(H,\gamma_\varphi) - \ln\frac{\pi^2}{\gamma_\varphi}\right).$$

$$\mathcal{F}(\phi,\gamma_\varphi) = \cos\left(2\pi\frac{\phi}{\phi_0}\right)\left[\psi\left(\frac{\pi}{4}\frac{\phi_0}{\phi} + \frac{1}{2} + \frac{\pi}{10}\frac{\phi}{\phi_0}\frac{E_T}{\Gamma}\right) - \psi\left(\frac{\gamma_\varphi}{4\pi}\frac{\phi_0}{\phi} + \frac{1}{2} + \frac{\pi}{10}\frac{\phi}{\phi_0}\frac{E_T}{\Gamma}\right)\right],$$

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# Limiting cases

- When the magnetic length is much larger than the grain size, the grain lattice is completely invisible for the field: "usual" behaviour
- Increasing the field, the grain lattice shows up itself by the appearance of the Thouless energy in the expression for the correction

$$\frac{\delta\sigma(H)}{\sigma_0} \approx \frac{2\pi}{3}\frac{1}{Zg_T}\left(\frac{\phi}{\phi_0}\right)^2 \propto H^2$$

$$\frac{\delta\sigma(H)}{\sigma_0} = \frac{2}{\pi Z g_T}\log\left(\sqrt{\frac{E_T}{\Gamma}}\frac{\phi}{\phi_0}\right).$$

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# Conclusions

In summary, we developed a new diagrammatic technique in a double-momentum representation for transport in granular metals. Using this technique, the weak localization corrections to the conductivity arise in a natural way and an explicit calculation shows the same low-temperature behavior as in bulk metals, but with the diffusion constant $\mathcal{D}_n$ replaced by the effective tunnelling diffusion constant $\mathcal{D} = \Gamma d^2$ and the mean free path $\ell$ by the average grain diameter $d$. Our result agrees with Eq. (13) of Ref. [8] in the $\mathbf{Q} \to 0$ limit; however, our technique underlines the presence of the grain lattice, represented by the cosine factor in Eq. (8), reminiscent of the lattice structure factor $\gamma_{\mathbf{Q}}$. We also give an estimate of the magnetoresistance correction for very weak fields.

# Perspectives

- Non-Linear Sigma Model in the double-momentum representation (non-perturbative technique)
- e-e interactions
  - Coulomb: screening length, Coulomb blockade in low dimensions
  - SC: s-wave and d-wave instablities, relations with HTCS